EE 3302 - Busso FA2020

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The University of Texas at Dallas

10/26/20

Outline

Fourier Analysis & Sound

Frequency Domain
- Magnitude Spectrum
- Phase Spectrum

Fourier Transform

FT Tricks

Fourier Series

FS vs FT

Common Fourier Transforms

Examples

Fourier Analysis is Intuitive

- Fourier analysis can look scary.
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Table 1: Qualitative descriptors of sounds.

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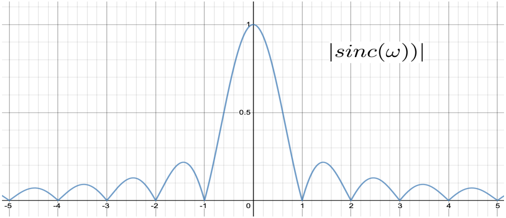
Table 1: Qualitative descriptors of sounds.

- The **magnitude and phase spectrums** lets us talk **quantitatively** about the different **frequencies** of a signal and how they will interact

Magnitude Spectrum



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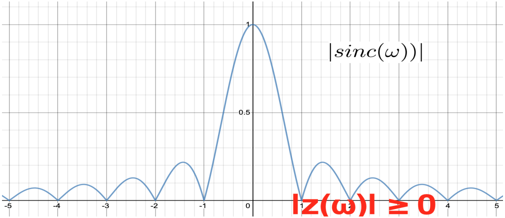

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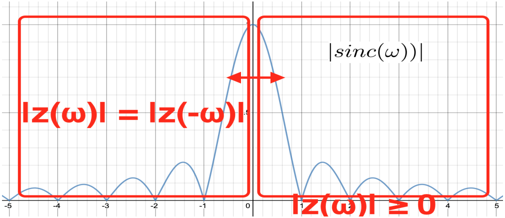

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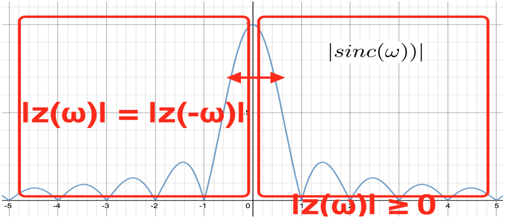

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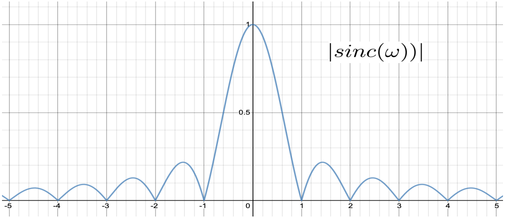
- Magnitude spectrum will be an even function

Magnitude Spectrum and Bandwidth



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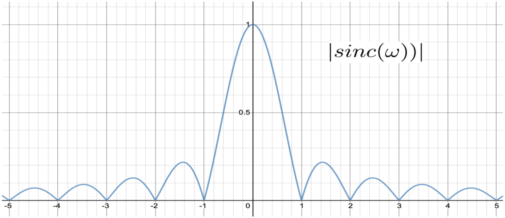

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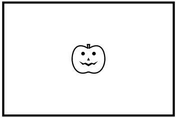
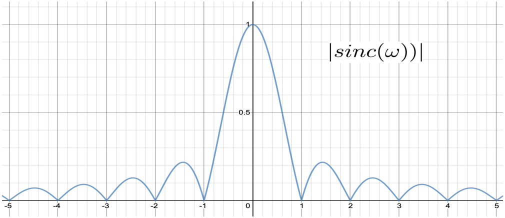


System

Magnitude Spectrum and Bandwidth



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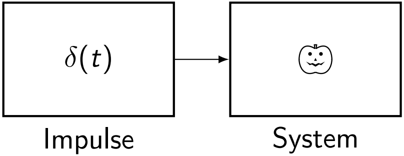

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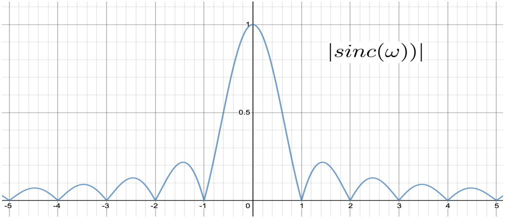

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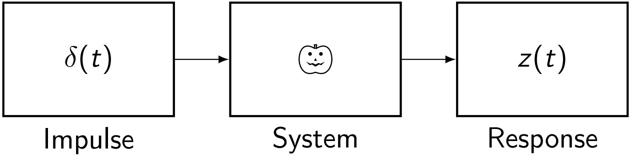

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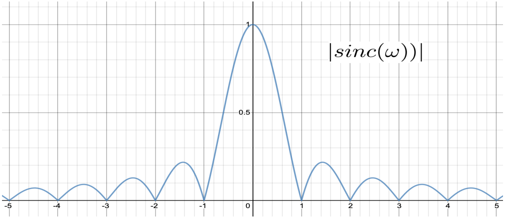

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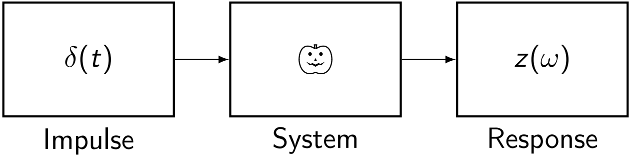
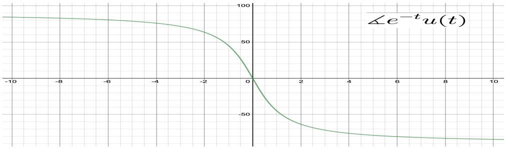


Systems typically specified by their **bandwidth** (BW) or gain

(I) For baseband signals: BW is the **half** width

(II) Otherwise: BW is the **full** width

Phase Spectrum



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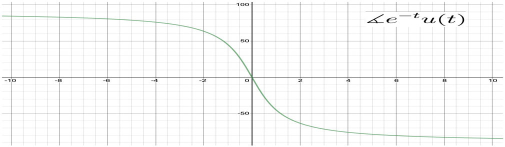

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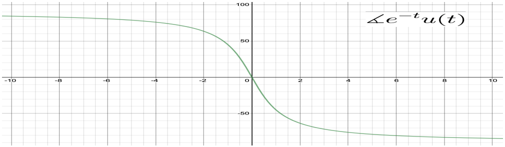

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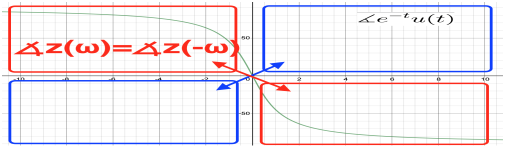

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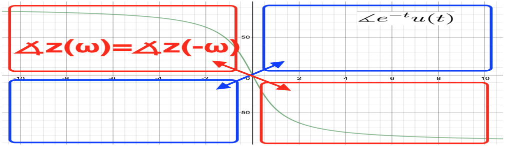

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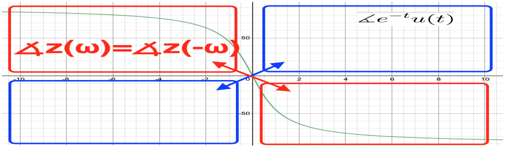

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- Phase spectrum will be an odd function.
- Sometimes omitted if $f(t)$ is real & odd or real & even

Misc. Frequency Domain Facts

- $f(t)$ odd & purely real $\iff$ z(ω) odd & purely imaginary
- $f(t)$ even & purely real $\iff$ z(ω) even & purely real

Fourier Transform

The Fourier Transform is an invertible, integral transform $\mathcal{F} : \mathbb{C}(t) \mapsto \mathbb{C}(t)$, defined by,

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In practice:

$$\boxed{f(t)}$$

Real Function

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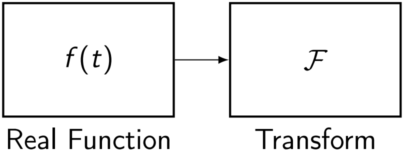

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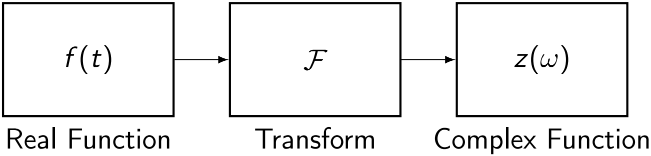

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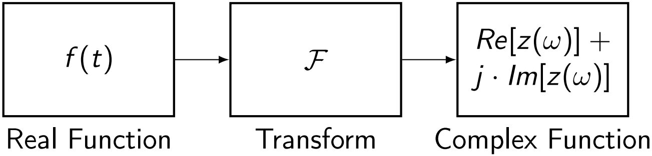

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In practice:

$$\underset{\text{Real Function}}{\boxed{f(t)}} \longrightarrow \underset{\text{Transform}}{\boxed{\mathcal{F}}} \longrightarrow \underset{\text{Complex Function}}{\boxed{|z(\omega)|e^{j\measuredangle z(\omega)}}}$$

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Why Magnitude and Phase Spectrums?

Plotting the magnitude ($|z(\omega)|$) and argument ($\measuredangle z(\omega)$) across all frequencies allow us to easily interpret our transformed data.

Duality & The Convolution Theorem

Recall the convolution theorem: convolution in TD is multiplication in FD

Theorem

Given signals $x(t), y(t) \in \mathbb{C}(t)$ *then,*

$$\mathcal{F}[x(t) * y(t)] = X(\omega)Y(\omega)$$

This is subject to duality

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- Parseval's Theorem: Energy is unitary

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

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- For purely real signals: Compact form

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n cos(n\omega_0 t + \theta_n)$$

Getting FS coefficients

DON'T do integrals if possible. Try by inspection first. Use these:

$$cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta} \;\;\&\;\; sin(\theta) = \frac{1}{2j}e^{j\theta} + \frac{1}{2j}e^{-j\theta}$$

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Getting FS coefficients

If necessary, here are the integrals for the coefficients.

- Important if you are given a periodic function with discontinuities. e.g. triangle, square, sawtooth waves
- These can be useful

$$cos^2(t) = \frac{1 + cos(2t)}{2} \;\; \& \;\; sin^2(t) = \frac{1 - cos(2t)}{2}$$

- Exponential form

$$c_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}$$

- Trigonometric form

$$a_n = \begin{cases} \frac{1}{T}\int_0^T x(t)dt; n = 0 \\ \frac{2}{T}\int_0^T x(t)cos(n\omega_0 t)dt \end{cases} \;\; \& \;\; b_n = \frac{2}{T}\int_0^T x(t)sin(n\omega_0 t)dt$$

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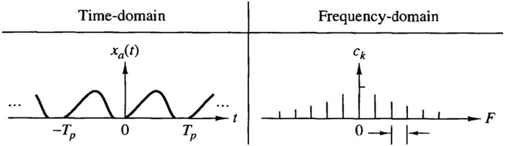

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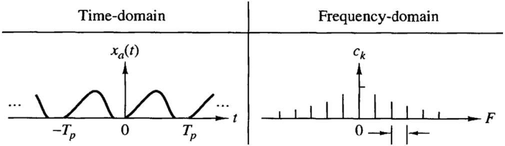

FT is for aperiodic signals

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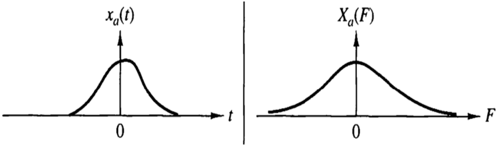

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(III) $sin(\omega_0 t) \rightleftharpoons \frac{\pi}{j}[\delta(\omega - \omega_0) + [\delta(\omega + \omega_0)]$

Common Fourier Transforms

Several Fourier Transforms / Fourier Series are useful for solving problems:

(I) $e^{j\omega_0 t} \rightleftharpoons \delta(\omega - \omega_0)$

(II) $cos(\omega_0 t) \rightleftharpoons [\delta(\omega - \omega_0) + [\delta(\omega + \omega_0)]$

(III) $sin(\omega_0 t) \rightleftharpoons \frac{\pi}{j}[\delta(\omega - \omega_0) + [\delta(\omega + \omega_0)]$

(IV) $\Pi(t) = \begin{cases} 1 & |t| < \frac{1}{2} \\ 0 & otherwise \end{cases} \rightleftharpoons sinc(\frac{\omega}{2\pi})$

Examples