Assignment 3

Due December 6, 2015

A. [10] Design an algorithm that can identify cycles in a *directed* graph.

B. *Coin denomination problem*

You have large amount of coins of each denominations: 1 cent, 5-cent, 10-cent, 25-cent, and 1 dollar. The optimal denomination problem is defined as:

PROBLEM: a total amount x cents to be made up in coins.

SOLUTION: Generate N_1, N_5, N_{10}, N_{25}, N_{100} such that

1. We minimize $N_1+N_5+N_{10}+N_{25}+N_{100}$
2. while subject to the constraint $N_1 + 5*N_2 + 10*N_{10} + 25*N_{25} + 100*N_{100} = x$.

Devise an algorithm that solves the optimal denomination problem using dynamic programming by following the these steps.

1. [10] How do you generate sub-problems?

2. [10] How do you synthesize the solution of a larger problem based on the solutions of the subproblems.

3. [10] Write down the recursive solution.

4. [10] What is the runtime of the recursive solution?

5. [20] Formulate the bottom-up computation so that it runs in polynomial time complexity.

6. [10] Formulate the memoization version of the recursive solution so that it runs in polynomial time complexity.

C. Solve the optimal denomination problem using a greedy algorithm.

1. [10] What is the complexity of the greedy algorithm.

2. [10] For the following values of x, compute the optimal number of coins used by both dynamic programming and the greedy algorithm.

x	by dynamic programming	by greedy
104		
122		
141		
156		
157		
167		
188		
189		
200		

Submission:

- Report.pdf