Large-Scale Galactic Magnetic Fields

Jeremiah Lübke

Ruhr-Universität Bochum, July 2019

Table of Contents

1. MHD
2. Dynamos
3. Spatial Structures of Dynamo Solutions
4. Observing Magnetic Fields

Magnetohydrodynamics (MHD)

Plasma

continuity:

$$\partial_t \rho + \nabla \cdot (\rho \underline{u}) = 0$$

momentum:

$$\begin{aligned}\partial_t(\rho \underline{u}) + \nabla \cdot (\rho \underline{u}\, \underline{u}) \\ = \underline{j} \times \underline{B} - \nabla p\end{aligned}$$

adiabatic closure:

$$p = 2nkT$$

Magnetic Field

Ohm's Law:

$$\underline{E} + \underline{u} \times \underline{B} = \eta \underline{j}$$

Faraday's Law:

$$\begin{aligned}\partial_t \underline{B} &= -\nabla \times \underline{E} \\ &= \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B}\end{aligned}$$

- Plasma *induces* Magnetic Field
- Magnetic Field *accelerates* Plasma

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Splitting up the involved quantities into regular average and random fluctuations:

$$X = \langle X \rangle + X' \quad \text{with} \quad \langle\langle X \rangle\rangle = \langle X \rangle, \quad \langle X' \rangle = 0$$

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Splitting up the involved quantities into regular average and random fluctuations:

$$X = \langle X \rangle + X' \quad \text{with} \quad \langle\langle X \rangle\rangle = \langle X \rangle, \quad \langle X' \rangle = 0$$

- And working out (1):

$$\implies \partial_t \langle \underline{B} \rangle = \nabla \times (\langle \underline{u} \rangle \times \langle \underline{B} \rangle) + \eta \nabla^2 \langle \underline{B} \rangle + \underbrace{\nabla \times \langle \underline{u}' \times \underline{B}' \rangle}_{\equiv \mathcal{E}}$$

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Splitting up the involved quantities into regular average and random fluctuations:

$$X = \langle X \rangle + X' \quad \text{with} \quad \langle\langle X \rangle\rangle = \langle X \rangle, \quad \langle X' \rangle = 0$$

- And working out (1):

$$\implies \partial_t \langle \underline{B} \rangle = \nabla \times (\langle \underline{u} \rangle \times \langle \underline{B} \rangle) + \eta \nabla^2 \langle \underline{B} \rangle + \underbrace{\nabla \times \langle \underline{u}' \times \underline{B}' \rangle}_{\equiv \mathcal{E}}$$

- The large-scale average is influenced by the small-scale fluctuations!

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

$$\mathcal{E} = 0 \implies \begin{cases} \partial_t B_r = \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{2}$$

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

$$\mathcal{E} = 0 \implies \begin{cases} \partial_t B_r = \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{2}$$

  - The magnetic field dilutes and the dynamo dies
    ($\rightarrow$ Antidynamo theorem)

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

$$\mathcal{E} \approx \alpha \langle \underline{B} \rangle \implies \begin{cases} \partial_t B_r = -\mathcal{R}_\alpha\, \partial_z(\alpha B_\phi) + \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{3}$$

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

$$\mathcal{E} \approx \alpha \langle \underline{B} \rangle \implies \begin{cases} \partial_t B_r = -\mathcal{R}_\alpha\, \partial_z(\alpha B_\phi) + \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{3}$$

  - small-scale turbulence ($\alpha \neq 0$) sustains the magnetic field ($\rightarrow$ α- and Ω-effect)

Dynamos

- Consider the equation governing the magnetic field:

$$\partial_t \underline{B} = \nabla \times (\underline{u} \times \underline{B}) + \eta \nabla^2 \underline{B} \tag{1}$$

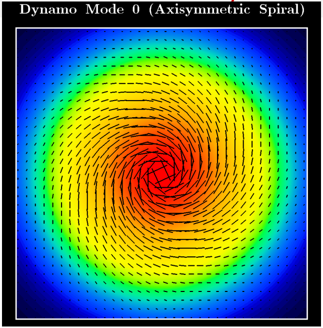
- Working out (1) in cylindrical coordinates, with $\underline{u} = r\,\Omega\,\underline{e}_\phi$ and some galaxy-related assumptions, one finally arrives at:

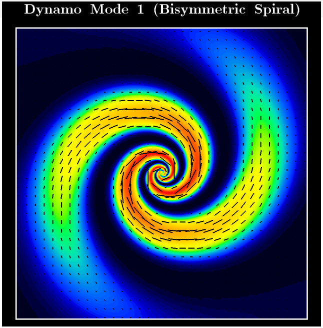
$$\mathcal{E} \approx \alpha \langle \underline{B} \rangle \implies \begin{cases} \partial_t B_r = -\mathcal{R}_\alpha\, \partial_z(\alpha B_\phi) + \partial_{zz} B_r \\ \partial_t B_\phi = \mathcal{R}_\Omega\, B_r\, r\, \partial_r \Omega + \partial_{zz} B_\phi \end{cases} \tag{3}$$

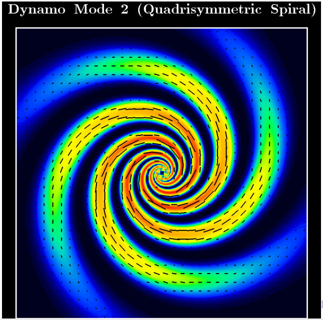
  - small-scale turbulence ($\alpha \neq 0$) sustains the magnetic field ($\to$ α- and Ω-effect)

▶ Order out of Chaos

Spatial Structures of Dynamo Solutions



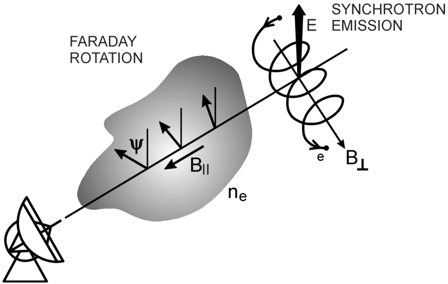




Observing Magnetic Fields

- Synchrotron radiation:
  polarization $\perp$ field orientation
- Polarization is changed when passing through magnetized plasma (*Faraday Rotation*):

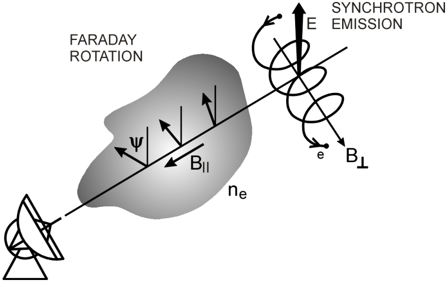
$$\beta = \mathrm{RM}(n_e, B_{\|})\,\lambda^2$$

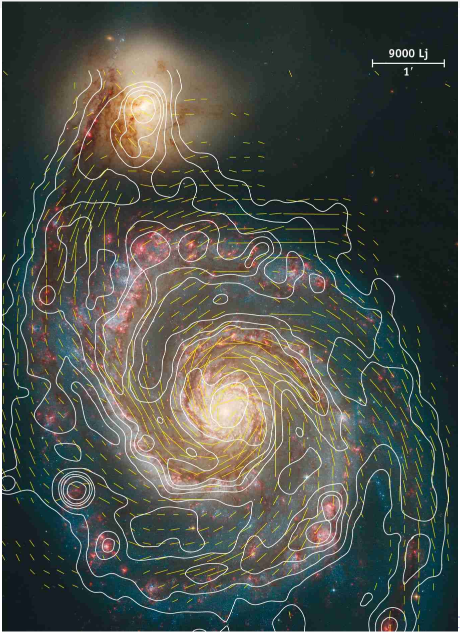


Observing Magnetic Fields

- Synchrotron radiation: polarization $\perp$ field orientation
- Polarization is changed when passing through magnetized plasma (*Faraday Rotation*):

$$\beta = \mathrm{RM}(n_e, B_{||})\,\lambda^2$$





References

S. B. F. Dorch.
Magnetohydrodynamics.
in: *Scholarpedia*, revision #186287.

R. Beck.
Galactic Magnetic Fields.
in: *Scholarpedia*, revision #185129.

U. Klein, A. Fletcher.
Galactic and Intergalactic Magnetic Fields.
Springer Praxis Books, 2015.

R. Beck.
Magnetic Fields in Spiral Galaxies, Part I: Disks.
https://www.mpifr-bonn.mpg.de/1281929/Beck_Galaxies.pdf