

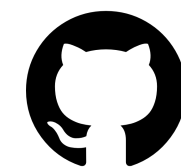
The anatomy of an order-preserving k-mer dictionary

Giulio Ermanno Pibiri

Ca' Foscari University of Venice



@giulio_pibiri



@jermpp

RIKEN-AIP Tutorial: Compressed Data Structures for Advanced Data Analysis
Tokyo, Japan, 2 July 2025

Agenda

1. Context, motivations, and problems
2. Tools:
 - Spectrum-preserving string sets
 - Minimizers
 - Minimal perfect hashing
3. Sparse and skew hashing of k-mers

1. Context, motivations, and problems

Massive DNA Collections

- **Peta bytes** of data available:
 - ENA (European Nucleotide Archive)
 - SRA (Sequence Read Archive)
 - RefSeq (Reference Sequence Database)
 - Ensembl
- For example: as of June 2025, ENA has 5.7 billions of assembled sequences, for 30 trillion bases.
<https://www.ebi.ac.uk/ena/browser/about/statistics>
- These collections are paving the way to answer fundamental questions regarding biology and evolution.



k-mers

- **Q.** But how do we exploit such potential?

We need efficient methods to index and search data at this scale.

- One popular strategy: “reduce” a DNA sequence to a set of short substrings of fixed length k — the so-called ***k-mers***.

ACGGTAGAACCGATTCAAATTCGACGTAGC...

A**CGGTAGAACCGA**

CGGTAGAACCGAT

GGTAGAACCGATT

GTAGAACCGATT

TAGAACCGATTCA

AGAACCGATTCAA

GAACCGATTCAAA

AACCGATTCAAAT

...

← Example for $k=13$.

k-mer applications

- Software tools based on k-mers are predominant in bioinformatics.
- Many applications:
 - genome assembly
 - variant calling
 - pan-genome analysis
 - meta-genomics
 - sequence comparison/alignment
 - ...

A world of k-mer indexes

- Huge research effort produced many types of indexes based on k-mers, with different:
 - representations (hashing, BWT-based, exact vs. approximate),
 - features (e.g., static vs. dynamic),
 - space/time trade-offs,
 - types of supported queries, etc.
- Recent surveys on this topic:
 - [Data Structures based on \$k\$ -mers for Querying Large Collections of Sequencing Data Sets](#)
Marchet et al., Genome Research, 2020.
 - [Data Structures to Represent a Set of \$k\$ -long DNA Sequences](#)
Chikhi et al., ACM Computing Surveys, 2021.

The k-mer dictionary problem

- We are given a large DNA string S (e.g., a genome or a pan-genome) and let K be the set of all its n distinct k-mers.

Example: The human genome (GRCh38) has >2.5B distinct k-mers for $k=31$.

- **Problem.** We want to build a dictionary for K so that the following operations are efficient:
 - Lookup(x) returns $1 \leq i \leq n$ if the k-mer $x \in K$ or \perp otherwise;
 - return the k-mer $x = \text{Access}(i)$ if $1 \leq i \leq n$.
- Other operations of interest are **streaming queries**, iteration, navigational queries.

Do we need an *ad-hoc* solution?

- The algorithmic literature about (*compressed*) *string dictionaries* is rich of solutions [[Martínez-Prieto et al., 2016](#)] (e.g., Front-Coding, path-decomposed tries, double-array tries).
- But they are relevant for “generic strings”:
 - variable-length,
 - larger alphabets (e.g., ASCII),
 - (usually) no particular properties of the strings to aid compression.
- Since k-mers are extracted *consecutively* from DNA, a k-mer following another one shares k-1 bases (very low entropy).

ACGGTAGAACCGATTCAAATTCGACGTAGC...

A**CGGTAGAACCGA**

CGGTAGAACCGAT

GGTAGAACCGATT

GTAGAACCGATT

TAGAACCGATTCA

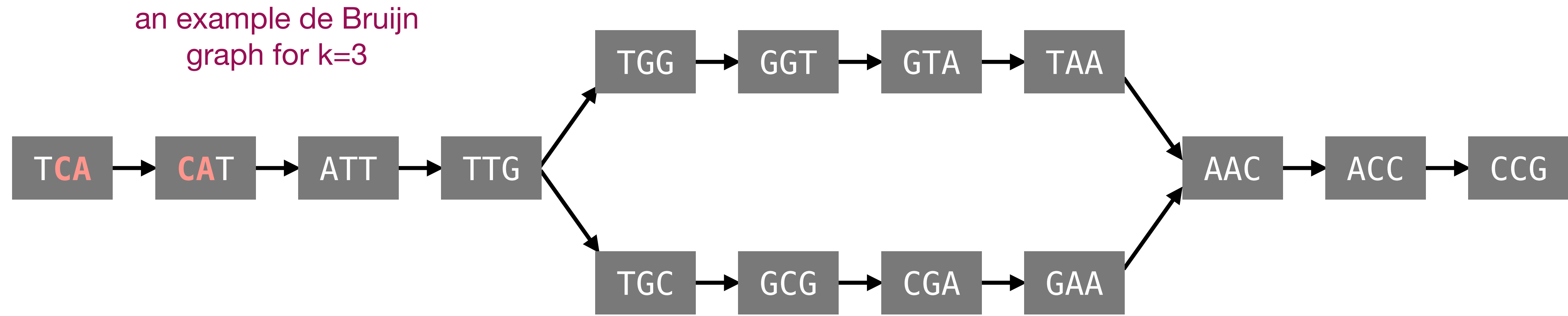
...

← Example for k=13.

2. Tools

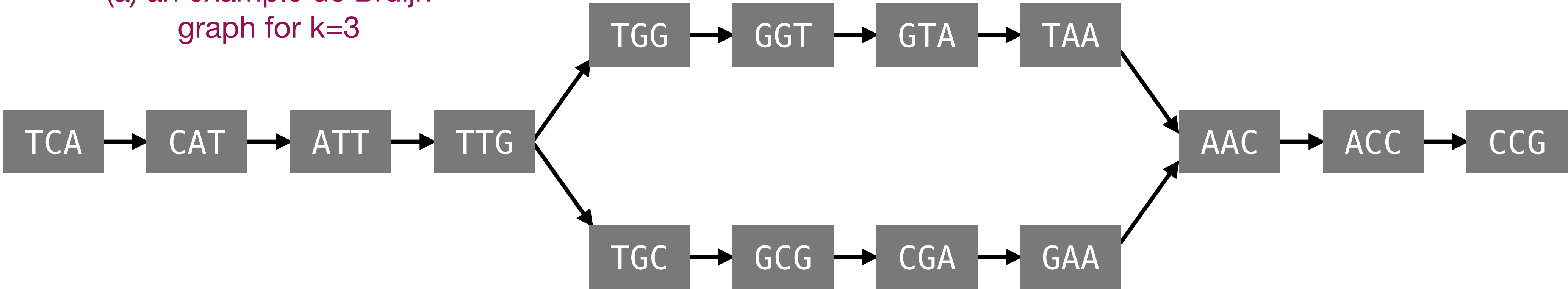
de Bruijn graphs

- **de Bruijn graph.** A (node centric) de Bruijn graph (dBG) of order k for a sequence S is a directed graph where nodes are the distinct k -mers of S and there exists a directed edge from x to y if $x[2..k] = y[1..k-1]$.
- **Fact.** Equivalence between a set of k -mers and a de Bruijn graph.



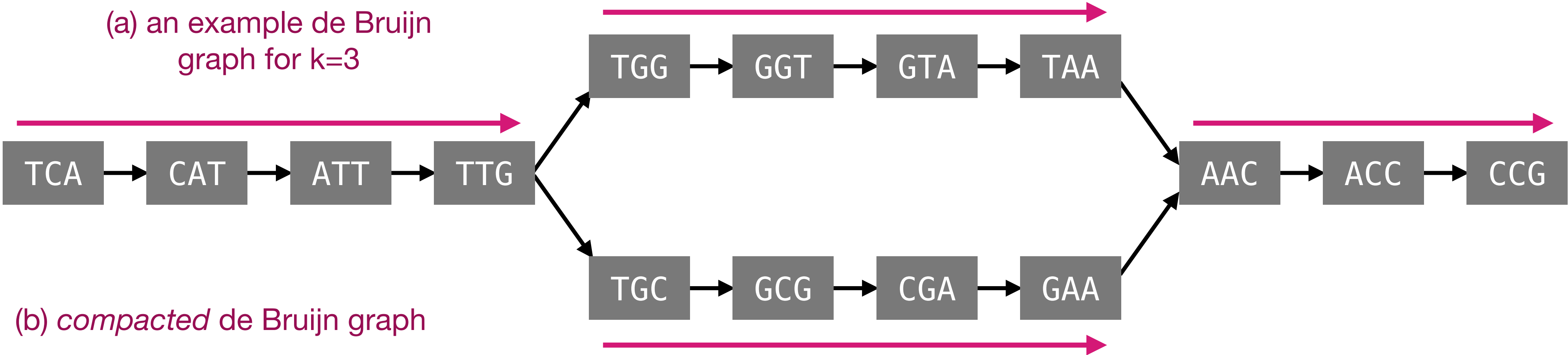
de Bruijn graphs

(a) an example de Bruijn graph for k=3

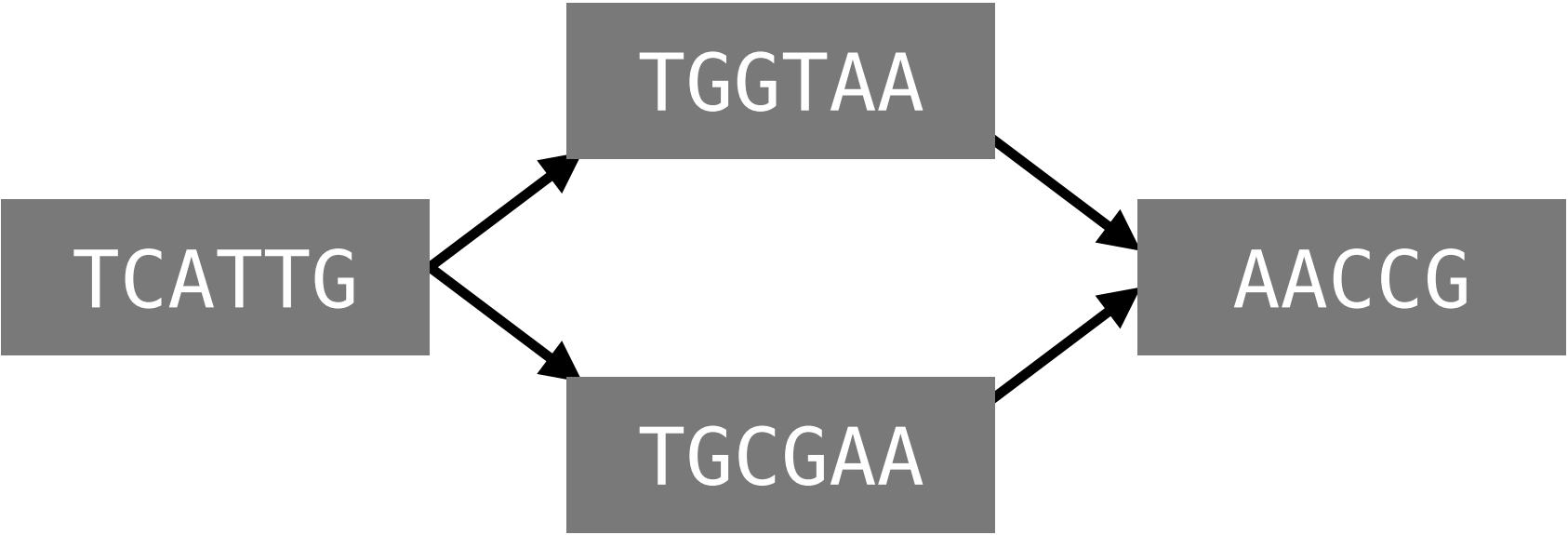


de Bruijn graphs

(a) an example de Bruijn graph for $k=3$

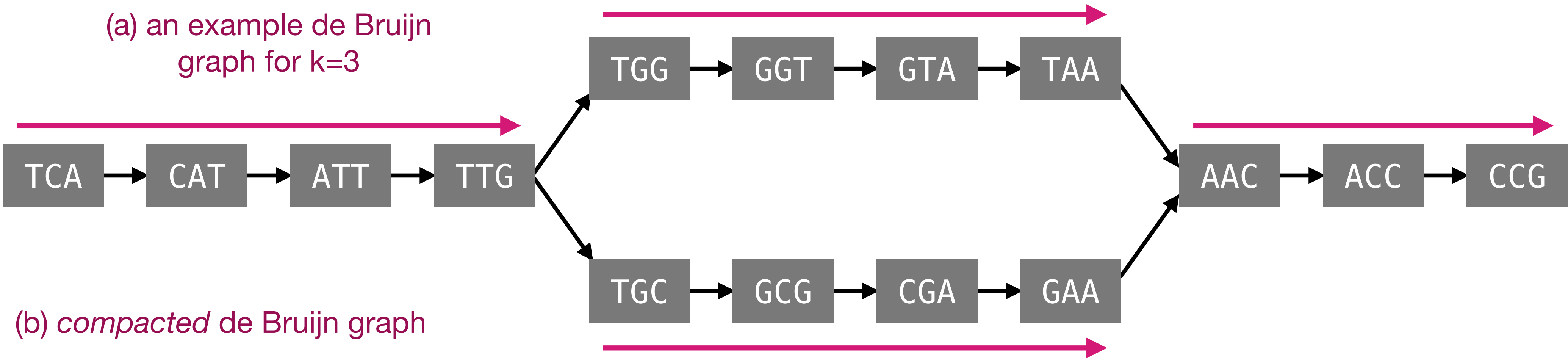


(b) compacted de Bruijn graph

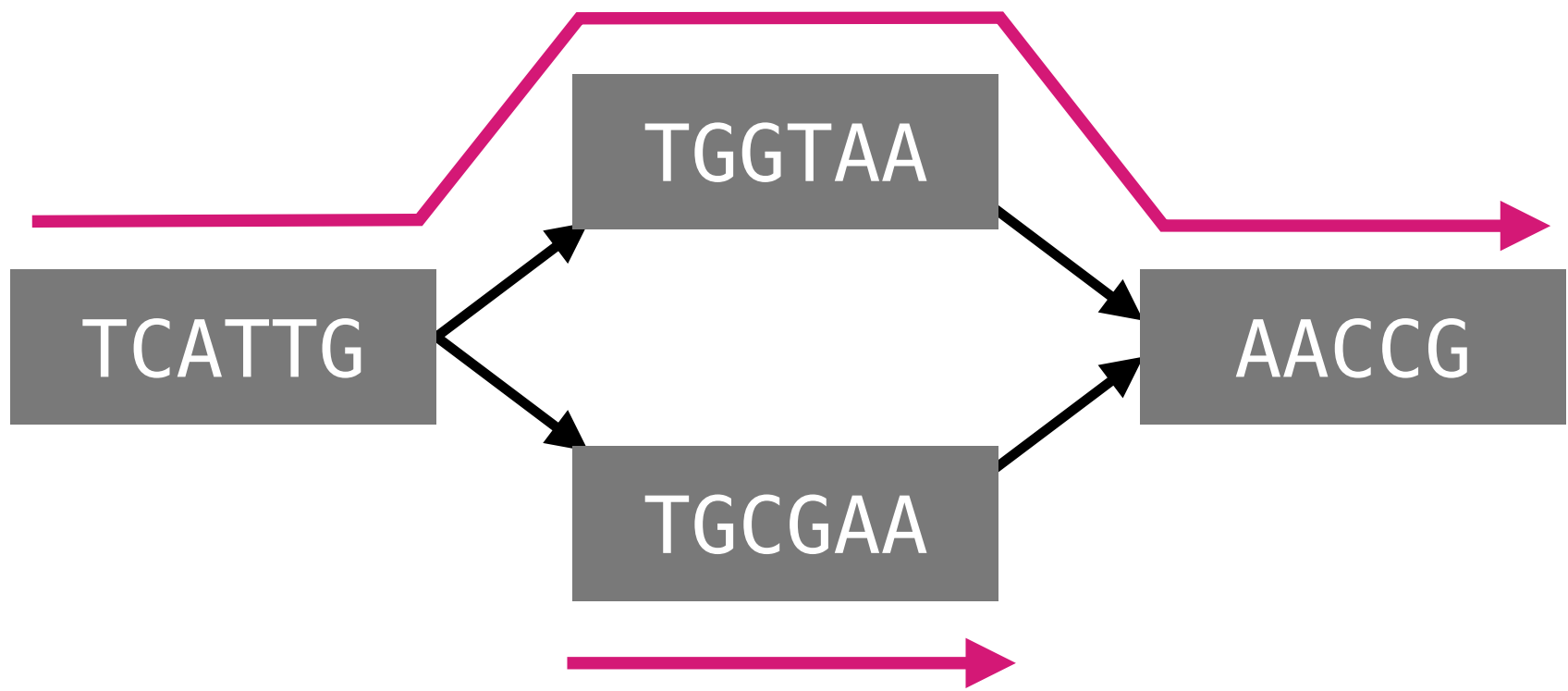


de Bruijn graphs

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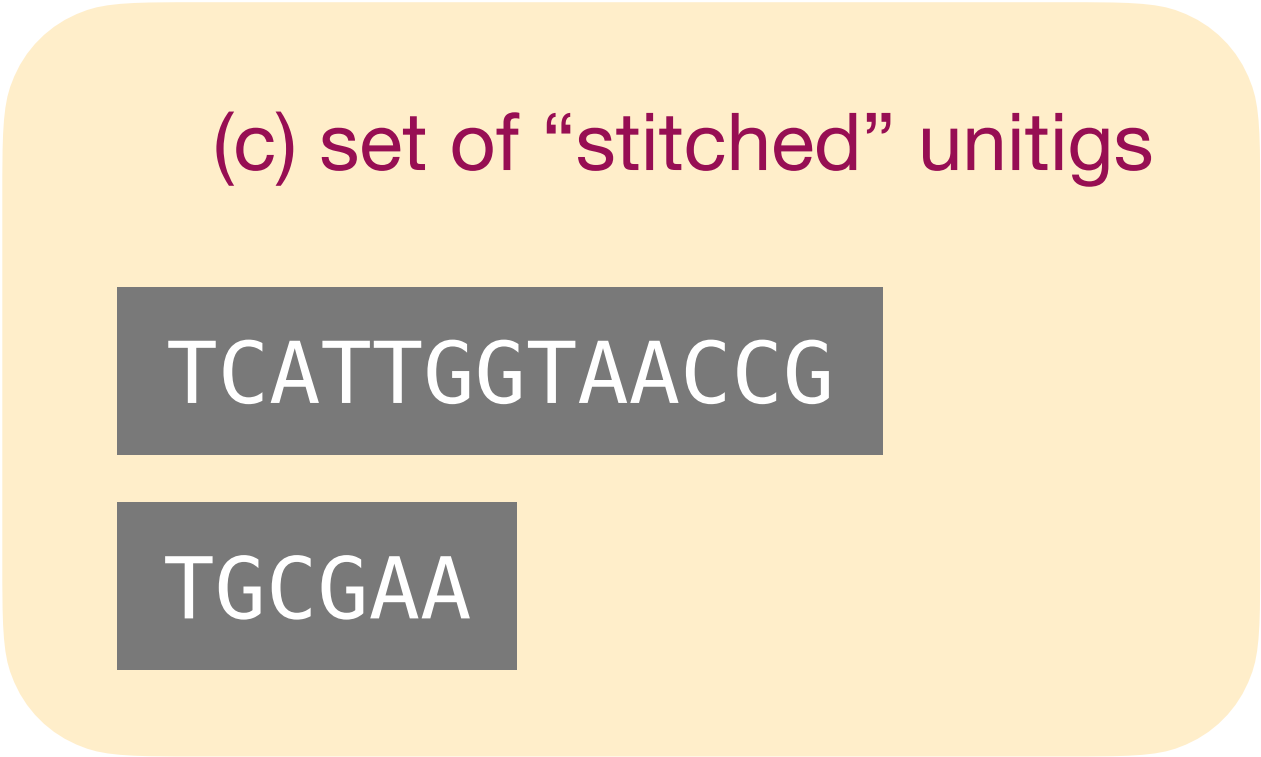
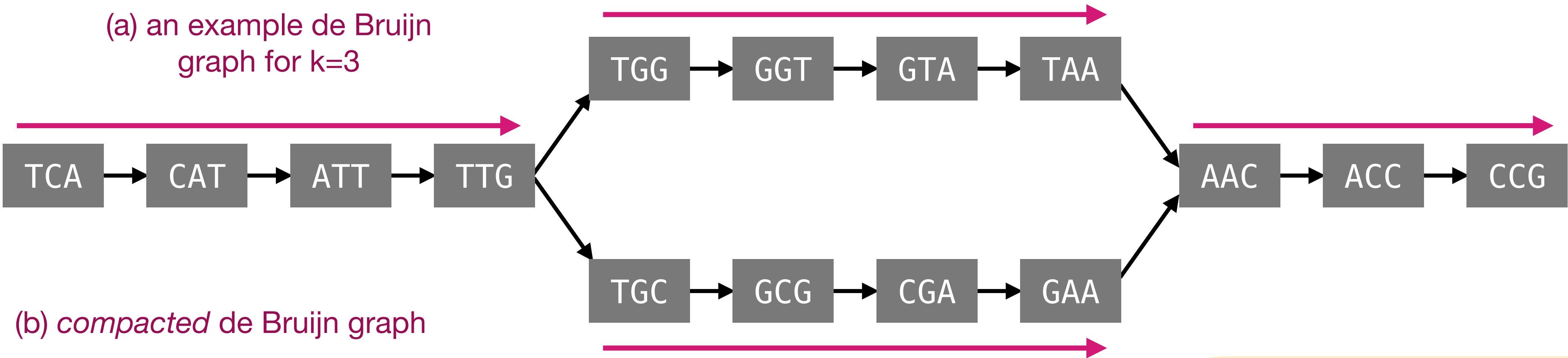
(b) compacted de Bruijn graph



(c) set of “stitched” unitigs

TCATTGGTAACCG
TGCGAA

de Bruijn graphs



this is an example of a
spectrum-preserving string set
(see next)

Spectrum-preserving string sets

- **k-mer spectrum.** The spectrum of S , say $spect(S)$, is the set of all the distinct k-mers of S .
- **Spectrum-preserving string set (SPSS).** A SPSS for S is a set of strings $\mathcal{U} = \{U_1, \dots, U_m\}$ such that $spect(S) = spect(U_1) \cup \dots \cup spect(U_m)$. Usually $spect(U_i) \cap spect(U_j) = \emptyset$ for any $i \neq j$.
- For example, the set of unitigs or “stitched” unitigs of the DBG of S are possible SPSS for S .

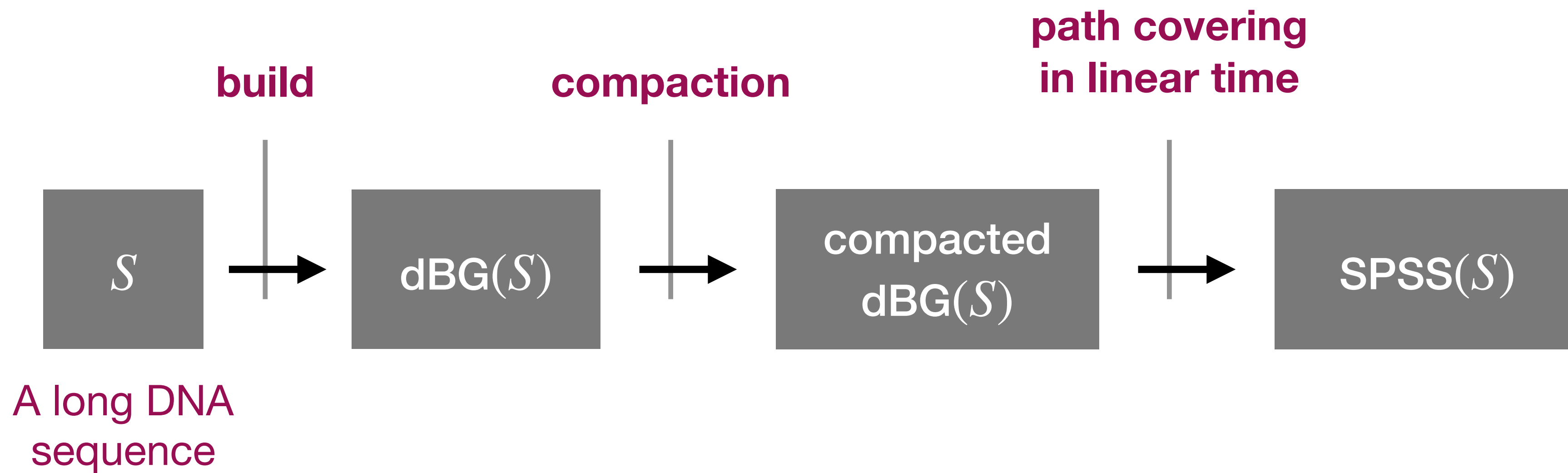
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- For example, the set of unitigs or “stitched” unitigs of the dBG of S are possible SPSS for S .
- In general, we want to **minimise** the cumulative length of \mathcal{U} , i.e., the number of characters in the strings of \mathcal{U} .
- A **general framework**: compute a minimum-size **path cover** for the (compacted) dBG of S . Usually the cover is **disjoint-node**, so that each k-mer of S appears exactly once in \mathcal{U} .

Spectrum-preserving string sets

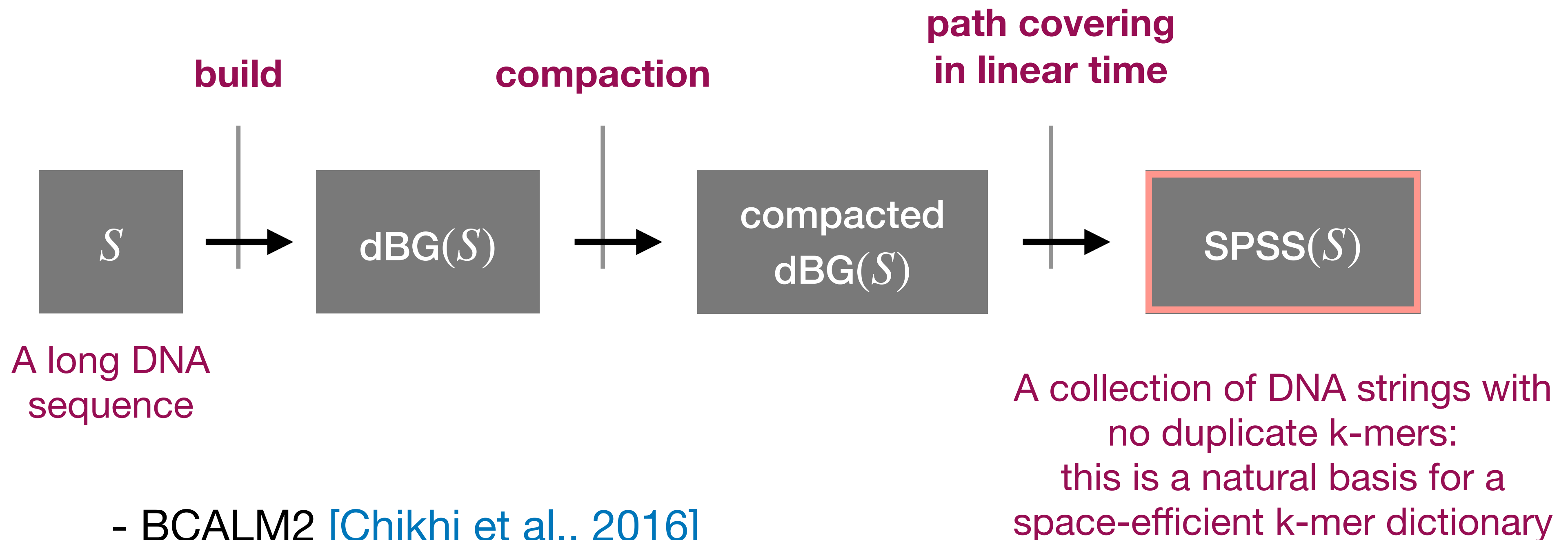
- Many SPSS available in the literature:
 - unitigs (folklore);
 - stitched unitigs [Rahman and Medvedev, 2020];
 - simplitigs [Brinda et al., 2020];
 - eulertigs [Schmidt and Alanko, 2022, 2023] (optimal: smallest num. of characters);
 - matchtigs [Schmidt, Khan, Alanko, P., Tomescu, 2023];
 - masked super strings [Brinda et al., 2025].
- Some of them have slightly different properties.
- For example: unitigs, stitched unitigs, simplitigs, and eulertigs do not allow repetitions of k-mers, whereas matchtigs and masked super strings do.

de Bruijn graphs and spectrum-preserving string sets



- BCALM2 [Chikhi et al., 2016]
- TwoPaCo [Minkin et al., 2017]
- Cuttlefish1 [Khan and Patro, 2021]
- Cuttlefish2 [Khan et al., 2022]
- GGCAT [Cracco and Tomescu, 2023]
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Indexing SPSS

- Now that we have an SPSS where each k-mer of S appears once, the question is:
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Indexing SPSS

- Now that we have an SPSS where each k-mer of S appears once, the question is:
Q. How do we index it so that Lookup and Access are efficient?
- Possible answers:
 - Compute the **BWT** of the strings in the SPSS.
 - We are going to see a solution based on **hashing**.
(We need two more tools.)

Sketching with minimizers

- Consider each k -mer x of S : sample one m -mer of x out of its $k - m + 1$ m -mers and call it the “representative” of x — or its *minimizer*.

Example for $k = 10$ and $m = 7$.

ACGGTAGAACCGATTCAAATTCGAT...

ACGGTAGAAC
CGGTAGAAC
GGTAGAACCG
GTAGAACCGA
TAGAACCGAT
AGAACCGATT
GAACCGATTTC
AACCGATTCA
...

Sketching with minimizers

- Consider each k-mer x of S : sample one m-mer of x out of its $k - m + 1$ m-mers and call it the “representative” of x — or its *minimizer*.
- We would like to sample the **same minimizer** from consecutive k-mers so that the **set of distinct minimizers** forms a succinct sketch for S .
- This reduces the memory footprint and comput. time of countless applications in Bioinformatics.

Example for $k = 10$ and $m = 7$.

ACGGTAGAACCGATTCAAATTCGAT...

ACGGTAGAAC
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Sketching with minimizers

- **Q.** How do we compare different sampling algorithms?

A. We define the *density* of a sampling algorithm as the fraction between the number of (distinct) minimizers and the total number of m -mers of S (i.e., $|S| - m + 1$).

The lower the density, the better!

Sketching with minimizers

- **Q.** How do we compare different sampling algorithms?

A. We define the *density* of a sampling algorithm as the fraction between the number of (distinct) minimizers and the total number of m -mers of S (i.e., $|S| - m + 1$).

The lower the density, the better!

- Call $w = k - m + 1$. Since the same m -mer cannot be a minimizer for more than w consecutive k -mers, we immediately have a **lower bound** of $1/w$ on the density of any sampling algorithm.

TAGAACCGAT
AGAACCGATT
GAACCGATTTC
AACCGATTCA
...

Example for $w = 4$ and $m = 7$.

The “folklore” minimizer

- **Minimizer.** [Schleimer et al. 2003, Roberts et al., 2004] Given a k -mer x and an order \mathcal{O} over all m -mers, the *minimizer* of length $m \leq k$ is the (leftmost) *smallest* m -mer of x according to \mathcal{O} .
- Example. Given $x = \text{ACGGTAGAACCGA}$ ($k = 13$) and $m = 4$:

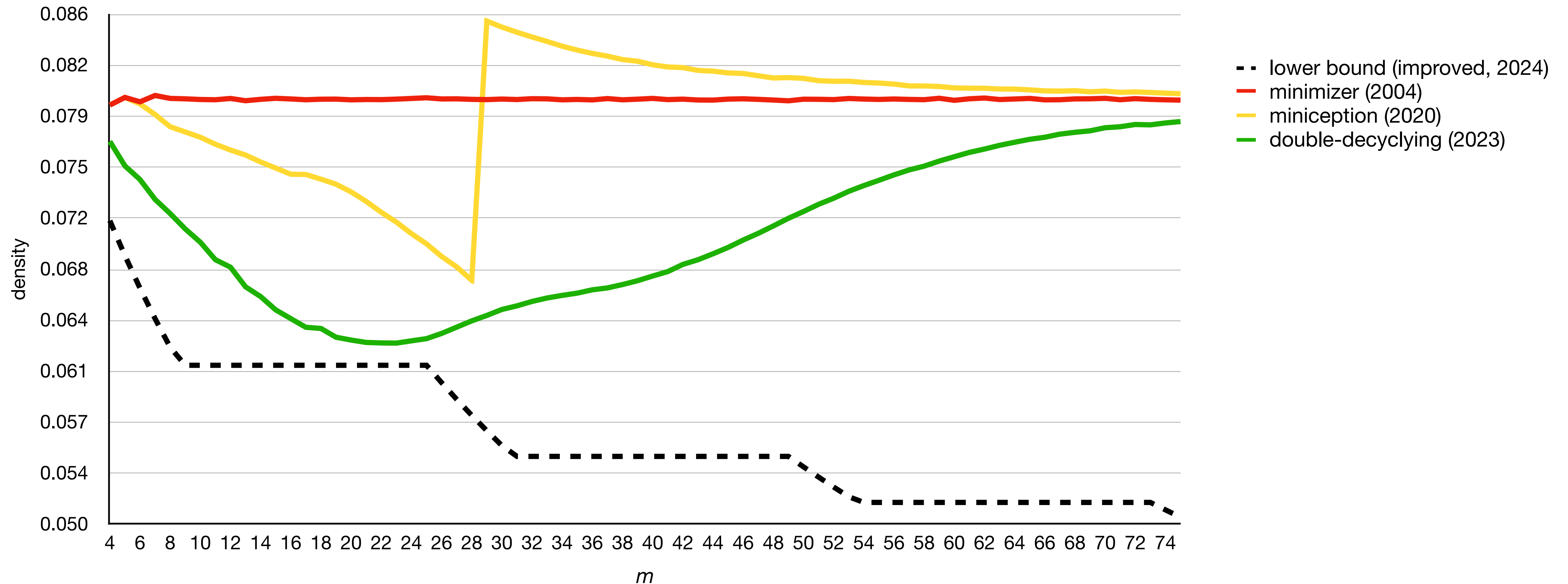
ACGG	$h(\text{ACGG}) = 9842978325$	
CGGT	$h(\text{CGGT}) = 817612312$	
GGTA	$h(\text{GGTA}) = 8265731$	← <i>smallest hash code</i>
GTAG	$h(\text{GTAG}) = 478491248$	
TAGA	$h(\text{TAGA}) = 17491411$	
AGAA	$h(\text{AGAA}) = 17148914$	
GAAC	$h(\text{GAAC}) = 91815379$	
AACC	$h(\text{AACC}) = 645793914$	
ACCG	$h(\text{ACCG}) = 918417644$	
CCGA	$h(\text{CCGA}) = 814188124$	

In this case, the density is $2/(w + 1)$: almost a factor of 2 away from the lower bound for large w .

\mathcal{O} is the *lexicographic* order.

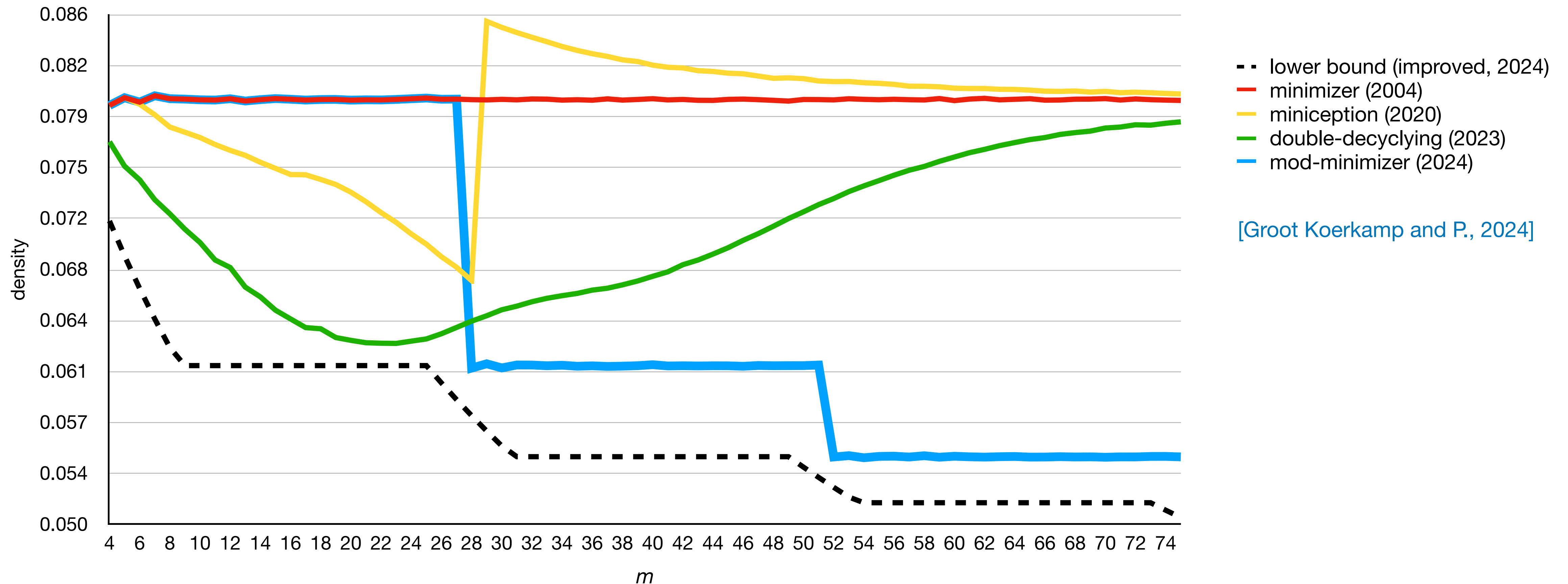
\mathcal{O} is defined by a random hash function h .

Density by varying m



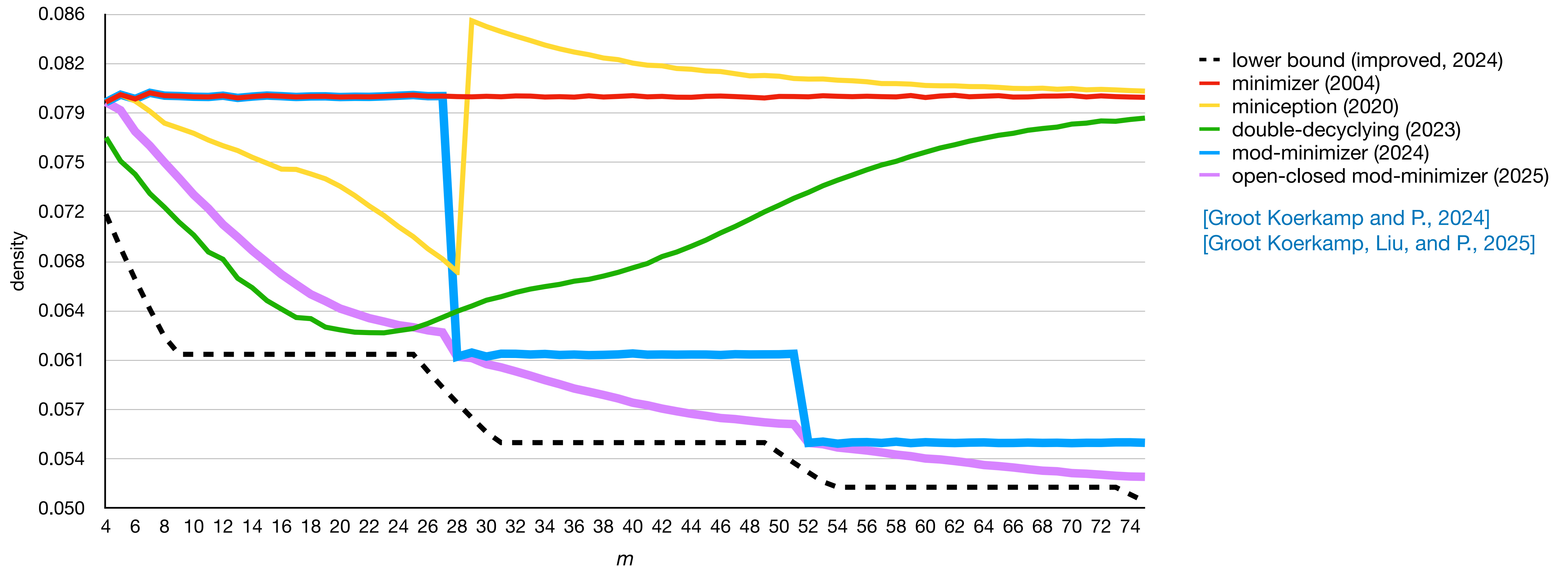
- Example for $w = 24$, so $k = 24 + m - 1$.
- Measured over a string of 10 million i.i.d. random characters with an alphabet size of 4.
- <https://github.com/jermp/minimizers>

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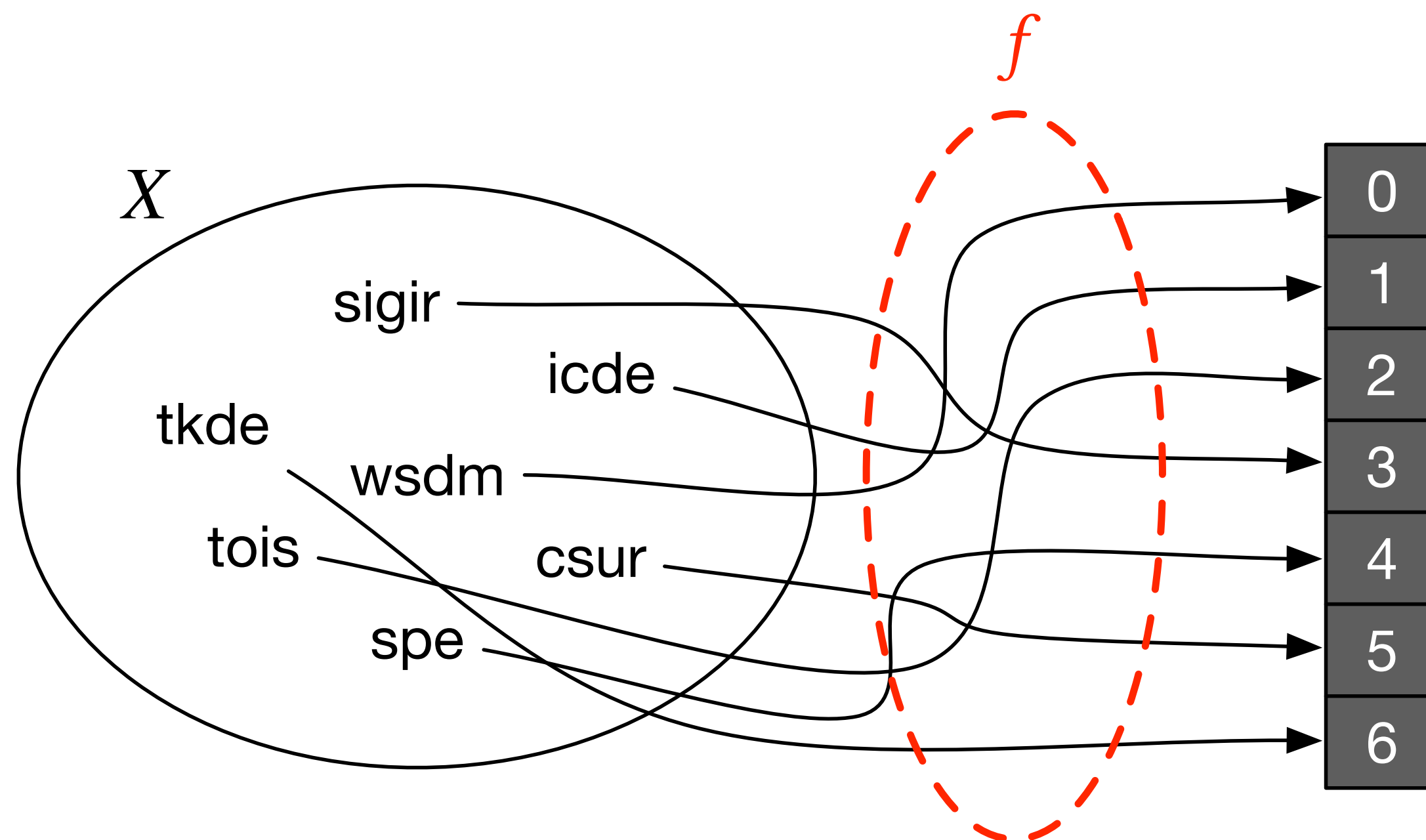
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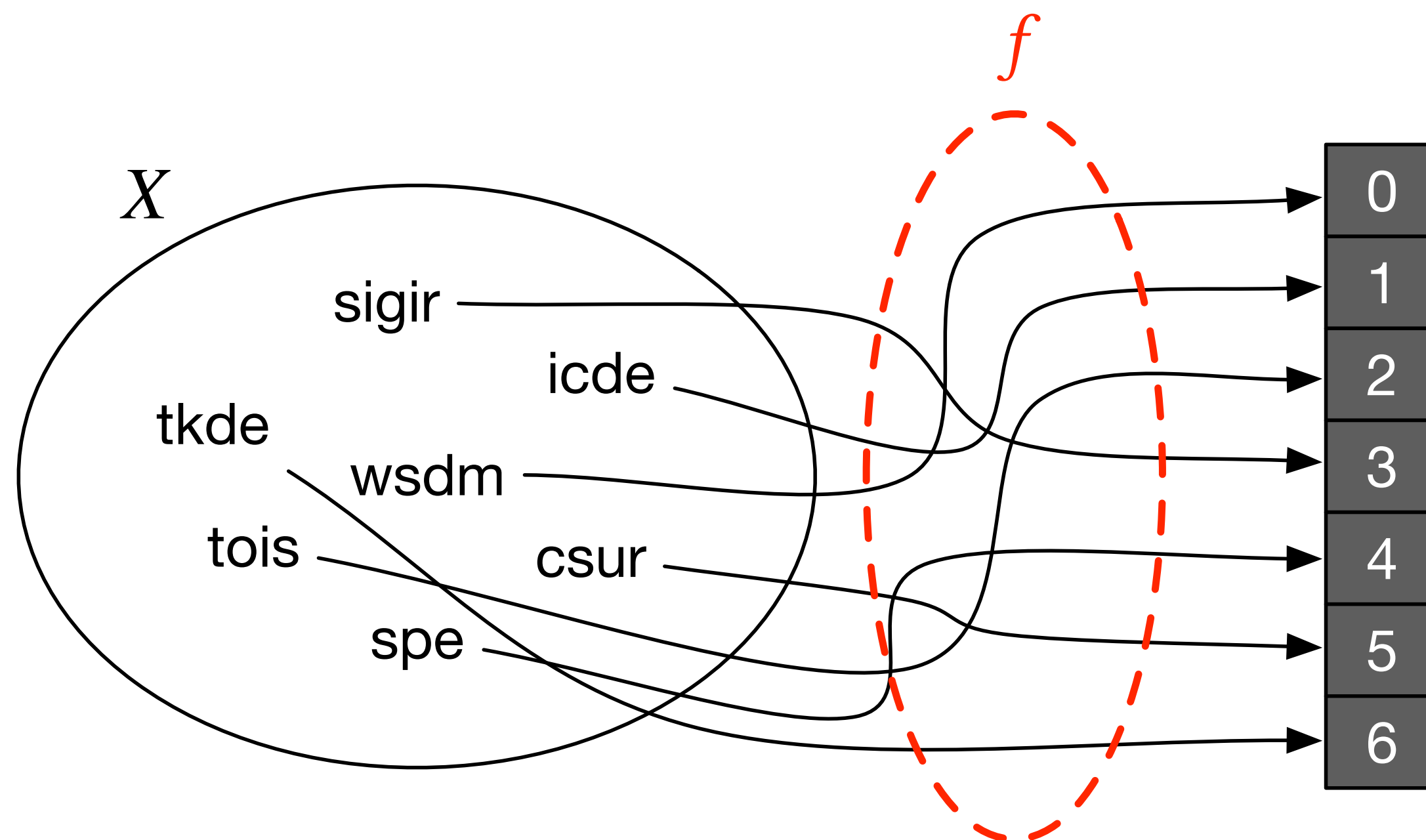
Minimal Perfect Hashing

MPHF. Given a set X of n distinct keys, a function f that *bijectively* maps the keys of X into the range $\{1, \dots, n\}$ is called a *minimal perfect hash function* (MPHF) for X .



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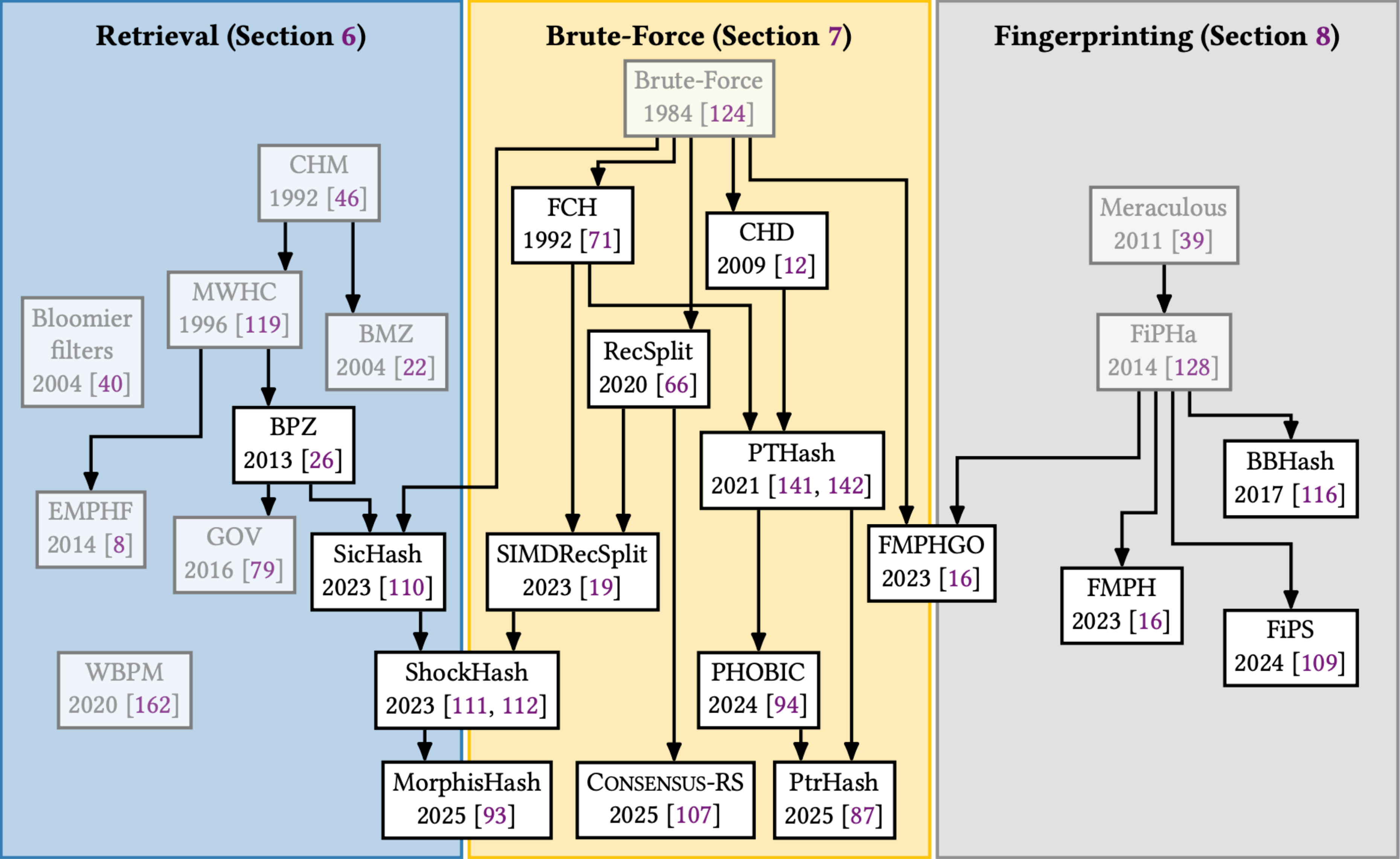
- Space lower bound of $\log_2(e) \approx 1.443$ bits/key [Mehlhorn, 1982].
- Many approaches available (see next).
- Most of them have:
 - Constant-time evaluation.
 - Expected linear-time construction.
 - Take 1.8 — 3 bits/key.

Modern Minimal Perfect Hashing: A Survey

Lehmann, Mueller, P., Sanders, Vigna, Walzer, 2025

<https://arxiv.org/pdf/2506.06536>

Minimal Perfect Hashing



The “PTHash family”

- The **fastest** functions for lookup time: 30-50 ns/key.
- Also very fast to build and space-efficient: they take from 1.7 to 3.0 bits/key.
- **PTHash**/PHOBIC [P. and Trani, 2021, 2023; Hermann et al., 2024]



<https://github.com/jermp/pthash>

- PtrHash [Groot Koerkamp, 2025]
- PHast [Beling and Sanders, 2025]


3. Sparse and skew hashing of k-mers

Super-k-mers

- **Property.** Consecutive k-mers are likely to have the same minimizer.

Example for $k=13$ and $m=4$:

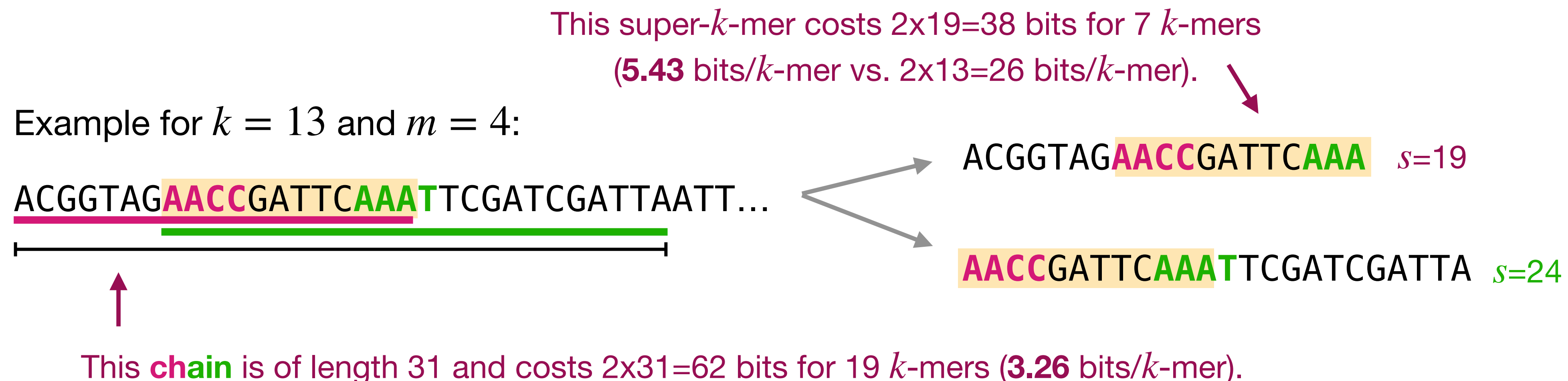
ACGGTAG**AACC**GATTCAAATTCGATCGATTAATTAGAGCGATAAC...
ACGGTAG**AACC**GA
CGGTAG**AACC**GAT
GGTAG**AACC**GATT
GTAG**AACC**GATTC
TAG**AACC**GATTCA
AG**AACC**GATTCAA
G**AACC**GATTCAAA
AACCGATTCA**AAAT**
...



- **Super-k-mer.** Given a string, a super-k-mer is a **maximal** sequence of consecutive k-mers having the same minimizer.

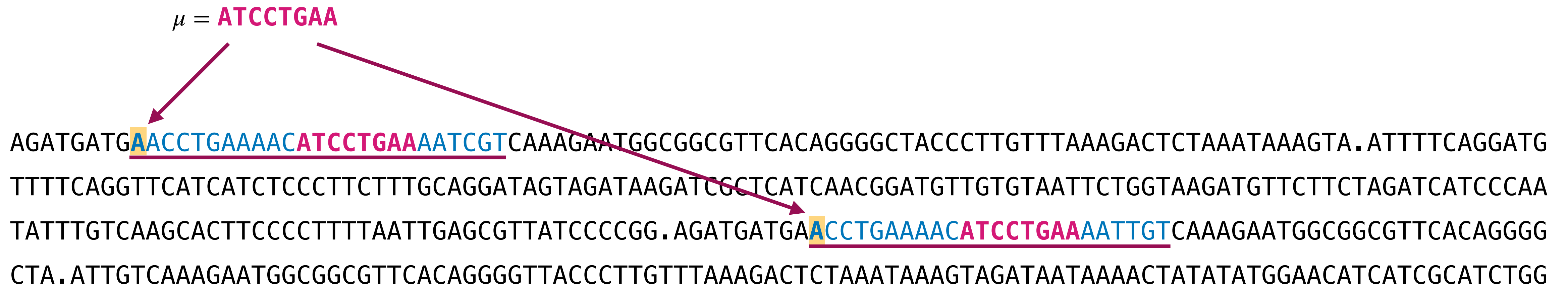
Super-k-mers

- **Observation 1.** Since minimizers are sparse, there are *far fewer* super-k-mers than k-mers — approx. $(k - m + 2)/2$ times less for *random* minimizers → **sparse** indexing.
- **Observation 2.** A super-k-mer of length s is a **space-efficient** representation of the set of its constituent $s - k + 1$ k-mers: $2s/(s - k + 1)$ vs. $2k$ bits/k-mer. If s is *sufficiently large and/or we have long chains* of super-k-mers, the cost becomes approx. 2 bits/k-mer.



Sparse hashing

- **Q.** How to index super- k -mers?
- Do **not** break the chains of super- k -mers to avoid wasting $2(k - 1)$ bits per super- k -mer.
- Locate super- k -mers with an array of offsets into the strings, indexed by a **minimal perfect hash function** (MPHF) on the minimizers.
- Upon Lookup(x): if μ is the minimizer of x , locate and scan the “bucket” of μ — the set of super- k -mers that have minimizer μ .



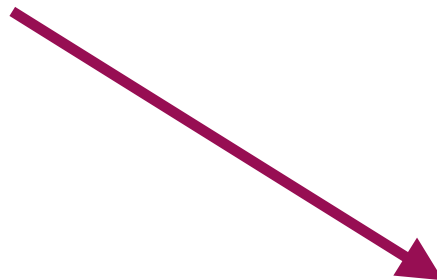
Sparse hashing — Example

A collection of 4 stitched unitigs:
285 k-mers for $k=31$, $N = 408$ bases in total



AGATGATGAACCTGAAAACATCCTGAAAATCGTCAAAGAATGGCGG
CGTTCACAGGGGCTACCCTTGTTTAAAGACTCTAAATAAAGTA.AT
TTTCAGGATGTTTTTCAGGTTCATCATCTCCCTTCTTTGCAGGATAG
TAGATAAGATCGCTCATCAACGGATGTTGTGTAATTCTGGTAAGAT
GTTCTTCTAGATCATCCCAATATTTGTCAAGCACTTCCCCTTTTAA
TTGAGCGTTATCCCCGG. AGATGATGAACCTGAAAACATCCTGAAA
ATTGTCAAAGAATGGCGGCGTTACAGGGGCTA. ATTGTCAAAGAA
TGGCGGCGTTACAGGGGTTACCCTTGTTTAAAGACTCTAAATAAA
GTAGATAATAAACTATATATGGAACATCATCGCATCTGG

24 minimizers, for $m = 8$

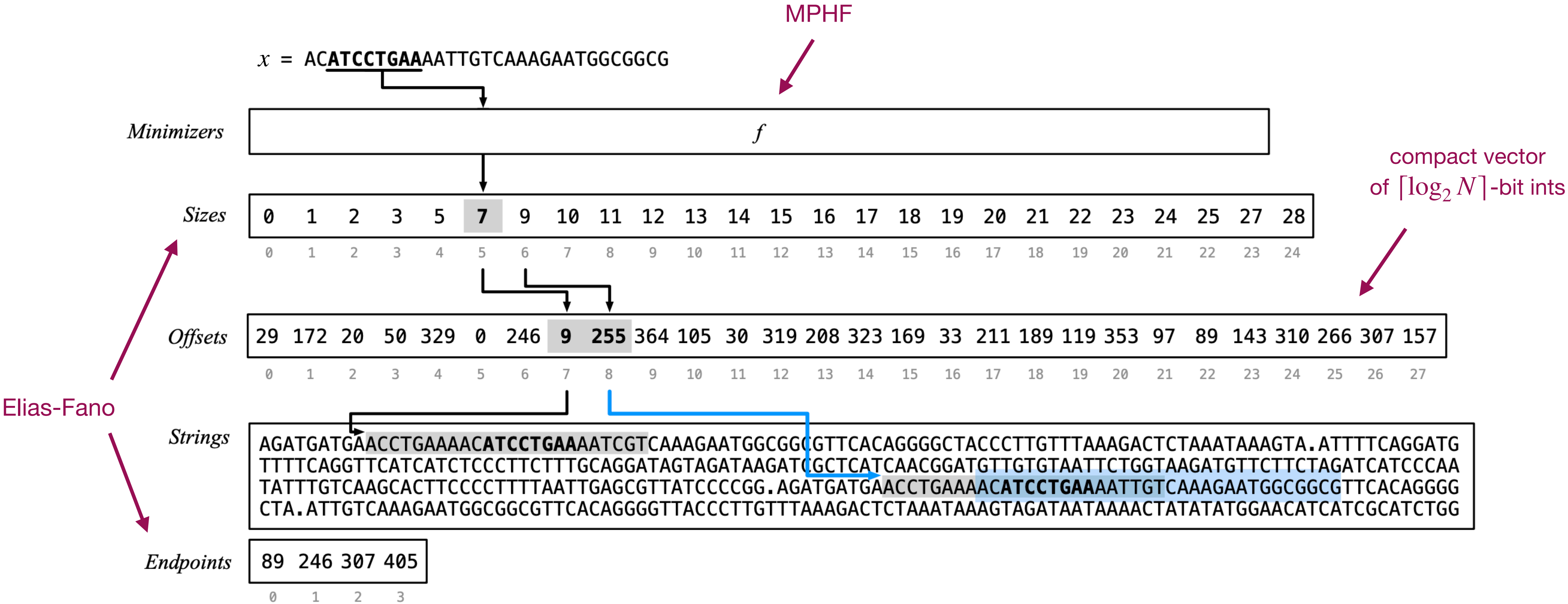


TCGTCAA:	29	
CATCCCA:	172	
ATCGTCA:	20	
GACTCTA:	50	329
AACCTGA:	0	246
ATCCTGA:	9	255
GAACATC:	364	
GCAGGAT:	105	
AGGGGCT:	30	
CTTGTTT:	319	
GAGCGTT:	208	
TTTAAAG:	323	
CTTCTAG:	169	
GGCTACCC:	33	
CGTTATCC:	211	
AGCACTTC:	189	
AAGATCGC:	119	
AACTATAT:	353	
CCTTCTTT:	97	
TTCAGGTT:	89	
ACGGATGT:	143	
ACAGGGGT:	310	
TGTCAAAG:	266	307
TAATTCTG:	157	



offsets

Data structure



The order of the k-mers in the SPSS is preserved

- Lookup : $\Sigma^k \rightarrow \{1, \dots, n\}$, where $1 \leq \text{Lookup}(x) \leq n$ if $x \in \text{SPSS}(S)$ and $\text{Lookup}(x) = \perp$ if $x \notin \text{SPSS}(S)$.
- So the hash code $i = \text{Lookup}(x)$ can be directly used to **associate** some satellite information to the k-mer x , e.g., its abundance, color set, etc.
- **Order-Preserving Property.** If $x[2..k] = y[1..k-1]$, i.e., y is the “successor” of x , then: $\text{Lookup}(y) = \text{Lookup}(x) + 1$.
- Any order on the strings of $\text{SPSS}(S)$ uniquely determines an order $i = 1, \dots, n$ for the k-mers $x_i \in \text{SPSS}(S)$, thus: $\text{Lookup}(x_i) = i$.

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- Any order on the strings of $\text{SPSS}(S)$ uniquely determines an order $i = 1, \dots, n$ for the k-mers $x_i \in \text{SPSS}(S)$, thus: $\text{Lookup}(x_i) = i$.
- This property makes **compression of satellite information** easy and effective. We will see another example on **4th July**.

Skew hashing

- **Problem.** Some buckets can be very large.

For example on the human genome (GRCh38), for $k = 31$ and $m = 20$: largest bucket size can be as large as 3.6×10^4 .

- **Property.** Minimizers have a (very) **skew** distribution for sufficiently-long length m .

Bucket size distribution (%) for $k = 31$ and the first $n = 10^9$ k -mers of the human genome, by varying minimizer length m .

size / m	11	12	13	14	15	16	17	18	19	20	21
1	13.7	19.8	29.7	42.4	61.5	79.5	89.8	94.4	96.3	97.1	97.5
2	7.5	10.6	14.4	17.7	19.4	13.6	7.3	3.9	2.4	1.7	1.4
3	5.2	7.3	8.8	10.4	8.4	3.7	1.4	0.8	0.5	0.4	0.4
4	4.0	5.5	6.0	7.0	4.1	1.3	0.5	0.3	0.2	0.2	0.2
5	3.2	4.4	4.5	5.0	2.2	0.6	0.3	0.2	0.1	0.1	0.1

On the **full** human genome (GRCh38),
for $k = 31$ and $m = 20$:
2,505,445,761 k -mers
421,845,806 minimizers
388,018,280 (91.98%) only appear **once!**

Skew hashing

- We fix an integer ℓ : by virtue of the skew distribution, the fraction of buckets having **more than 2^ℓ super-k-mers** is **small**.
- So, we can afford a MPHF over the set of k-mers that belong to such super-k-mers. The output of the MPHF for a k-mer x is the **identifier** of the super-k-mer where x is present.
- Upon Lookup, we will scan **one** super-k-mer only.

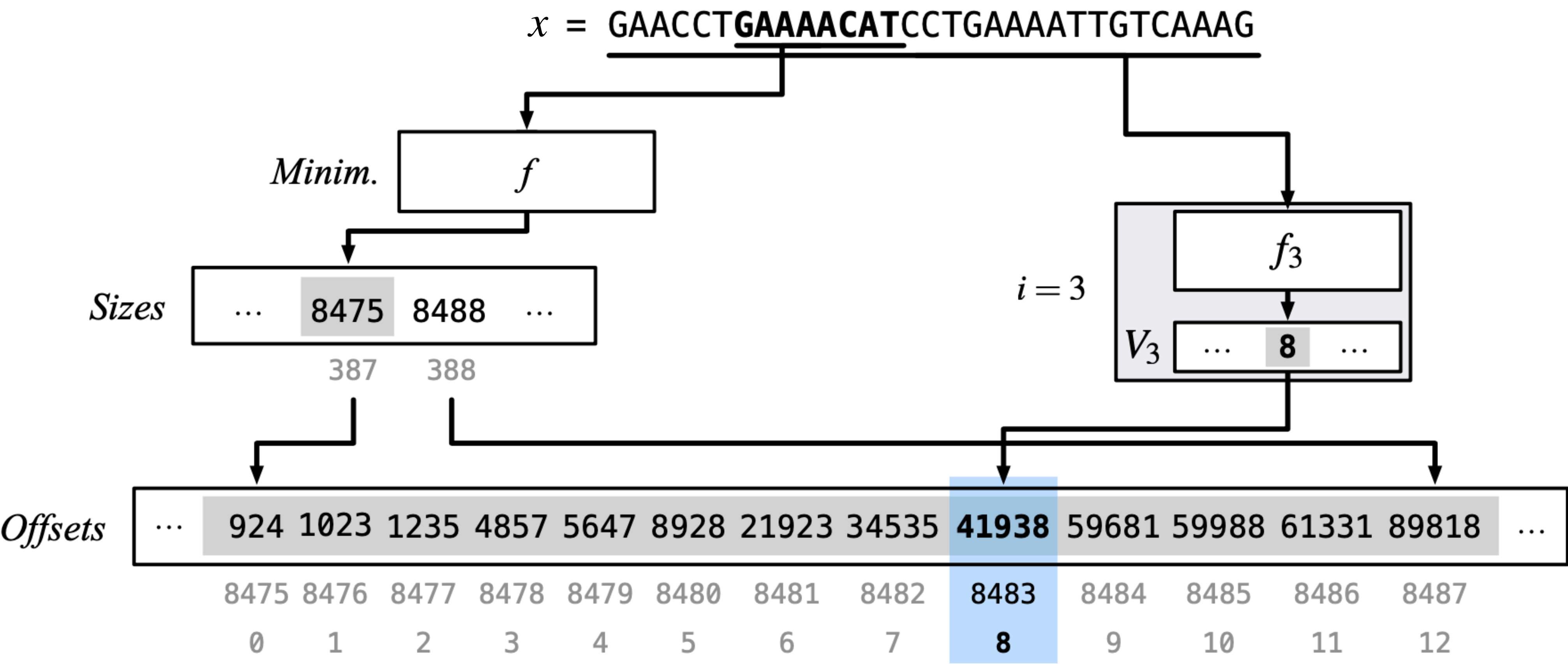
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2	7.5	10.6	14.4	17.7	19.4	13.6	7.3	3.9	2.4	1.7	1.4
3	5.2	7.3	8.8	10.4	8.4	3.7	1.4	0.8	0.5	0.4	0.4
4	4.0	5.5	6.0	7.0	4.1	1.3	0.5	0.3	0.2	0.2	0.2
5	3.2	4.4	4.5	5.0	2.2	0.6	0.3	0.2	0.1	0.1	0.1

For $\ell = 2$, just
 $100.0 - (97.1 + 1.7 + 0.4 + 0.2)\% = 0.6\%$ of
buckets with more than $2^{\ell=2} = 4$ super- k -mers.

Skew hashing — Example

Example for $\ell = 3$.



Implementation and results

- These ideas have been implemented in a software tool (C++17):



<https://github.com/jermp/sshash>

New benchmarks

<https://github.com/jermp/sshash/tree/master/benchmarks>

Ubuntu 18.04.6; gcc 9.4; Intel Xeon W-2245 CPU @ 3.90GHz; Results taken on 18/06/2025

Whole genome	Space		Building time (8 threads, 16 GB of RAM)	Positive random Lookup	Negative random Lookup	Streaming Lookup high-hit		
	bits/kmer	total GB				ns/kmer	hit-rate (%)	extension-rate (%)
k = 31								
Cod, regular, m = 20	7.79	0.49	0:28	1124	1078	87	81	94
Cod, canonical, m = 19	8.76	0.55	0:40	899	631	69		
Kestrel, regular, m = 20	7.44	1.07	0:50	1099	1220	130	76	98
Kestrel, canonical, m = 19	8.43	1.21	1:06	873	701	97		
Human, regular, m = 21	8.65	2.71	2:12	1584	1365	208	92	92
Human, canonical, m = 20	9.70	3.04	2:50	1204	764	143		
k = 63								
Cod, regular, m = 24	4.24	0.29	0:25	1311	1058	344	69	49
Cod, canonical, m = 23	4.84	0.34	0:30	1140	730	280		
Kestrel, regular, m = 24	3.73	0.54	0:27	1135	1233	244	64	49
Kestrel, canonical, m = 23	4.23	0.61	0:40	1008	811	230		
Human, regular, m = 25	4.63	1.60	1:40	1658	1372	491	85	46
Human, canonical, m = 24	5.31	1.84	2:24	1357	875	387		

To sum up

- SSHash is an **order-preserving** k-mer dictionary.
- Three important tools:
 1. spectrum-preserving string sets;
 2. minimizers; and
 3. minimal perfect hashing.
- Ingredients:
 - **Sparse indexing** to obtain good space effectiveness;
 - **Skew hashing** to guarantee fast lookup for “heavy” buckets.
- Code in C++17 is available at: <https://github.com/jermp/sshash>.

Extensions

- **k-mer abundances** [P. 2022, 2023]
- **sequence membership**: a sequence S is considered as present in the dictionary if at least a given fraction of its k-mers is found in the dictionary [Schmidt, Khan, Alanko, P., Tomescu, 2023]
- reference indexing: store also **positional** information for each k-mer [Fan, Khan, P., Patro, 2023]
- **colored de Bruijn graphs**: annotate each k-mer with the set of its “colors” (i.e., the references where it appears) [Fan, Khan, Singh, P., Patro, 2023, 2024; Fan, P., Patro, 2024; Campanelli, P., Fan, Patro, 2024; Campanelli, P., Patro, 2025]

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