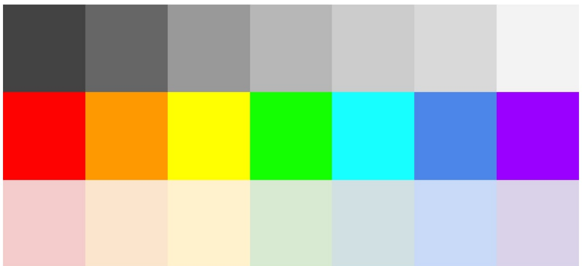
MATH 282.1: Computer Vision **Due Date: Nov. 18, 2024**
First Semester, 2024–2025

Instructions: Submit your solutions in A4 sheets of paper. Show all relevant solutions.

1. Consider the following image: [**6**]



   (**a**) Sketch the RGB components of the following image as they would appear on a monochrome monitor. Make sure to state all your assumptions on the nature of the intensity and saturation of the colors in the image.

   (**b**) Under the same assumptions, sketch the HSI components of the image as they would appear on a monochrome monitor.

2. (**a**) Let C be a constant. Show that [**8**]

$$\mathcal{F}\left\{e^{-Ct^2}\right\} = e^{-\omega^2/(4C)}\sqrt{\pi/C}$$

   where $\omega = 2\pi\mu$.

   (**b**) Show that

$$\mathcal{F}\{\cos(2\pi\mu_0 t)\} = \frac{1}{2}\left(\delta(\mu - \mu_0) + \delta(\mu + \mu_0)\right).$$

3. Prove the validity of the following properties of the Radon transform: [**10**]

   (**a**) (*Linearity*) The Radon transform is a linear operator.

   (**b**) (*Translation property*) The Radon transform of $f(x - x_0, y - y_0)$ is

$$g(\rho - x_0\cos\theta - y_0\sin\theta, \theta).$$

   (**c**) (*Convolution property*) The Radon transform of the convolution of two functions is equal to the convolution of the Radon transforms of the two functions.

4. Recall that the representation in vector-matrix form of a degraded image (which we will assume has size $M \times N$) is given by [**6**]

$$\mathbf{g} = \mathbf{Hf} + \eta.$$

   where $\mathbf{g}, \mathbf{H}, \mathbf{f}, \eta$ are the vector or matrix representations of the Fourier transforms of the degraded image, the spatial degradation, the undegraded image, and the noise functions, respectively.

   One of the methods we discussed for image restoration, especially in the presence of noise, was the *constrained least square (CLS) filtering*. To obtain our estimate for f, we introduced a smoothness term in our criterion, which gave rise to the following constrained optimization problem:

$$\text{Minimize} \quad ||\mathbf{Qf}||^2 \quad \text{subject to} \quad \mathbf{Hf} = \mathbf{g}$$

Use Lagrange multipliers to show that the solution (for $\mathbf{f}$) to the CLS optimization problem is given by

$$\hat{\mathbf{f}} = \left(\mathbf{H}^T\mathbf{H} + \lambda\mathbf{Q}^T\mathbf{Q}\right)^{-1}\mathbf{H}^T\mathbf{g}.$$

5. Refer to the notebook for the specifications. To be released at a later date [**10**]