DIFFERENCING TIME SERIES

ORIGINAL SERIES $X_t \quad t = 1, 2, \ldots$

DIFFERENCE SERIES $[X_t - X_{t-1}] \quad t = 2, 3, \ldots$

CONSIDER SIMPLE REG. EQUATION

$$Y_t = b_0 + b_1 X_t$$

$$Y_{t-1} = b_0 + b_1 X_{t-1}$$

SUBTRACT: $$[Y_t - Y_{t-1}] = b_1 [X_t - X_{t-1}]$$

↑ CHANGE IN Y

↑ CHANGE IN X

ADD BACK A CONSTANT:

$$[Y_t - Y_{t-1}] = b_0 + b_1 [X_t - X_{t-1}]$$

"DIFFERENCED" REGRESSION EQ.

- ALLOWS FOR TREND NOT ASSOCIATED WITH CHANGES IN X_t

WHAT IF ORIGINAL VARIABLES ARE LOGGED?

$$LN(X_t) - LN(X_{t-1})$$

$$= LN\left(\frac{X_t}{X_{t-1}}\right) \approx \frac{X_t}{X_{t-1}} - 1$$

IF $\frac{X_t}{X_{t-1}}$ NOT TOO FAR FROM 1.0

CONCLUSION: INSTEAD OF WORKING WITH DIFF OF LN, JUST WORK WITH

RATIO

- EASIER TO INTERPRET

GENERALIZATION OF DIFFERENCING:

CONSIDER A SERIES OF FORM:

$$[X_t - \phi X_{t-1}] \qquad 0 \le \phi \le 1$$

"PHI"

"ϕ DIFF" OF SERIES

- OPERATION CAN BE MOTIVATED BY ASSUMPTION ABOUT AUTOCORRELATION OF ERRORS IN ORIGINAL SERIES
- PROVIDES A "DEGREE OF FREEDOM" THAT CAN BE USEFUL IN REMOVING AUTOCORRELATION IN RESIDUALS