NON LINEAR OPTIMIZATION

OBJECTIVE AND/OR CONSTRAINTS INCLUDE NON LINEARITIES

x^2, $\frac{1}{x}$, $\ln(x)$, e^x, $\sin(x)$ ....

GETS A LOT HARDER (SOMETIMES...)

EX EOQ MODEL

"ECONOMIC ORDER QUANTITY"

INVENTORY MODEL

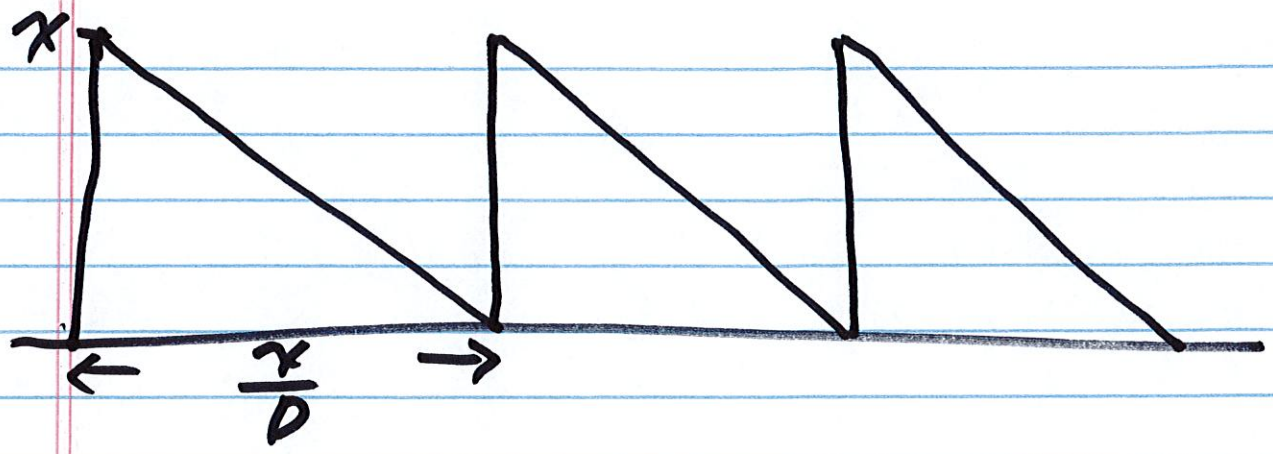
DEMAND RATE D [UNITS/TIME]

ORDER QUANTITY x

INVENTORY HOLDING COST H

[$/UNIT-TIME]

FIXED ORDER/PRODUCTION COST C



COST PER CYCLE: $C + H\frac{x^2}{2D}$

↑ AVERAGE INVENTORY

LENGTH OF CYCLE $\frac{x}{D}$

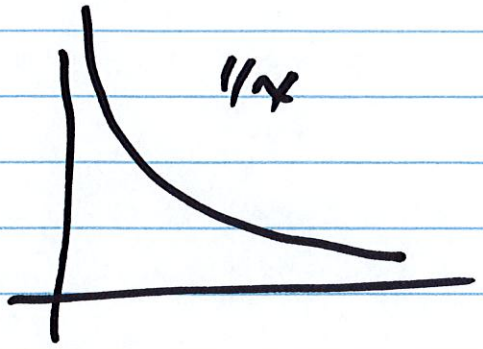
$$\frac{\text{AVE COST}}{\text{TIME}} = \frac{C + H\frac{x^2}{2D}}{\frac{x}{D}}$$

$$= \boxed{\frac{Hx}{2} + \frac{CD}{x}}$$

NOT LINEAR

LINEAR

WANT TO MINIMIZE



CONSIDER $D = 4000/YR$

$H = \$20$

$C = \$100$

$\Rightarrow$ EXCEL SOLVER USING GRG

NOTE: USING CALCULUS, CAN SHOW THAT OPT ORDER QUANTITY IS

$$x = \sqrt{\frac{2CD}{H}}$$

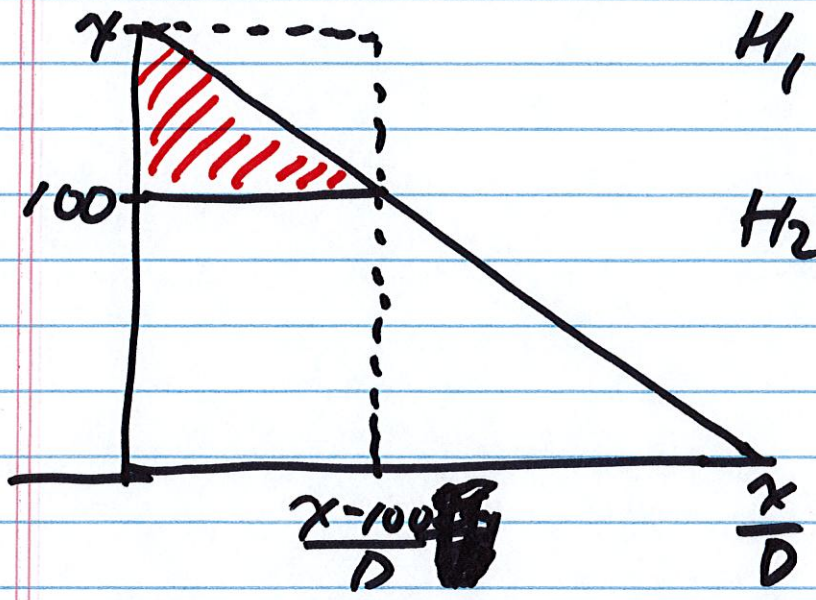
$$= \sqrt{\frac{2(100)4000}{20}}$$

$$= 200 \quad \checkmark\checkmark$$

"20 TURNS PER YEAR"

SLIGHTLY MORE COMPLICATED:

ASSUME ADDITIONAL CHARGE FOR INVENTORY ABOVE 100



H_1: ORIGINAL CHARGE

H_2: EXTRA CHARGE

ASSUMING $x > 100$, COST PER CYCLE IS NOW

$$H_1\left[\frac{x^2}{2D}\right] + H_2\left[\frac{(x-100)^2}{2D}\right] + C$$

$$H_1\frac{x^2}{2D} + H_2\left[\frac{x^2-200x+10000}{2D}\right] + C$$

TO GET LONG-RUN AVE COST, DIVIDE BY $\frac{x}{D}$

...

$$(H_1+H_2)\frac{x}{2} + \frac{5000H_2+CD}{x} - 100H_2$$

[SANITY CHECK: $H_2=0$ GIVES COST FOR ORIGINAL EOQ MODEL]

EX: $D = 4000$
$C = 100$
$H_1 = 20$
$H_2 = \underline{10}$ [FOR INVENTORY ABOVE 100]

COMPUTE ORIGINAL MODEL WITH $x \leq 100$ COST = 5000

TO MIN MODEL WITH $x \geq 100$

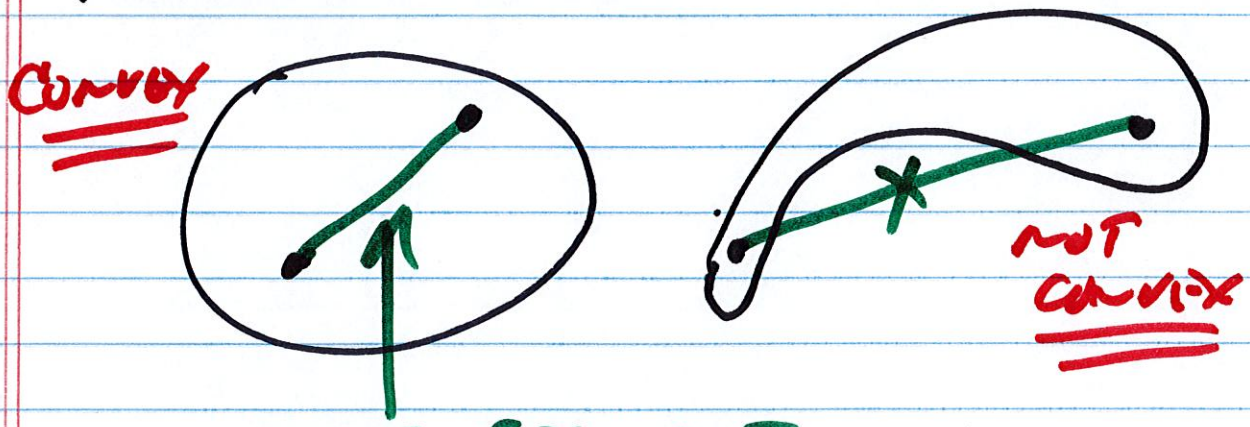
COST = 4196
$(x \approx 173)$

WHAT CAN GO WRONG?

- EOQ (AND MORE ELABORATE VERSION) SOLVED FINE
- MAX $(x-1)^2$ TROUBLE

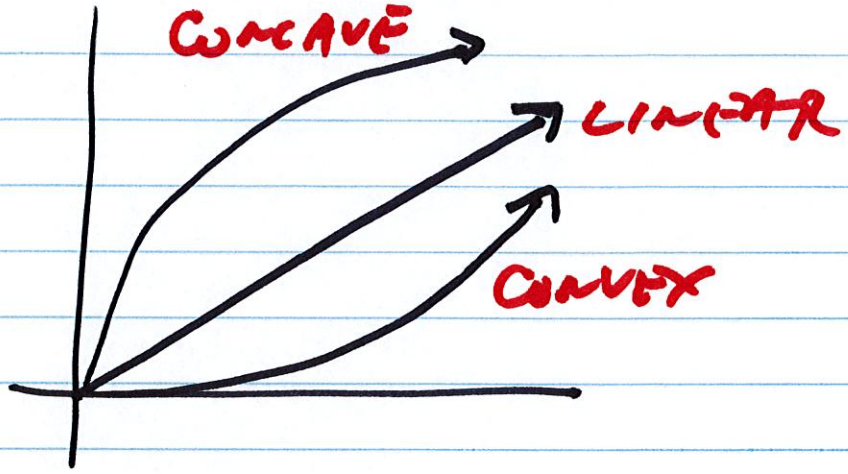
ISSUE IS CONVEXITY

FOR FEASIBLE REGION:



LINE SEGMENT CONNECTING TWO FEASIBLE POINTS IS FEASIBLE

ONE VARIABLE...



TO GET CONVEX FEASIBLE REGION, WANT

"THE FUNCTION f"

$f(x) \leq b$ WITH $f(\cdot)$ CONVEX ✓

AND/OR $f(x) \geq b$ WITH $f(\cdot)$ CONCAVE ✓

AND/OR $f(x) = b$ WITH $f(\cdot)$ LINEAR ✓

(ONLY INTERESTING WITH MULTIPLE VARIABLES)

BUT IF $f(\cdot)$ IS CONCAVE AND CONSTRAINT IS

$f(x) \leq b$ TROUBLE

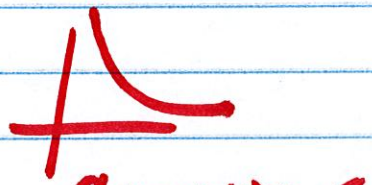
FOR MULTIPLE VARIABLES, MAY BE DIFFICULT TO DETERMINE CONVEXITY/CONCAVITY. BUT MANY KNOWN SPECIAL CASES

FOR OBJECTIVE WANT TO CHECK

MIN $f(x)$, $f(\cdot)$ CONVEX

OR MAX $f(x)$, $f(\cdot)$ CONCAVE

EOQ $\frac{Hx}{2} + \frac{CD}{x}$



LINEAR ✓

CONVEX ✓

OTHER EXAMPLE

MAX $(x-1)^2$

CONVEX ✓ $0 \le x \le 5$

CONVEX, BUT OBJ IS MAX