Large-Scale Galactic Magnetic Fields

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Magnetohydrodynamics (MHD)

Plasma

continuity:

$$\partial_t \rho + \nabla \cdot (\rho \underline{u}) = 0$$

momentum:

$$\partial_t(\rho \underline{u}) + \nabla \cdot (\rho \underline{u}\, \underline{u}) = \underline{j} \times \underline{B} - \nabla p$$

adiabatic closure:

$$p = 2nkT$$

Magnetic Field

Ohm's Law:

$$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$$

Faraday's Law:

$$\partial_t \underline{B} = -\nabla \times \underline{E} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B}$$

- Plasma *induces* Magnetic Field
- Magnetic Field *accelerates* Plasma

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> - The large-scale average is influenced by the small-scale fluctuations!

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$$\mathcal{E} = 0 \implies \begin{cases} \partial_t B_r = \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{2}$$

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  - The magnetic field dilutes and the dynamo dies ($\rightarrow$ Antidynamo theorem)

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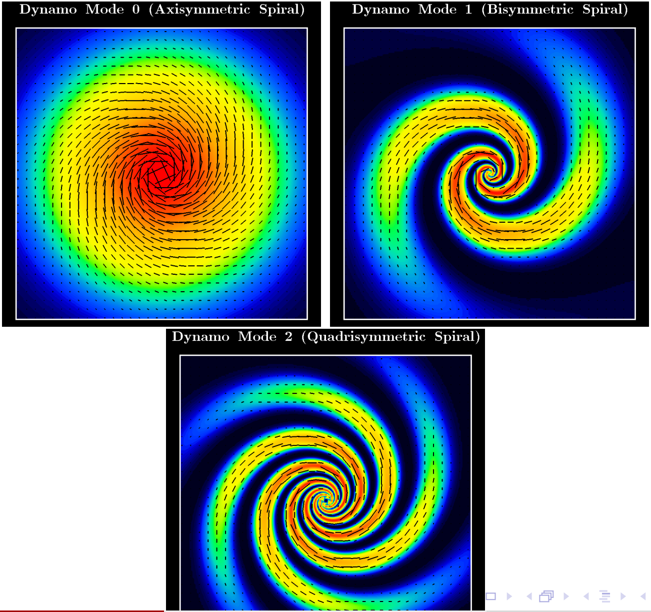
- Order out of Chaos

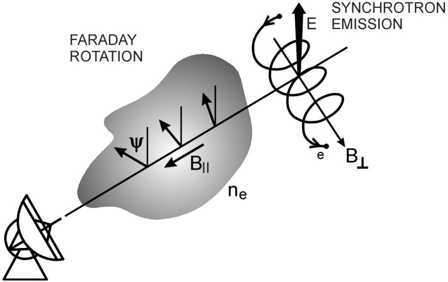
Spatial Structures of Dynamo Solutions



Observing Magnetic Fields

- Synchrotron radiation: polarization $\perp$ field orientation
- Polarization is changed when passing through magnetized plasma (*Faraday Rotation*):

$$\beta = \mathrm{RM}(n_e, B_{\|})\,\lambda^2$$



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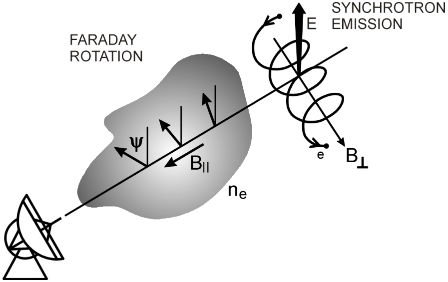

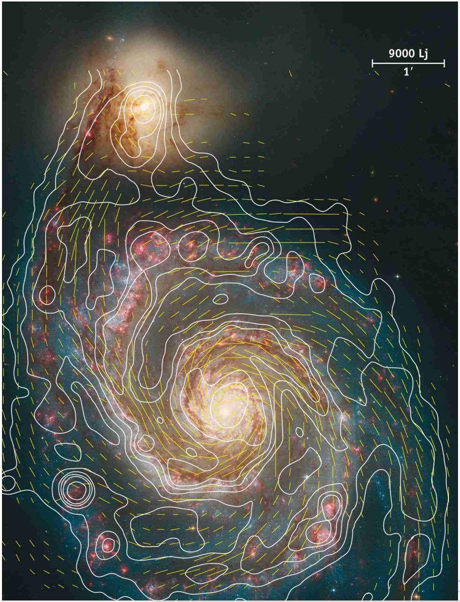

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