Time Series Forecasting using ARMA-GARCH Models: Sto. Niño, Marikina River Water Level Prediction

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Abstract

Forecasting water levels in the frequently flooded Marikina River Basin (MRB) is crucial for disaster risk reduction. This study aims to create an ARMA-GARCH time series model to accurately predict the water level at the Sto. Niño water level station in the MRB. Hourly water level data of the Sto. Niño station for the entire year 2017 was used. After checking that the data was stationary and autocorrelated, an AR(3) model was chosen as the best mean model based on both performance and parsimony. Using the Ljung-Box Test, the residuals of the mean model were found to exhibit the ARCH effect. Using a grid search, the best joint model was determined to be AR(3)-GARCH(3,1). After model refinement, the final model AR(1)-GARCH(1,1) was used to predict the water level over the next 24 hours. After comparing the predicted and actual values, the model had a mean absolute percent error of 0.7952%, indicating promising predictive capability. Lastly, the standardized residuals of the joint model were found to be correlated, which indicates that future studies can explore more complicated GARCH models to more completely account for the volatility of the time series.

1 Introduction

1.1 Background of the study

Flooding is one of the most prevalent natural disasters worldwide. With rapid urbanization and climate change, rainfall events have become increasingly likely to cause flooding [1] and reduce the environment's capacity to absorb rainfall, heightening flood risk in low-lying city areas. Low-income countries are more vulnerable to flooding as over 89% of all flood-exposed people worldwide live in lower and middle income countries [7]. In the Philippines, Marikina City, which is located downstream of the Marikina River Basin is one of the most flood-prone urban cities in the entire country [6]. Only recently, in 2022, Typhoon Karding brought rainfalls that highly increased the water level of the Marikina River, exceeding the critical point of 18 m, and caused severe flooding in parts of Marikina that rendered some roads inaccessible [10]. Because of climate change and improper land use, this phenomenon has become more frequent and not only threatens human lives but also poses significant risks to infrastructure, livelihoods, and ecosystems [5].

Recognizing the urgency and magnitude of these challenges, effective strategies are devised to anticipate and mitigate the effects of flood-related hazards. One of the advancements in flood management is the development of flood forecasting and early warning systems [9]. These systems are particularly crucial for urban areas due to their susceptibility to severe flooding. Through meteorological data, hydrological insights, and computational models, these systems aim to provide timely and accurate predictions of potential flood events.

Currently, Marikina City employs an alarm level system [8], where the water level of the Marikina River under the Sto. Niño bridge is tracked. This is used to monitor flooding and propose necessary

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actions such as preparation and evacuation to be undertaken by the residents near the river. While the current alarm level system relies on the water level in Sto. Niño has been of great use, improvements can still be made in the form of more robust flood forecasting methods that can predict future water levels in real-time with minimal data requirements.

1.2 Statement of the problem

Due to the lack of forecasting capabilities along the Marikina River Basin, this study aims to conduct a time series analysis in forecasting Marikina River water levels. Specifically, the study aims to:

1. gather and analyze 2017 data on the Marikina River water levels at the Sto. Niño station from the Metropolitan Manila Development Authority (MMDA);
2. develop and implement linear time series models – Autoregressive (AR), Moving Average (MA), and Autoregressive-Moving Average (ARMA) – in predicting the water levels;
3. fit volatility models – Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) to further capture the variation of the water levels; and
4. assess the performance of the best linear time series model combined with the best volatility model and identify the best model to capture water level dynamics and generate reliable forecasts.

1.3 Scope and limitations

This study focuses solely on the Sto. Niño Water Level Station of the Marikina River Basin (MRB). While this station provides valuable data for analysis, it is important to note that other water level stations along the MRB may require different modeling approaches due to variations in local conditions and data characteristics. Furthermore, the dynamic nature of the MRB's physical characteristics suggests that the time series models developed in this study may have limited accuracy for forecasts extending beyond 2017.

In terms of methodology, this research employs a set of simple time series models for both mean and volatility modeling. Specifically, the study utilizes basic linear time series models, including AR, MA, and ARMA models for the mean equation. For volatility modeling, the study incorporates basic ARCH and GARCH models.

It is also worth noting that the scope of this study does not extend to more advanced modeling techniques that have been employed in related literature. For instance, some studies have utilized Self-Exciting Threshold Autoregressive GARCH (SETAR-GARCH) models, which provide interval predictions, offering a potentially more nuanced approach to forecasting [3]. Additionally, other research has incorporated multivariate GARCH (MGARCH) models to account for the influence of multiple hydrologic variables, such as rainfall, in their predictions [4]. These more complex modeling approaches fall outside the scope of the current study but represent potential avenues for future research in this domain.

2 Methodology

2.1 Mathematical theorems, definitions, and tests

Definition 1.1. $\{X_t\}$ is an **ARMA(p, q)** process if $\{X_t\}$ is stationary and if for every t,

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \cdots + \theta_q Z_{t-q},$$

where $\{Z_t\} \sim \text{WN}(0, \sigma^2)$ and the polynomials $(1 - \phi_1 z - \cdots - \phi_p z^p)$ and $(1 + \theta_1 z + \cdots + \theta_q z^q)$ have no common factors.

Definition 1.2 Let **AR (*p*)** be an autoregressive process of order *p* and **MA(*q*)** be a moving average process of order q. Then,

1. PACF cuts off at lag *p*
2. ACF cuts off at lag *q*

Definition 1.3 Let $\{X_t\}$ be a time series process. Then, $\{Y_t\}$ are the **residuals** of the time series process with the equation

$$Y_t = X_t - \hat{X}_t,$$

where X_t is the observed value and $\hat{X}_t$ is the predicted value.

Definition 1.4 The series $\{Y_t\}$ is an autoregressive conditionally heteroscedastic model of order m, denoted by **ARCH** (m), if it is of the form

$$\begin{cases} Y_t = \sigma_t \epsilon_t, \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i Y_{t-i}^2, \end{cases}$$

where $\epsilon_t \overset{\text{iid}}{\sim} (0,1), \alpha_0 > 0, \alpha_i \geq 0$, and $\sum_{i=1}^{m} \alpha_i < 1$.

Definition 1.5 The series $\{Y_t\}$ is a generalized autoregressive conditionally heteroscedastic model of order (m, s), denoted by **GARCH** (m, s), if it is of the form

$$\begin{cases} Y_t = \sigma_t \epsilon_t, \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i Y_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2, \end{cases}$$

where $\epsilon_t \overset{\text{iid}}{\sim} (0,1), \alpha_0 > 0, \alpha_i, \beta_j \geq 0$, and $\sum_{i=1}^{\max(m,s)} (\alpha_i + \beta_i) < 1$.

Definition 1.6 Given an ARCH model, let $\widetilde{Y}_t$ be the standardized residuals of the model, which is given by the equation:

$$\widetilde{Y}_t = \frac{Y_t}{\sigma_t},$$

where Y_t are the model's residuals and σ_t is the conditional standard deviation.

Augmented Dickey-Fuller Test (1979): Test for Stationarity. Consider the characteristic equation of an AR process.

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$$

1. $H_0 : \phi_i = 1$ for some $i \in \{1, \ldots, p\}$
   $H_1 : \phi_i \neq 1$ for all $i \in \{1, \ldots, p\}$
2. Test Statistic: $\hat{\tau}_\mu = \hat{\phi}_1^* / \hat{SE}(\hat{\phi}_1^*)$
3. Given $\alpha = 0.05$. If $\hat{\tau}_\mu < -2.86$, reject H_0. We conclude that the process is stationary.

Ljung-Box (1978): Test for Autocorrelation

1. $H_0 : \rho_1 = \rho_2 = \cdots = \rho_m = 0$
   $H_1 : \rho_i \neq 0$ for some $i \in \{1, \ldots, m\}$where $m \approx lnT$
2. $Q(m) = T(T+2) \sum_{h=1}^{m} \frac{\hat{\rho}_h^2}{T-h}$
3. Reject H_0 if $Q(m) > \chi^2(m, 1-\alpha)$

2.2 Dataset

This study used hourly water level data in meters (m) at the Sto. Niño station for the entire year of 2017. The data was obtained from the MMDA, and it was stored in a CSV file. A snippet of the dataset is found in Table 3 in Appendix B, while the plot is illustrated in Fig. 4 in Appendix B.

In general, it can be observed that the water level was typically low and stable during the first half of the year, with most of the spikes occurring in the latter half of the year. This behavior corresponds to the different seasons in the Philippines, wherein the second half of the year experiences more rainfall which contributes to a rise in water level.

However, further inspection reveals that there may be an inaccuracy in the reading or recording of the data. For one, the data for most of January is constant, then it is followed by a sudden drop in water level. In spite of this, the study proceeded with using the dataset obtained without changing its values.

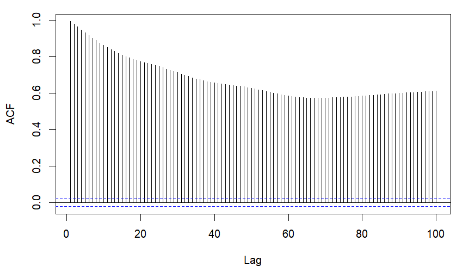
2.3 Checking stationarity and autocorrelation

The Augmented Dickey-Fuller test was used through the '`adf.test`' function in `R` to check the stationarity of the data. With an obtained $p-$value of 0.01, the null hypothesis is rejected and the conclusion is that the data is stationary.

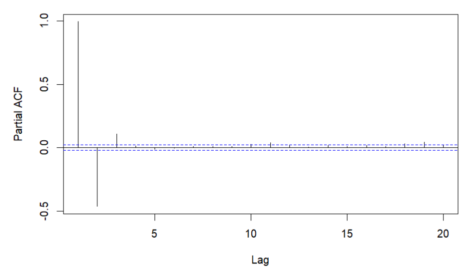
On the other hand, the Ljung-Box test was conducted using '`Box.test`' in `R` to check whether the data is serially correlated or not. A $p-$value of less than 2.2×10^{-16} was obtained, and so the null hypothesis is rejected. It was concluded that the time series is serially correlated.

2.4 Identifying candidate models for the mean model

In identifying candidate mean models, possible lag orders must first be determined. This is done by taking the ACF and PACF correlograms of the data, which are illustrated in Figs. 1a and 1b.



(a) ACF



(b) PACF

Figure 1: Correlograms.

It is observed that the ACF does not seem to cut off, while the PACF cuts off at different lag orders. Hence, the model to be used in this study is an AR model. Furthermore, it was determined that AR(2), AR(3), and AR(11) will be tested. The lag orders 3 and 11 were chosen because the PACF cuts off at those lag orders, and lag order 2 was chosen to check if a more parsimonious model is possible.

After modeling the data on these AR models, the metrics for each were recorded and are found in Table 1.

Table 1: Model performance for candidate models

Model	Log Likelihood	AIC	AICc	BIC
AR(2)	12367.56	-24727.13	-24727.12	-24698.81
AR(3)	12417.99	-24825.97	-24825.96	-24790.58
AR(11)	12431.05	-24836.10	-24836.06	-24744.09

The AR(2) model performed the worst in all metrics, while the AR(11) model performed the best in all but the BIC. However, given that there is a large difference in the number of coefficients to be estimated between the AR(11) and AR(3) models, it was determined that the more parsimonious model was to be used at the cost of worse performance in log-likelihood, AIC, and AICc. Moreover, the performance of AR(11) was also checked later on and the results can be seen in Appendix C.

2.5 Mean model and residual diagnostic

Using the '`arima`' function in `R`, the coefficients of the AR(3) model and their corresponding standard errors were obtained. These are listed in Table 4 in Appendix B.

Constructing 95% confidence intervals for each coefficient through Value $\pm$ $1.96 \cdot$ SE revealed that all coefficients are significant, and so the AR(3) model is given by

$$X_t = 0.1168 + 1.5035X_{t-1} - 0.6200X_{t-2} + 0.1070X_{t-3} + Y_t,$$

where $0.1168 = \phi_0 = \mu(1 - \phi_1 - \phi_2 - \phi_3)$.

The residuals were then tested to determine if they exhibited the ARCH effect. Using the Ljung-Box test on the residuals, a $p-$value of 0.9028 was obtained, which means that the null hypothesis is not rejected and that the residuals are uncorrelated. The Ljung-Box test was also used on the squared residuals, which yielded a $p-$value of less than 2.2×10^{-16}, and so the squared residuals are autocorrelated. Hence, the residuals exhibit the ARCH effect.

2.6 Identifying candidate models for the volatility model

A grid search using the joint estimation '`garchFit`' in `R` was implemented to find suitable values for m and s, the parameters of the GARCH model. Values of $1, \ldots, 4$ were assigned to m, while $0, \ldots, 4$ were assigned to s. The top three models are outlined in Table 2 and the best model was identified as AR(3)-GARCH(3,1).

Table 2: Top 3 AR(3)-GARCH(m, s) models

(m, s)	LLH	AIC	BIC	SIC	HQIC
(3,1)	16104.00	-3.6747	-3.6674	-3.6747	-3.6722
(1,0)	14599.49	-3.3318	-3.3270	-3.3318	-3.3302
(4,2)	13994.79	-3.1926	-3.1838	-3.1926	-3.1896

However, upon checking the significance of the coefficients of the AR(3)-GARCH(3,1) using Table 5 found in Appendix B, only the μ, ϕ_1, α_1, and β_1 are significant. Hence, a reduced model AR(1)-GARCH(1,1) was evaluated.

2.7 Refining the model

In deciding between the two models, the metrics were again checked as shown in Table 6 in Appendix B. With a minimal difference in the information criteria, the AR(1)-GARCH(1,1) model was chosen as the final model in the essence of parsimony. Finally, upon checking the significance of the coefficients of the final model as illustrated in Table 7 in Appendix B, all of the coefficients are now significant.

3 Result and discussion

3.1 Final model equation

The final model is AR(1)-GARCH(1,1) with equation:

$$\begin{cases} X_t = 0.0312 + 0.9495 X_{t-1} + Y_t \\ Y_t = \sigma_t \epsilon_t \\ \sigma_t^2 = 0.0006 + 0.4580 Y_{t-1}^2 + 0.5495 \sigma_{t-1}^2 \end{cases}$$

where $\epsilon_t \overset{\text{iid}}{\sim} (0, 1)$.

3.2 Checking standardized residuals

Given the final model, the standardized residuals were tested using the Ljung-Box Test to check if the standardized residuals were serially correlated or conditionally heteroscedastic, meaning the squares of the residuals were correlated. The standardized residuals of an ARMA-GARCH model should ideally be independent and identically distributed. The plot of the standardized residuals seen in Fig. 5 in Appendix B shows that the mean of the residuals is roughly zero.

For the standardized residuals, a p-value of less than 2.2×10^{-16} for the Ljung-Box test was obtained. This means that the standardized residuals are correlated. Meanwhile, for the squared standardized residuals, a p-value of 0.9178 was obtained in the Ljung-Box test. This means that the squared standardized residuals are not correlated.

Since the standardized residuals of the final model were correlated, the GARCH model was unable to fully capture some patterns in the data. The model's residuals were not independent in the final model, so this could lead to less accurate forecasts. Possible extensions to the GARCH model are discussed in Section 4.

3.3 Forecasting

A forecast horizon of 24 hours or 1 day was used in this study. Using the '`predict`' function in `R`, the plot of the forecasts can be seen in the following figure. The plot shows confidence intervals of the predictions given by meanForecast $\pm 1.96(\text{meanError})$.

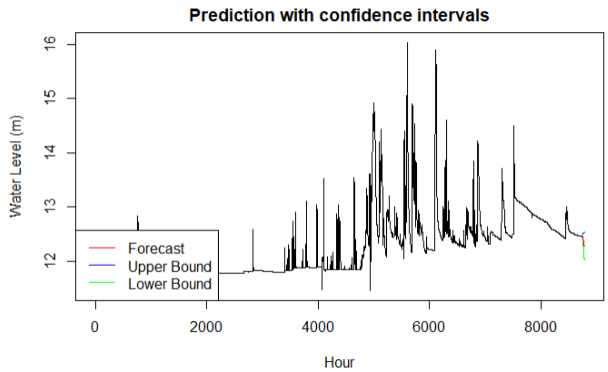


Figure 2: 24 Hour Forecasts of the Sto. Niño Water Level

A comparison of the forecasted water level and the actual values taken from the MMDA can be seen in Table 8 found in Appendix B. The 1-hour ahead forecast for the water level is 12.4283 meters with a forecasted variance of 0.0015. According to the refined model's predictions, the water level of Sto. Niño will slowly but continuously decrease in the next 24 hours. Since the predicted values continually decrease, the model may be unable to predict the water level over a long forecast horizon.

The 24-hour forecasts had a mean absolute percent error (MAPE) of 0.7952%. The mean absolute percent error is a measure of prediction accuracy expressed as the average percentage difference between the predicted and actual values. Since the final model obtained a MAPE of less than 1%, the model has sufficient predictive capability for the water level height 1 day ahead. A MAPE of below 10% is generally considered excellent for ARMA forecasting models [2]. However, since the refined model's standardized residuals are still correlated, the model may be lacking in its ability to predict future variance.

4 Recommendations

For future work, the proponents of this study suggest the following considerations:

1. Incorporate additional GARCH or ARCH terms to capture more complex patterns in the volatility of water levels. This approach may improve the model's ability to account for longer-term dependencies and more nuanced fluctuations in the data.
2. Introduce relevant exogenous variables, such as rainfall data or upstream flow rates to provide additional context and predictive power to the model. This inclusion will allow the model to account for external factors that influence water levels, potentially leading to more accurate forecasts.
3. Explore more advanced GARCH-family models, such as EGARCH or GJR-GARCH to capture asymmetric effects and other sophisticated volatility dynamics in the water level data. These models may offer improved performance in scenarios where standard GARCH models fall short, particularly in capturing non-linear relationships or changes in the volatility process.

References

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A Appendix: Grid Search

See the attached R Notebook for codes.

Model (p,m,s)	LLH	AIC	BIC	SIC	HQIC	Rank	Box.test(res)	Box.test(res^2)
3,3,1	16104	-3.674657	-3.667385	-3.674659	-3.67218	1	2.20E-16	0.3045
3,1,0	14599.49	-3.331846	-3.326998	-3.331847	-3.330194	2	2.20E-16	0.9478
3,4,2	13994.79	-3.192646	-3.183758	-3.192649	-3.189618	3	2.20E-16	0.9319
3,3,4	13620.5	-3.106964	-3.097269	-3.106968	-3.103661	4	2.20E-16	0.6002
3,2,1	13154.61	-3.001508	-2.995044	-3.00151	-2.999306	5	2.20E-16	0.3484
3,4,1	11990.98	-2.735383	-2.727303	-2.735385	-2.73263	6	2.20E-16	0.0182
3,1,3	9918.787	-2.262508	-2.255236	-2.26251	-2.260031	7	2.20E-16	0.4458
3,4,4	7708.182	-1.756891	-1.746387	-1.756895	-1.753312	8	2.20E-16	0.001406
3,4,0	7507.957	-1.712091	-1.704819	-1.712093	-1.709613	9	2.20E-16	0.9909
3,2,2	6996.736	-1.595373	-1.588102	-1.595376	-1.592896	10	2.20E-16	2.98E-06
3,3,3	6348.225	-1.446855	-1.437967	-1.446858	-1.443827	11	2.20E-16	0.8622
3,2,4	6296.339	-1.435009	-1.426121	-1.435012	-1.43198	12	2.20E-16	0.7871
3,4,3	5718.733	-1.302907	-1.293211	-1.302911	-1.299603	13	2.20E-16	0.09551
3,1,4	4354.326	-0.9918553	-0.9837754	-0.9918579	-0.9891022	14	2.20E-16	0.908
3,2,3	4045.627	-0.9213761	-0.9132963	-0.9213787	-0.9186231	15	2.20E-16	0.9718
3,1,2	3688.438	-0.8402827	-0.8338188	-0.8402843	-0.8380802	16	2.20E-16	0.9422
3,3,2	2967.208	-0.6751617	-0.6670819	-0.6751643	-0.6724086	17	2.20E-16	0.5635
3,1,1	2542.562	-0.5788954	-0.5732395	-0.5788967	-0.5769682	18	2.20E-16	0.6257
3,2,0	1652.251	-0.3756282	-0.3699723	-0.3756295	-0.373701	19	2.20E-16	0.8298
3,3,0	1533.138	-0.3482051	-0.3417412	-0.3482067	-0.3460026	20	2.20E-16	0.8581

Figure 3: Entire results of the grid search

B Appendix: Tables and Figures

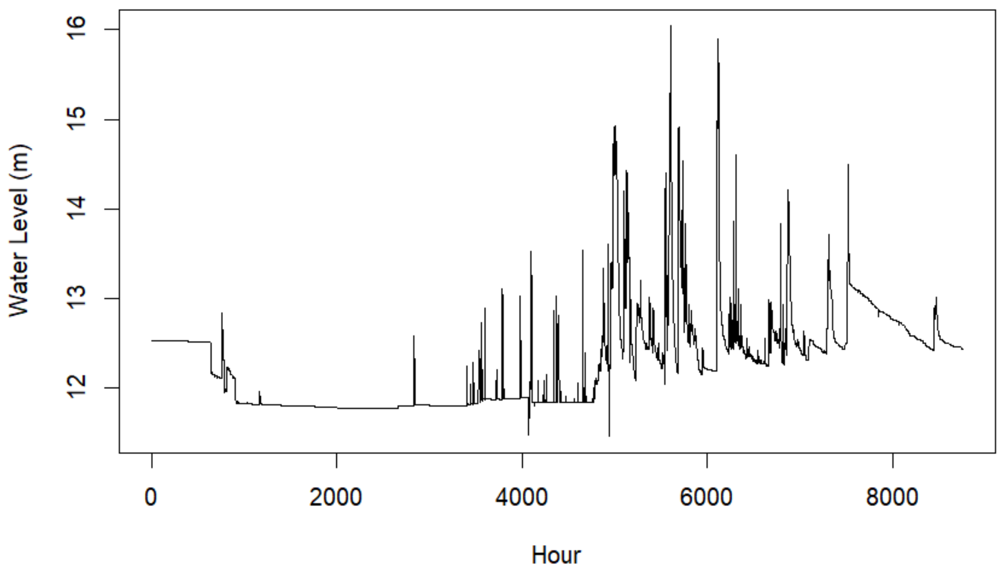

Figure 4: 2017 Sto. Niño water levels

Table 3: Snippet of the dataset

Waterlevel_Sto_Niño	datetime
11.87	02/07/2017 13:00
11.85	02/07/2017 14:00
11.84	02/07/2017 15:00

Table 4: Coefficients of the mean model

Coefficient	ϕ_1	ϕ_2	ϕ_3	μ
Value	1.5035	-0.6200	0.1070	12.2958
S.E.	0.0106	0.0181	0.0106	0.0658

Table 5: The p-value of the AR(3)-GARCH(3,1) coefficients

Coefficient	$p-$value
μ	$< 2.2 \times 10^{-16}$
ϕ_1	$< 2.2 \times 10^{-16}$
ϕ_2	0.329
ϕ_3	0.849
α_0	NaN
α_1	$< 2.2 \times 10^{-16}$
α_2	1
α_3	1
β_1	$< 2.2 \times 10^{-16}$

Table 6: Comparison between the original and refined models

Metric	AR(3)-GARCH(3,1)	AR(1)-GARCH(1,1)
LLH	16104.00	14039.95
AIC	-3.6747	-3.2043
BIC	-3.6674	-3.2003
SIC	-3.6747	-3.2043
HQIC	-3.6722	-3.2029

Table 7: The p-value of the AR(1)-GARCH(1,1) coefficients

Coefficient	$p-$value
μ	$< 2.2 \times 10^{-16}$
ϕ_1	$< 2.2 \times 10^{-16}$
α_0	$< 2.2 \times 10^{-16}$
α_1	$< 2.2 \times 10^{-16}$
β_1	$< 2.2 \times 10^{-16}$

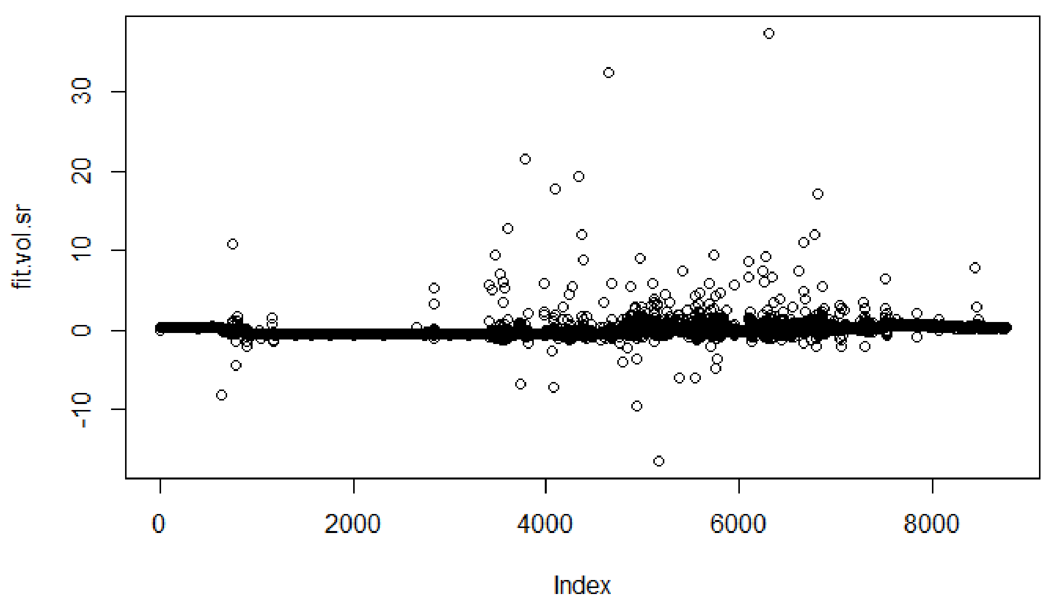

Figure 5: Standardized residuals

Table 8: AR(1)-GARCH(1,1) Forecasts

DateTime	Actual	Prediction	Mean Error	σ^2
2018-01-01 00:00:00	12.44	12.4283	0.0384	0.0015
2018-01-01 01:00:00	12.44	12.4173	0.0584	0.0021
2018-01-01 02:00:00	12.44	12.4067	0.0760	0.0027
2018-01-01 03:00:00	12.44	12.3967	0.0924	0.0033
2018-01-01 04:00:00	12.44	12.3873	0.1079	0.0040
2018-01-01 05:00:00	12.44	12.3783	0.1228	0.0046
2018-01-01 06:00:00	12.44	12.3697	0.1372	0.0052
2018-01-01 07:00:00	12.44	12.3616	0.1511	0.0059
2018-01-01 08:00:00	12.44	12.3539	0.1646	0.0065
2018-01-01 09:00:00	12.44	12.3466	0.1777	0.0072
2018-01-01 10:00:00	12.44	12.3396	0.1905	0.0078
2018-01-01 11:00:00	12.44	12.3330	0.2029	0.0085
2018-01-01 12:00:00	12.44	12.3268	0.2151	0.0091
2018-01-01 13:00:00	12.44	12.3208	0.2269	0.0098
2018-01-01 14:00:00	12.44	12.3152	0.2386	0.0105
2018-01-01 15:00:00	12.44	12.3098	0.2499	0.0112
2018-01-01 16:00:00	12.43	12.3047	0.2611	0.0118
2018-01-01 17:00:00	12.43	12.2999	0.2720	0.0125
2018-01-01 18:00:00	12.43	12.2953	0.2827	0.0132
2018-01-01 19:00:00	12.43	12.2909	0.2932	0.0139
2018-01-01 20:00:00	12.43	12.2868	0.3036	0.0146
2018-01-01 21:00:00	12.43	12.2829	0.3137	0.0154
2018-01-01 22:00:00	12.43	12.2791	0.3237	0.0161
2018-01-01 23:00:00	12.43	12.2756	0.3336	0.0168

C Appendix: AR(11) Modeling

For the GARCH model to be used with the AR(11) mean model, m was assigned values of 1 and 2 while s was assigned values of 0 and 1 so as to keep the number of variables as small as possible. Fig. 6 contains the metrics for each of the 4 ARMA-GARCH models created for this instance.

Model (p,m,s)	LLH	AIC	BIC	SIC	HQIC	Rank	Box.test(res)	Box.test(res^2)
11,2,1	14492.39	-3.305113	-3.292185	-3.30512	-3.300708	1	2.20E-16	2.20E-16
11,2,0	13985	-3.189498	-3.177379	-3.189504	-3.185369	2	2.20E-16	2.20E-16
11,1,1	12375.72	-2.822082	-2.809962	-2.822088	-2.817952	3	2.20E-16	2.20E-16
11,1,0	3789.967	-0.8620929	-0.8507811	-0.862098	-0.8582386	4	2.20E-16	2.20E-16

Figure 6: Metrics of the AR(11)-GARCH(m, s) models

Additionally, the significance of coefficients were checked to see if refinements could be done on the AR(11)-GARCH(2,1) model. This is illustrated in Fig. 7.

```
Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)    
mu      3.516e-02   2.962e-05 1186.956  < 2e-16 ***
ar1     9.999e-01   3.785e-02   26.420  < 2e-16 ***
ar2    -5.278e-01   8.031e-02   -6.572 4.95e-11 ***
ar3     1.496e-01   8.381e-02    1.785  0.07425 .  
ar4     8.996e-02   7.478e-02    1.203  0.22895    
ar5     4.977e-02   6.586e-02    0.756  0.44984    
ar6     3.033e-02   6.045e-02    0.502  0.61585    
ar7     4.088e-02   5.224e-02    0.783  0.43389    
ar8     4.621e-02   4.687e-02    0.986  0.32413    
ar9     3.669e-02   4.313e-02    0.851  0.39501    
ar10    5.072e-03   4.070e-02    0.125  0.90082    
ar11    7.677e-02   2.458e-02    3.124  0.00179 ** 
omega   6.177e-04   1.146e-07 5391.558  < 2e-16 ***
alpha1  2.890e-01         NaN      NaN      NaN    
alpha2  2.448e-01         NaN      NaN      NaN    
beta1   4.783e-01   5.339e-05 8959.294  < 2e-16 ***
```

Figure 7: Significance of coefficients of the AR(11)-GARCH(2,1) model

Because the coefficient, ϕ_{11} is significant, the AR order remains at 11. Also, because m cannot be 0 and GARCH(1,1) was already tested, it was determined that the GARCH model will not be reduced. These mean that the model to be used is AR(11)-GARCH(2,1).

However, it was noted that the AR(11)-GARCH(2,1) model performs worse than the AR(3)-GARCH(3,1) model, which was the selected model, across all metrics. Moreover, the residuals were still found to be correlated through the Ljung-Box Tests. Hence, it was decided that the AR(3)-GARCH(3,1) model, before refinements, would be the one to be used in this study.