LOGS + REGRESSION

TERMINOLOGY:

POWER

$$\log_a(x) = p \iff a^p = x$$

"LOG TO THE BASE
a OF x EQUALS
p"

CHOICE FOR BASE a:

① $a = 10$ "LOG BASE 10"

② $a = 2$ COMPUTER SCIENCE

③ $a = e \approx 2.71828\ldots$

☆ $\log_e \iff \ln$
"NATURAL LOG"

$$\underset{\displaystyle \ln(x)}{\underset{\parallel}{\log_e(x)}} = p \iff \underset{\displaystyle \exp(p)}{\underset{\parallel}{e^p}} = x$$

"EXPONENTIAL
FUNCTION"

PROPERTIES (RULES)

INVERSE $e^{\ln(x)} = x \quad \ln(e^x) = x$

PRODUCT $e^{x+y} = e^x e^y$

$\ln(xy) = \ln(x) + \ln(y)$

POWER $(e^x)^y = e^{xy}$

$\ln(x^y) = y\ln(x)$

NICE PROPERTY [ONLY FOR BASE $a = e$]

$e^x \approx 1 + x \qquad \log(1+x) \approx x$

IF $|x|$ NOT TOO BIG

(SAY $-.2 \leq x \leq .2$)

EX: $e^{.05} = 1.0513$

EXAMPLE OF EXP(·)

EXPONENTIAL GROWTH

START WITH DEPOSIT OF AMOUNT D AT TIME $t=0$, GROWS AT RATE b, WITH "CONTINUOUS COMPOUNDING"

⇒ AMOUNT AT TIME t

$$= De^{bt}$$

EX $b = .05$ [5% RATE]

$t=1$: $e^{b} = e^{.05} = 1.0513$

ANNUAL "YIELD" = 5.13%

DOUBLING TIME

$$e^{bt} = 2$$

$$\ln(e^{bt}) = bt = \ln(2) = .693$$

$$t = .693/b$$

i.e. ABOUT 70/RATE (IN % PTS)

EXPONENTIAL DECAY

SAME IDEA, B-T $b < 0$

DECAY RATE $|b|$, INITIAL QUANTITY Q, AMOUNT LEFT AT TIME τ

$$= Q e^{b\tau}$$

HALF-LIFE: WANT

$$e^{b\tau} = \frac{1}{2}$$

$$\ln(e^{b\tau}) = b\tau = \ln\left(\tfrac{1}{2}\right) = -.693$$

$$\tau = \frac{.693}{|b|}$$

i.e. HALF LIFE IS ABOUT 70/DECAY RATE

REGRESSION

ASSUME A RELATIONSHIP OF FORM

$\boxed{Y = Ae^{bX}}$ EXPONENTIAL

$$\ln(Y) = \ln(A) + \ln(e^{bX})$$
$$= a + bX$$
$$a = \ln(A)$$

$$\rightarrow \ln(Y) = a + bX$$

LOOKS LIKE SIMPLE REGRESSION EQUATION, DEP VAR $\ln(Y)$

EX: CPI, 1946-1999

$$\ln CPI = \underline{2.8} + \boxed{.0436}\,t$$

$t = 0$: $\ln CPI = 2.8$
$$CPI = e^{2.8} = 16.45$$

INTERPRETATION OF b:

$$Y = Ae^{bt}$$

$$t \to t+1 \quad Y = Ae^{b(t+1)}$$

$$= Ae^{bt+b}$$

$$= [Ae^{bt}]e^{b}$$

$$\approx 1+b$$

$$e^{b} \approx 1+b$$

b = GROWTH RATE, CONT. COMPOUNDING

$$e^{.0436} = 1.0445$$

IN GENERAL b GIVES APPROX % CHANGE IN Y PER UNIT INCREASE IN X. APPROX IS REASONABLE UP TO CHANGES OF ± 20%

ANOTHER POSSIBILITY....

ASSUME THAT

$$Y = AX^{b} \quad \text{POWER}$$

$$\ln(Y) = \ln(A) + \ln(X^{b})$$

$$ = \underset{a}{\overset{\shortparallel}{}} + b\ln(X)$$

$$\Rightarrow \ln(Y) = a + b\ln(X)$$

"LOG-LOG RELATIONSHIP"

EX: LN BUCHANAN ON LN TOTAL, FLORIDA 2000 ELECTION

$$\ln(Y) = -2.5 + .7035X$$

$$Y = e^{-2.5} X^{.7035}$$

$$\downarrow$$

$$.082$$

NOTE $(10X)^{.7035} = X^{.7035}\, 10^{.7035}$

$$\approx 5X^{.7035}$$

$\Rightarrow$ SHARE CUT IN HALF

MULTIPLE REGRESSION

$$\ln(Y) = a + b_1 \left\langle \begin{matrix} X_1 \\ \ln(X_1) \end{matrix} \right\rangle + b_2 \left\langle \begin{matrix} X_2 \\ \ln(X_2) \end{matrix} \right\rangle + \dots + b_k \left\langle \begin{matrix} X_k \\ \ln(X_k) \end{matrix} \right\rangle$$

THINK ABOUT X_1, HOLD OTHERS <u>FIXED</u> ...

$$\ln(Y) = \hat{a} + b_1 \left\langle \begin{matrix} X_1 \\ \ln(X_1) \end{matrix} \right\rangle$$

THINK ABOUT EFFECTS OF VARIABLES ONE AT A TIME, JUST LIKE IN SIMPLE REGRESSION