TIME SERIES

X_i $i = 1 \ldots n$

OBSERVATION IN TIME PERIOD i

DAYS
WEEKS
MONTHS
QUARTERS
YEARS
⋮
ETC.

TODAY

- COMPONENTS OF A TIME SERIES
- SMOOTHING METHODS
  - EXPONENTIAL SMOOTHING
  - MOVING AVERAGES
- SEASONALITY
  - SEASONAL ADJUSTMENT
    - RATIO TO M.A.
  - REGRESSION APPROACH
    - DUMMY VARIABLES

COMPONENTS OF A T.S.

- LEVEL/TREND
  AVERAGE VALUE / CHANGE IN VALUE

- SEASONAL
  PERIODIC VARIATION, KNOWN PERIODICITY

- CYCLICAL
  PERIODIC, BUT PERIODICITY NOT KNOWN (AND MAY CHANGE)

- RANDOM
  IRREGULAR VARIATION "NOISE"

SMOOTHING METHODS

- CAROFCY "AUTOMATIC" - DON'T REQUIRE EXPLICIT STRUCTURAL ASSUMPTIONS
- DAMP NOISE / CYCLICAL COMPONENTS
- DAMP OR ELIMINATE SEASONAL COMPONENT
- MODEL LEVEL/TREND - FORECASTING N OTHER GOAL

VERY COMMON EX: DEMAND FORECASTING

EXPONENTIAL SMOOTHING

ORIGINAL SERIES X_i
"SMOOTHED" SERIES S_i

START WITH $S_1 = X_1$

THEN $S_i = \alpha X_i + (1-\alpha) S_{i-1}$ $\quad i \geq 2$

↑
"ALPHA"

SMOOTHING CONSTANT
$0 < \alpha < 1$

IF LAST VALUE IS X_M, THEN
S_M IS FORECAST FOR
FUTURE VALUES

ASSUMES SERIES IS LEVEL
(NO TREND)
AND NO SEASONALITY

$$S_1 = X_1$$

$$S_2 = \alpha X_2 + (1-\alpha) S_1$$

$$S_3 = \alpha X_3 + (1-\alpha) S_2$$

ETC

CAN SUBSTITUTE OUT ALL THE S_i VALUES ... GET

$$S_i = \alpha \left[\sum_{j=0}^{i-2} (1-\alpha)^j X_{i-j} \right] + (1-\alpha)^{i-1} X_1$$

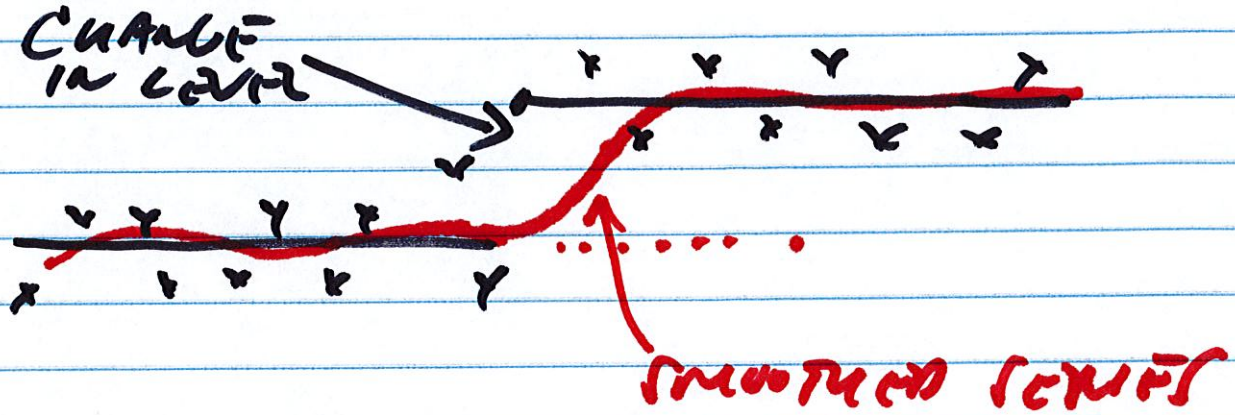
WEIGHT

BACK IN TIME

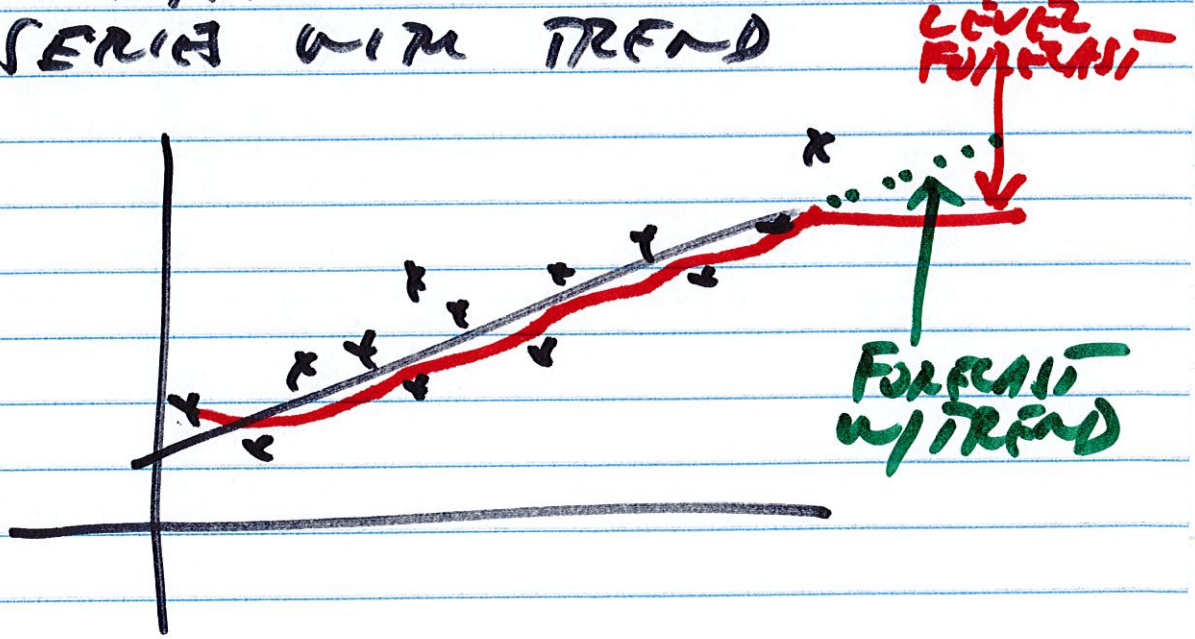
CONCLUDE: LARGER α GIVES MORE WEIGHT TO RECENT OBSERVATIONS

- LESS DAMPING OF NOISE
- PICK UP CHANGES IN LEVEL MORE QUICKLY

TYPICAL α: [.3, .6]



- DAMP NOISE
- PICKING UP CHANGES IN LEVEL
- NOT APPROPRIATE FOR ORIGINAL SERIES WITH TREND



WHAT ABOUT TREND?

DOUBLE EXPONENTIAL SMOOTHING

S_i : SMOOTHED VALUE

T_i : SMOOTHED TREND

START WITH $S_1 = X_1$

$T_1 = 0$

$$S_i = \alpha X_i + (1-\alpha)\left[S_{i-1} + T_{i-1}\right]$$

↑ ESTIMATE FOR ONE-PERIOD CHANGE

$$T_i = \beta (X_i - X_{i-1}) + (1-\beta) T_{i-1}$$

↑ OBSERVED CHANGE

FORECAST FOR X_{i+k}

$k = 1, 2, 3 \ldots$

IS $S_i + kT_i$

SEASONALITY?

- HOLT-WINTERS SMOOTHING
- SEASONAL ADJUSTMENT:
  RATIO TO MOVING AVERAGE
  - REGRESSION - DUMMY VARIABLES

MOVING AVERAGES

- DAMP RANDOM + CYCLICAL COMPONENTS
- CAN BE USED TO ELIMINATE SEASONAL COMPONENT

K-PERIOD TRAILING M.A.

$$M_i = \frac{X_i + X_{i-1} + X_{i-2} + \ldots + X_{i-(K-1)}}{K}$$

EX: • STOCK PRICE DATA $K = 30$
$K = 60$

- K-PERIOD SEASONALITY

CENTERED MOVING AVERAGE

2k+1 CMA [ODD #]

Ex: $k=3$, $2(3)+1 = 7$

M T W Th F S Su M T W

$$\frac{X_i + [X_{i+1} + \dots + X_{i+k}] + [X_{i-1} + \dots + X_{i-k}]}{2k+1}$$

$= M_{i+k}$, WHERE M_i IS A 7 PERIOD TRAILING MA

ADVANTAGE: BETTER REPRESENT TREND [ORDINARY MA WILL LAG]

DISADVANTAGE: CANNOT BE USED FOR FORECASTING

2K CMA [EVEN #]

3.5 4.5

EX: 1 [2 [3 4 1] 2] 3 4

PROBLEM: CANNOT "CENTER" ON OBSERVATION

SOLUTION: TAKE TWO PTS AND AVERAGE

$$= \frac{M_{i+k} + M_{i+(k-1)}}{2}$$

WHERE M_i IS A TRAILING 2K MA

APPLICATION: SEASONAL ADJUSTMENT OF CPI