OPTIMIZATION WITH 0/1 VARIABLES

EX: TRANSPORTATION WITH FACILITY COST

"FACILITY LOCATION"

X_{ij} = AMOUNT SENT FROM SOURCE i TO DESTINATION (FACILITY) j

$(x_{ij} \geq 0)$

$$Y_j = \begin{cases} 0 & \text{DON'T USE FACILITY } j \\ 1 & \text{USE FACILITY } j \end{cases}$$

$$\text{OBJ: MIN } \sum_{ij} \underbrace{c_{ij}}_{\text{TRANSPORTATION COST}} x_{ij} + \sum_j \underbrace{d_j}_{\text{FACILITY COST}} y_j$$

$$\sum_j x_{ij} = a_i \quad \text{EACH } i$$

CAPACITY IF USED

$$\sum_i x_{ij} \leq b_j y_j \quad \text{EACH } j$$

MORE GENERALLY...

$$\underbrace{l_j y_j}_{\text{LOWER BOUND}} \leq \sum_i x_{ij} \leq \underbrace{u_j y_j}_{\text{UPPER BOUND}}$$

EX: AT&T TELEMARKETING
SITE SELECTION

McMARTIN CATALOG

FACILITY SIZE $\leq$ 200,000 HR/YR

MIN COST $5,897,485

HOW ARE PROBLEMS
LIKE THIS SOLVED?
– "BRANCH AND BOUND"

LP RELAXATION: $5,539,069

6% GAP

ADD $\sum_j y_j \geq 2$ $5,807,377

1.5%

RECALL x_{ij} = FRACTION OF CALL VOLUME FROM i ASSIGNED TO j

COULD ADD $x_{ij} \leq y_j$ FOR ALL i, j

170 CONSTRAINTS

GAP 4.5%

VALID FOR ANY BINARY VALUES OF y_j

BRANCH AND BOUND

START WITH "LP RELAXATION"

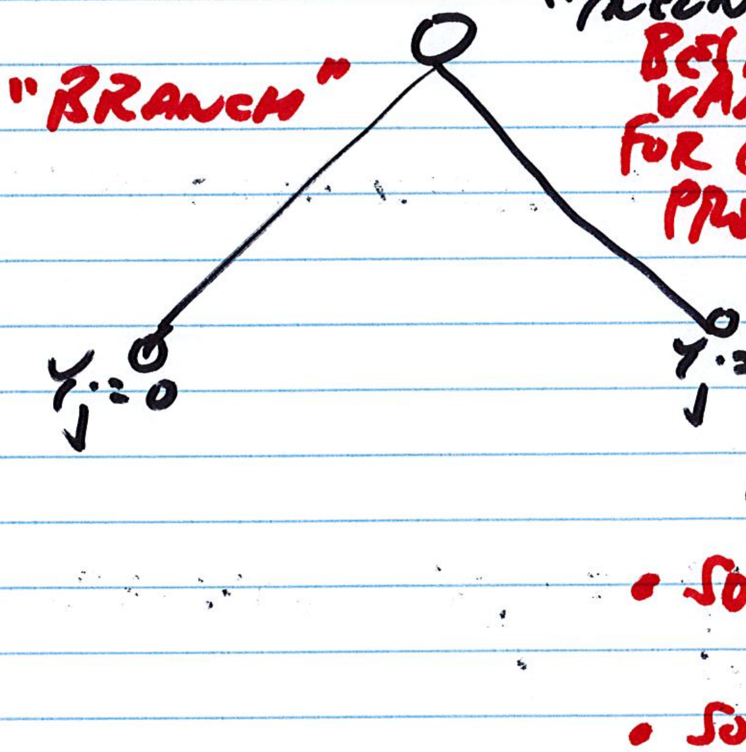
⇒ REPLACE $y_j \in \{0, 1\}$

WITH $0 \le y_j \le 1$

LP RELAXATION

"INCUMBENT VALUE" V

BEST KNOWN OBJ VALUE FOR ORIGINAL PROBLEM [+∞ IF NONE]



SOLVE RELAXATION

GET SOLUTION VALUE Z

• SOLUTION BINARY – Z < V UPDATE INCUMBENT

• SOLUTION NOT BINARY

→ Z ≥ V "PRUNE" "FATHOM"

→ Z < V BRANCH AGAIN

ETC.

KEY ISSUE: HOW BIG DOES TREE GET?

WITH FULL ENUMERATION, NUMBER OF END NODES IS 2^m WITH m BINARY VARIABLES

$m = 10 \quad 2^m = 1024$

$m = 20 \quad 2^m > 10^6$

$m = 50 \quad 2^m > 10^{15}$ (!)

- THIS CAN GET OUT OF HAND!
- RELATED TO "P=NP" PROBLEM
- TO KEEP IT UNDER CONTROL, WANT "GAP" AT ROOT OF TREE TO BE REASONABLE

ADDING VALID CONSTRAINTS CAN HAVE BIG EFFECT ON GAP...

DO <u>BOTH</u>

$$\sum_j y_j \geq 2 \quad (1)$$

$$x_{ij} \leq y_j \quad \text{ALL } i,j \quad (170)$$

LP SOLUTION VALUE
5.885M $\boxed{.2\%}$

THESE TECHNIQUES ARE IMPORTANT!

- MANUALLY (EXPERT KNOWLEDGE)

- HIGH-PERFORMANCE SOFTWARE
  - CPLEX
  - GUROBI
  - SAS
  - ⋮

ANOTHER EXAMPLE:

PRODUCTION/INVENTORY PROBLEM

TIME PERIODS

$t = 1, 2, \ldots T$

DEMANDS D_t DATA

INVENTORY I_t (START OF PERIOD)

PRODUCTION X_t VARIABLES

COSTS: UNIT ONE-PERIOD INVENTORY HOLDING COST h

FIXED COST OF PRODUCTION C

$Y_t = \begin{cases} 0 & \text{OTHERWISE} \\ 1 & \text{PRODUCTION OCCURS IN TIME PERIOD } t \end{cases}$

L, U ARE LOWER & UPPER LIMITS ON PRODUCTION, IF IT OCCURS

PROBLEM:

$$\min \sum_{t=1}^{T} c\,Y_t + \sum_{t=1}^{T+1} h I_t$$

FIXED COST OF PRODUCTION

INVENTORY

$$I_{t+1} = I_t + X_t - D_t \qquad t=1,2,\ldots T$$

MATERIAL BALANCE

I_1 IS DATA

VARIABLES $Y_t \in \{0,1\}$

$$L Y_t \le X_t \le U Y_t$$

LIMITS ON PRODUCTION

$I_t \ge 0$ ALL t

NO BACKORDERS

EX: $T = 26$ WEEKS

LP RELAXATION
MIN COST = 425

FIXED BATCH HEURISTIC HAS
MIN ≈ 1695

- GAP LOOKS HUGE!
- SOLUTION OF LP RELAXATION <u>NOTHING LIKE</u> BINARY SOLUTION

NOW FOR SOMETHING
COMPLETELY DIFFERENT....

CONSIDER ASSIGNMENT-TYPE
FORMULATION

DEMAND (i) 1 2 3 4 5 6...

PRODUCTION (j)

X_{ij} = FRACTION OF PERIOD i
DEMAND PRODUCED IN
PERIOD j

$$\underline{\underline{j \le i}}$$

$$\sum_{j \le i} X_{ij} = 1$$

(ALL PRODUCED SOMEWHERE)

BINARY VARIABLES Y_j AS BEFORE

CONSTRAINTS:

TOTAL PRODUCTION IN TIME PERIOD j BETWEEN UPPER + LOWER LIMITS

$$L y_j \le \underbrace{\sum_{i \ge j} d_i x_{ij}}_{\text{AMOUNT PRODUCED IN TIME PERIOD } j} \le U y_j$$

OBJECTIVE

$$\underbrace{c \sum_j y_j}_{\text{PRODUCTION COSTS}} + \underbrace{h \sum_i \sum_{j \le i} \left[d_i x_{ij} \overbrace{(i-j)}^{\text{HELD FOR } i-j \text{ PERIODS}} \right]}_{\text{INVENTORY HOLDING COST}}$$

ADDITIONAL CONSTRAINTS:

$$x_{ij} \le y_j \quad \text{BINARY}$$

ORIGINAL LP 425

LP RELAXATION FOR THIS VERSION: 1436.3

ACTUAL BINARY SOLUTION VALUE $\boxed{1443}$

REMOVE $X_{ij} \leq Y_j$ CONSTRAINTS,
GET 425 AGAIN!!