FROM MILITARY

OPTIMIZATION

LINEAR $\boxed{\text{PROGRAMMING}}$

LINEAR FUNCTION OF m VARIABLES

$x_1, x_2 \dots x_m$

IS OF THE FORM

$$b_1 x_1 + b_2 x_2 + \dots + b_m x_m$$

SLOPE COEFF (#)

VARIABLE

AN LP IS AN OPTIMIZATION PROBLEM WITH AN OBJECTIVE, AND CONSTRAINTS THAT ARE ALL LINEAR FUNCTIONS.

EQUALITIES AND/OR INEQ.

EX: TRANSPORTATION PROBLEM

[BUILDING BLOCK FOR LOGISTICS]

IDEA: m SOURCES

n DESTINATIONS

HAVE AN AMOUNT

x_{ij} MOVED FROM SOURCE i TO DESTINATION j

OBJECTIVE

$$\text{MIN} \sum_{i,j} c_{ij}\, x_{ij}$$

UNIT COST FOR UNIT MOVED FROM SOURCE i TO DEST. j

CONSTRAINTS

VERSION 1

$$\sum_j x_{ij} = a_i \quad \text{FOR EACH } i$$

MOVE IT ALL OUT

$$\sum_i x_{ij} \le b_j \quad \text{FOR EACH } j$$

CAPACITY AT DESTINATIONS

VERSION 2

WAREHOUSES

$$\sum_j x_{ij} \le a_i$$

CANNOT EXCEED SUPPLY

DISTRIBUTION NETWORK

$$\sum_i x_{ij} = b_j$$

SATISFY DEMAND

RETAILERS

"NON-NEGATIVE"

$$\boxed{x_{ij} \ge 0 \quad \text{ALL } i,j}$$

LP: SENSITIVITY REPORT

EX: TROPIC SUN

VARIABLES

	FINAL VALUE	REDUCED COST	OBJ COEFF	ALLOW +	ALLOW −
MT DURA → UCAGA	200,000 ✓	0	21 ✓	27	∞

↑ SKIP FOR NOW...

ALLOWABLE INCREASE/DECREASE:
RANGE IN WHICH CHANGES TO A $\boxed{\text{SINGLE}}$ OBJ COEFF DO NOT CHANGE $\boxed{\text{SOLUTION}}$ OF PROBLEM

↑ VALUES OF VARIABLES

WHAT ABOUT CHANGING MORE THAN ONE OBJ. COEFF?

CAN APPLY "100% RULE"

$$\text{IF} \sum_{\text{VARIABLES}} \frac{\text{CHANGE}}{\text{RANGE}} \le 1$$

OBJ COEFF

CORRESPONDING ALLOWABLE +/−

THEN SOLUTION STAYS SAME

EX DISTANCE
MT DORA → OCALA
21 → 24

MT DORA → ORLANDO
50 → 49

$$\sum \frac{\text{CHANGE}}{\text{RANGE}} = \frac{+3}{27} + \frac{1}{2} < 1$$

INCREASES DECREASES

⇒ SOLUTION SAME ✓

EFFECT ON OBJECTIVE?

3 [MT DORA → OCALA] − 1 [MT DORA → ORLANDO]

= 3 [200,000] − 1 [0] = $\underline{\underline{600,000}}$

SECOND PART... <u>CONSTRAINTS</u>

	VALUE	SHADOW PRICE	CONSTRAINT RHS	ALLOW +	ALLOW −
RECEIVED → OCALA	200,000 ✓	−27	200,000 ✓	75,000	50,000

<u>SHADOW PRICE</u>: CHANGE IN OBJ VALUE PER UNIT INCREASE IN CONSTRAINT RHS

<u>ALLOW +/−</u> : RANGE FOR MODIFYING <u>ONE</u> RHS OVER WHICH SHADOW PRICE HOLDS.

WHAT ABOUT CHANGING MORE THAN ONE RHS?

100% RULE:

CONSTRAINT RHS

$$\text{IF} \sum_{\text{CONSTRAINTS}} \frac{\text{CHANGE}}{\text{RANGE}} \leq 1$$

CORRESPONDING ALLOWABLE +/−

THEN SHADOW PRICES HOLD, AND <u>EFFECTS</u> <u>ARE</u> <u>ADDITIVE</u>

EX: MT DORA SHIPPED

$$275{,}000 \rightarrow 300{,}000$$

OCALA CAPACITY

$$200{,}000 \rightarrow 190{,}000$$

$$\sum \frac{\text{CHANGE}}{\text{RANGE}} = \frac{25{,}000}{50{,}000} + \frac{10{,}000}{50{,}000} < 1 \quad \checkmark\checkmark$$

ALLOW + ALLOW −

EFFECT ON OBJECTIVE

$$48(25{,}000) - 27(-10{,}000)$$

$$\underline{\underline{1{,}470{,}000}} \quad \text{BC-MT}$$

REDUCED COSTS?

SHADOW PRICE FOR A TIGHT <u>BOUND</u> <u>CONSTRAINT</u> [USUALLY NON-NEGATIVITY]

SPECIAL CASE OF
TRANSPORTATION PROBLEM

$m = n$

$a_i = 1, \; i = 1 \ldots m$
$b_j = 1 \quad j = 1 \ldots n$

ASSIGNMENT, OR MATCHING
PROBLEM

EX:
WORKERS → JOBS
2-SIDED
MEN ↔ WOMEN
"MARRIAGE"
MEDICAL RESIDENTS ↔ RESIDENCY SLOTS

PISTONS → BORES

→ CB SIMULATION –
BIG EFFECT OF
ORDERING PISTONS
& BORES
CAN THIS BE EXPLAINED??

YES

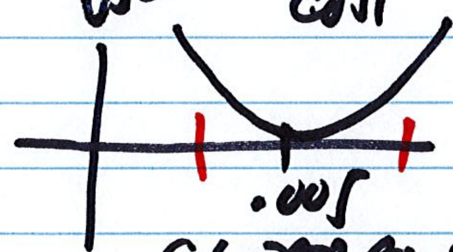
CONSIDER AN ASSIGNMENT PROBLEM WITH NUMBERS

$p_1 \cdots p_m$ EX: PISTON DIAMETER

$q_1 \cdots q_m$ EX: BORE DIAMETER $-.005$

$$C_{ij} = (p_i - q_j)^2$$

IDEA:



COULD SOLVE ASSIGNMENT PROBLEM TO GET MATCHING WITH MIN $\sum_{ij} (p_i - q_j)^2 x_{ij}$

FACT CAN PROVE THAT SOLUTION IS TO ORDER

$$p_1 \le p_2 \le \cdots \le p_m$$
$$q_1 \le q_2 \le \cdots \le q_m$$

SET $x_{ii} = 1 \quad i = 1 \ldots m$