Module 3 Lab - Markov models

Markov Chains and Education

We analyze the situation of re-opening colleges under Covid-19, using a Markov Chain defined over a nine element state space. The proposed Markov Chain models a non-recurrent process that moves through a number of transient states (1 through 7) eventually leading to one of two absorbing states (8 & 9: Expulsion or Coursework Completion) where students remain forever. Absorbing (or recurring) states, like their name implies, are states wherein once entered, never leaves; whereas transient states are non-absorbing states. Because the state space contains both transient and absorbing states, instead of a steady state solution we obtain the long run probabilities of Expulsion or Completion the Course, when starting in any of the transient states. Their corresponding expected times are irrelevant here.

Consider a Markov Chain over a nine-element State Space defined as follows:

(1) Arrival to Campus and Covid-19 testing;
(2) Infected students go into Isolation units;
(3) Some students are placed in face to face courses;
(4) Other students are placed in online courses;
(5) Some students who violated Code are placed in Suspension;
(6) Some students become infected with Covid-19, but are not detected;
(7) Some students violate code but are not detected;
(8) Absorption: Some students are Expelled from College
(9) Absorption: Other students Complete their Semester

The Transition Probability Matrix A for this Markov Chain is:

State	1	2	3	4	5	6	7	8	9
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.05	0.70	0.00	0.00	0.00	0.50	0.00	0.00	0.00
3	0.35	0.20	0.80	0.00	0.10	0.00	0.00	0.00	0.00
4	0.60	0.00	0.00	0.80	0.10	0.00	0.00	0.00	0.00
5	0.00	0.05	0.00	0.00	0.70	0.00	0.70	0.00	0.00
6	0.00	0.00	0.05	0.05	0.00	0.30	0.10	0.00	0.00
7	0.00	0.00	0.05	0.05	0.00	0.00	0.00	0.00	0.00
8	0.00	0.05	0.00	0.00	0.10	0.00	0.20	1.00	0.00
9	0.00	0.00	0.10	0.10	0.00	0.20	0.00	0.00	1.00

Problem 1. Using any python graphing package (or online diagramming tool), visualize the state space transition diagram of A. You could either explore other python packages, or use cloud based diagramming sites (such as http://www.lucidchart.com or http://app.diagrams.net)

The Markov Chain unit time is a day. Transitions refer to the State changes that occur from one morning to the following morning. Students arrive (1), are tested and distributed into isolation (2) if infected (5%), face to face (3), and online courses (4). Some students become infected (6) and are again Isolated (if detected) or remain, asymptomatic, until the end of the semester (possibly spreading the disease). Some students violate Code by partying, gathering, etc. (7), and some get caught and are Suspended (5) or become infected (6), while others are not detected and return to their courses. Some isolated students break their status, are caught and are Suspended (5). Some Code violations are so blatant that students caught are Expelled (8), as also are some students that break their Suspension. Students Expelled do not return to Campus. All other students finish their semester (9) after completing Isolation, Suspension, and online or face to face coursework. The transition rates and transitions between states depend on environmental conditions (e.g. infection rates, social distancing, mask compliance), based on arbitrary values.

Problem 2. Define a MarkovChainLab class whose constructor accepts an n x n transition probability matrix A and, optionally, a list of state labels. If A is not column stochastic, raise a ValueError.

Similar to the MarkovChain class in the module, construct a dictionary mapping the state labels to the row/column index that they correspond to in A (given by order of the labels in the list), and save A, the list of labels, and this dictionary as attributes. Use the transition probability matrix table above, but EXCLUDE ALL the absorption states.

Add the following to the MarkovChainLab class.

- an attribute S, to represent the inverse matrix of (I- A), that is computed by $(I – A)^{-1}$, where I is the identity matrix.

- get() methods for both attributes.

We calculate the probabilities of Expulsion and Coursework Completion, starting from any of the transient states. We calculate these probabilities using Matrix B, a sub-matrix of all transient states. B is constructed by taking the rows and columns of A, corresponding to the two Absorbing states (last two states).

Problem 3. Using the S attribute from the MarkovChainLab class, multiplied with matrix B indicated from the paragraph above, calculate and display a new matrix G.

Such Matrix G, yields the probability of ever reaching absorbing states of Expulsion, or Coursework Completion, starting from any given transient state.

This Markov model, due to its specific State Spaces and its transition probabilities is particularly useful to assess the effectiveness of reopening plans.

Problem 4. If you were the dean / head of this Campus, what is your recommendation for the reopening of the school? Justify your answer by analyzing and providing insights from the simulation results.