ATENEO DE MANILA UNIVERSITY

SYLLABUS FOR GRADUATE COURSES

A. COURSE INFORMATION

COURSE NUMBER	MATH 240.5			**NO. OF UNITS**	3 units
COURSE TITLE	Applied Linear Algebra				
PREREQUISITE/S	MATH 40.1				
DEPARTMENT/ PROGRAM	Department of Mathematics			**SCHOOL**	SOSE
SCHOOL YEAR	2024 – 2025			**SEMESTER**	Second
INSTRUCTOR/S	Mark L. Loyola, PhD				
VENUE	SECA-303A	**SECTION**	L	**SCHEDULE**	TF 11:00 – 12:30

B. COURSE DESCRIPTION

This is a course on linear algebra aimed at both pure and applied mathematics students who wish to employ linear algebra methods in a variety of applications. Key topics include factorization of matrices; linearity; inner products and orthogonality; least squares approximations and interpolations; singular values and principal component analysis; and linear iterations and other numerical solution schemes. While the core concepts in the course are treated and handled with mathematical rigor, the use of software to perform and implement computations and algorithms are greatly emphasized. An exposition of new methods and techniques from journal articles are surveyed for graduate students.

WHERE IS THE COURSE SITUATED WITHIN THE FORMATION STAGES IN THE FRAMEWORK OF THE LOYOLA SCHOOLS CURRICULA	
	FOUNDATIONS: Exploring and Equipping the Self
X	ROOTEDNESS: Investigating and Knowing the World
	DEEPENING: Defining the Self in the World
	LEADERSHIP: Engaging and Transforming the World

C. PROGRAM LEARNING OUTCOMES

BS in Applied Mathematics – M in Data Science Program Learning Outcomes	
PLO 1	Implement basic mathematical, statistical, and computational methods in data analysis and problem solving.

PLO 2	Understand the limitations and implications of mathematical models as applied in real-life contexts.
PLO 3	Develop creativity, rigor, and discipline in dealing with various mathematical problems.
PLO 4	Utilize appropriate data visualization techniques in exploratory data analysis to extract meaningful insights from complex datasets and identify correlations, outliers, patterns, and trends.

	PLO1	**PLO2**	**PLO3**	**PLO 4**
CLO 1.1			X	
CLO 1.2	X	X		X
CLO 1.3	X	X		X
CLO 2.1			X	
CLO 2.2	X	X		X
CLO 2.3	X	X	X	X
CLO 3.1			X	
CLO 3.2		X		
CLO 3.3	X	X		

D. COURSE LEARNING OUTCOMES

By the end of this course, students should be able to:

Competence	**Course Learning Outcomes [Knowledge · Skills · Attitude]**
Competence 1: Employ applied linear algebra concepts and procedures to analyze or solve a real-world task.	**CLO 1 [K]:** Discuss important concepts, theorems, objects, or methods related to linear algebra in a mathematically rigorous manner.
	CLO 2 [S]: Implement fundamental procedures and methods (matrix factorizations and decompositions, linear approximations and interpolations) in a numerical software package.
	CLO 3 [A]: Realize the power of linear algebra in analyzing or solving a real-world task.
Competence 2: Design linear algebra-based procedures and algorithms to accomplish a task methodically and systematically.	**CLO 1 [K]:** Explain underlying principles behind a linear algebra-based procedure or algorithm.
	CLO 2 [S]: Translate linear algebra concepts, procedures, or algorithms to computer readable codes.

	CLO 3 [A]: Develop the ability to structure algorithms so that they are mathematically sound and efficient in computational tasks.
Competence 3: Critique novel methods of factorization and algorithms introduced in recent research journals.	**CLO 1 [K]:** Explain the theoretical foundations of the new methods.
	CLO 2 [S]: Compare the strengths and weaknesses between new and established methods.
	CLO 3 [A]: Exemplify innovative strategies in crafting new algorithms.

E. COURSE OUTLINE and LEARNING HOURS

Course Outline	CLOs	Estimated Learning Hours
Module 1 Linear Algebraic Systems and Vector Spaces 1.3. Gaussian Elimination – Regular Case ▪ The LU Factorization ▪ Forward and Back Substitution 1.4. Pivoting and Permutations ▪ The Permuted LU Factorization 1.5. Matrix Inverses ▪ Gauss-Jordan Elimination ▪ The LDV Factorization 1.6. Transposes and Symmetric Matrices ▪ Factorization of Symmetric Matrices 1.7. Practical Linear Algebra ▪ Tridiagonal Matrices ▪ Pivoting Strategies 1.8. General Linear Systems ▪ Homogeneous Systems 2.5. The Fundamental Matrix Subspaces ▪ Kernel and Image ▪ The Superposition Principle ▪ Adjoint Systems, Cokernel, and Coimage ▪ The Fundamental Theorem of Linear Algebra	1.1 – 3.3	10
Module 2 Inner Products, Norms, and Orthogonality 3.3. Norms ▪ Unit Vectors ▪ Equivalence of Norms ▪ Matrix Norms 3.4. Positive Definite Matrices ▪ Gram Matrices 3.5. Completing the Square ▪ The Cholesky Factorization 3.6. Complex Vector Spaces ▪ Complex Numbers ▪ Complex Vector Spaces and Inner Products 4.3. Orthogonal Matrices ▪ The QR Factorization ▪ Ill-Conditioned Systems and Householder's Method	1.1 – 3.3	20

4.4. Orthogonal Projections and Orthogonal Subspaces ▪ Orthogonal Projection ▪ Orthogonal Subspaces ▪ Orthogonality of the Fundamental Matrix Subspaces and the Fredholm Alternative		
Module 3 Minimization and Least Squares 5.1. Minimization Problems ▪ Equilibrium Mechanics ▪ Solution of Equations ▪ The Closest Point 5.2. Minimization of Quadratic Functions 5.3. The Closest Point 5.4. Least Squares 5.5. Data Fitting and Interpolation ▪ Polynomial Approximation and Interpolation ▪ Approximation and Interpolation by General Functions ▪ Least Squares Approximation in Function Spaces	1.1 – 3.3	20
Module 4 Linearity, Eigenvalues, and Singular Values 7.4. Linear Systems ▪ The Superposition Principle ▪ Inhomogeneous Systems ▪ Superposition Principles for Inhomogeneous Systems 7.5. Adjoints, Positive Definite Operators, and Minimization Principles ▪ Self-Adjoint and Positive Definite Linear Functions ▪ Minimization 8.2. Eigenvalues and Eigenvectors ▪ The Gerschgorin Circle Theorem 8.3. Eigenvector Bases ▪ Diagonalization 8.4. Invariant Subspaces 8.5. Eigenvalues of Symmetric Matrices ▪ The Spectral Theorem ▪ Optimization Principles for Eigenvalues of Symmetric Matrices 8.6. Incomplete Matrices ▪ The Schur Decomposition ▪ The Jordan Canonical Form 8.7. Singular Values ▪ The Pseudoinverse ▪ The Euclidean Matrix Norm ▪ Condition Number and Rank 8.8. Principal Component Analysis ▪ Variance and Covariance ▪ The Principal Components	1.1 – 3.3	25

*Sections here refer to chapter numbers in our main reference (see **Section H**).

F. ASSESSMENTS AND RUBRICS

Assessment Task	Tentative Date and Time	Assessment Weight	CLOs
Problem Set No. 1 (Modules 1 and 2)	February 21 (05:00 PM)	55 points	1.1 – 3.3
Programming Task No. 1 (Modules 1 and 2)	February 28 (05:00 PM)	45 points	1.2, 1.3, 2.2, 2.3, 3.3
Problem Set No. 2 (Module 3)	March 21 (05:00 PM)	55 points	1.1 – 3.3
Programming Task No. 2 (Module 3)	March 28 (05:00 PM)	45 points	1.2, 1.3, 2.2, 2.3, 3.3
Problem Set No. 3 (Module 4)	April 25 (05:00 PM)	55 points	1.1 – 3.3
Programming Task No. 3 (Module 4)	May 2 (05:00 PM)	40 points	1.2, 1.3, 2.2, 2.3, 3.3
Class Participation		10 points	1.1 – 3.3

Criteria for Grading Programming Tasks	Weight
Code Accuracy and Efficiency	5 points
Code Output	3 points
Code Readability	2 points

Criteria for Grading Proofs	Weight
Statement of the problem and correct use of notations	2 points
Appropriate choice and use of proving technique	2 points
Correct use of definitions and reference to previous results	2 points
Concise writing	2 points
Logical Flow	2 points

G. TEACHING AND LEARNING METHODS

TEACHING & LEARNING METHODS and ACTIVITIES	CLOs
Synchronous Lecture Sessions	1.1 – 3.3
Self-paced Study/Reading Assignments	1.1 – 3.3
Problem Solving Sessions	1.1 – 2.3

H. MAIN REFERENCE

Olver, P.J. and Shakiban, C. *Applied Linear Algebra*, 2nd Ed. Springer, 2018.

I. SUGGESTED READINGS

1. Agrawal, C.C. Linear Algebra and Optimization for Machine Learning. Springer, 2020.
2. Boyd, S. and Vandenberghe L. Introduction to Applied Linear Algebra. Cambridge University Press, 2018.
3. Cohen, M.X. Practical Linear Algebra for Data Science: From Core Concepts to Applications Using Python. O'Reilly, 2022.
4. DeBonis, M.J. Introduction to Linear Algebra: Computation, Application, and Theory. CRC Press, 2022.
5. Deisenroth, M.P., Faisal, A.A., and Ong, C.S. Mathematics for Machine Learning. Cambridge University Press, 2020.
6. Meyer, C.D. Matrix Analysis and Applied Linear Algebra. SIAM, 2010.
7. Miolane, L. Optimization and Computational Linear Algebra for Data Science Lecture Notes. https://leomiolane.github.io/linalg-for-ds.html
8. Mitran, S. Linear Algebra for Data Science. http://mitran-lab.amath.unc.edu/.../textbook.pdf
9. Olver, P.J. and Shakiban, C. Applied Linear Algebra, 2nd Ed. Springer, 2018.
10. Shores, T.S. Applied Linear Algebra and Matrix Analysis. Springer, 2007.
11. Strang, G. Introduction to Linear Algebra, 5th Ed. Wellesley-Cambridge Press, 2016.
12. Strang, G. Linear Algebra and Learning from Data. Wellesley-Cambridge Press, 2019.

J. GRADING SYSTEM

Each assessment task listed in **Section F** will be individually marked. The final numeric mark will be the average of the scores in these assessments. The following conversion table will be used to determine the corresponding letter grade.

Percentage of the Total Score	Letter Grade
92 % – 100 %	A
80 % – < 92 %	A-
66 % – < 80 %	B+
50 % – < 66 %	B
40 % – < 50 %	C
< 40%	F

K. CLASS POLICIES

1. **Learning Management System.** All learning modules for this course are found in the official MATH 240.5-L Canvas account in the Ateneo Blue Cloud.

   Only students who are officially enrolled will be included in the Canvas class and will be allowed to take and submit graded assessments. Students who join the course in Canvas but whose names do not appear in the official class list provided by the registrar will be removed in the Canvas course. The student's Canvas account should bear the student's full name. The student's recent ID picture should be used as profile photo.

2. **Learning Mode.** All lecture sessions will be delivered onsite on the indicated schedule. However, if, due to an unforeseen event (e.g. weather disturbance, health crisis), the course has to switch to a fully-online mode of delivery, then all remaining onsite sessions will now be online synchronous sessions. In such event, we shall use the Zoom platform. The recurring meeting details are indicated below:

   Link: https://ateneo-edu.zoom.us/j/96449618017?pwd=lkhwn7a2RWhIW3E2kPBqB0d5bkxAVu.1

Meeting ID: 964 4961 8017
Passcode: MATH240.5L

Note that you are expected to have read and understood all relevant sections of pre-assigned modules before attending the lecture session. For an online session, an unedited recording of the session including the materials used/produced during that session will be uploaded to the following google drive at most two days after the session:

https://drive.google.com/drive/folders/1tSe9ODdz-ZzItkGaL459cgMUQAqZdzFY?usp=sharing

3. **Attendance.** Attendance in each lecture session will be checked two minutes after the start of the session. A student will be marked absent if they are not present when their name is called. Note that a student's absence may be excused depending on the circumstances. In such a case, a student must prepare an excuse letter explaining the circumstance of the absence duly noted and signed by their parent/guardian. Only a limited number of absences is allowed. A student who exceeds six absences will automatically get a grade of W.

   Students are expected to participate fully in all sessions. Students who cannot participate in a graded assessment shall inform the instructor in advance or immediately after through Canvas or email.

4. **Communication.** If you have non-time sensitive concerns, questions, or clarifications, you may send me an email during office hours (weekdays, 08:00 AM – 05:00 PM) or schedule a consultation. My consultation hours and contact details are indicated in **Section L**.

5. **Beadle.** During the first synchronous session, a class beadle shall be selected. The adjusted functions of the beadle are as follows:

   a) create and manage alternative platforms for purposes of communication and community;
   b) serve as liaison between the class and the instructor;
   c) remind the instructor to record synchronous online class sessions; and
   d) report class concerns to the department chair and school dean (e.g. deviations from the syllabus, difficulties in contacting the instructor, etc.)

6. **Computing Tools.** Computer activities to enrich the understanding of certain lessons will also require the use Jupyter noteboks using the Python programming language as the kernel. I recommend Visual Studio Code as integrated development environment.

7. **Problem Sets.** Use A4 size bond papers for problem sets. LaTeX typesetting is required. The full set of problems will be posted at least a week before the set deadline.

8. **Late/Non-compliant Submissions.** Penalties will be imposed on any submission that is either completed after the deadline or not compliant to the guidelines set by the instructor. As a general rule, there will be a deduction of 1 point for every unfollowed instruction or for every two minutes of late submission.

9. **Corrections in Checking.** Should there be any mistake in the checking of any course requirement, a student is given two days after the said requirement is returned to report the oversight to the instructor.

10. **Canvas Grades.** Grade averages reflected on Canvas are NOT the official grades due to differences in weights and other factors. The class will be informed of official marks after each assessment. The official grade of the student in the course is the grade that is posted by the Registrar and reflected in the student's AISIS account.

11. **Copyright and Data Protection.** All materials in the course modules and recordings of the synchronous sessions should not be posted anywhere on the internet nor shared with individuals who are not enrolled in the class. Infringements will result in disciplinary actions. If any of the course material is caught to have been uploaded in any form to any platform or website online or shared offline, the person/s responsible for it will be reprimanded. In the case that any assessment item is

caught to have been uploaded in any form to any platform or website online or shared offline, the person/s responsible will get a zero for the said assessment.

12. **Privacy.** Taking photos or recording videos of class lectures, regardless of modality, is subject to the approval of the course instructor. However, in case the instructor allows for photos or videos to be taken, these should not be made publicly available (e.g. posted on social media) without the consent and approval of the instructor. For non-lecture photos or videos taken in class (e.g. group photo), first seek the consent of your instructor (and of classmates involved) prior to posting online. Posting without the prior consent of the instructor (and of classmates involved) will be considered a breach of privacy and will result in disciplinary actions.

13. **Academic Integrity.** Students are expected to exercise the highest level of academic integrity. Cheating or plagiarism will not be tolerated and will be treated as a grave offense. Disciplinary action will be pursued, following the process set by the university. Cheating during any graded activity includes, but is not limited to, posting of answers or hints related to any graded work in any online or offline platform.

14. **Use of Generative AI Policy.** The use of generative AI (GenAI) for written exams is strictly prohibited. However, it may be used as an exploratory tool for other types of assessments. If GenAI was used for at least 10% of a task, the student must explicitly state how it was used. In this case, the use of GenAI may affect the student's final score on the applicable task.

15. **Academic Conduct.** Students are expected to treat their peers and instructor with respect at all times even in an online environment. Committing any form of discourtesy or misbehavior may lead to disciplinary action.

16. **Non-discrimination.** The school does not discriminate on the basis of sex, gender, marital or parental status, sexual orientation, or gender identity or expression.

17. **Useful Links for Students.** The following are useful links for students:

    - Higher Education Gender Policy
    - Code of Decorum and Administrative Rules on Sexual Harassment, Other Forms of Sexual Misconduct, and Inappropriate Behavior
    - Graduate Student Handbook, 2024 edition

L. CONSULTATION HOURS

NAME OF FACULTY	EMAIL	DAY/S	TIME
Mark L. Loyola	mloyola@ateneo.edu	Tue, Fri	12:30 – 01:50 PM