

# Introduction to Algorithms

IIS A. Pacinotti, Mestre (Venice)

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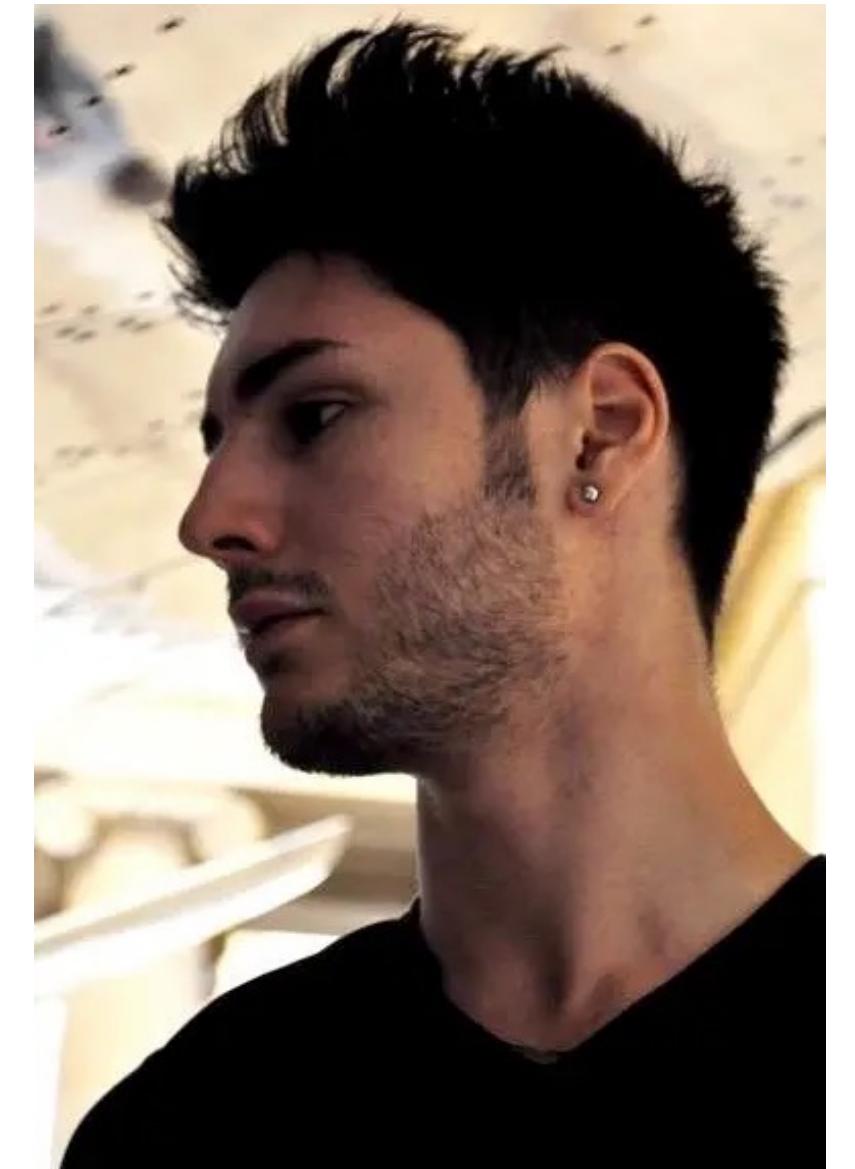
**Giulio Ermanno Pibiri**

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# About me

- As of June 2022:  
Assistant Prof. of Computer Science at Ca' Foscari University of Venice.
- Before:
  - post-doctoral researcher at CNR, Pisa (March 2019 - June 2022)
  - Ph.D. in Computer Science from University of Pisa (Jan. 2016 - March 2019).
- Research interests:  
Algorithms and compressed data structures with applications to real-world problems, for example, in Information Retrieval and Computational Biology.
- Contact:
  - Web page: <https://jermp.github.io>
  - Email: [giulioermanno.pibiri@unive.it](mailto:giulioermanno.pibiri@unive.it)



# What is this lecture about?

- This is an introductory lecture to the field of *Algorithms and Data Structures*.
- **Our goal for today:** understand **why algorithms are fundamental** to solve large-scale problems.

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- **Algorithms:** methods (recipes) to solve a problem.
- **Data Structures:** ways to **organise the data** that it is accessed by an algorithm to solve a problem.
- **Data Compression:** **better data representation** to enable more efficient algorithms (we will not talk about this today, though).

# Overview

- 9:00 – 10:00

**Part 1** – Basic definitions, warm-up

- 10:10 – 11:00

**Part 2** – Motivations, analysis of algorithms, same applications

- 11:10 – 12:00

**Part 3** – Some example problems: integer search and sub-string search

# **Part 1 – Basic definitions, warm-up**

# Basic definitions – Algorithm

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Recipe to make bread (simplified):

1. Stir together water, yeast, and flour.
2. Add oil and salt.
3. Knead the dough.
4. Let the dough rest for 1 h.
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input

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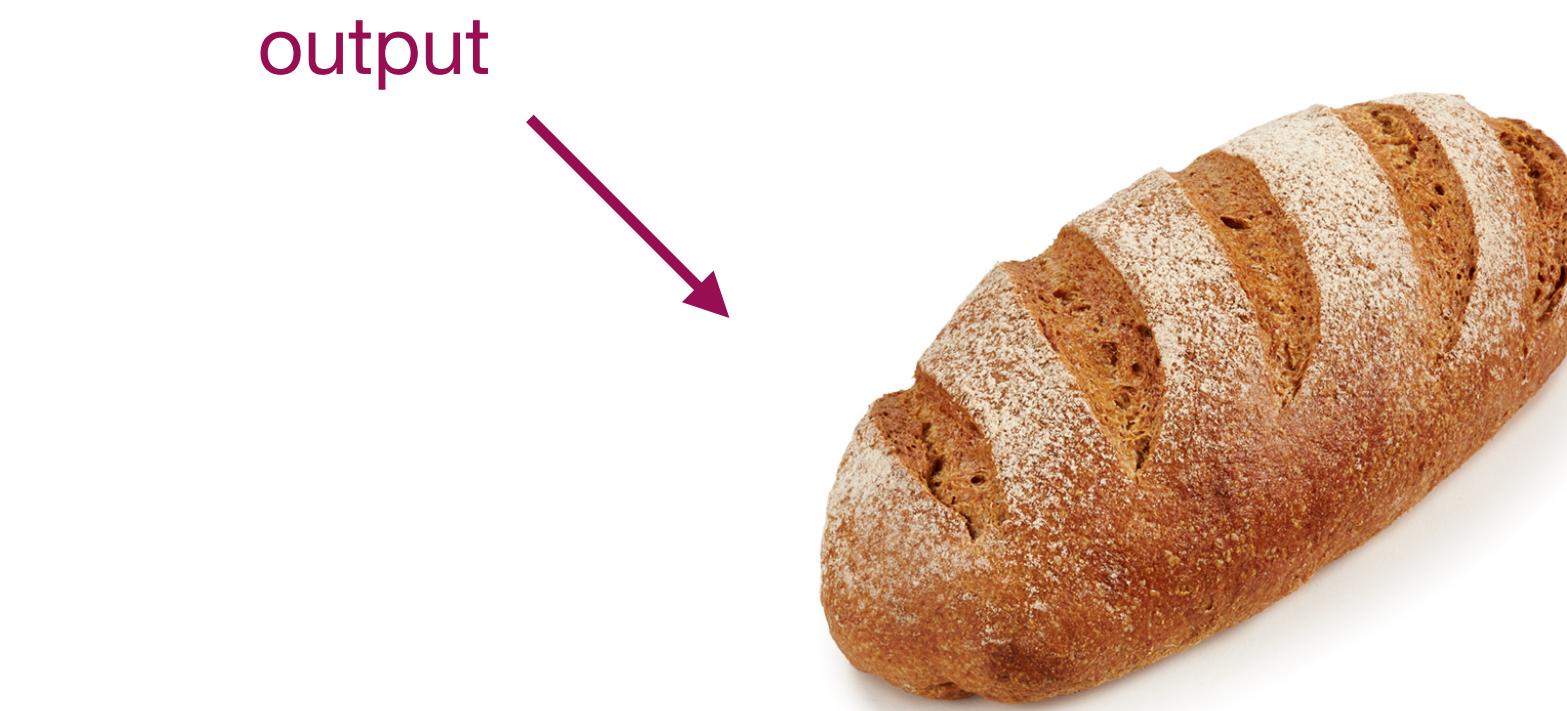


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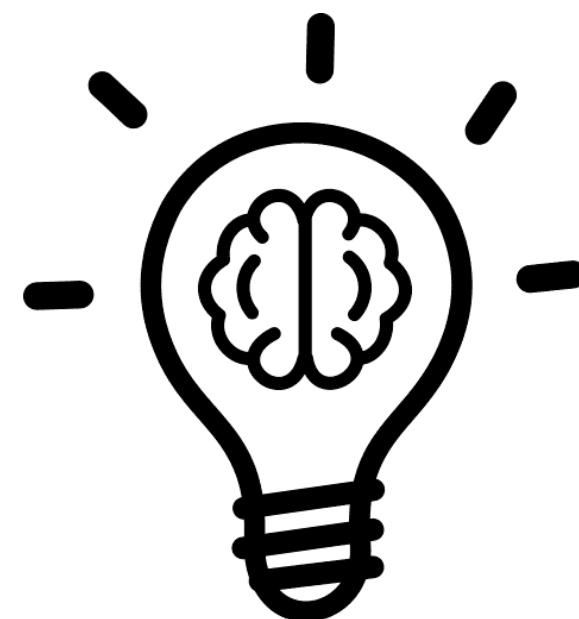
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- In this lecture, we care about the algorithms that can be implemented on a **computer**.

# Programming languages

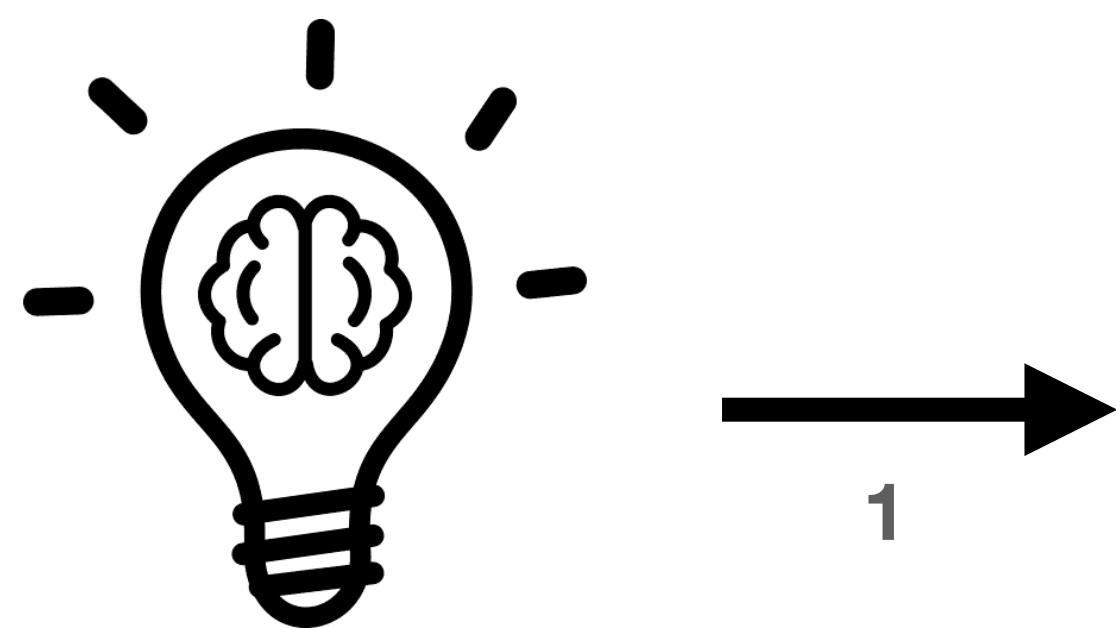
- **Implementation:** write the sequence of steps in a **programming language** (like C/C++, Java, Rust, Python, etc.) to let the algorithm be executed on a computer.



idea for a new  
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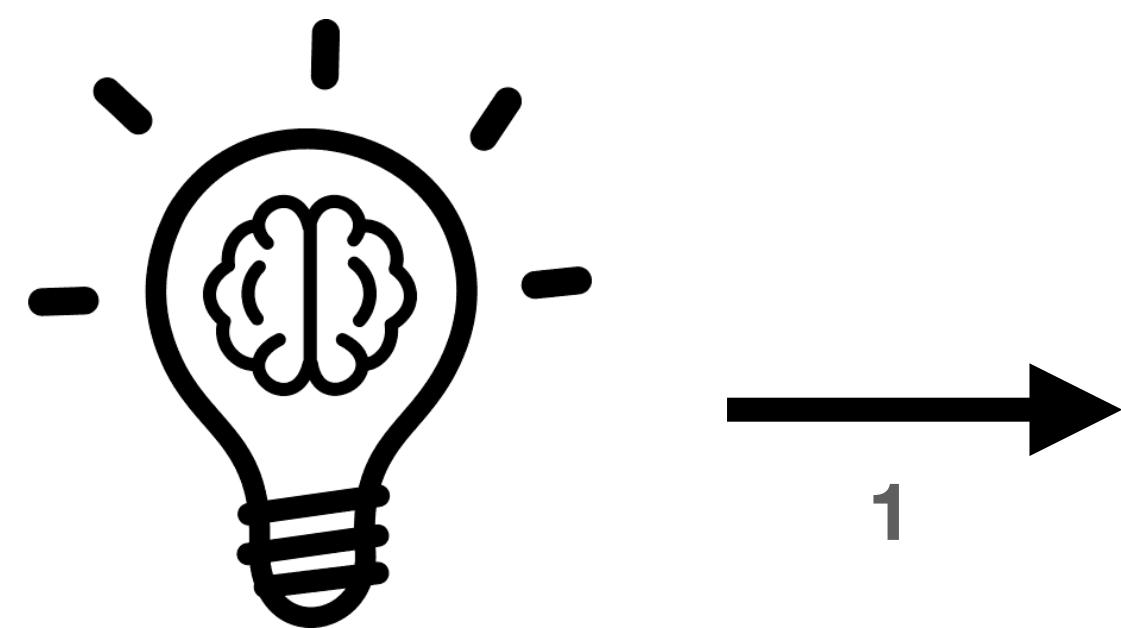
my\_algorithm.cpp

```
130     for (; processed_buckets < num_non_empty_buckets; ++processed_buckets, ++buckets) {
131         auto const& bucket = *buckets;
132         assert(bucket.size() > 0);
133
134         for (uint64_t pilot = 0; true; ++pilot) {
135             uint64_t hashed_pilot = PTHASH_LIKELY(pilot < search_cache_size)
136             ? hashed_pilots_cache[pilot]
137             : default_hash64(pilot, seed);
138
139             positions.clear();
140
141             auto bucket_begin = bucket.begin(), bucket_end = bucket.end();
142             for (; bucket_begin != bucket_end; ++bucket_begin) {
143                 uint64_t hash = *bucket_begin;
144                 uint64_t p = fastmod::fastmod_u64(hash ^ hashed_pilot, M, table_size);
145                 if (taken.get(p)) break;
146                 positions.push_back(p);
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148
149             if (bucket_begin == bucket_end) { // all keys do not have collisions with taken
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152                 auto it = std::adjacent_find(positions.begin(), positions.end());
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156                 pilots.emplace_back(bucket.id(), pilot);
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161                 if (config.verbose_output) log.update(processed_buckets, bucket.size(), pilot);
162             }
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code

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- **Implementation:** write the sequence of steps in a **programming language** (like C/C++, Java, Rust, Python, etc.) to let the algorithm be executed on a computer.



idea for a new  
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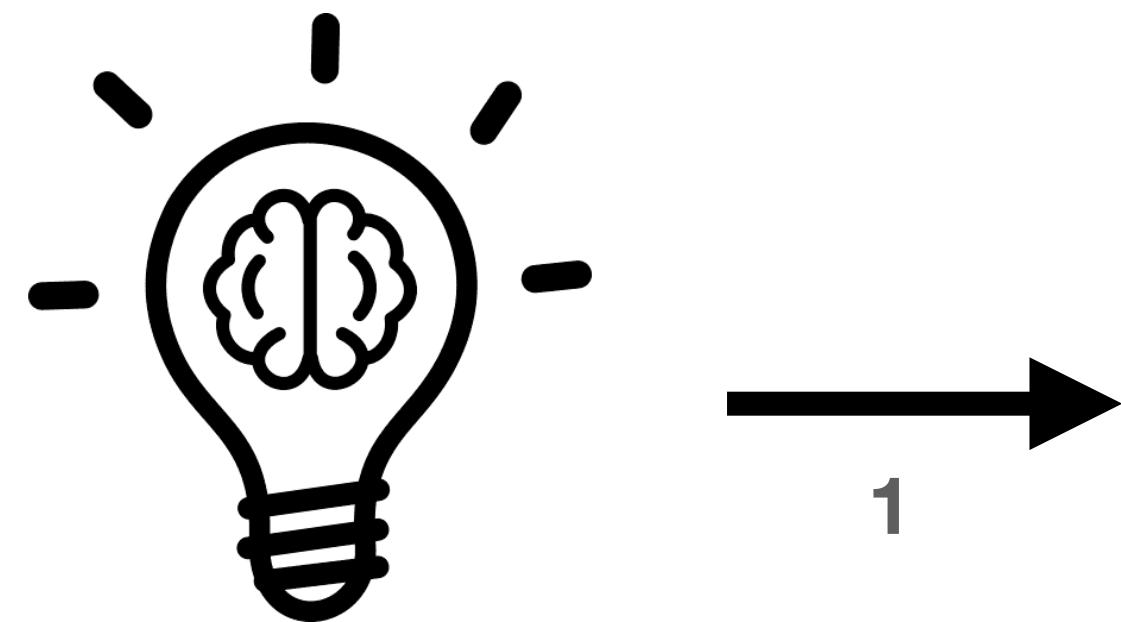
written in the C++ programming language

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written in the C++ programming language

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```
[→ ~ cd Desktop
[→ Desktop nano my_program.cpp
[→ Desktop g++ my_program.cpp -o my_program
[→ Desktop ./my_program
Your program was run with success!
→ Desktop ]
```

result

# Basic definitions – Data Structure

- **Data Structures** store the data that is accessed by an algorithm.
- Idea: the algorithm can read/write the data from/to a data structure to solve the problem **faster**.

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- **Data Structures** store the data that is accessed by an algorithm.
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- Let's introduce the most basic data structure in all Computer Science: the **array** – a sequence of items all of the same type.
- For example, a sequence of integer numbers, or a sequence of characters.

$N = [1, 4, 5, 13, 23, 0, -9, 34]$   
1 2 3 4 5 6 7 8

$S = ['p', 'a', 'c', 'i', 'n', 'o', 't', 't', 'i']$   
1 2 3 4 5 6 7 8 9

# Basic definitions – Arrays

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N = [1,4,5,13,23,0,-9,34]  
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```

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S = ['p','a','c','i','n','o','t','t','i']  
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```

- **Notation.** With  $|A|$  we indicate the number of items in the array A (its length) and with  $A[i]$  the  $i$ -th item of the array, for all  $i=1..|A|$ .
- For example,  $N[3]$  is the integer number 5 and  $S[7]$  is the character 't'.

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- If we do  $S[1]='t'$ , then we over-write the first character of S, so that now S is  
 $S = ['t','a','c','i','n','o','t','t','i']$ .
- If we do  $N[4]+=3$ , now  $N[4]$  is equal to  $13+3=16$ .

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- If we do  $N[4]+=3$ , now  $N[4]$  is equal to  $13+3=16$ .
- **Important note:**  $i$  must be an integer. It does not make any sense to refer to the element in position  $i=3.56\dots$

# Basic definitions – Arrays and memory

- In practice, an array is stored in the memory of your computer as a contiguous sequence of *bytes*.
- The "byte" is the smallest unit of memory on a computer and corresponds to a group of 8 *bits* – 8 binary digits.
- For example, these are 3 bytes.

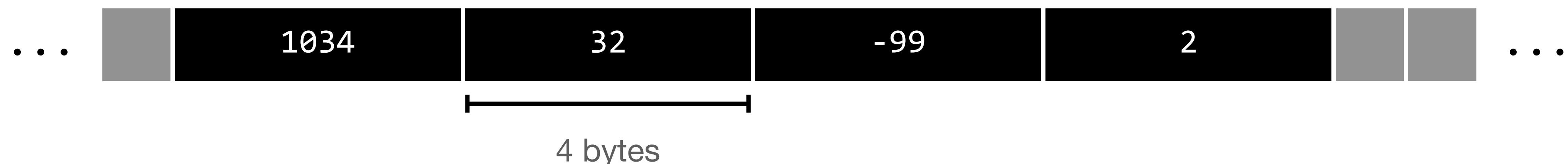
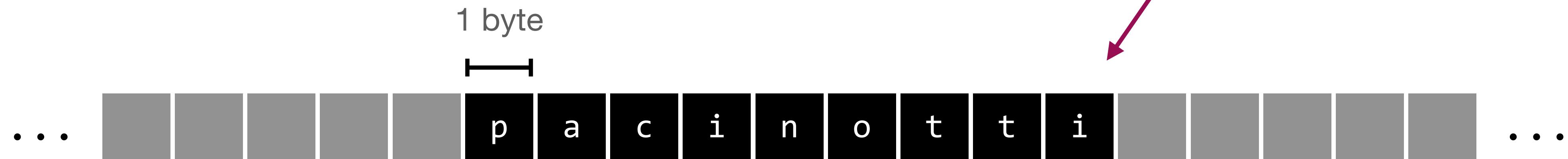
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**computer memory abstraction:**  
a sequence of memory cells,  
each holding **1 byte**



# Warm up – Counting occurrences

- **Problem 1.** Suppose we have a string  $S = \text{"abracadabraabracaba"}$  (an array of characters) and we want to count the **number of occurrences** of a given character  $x$  (which can be any character, like a, b, c, etc.).
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- Easy with a small string. What if the string is 1 billion (i.e., 1,000,000,000) characters? We need an algorithm to do the task for us!
- Our method: "*For each character of S, check if it is equal to x: if so, we have found an occurrence of x.*"
- **Input:** the string  $S$ .
- **Output:** an integer number, indicating the number of occurrences of the character  $x$ . (For example, we expect the answer to be 4 for  $x = 'b'$ .)

# Warm up – Counting occurrences

```
occ_count(S,x):
```

1. count = 0
2. for i = 1..|S|:
3. if S[i] is equal to x:
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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑  
count 0
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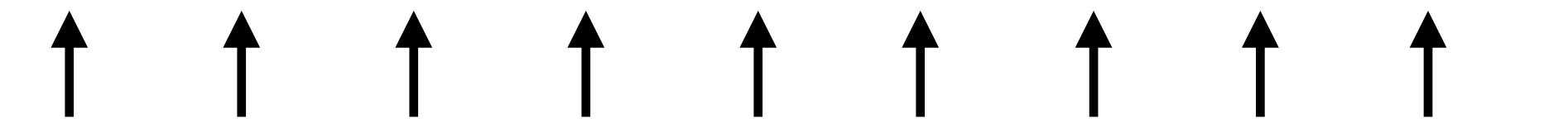


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```



" For each character of S, check if it is equal to x: if so, we have found an occurrence of x. "

x = 'b'

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
```

A sequence of 15 upward-pointing arrows above a horizontal axis. Below the axis, the word "count" is written, followed by a sequence of numbers: 0, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 3. The numbers 1, 2, and 3 are colored orange, while the others are black.

# Warm up – Counting occurrences

**occ\_count(S, x):**

- ```
1. count = 0
2. for i = 1..|S|:
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S = ['a','b','r','a','c','a','d','a','b','r','a','a','b','r','a','c','a','b','a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
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A horizontal sequence of 15 upward-pointing black arrows, each pointing towards the top edge of the frame. The arrows are evenly spaced and have a consistent height.

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S = [ 'a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a' ]																		
i → 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
count	0	1	1	1	1	1	1	1	2	2	2	3	3	3	3	3	3	3

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x = 'b'																		
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i → 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
count	0	1	1	1	1	1	1	1	2	2	2	3	3	3	3	3	3	4

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x = 'b'																		
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	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑	↑
count	0	1	1	1	1	1	1	1	2	2	2	3	3	3	3	3	4	4

# Warm up – Counting occurrences

- **Problem 2.** Suppose we have a string  $S = \text{"abracadabraabracaba"}$  (an array of characters) and we want to count the number of occurrences **of each character** appearing in the string.
- **Input:** the string  $S$ .
- **Output:** ('a',9) ('b',4) ('c',2) ('d',1) ('r',3).

# Warm up – Counting occurrences

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- **Input:** the string  $S$ .
- **Output:**  $(\text{'a'}, 9) (\text{'b'}, 4) (\text{'c'}, 2) (\text{'d'}, 1) (\text{'r'}, 3)$ .
- **Idea 1:** use the previous `occ_count(S, x)` algorithm.

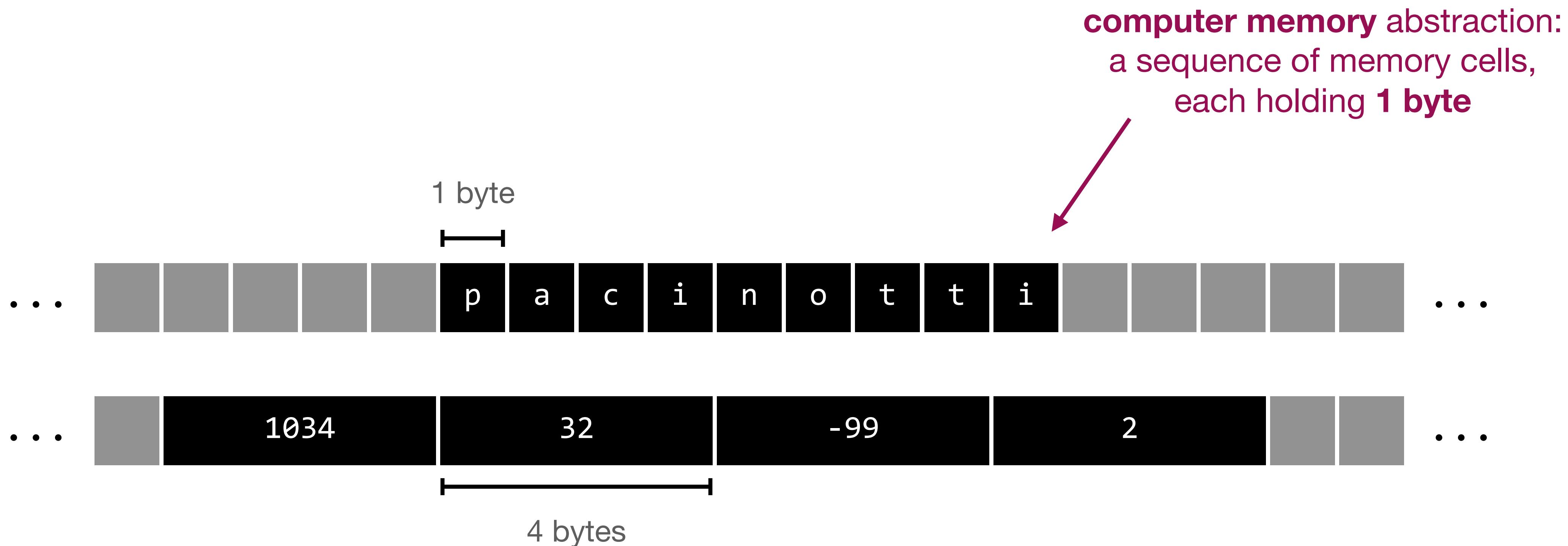
```
all_occ_count_v1(S):
1. for each character x in ['a', 'b', 'c', 'd', 'e', 'f', ..., 'z']:
2.     occ = occ_count(S, x)
3.     print(x, occ)
```

# Warm up – Counting occurrences

- **Idea 2:** exploit the fact that each character is actually a small integer (1 byte = 8 bits).

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  - With 2 bits: 00, 01, 10, 11. (4 integers)

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  - With 3 bits: 000, 001, 010, 011, 100, 101, 110, 111. (8 integers)
  - ...
  - With 8 bits:  $2^8 = 256$  integers.

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  - With 3 bits: 000, 001, 010, 011, 100, 101, 110, 111. (8 integers)
  - ...
  - With 8 bits:  $2^8 = 256$  integers.
- A character, when interpreted as an integer, can therefore be used **as an index** into an array of length 256. This is known as the **ASCII** representation.

# Warm up – Counting occurrences

- Idea 2: exploit the fact that each character is actually a small integer (1 byte = 8 bits).

**ASCII table**

Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char	Decimal	Hex	Char
0	0	[NULL]	32	20	[SPACE]	64	40	@	96	60	`
1	1	[START OF HEADING]	33	21	!	65	41	A	97	61	a
2	2	[START OF TEXT]	34	22	"	66	42	B	98	62	b
3	3	[END OF TEXT]	35	23	#	67	43	C	99	63	c
4	4	[END OF TRANSMISSION]	36	24	\$	68	44	D	100	64	d
5	5	[ENQUIRY]	37	25	%	69	45	E	101	65	e
6	6	[ACKNOWLEDGE]	38	26	&	70	46	F	102	66	f
7	7	[BELL]	39	27	'	71	47	G	103	67	g
8	8	[BACKSPACE]	40	28	(	72	48	H	104	68	h
9	9	[HORIZONTAL TAB]	41	29	)	73	49	I	105	69	i
10	A	[LINE FEED]	42	2A	*	74	4A	J	106	6A	j
11	B	[VERTICAL TAB]	43	2B	+	75	4B	K	107	6B	k
12	C	[FORM FEED]	44	2C	,	76	4C	L	108	6C	l
13	D	[CARRIAGE RETURN]	45	2D	-	77	4D	M	109	6D	m
14	E	[SHIFT OUT]	46	2E	.	78	4E	N	110	6E	n
15	F	[SHIFT IN]	47	2F	/	79	4F	O	111	6F	o
16	10	[DATA LINK ESCAPE]	48	30	0	80	50	P	112	70	p
17	11	[DEVICE CONTROL 1]	49	31	1	81	51	Q	113	71	q
18	12	[DEVICE CONTROL 2]	50	32	2	82	52	R	114	72	r
19	13	[DEVICE CONTROL 3]	51	33	3	83	53	S	115	73	s
20	14	[DEVICE CONTROL 4]	52	34	4	84	54	T	116	74	t
21	15	[NEGATIVE ACKNOWLEDGE]	53	35	5	85	55	U	117	75	u
22	16	[SYNCHRONOUS IDLE]	54	36	6	86	56	V	118	76	v
23	17	[END OF TRANS. BLOCK]	55	37	7	87	57	W	119	77	w
24	18	[CANCEL]	56	38	8	88	58	X	120	78	x
25	19	[END OF MEDIUM]	57	39	9	89	59	Y	121	79	y
26	1A	[SUBSTITUTE]	58	3A	:	90	5A	Z	122	7A	z
27	1B	[ESCAPE]	59	3B	;	91	5B	[	123	7B	{
28	1C	[FILE SEPARATOR]	60	3C	<	92	5C	\	124	7C	
29	1D	[GROUP SEPARATOR]	61	3D	=	93	5D	]	125	7D	}
30	1E	[RECORD SEPARATOR]	62	3E	>	94	5E	^	126	7E	~
31	1F	[UNIT SEPARATOR]	63	3F	?	95	5F	_	127	7F	[DEL]

# Warm up – Counting occurrences

```
all_occ_count_v2(S):  
1. C[1..256] = [0,0,...,0]  
2. for i = 1..|S|:  
3.     j = int(S[i])  
4.     C[j] += 1  
5. for i = 1..|C|:  
6.     print(char(i),C[i])
```

C = [0, 0, ..., 0, 0, 0, 0, ..., 0, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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```

C = [0, 0, ..., 3, 1, 0, ..., 1, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑ ↑ ↑ ↑ ↑

int	char
96	
97	a
98	b
99	c
100	d
101	e
102	f
103	g
104	h
105	i
106	j
107	k
108	l
109	m
110	n
111	o
112	p
113	q
114	r

# Warm up – Counting occurrences

```
all_occ_count_v2(S):  
1. C[1..256] = [0,0,...,0]  
2. for i = 1..|S|:  
3.     j = int(S[i])  
4.     C[j] += 1  
5. for i = 1..|C|:  
6.     print(char(i),C[i])
```

C = [0, 0, ..., 3, 1, 0, ..., 1, ...]  
0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑ ↑ ↑ ↑ ↑ ↑

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96	
97	a
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```

C = [0, 0, ..., 3, 1, 1, 1, ..., 1, ...]  
0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑ ↑ ↑ ↑ ↑ ↑

int	char
96	
97	a
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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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# Warm up – Counting occurrences

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```

C = [0, 0, ..., 4, 1, 1, 1, ..., 1, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑ ↑ ↑ ↑ ↑ ↑

int	char
96	
97	a
98	b
99	c
100	d
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```

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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
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```

C = [0, 0, ..., 4, 2, 1, 1, ..., 1, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
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```

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0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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  0 1 ... 97 98 99 100 ... 114 ...

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  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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int	char
96	
97	a
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```

C = [0, 0, ..., 6, 3, 1, 1, ..., 3, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

int	char
96	
97	a
98	b
99	c
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6.        print(char(i),C[i])
```

$$C = [0, 0, \dots, 6, 3, 1, 1, \dots, 3, \dots]$$

$\begin{matrix} 0 & 1 & \dots & 97 & 98 & 99 & 100 & \dots & 114 & \dots \end{matrix}$

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  

```

int	char
96	'
97	a
98	b
99	c
100	d
101	e
102	f
103	g
104	h
105	i
106	j
107	k
108	l
109	m
110	n
111	o
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114	r

# Warm up – Counting occurrences

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5.    for i = 1..|C|:
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```

$$C = [0, 0, \dots, 7, 3, 1, 1, \dots, 3, \dots]$$

$\begin{matrix} 0 & 1 & \dots & 97 & 98 & 99 & 100 & \dots & 114 & \dots \end{matrix}$

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']
i → 0   1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18
```



int	char
96	'
97	a
98	b
99	c
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101	e
102	f
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106	j
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112	p
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114	r

# Warm up – Counting occurrences

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```

$$C = [0, 0, \dots, 7, 3, 1, 1, \dots, 3, \dots]$$

0	1	...	97	98	99	100	...	114	...
---	---	-----	----	----	----	-----	-----	-----	-----

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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```

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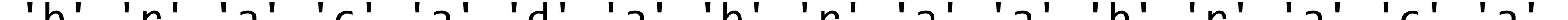
# Warm up – Counting occurrences

```
all_occ_count_v2(S):
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2.    for i = 1..|S|:
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4.        C[j] += 1
5.    for i = 1..|C|:
6.        print(char(i),C[i])
```

$$C = [0, 0, \dots, 7, 3, 2, 1, \dots, 3, \dots]$$

$\begin{matrix} 0 & 1 & \dots & 97 & 98 & 99 & 100 & \dots & 114 & \dots \end{matrix}$

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']
i → 0   1   2   3   4   5   6   7   8   9   10  11  12  13  14  15  16  17  18
```



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96	'
97	a
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C = [0, 0, ..., 7, 3, 2, 1, ..., 3, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
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↑ ↑

int	char
96	
97	a
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# Warm up – Counting occurrences

```
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3.     j = int(S[i])  
4.     C[j] += 1  
5. for i = 1..|C|:  
6.     print(char(i),C[i])
```

C = [0, 0, ..., 8, 3, 2, 1, ..., 3, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑

int	char
96	
97	a
98	b
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101	e
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109	m
110	n
111	o
112	p
113	q
114	r

# Warm up – Counting occurrences

```
all_occ_count_v2(S):  
1. C[1..256] = [0,0,...,0]  
2. for i = 1..|S|:  
3.     j = int(S[i])  
4.     C[j] += 1  
5. for i = 1..|C|:  
6.     print(char(i),C[i])
```

C = [0, 0, ..., 8, 3, 2, 1, ..., 3, ...]  
  0 1 ... 97 98 99 100 ... 114 ...

S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  
↑ ↑

int	char
96	
97	a
98	b
99	c
100	d
101	e
102	f
103	g
104	h
105	i
106	j
107	k
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```

$$C = [0, 0, \dots, 8, 4, 2, 1, \dots, 3, \dots]$$

$\begin{matrix} 0 & 1 & \dots & 97 & 98 & 99 & 100 & \dots & 114 & \dots \end{matrix}$

```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  

```

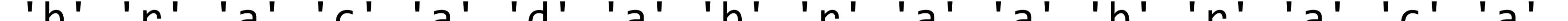
int	char
96	'
97	a
98	b
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102	f
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S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  

```

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$$C = [0, 0, \dots, 9, 4, 2, 1, \dots, 3, \dots]$$

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```
S = ['a', 'b', 'r', 'a', 'c', 'a', 'd', 'a', 'b', 'r', 'a', 'a', 'b', 'r', 'a', 'c', 'a', 'b', 'a']  
i → 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18  

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# We have two different algorithms for the same problem

```
occ_count(S,x):  
1. count = 0  
2. for i = 1..|S|:  
3.   if S[i] is equal to x:  
4.     count += 1  
5. return count
```

```
all_occ_count_v1(S):  
1. for each character x in ['a','b','c','d','e','f',..., 'z']:  
2.   occ = occ_count(S,x)  
3.   print(x,occ)
```

v1

```
all_occ_count_v2(S):  
1. C[1..256] = [0,0,...,0]  
2. for i = 1..|S|:  
3.   j = int(S[i])  
4.   C[j] += 1  
5. for i = 1..|C|:  
6.   print(char(i),C[i])
```

v2

- Algorithm v2 uses a data structure (an array), whereas algorithm v1 does not.
- **Q.** Which one should we use?
- To answer this question we need to **analyse** an algorithm.

# Basic definitions – Analysis of algorithms

- When developing a solution to a problem with an algorithm, we are concerned about two things:

# Basic definitions – Analysis of algorithms

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**Q.** After how many seconds will my algorithm terminate?

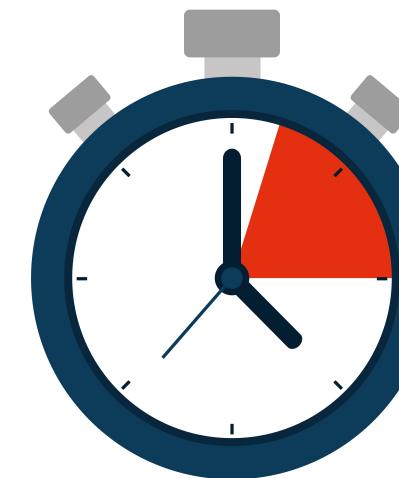


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- the **space** taken by the data structure(s) it uses.

**Q.** Can I run my algorithm on my computer with 4GB of RAM?

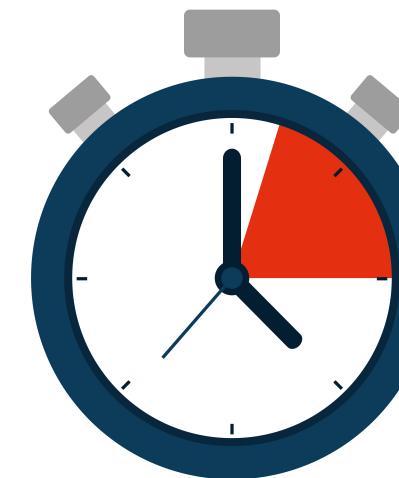


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- the **running time** of the algorithm;

**Q.** After how many seconds will my algorithm terminate?



- the **space** taken by the data structure(s) it uses.

**Q.** Can I run my algorithm on my computer with 4GB of RAM?



- **The less, the better. We strive for efficient algorithms.**
- Analysing an algorithm means understanding its running time and memory usage. We will talk more about this soon.

# Part 1 – Summary

- Definition of Algorithm and Data Structure
- Arrays and memory
- Warm up: two algorithms for counting the occurrences of characters in strings

## **Part 2 – Motivations, analysis of algorithms, same applications**

# Why algorithms?

- The romantic/philosophical view: **algorithms describe our life.**
- Fundamental questions:
  - **Q.** What problems can I solve?
  - **Q.** And how, i.e., what resources do I need?
  - **Q.** Can I do better (use less resources)?



# Why algorithms?

- The romantic/philosophical view: **algorithms describe our life.**
- Fundamental questions:
  - **Q.** What problems can I solve?
  - **Q.** And how, i.e., what resources do I need?
  - **Q.** Can I do better (use less resources)?
- Understanding if we can do something better has always been a primary question in the history of human evolution.
- There are many known algorithms. Yet, probably more need to be invented!
- **Democracy: can be invented by anyone, anywhere. You could be next!**



# Huffman's data compression algorithm



Robert Fano  
(1917 - 2016)



David Huffman  
(1925 - 1999)

- D. Huffman was a graduate student at MIT in 1951.
- He solved an open problem left by his teacher R. Fano, during a class on Information Theory.

1098

*PROCEEDINGS OF THE I.R.E.*

*September*

## A Method for the Construction of Minimum-Redundancy Codes\*

DAVID A. HUFFMAN<sup>†</sup>, ASSOCIATE, IRE

*Summary*—An optimum method of coding an ensemble of messages consisting of a finite number of members is developed. A minimum-redundancy code is one constructed in such a way that the average number of coding digits per message is minimized.

### INTRODUCTION

ONE IMPORTANT METHOD of transmitting messages is to transmit in their place sequences of symbols. If there are more messages which might be sent than there are kinds of symbols available, then some of the messages must use more than one symbol. It is natural to suppose that the best way to send a message is to choose the symbols so that the average number of symbols used per message is as small as possible. It is also natural to suppose that the best way to do this is to assign to each message a sequence of symbols which depends on the message itself. This is the basic idea of the present paper.

will be defined here as an ensemble code which, for a message ensemble consisting of a finite number of members,  $N$ , and for a given number of coding digits,  $D$ , yields the lowest possible average message length. In order to avoid the use of the lengthy term "minimum-redundancy," this term will be replaced here by "optimum." It will be understood then that, in this paper, "optimum code" means "minimum-redundancy code."

The following basic restrictions will be imposed on an ensemble code:

1. The ensemble consists of  $N$  different messages, denoted by  $m_1, m_2, \dots, m_N$ .

# Why algorithms?

- The practical view: to **solve problems** that are otherwise “impossible” to solve in a **reasonable amount of time**.
- **Example.** Sub-string search.  
**Q.** Does the following string contain "CGTGGTTAACACGAGC" and, if so, at what position?



TTGCTCATGCCCGGTGCAGTGAAC TGAGGAAAATAAGTTAACCGCCGTGGCGAGCGCGCACGTTGAGCGCCCTGACGCTATGGCGATAAAATGCCGCCATGCCGTGTACCA GGTATCAATGTCGCGAACAGCAGCGGGTAAATTTCGGCCAGTCAGATGACACTTGACAAGTGCAGTTGGCGAGGGCGCTGTTTCCAGACGTTGATGCGTCCGCGCTCGCACCTGATGCCCTGCTGACTTAGCGCAA GACCAGGTGCTGGCGATATAGCCGCTGGCGCCGAGAACCGAGAACATGCGTGCACGTTGCTCTCCTAGCGGGCTAAAAAGGCGGCCAGTGGCGACGACGTCGGTAAGCTGTCGAGAGAGACATCCAGATGCGTACCA GACGCAATCGCGCGCGTTAACAGGATATTCCGTTCCCACGTTAGTCGCAAGCGCGCCCTGCTGCTTCGCAACGCGAACAAACAGCATATTGTTGCGTATGACCTCCGCGCCGCTTCCGAAGCTGCTGCGCCAGCCAGG CGCGTTATCATGATCCTCTTGCAGACGCGCACGTTAGTCGCAAGCGCATACAGTCCGGCCGCTGCCAGAACATCCGGCCTGACGCATTCCGCGCCGACCATTTCAGCCAGCGCGTCCGCGTTAACGTAATCGCGTTGCCGACCA GCAAGTGAACCGACC GGCGTTCCGAGACCTTTGACAGGCAGATGGTAAAAGAGTCGAATACTGCGTAATCTCTTTAACTCACAGCGTAGGCAACCACCGCGTTAAAATCGGGCGCCGTCACGTCAGCGCCAGTCCACGTTGCG GGGTAAATGTC CAGGGCGTCTTCAGATA CGCGCGGGCAGCAGTTCCGTTATGCGTATTTCAGACTGAGCAAGCGCGTGC CGCGAAGTGGATGTCATCCGCTTAATCTCGCCGCCACGTTCTCCAGCGGAGCGTACCGTCCCG CGCGCGCGTCAATGGCTGGCTGCCAGCTGGCTGGCTGAATGCTGCGAGCACCGCCGCGCCAGCTTCAGCTGCGCCGCTCACAAATGGCTAACGAGCGCAGCAGATTGGCTGGGTGCCGG TGGGTAAAAAAAGCGCCGCTTACCGGAAAGGTGGCGGTAGCGCTGAAGGGCGTTAACAGTAGGGTACATCCCCGACCGGGCGGTACATCGCCTCGAGCATGGCGCCGGCTGGTAACGGTATCACTGCGTAAATCAA TCATGGCACATCCCTGGATTAAAGGTGATGTGCACTGTTTACCTAGCCAGTTGCTTCCAGTTGCGATCAGTCACCTCATCGCCGCGCCGCTAACATGGCGCTAGCATGTACAGGCTAACGCTTCTGCTGGCTAGTTGAT CTGCGCGGGATGCCAGCTCTTGCACGACCACGTCAACCGAGTACCGGGCGTCAATGGAAAACGCGCGCTGTAGCGCACCGTCTGCGCTTTCACGCGAATACGGTAATGCCGAGGCTCGCGATACGCGC GAAATTGGTGTGTCAGTCGGTACCGTCGGTAAGGTAGCCGGCTTCATTCCATGCCACAAAGCCCAGCAGCTGTTATTAAAGACGACGATTTCATGGCAGCTTCATCTGTACCGAGAGAAAATGCCCATCAGCAT ACTGAAGCCGCCATACCGCACATCGCGATAACCTGACGACCCGGCGCGTAGCCTGAGCGCCGAGCGCCTGCGGCTAGCGTTGGCATTGACCCGTGGTAAACGAGCCTAGCAGGGCGCTTGCCTGCGTACAGGCTGGCTGGTAAGTCAGTCCCTACGGCGTCCCAGTGT CGCCAGACGGTGGCGTGGACATCGCAGGTAAAAATAGCGTCGTAGCGGGAAATGACTAACATTGTTGCGCCAGATATTGTGGGTGGATGGCTTATCGCTGAGTTGGCTAACGCTAACGCTTACGGCGTCCCAGTGT CTCCAGAGCTTATCGAGGAATTACGATTGCTTTCTTCCACCAGCGGAGCAGGGCGGAAGCGTGGCTTAATATGCCCACTAGCGCCATGTCACCTTGCTGTGCGGCCAATACTGCCGGTTGATGTCAATCTGAATGAT TTTGGCATCGCTGGATAAAAGGCGCGATAGGGGAACTGGGTGCCGAGCAGGATCAGCGTATCGCGTTCATCGGTGTTGAGGCCAGAACGAGAAGCCAATCAGGCCGGTATTCCCACATCATAAGGGTTATCGTACTCAACGTGCTC TTTGCCCGCAGGGCATGAGCGATTGGCGCTTTAGTTGCCAACCGCACCATCCTCATCGCGCCCGCGCAGCCGCTACCGCACATCAATCGGATATTGCTGGAGTAGCGCAGTTGCCAGTTTCAG

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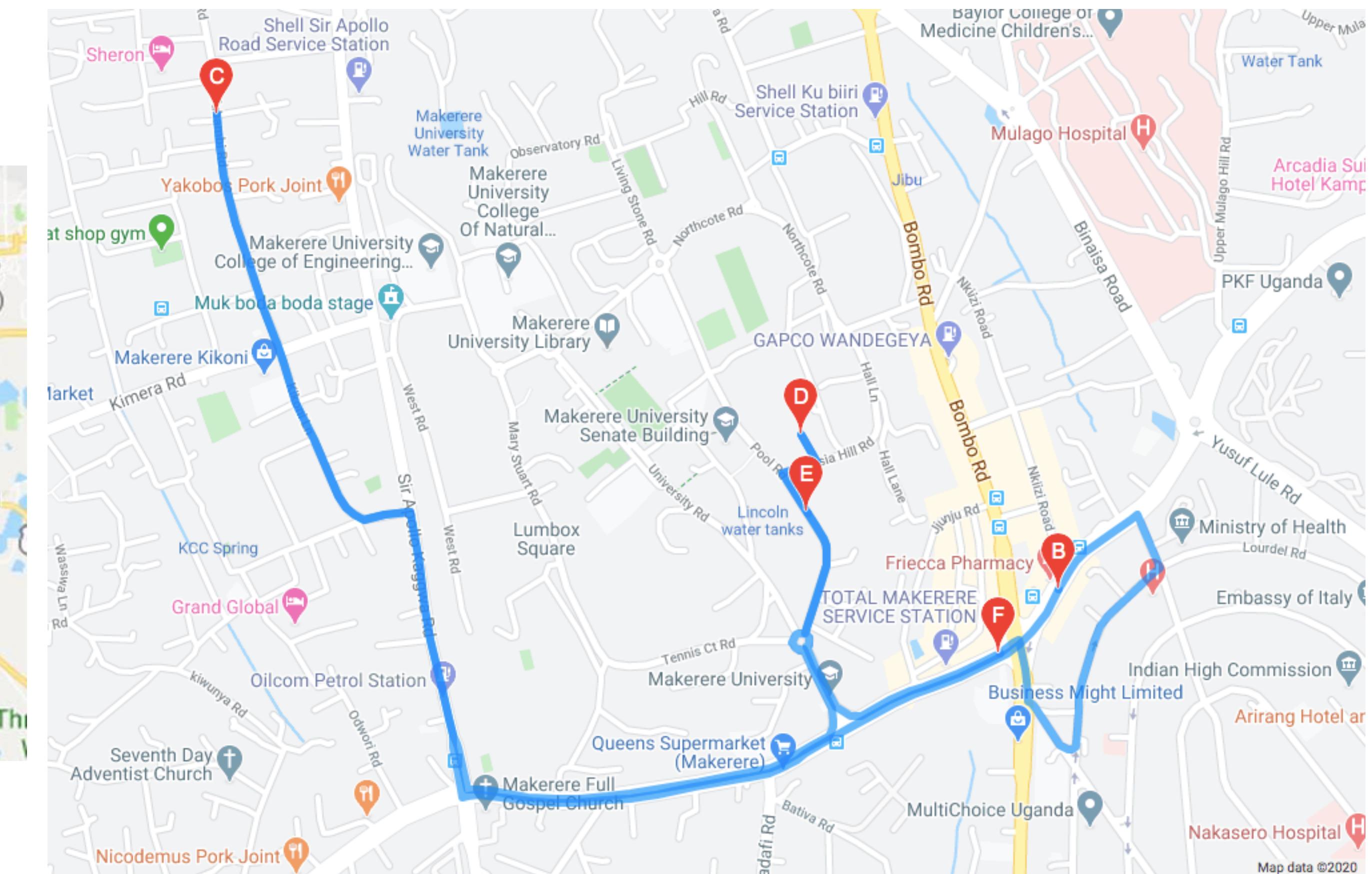
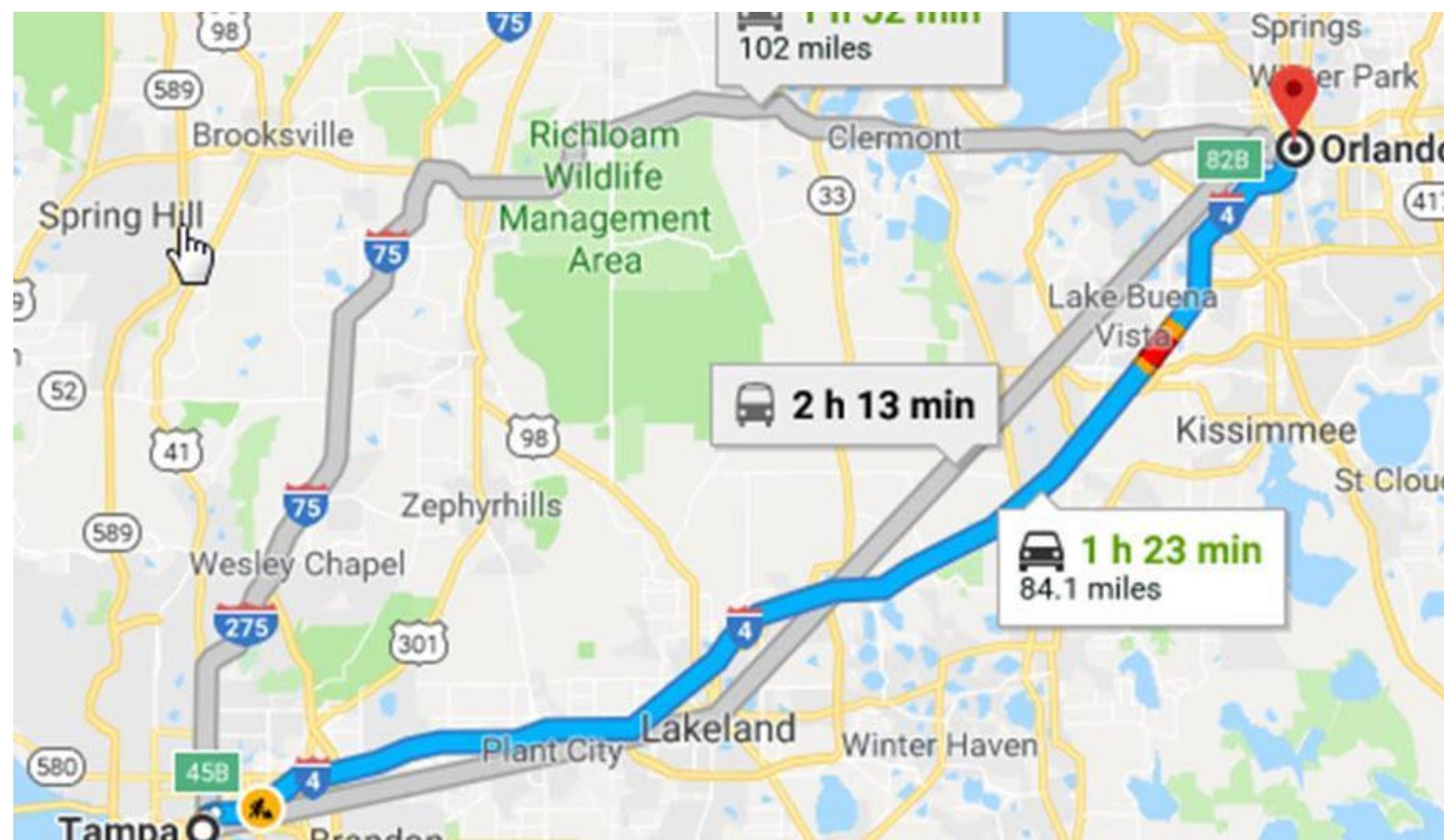


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In this case, the answer is "yes: at position 1896".

# Why algorithms?

- **Example.** Shortest path between two points in a map.



# Why algorithms?

- **Example.** Query suggestion.

The screenshot shows a search interface with a search bar containing the partial query "dream the|". Below the search bar is a list of suggestions:

- dream theater
- dream theater images and words
- dream theater discografia
- dream theater discography
- dream theater scenes from a memory
- dream theater awake
- dream theater another day
- dream theater logo
- dream theater through her eyes
- dream theater octavarium

To the right of the suggestions is a dropdown menu with the following items:

- All ▾ The Rust pro
- Buy Again the rust programming language
- the rust programming language, 3rd edition
- the rust programming language, 2nd edition
- the rust programming language 2023
- outdoor curtain rods for patio rust proof
- shower curtain hooks rust proof
- rust programming
- shower rings for curtain rust proof

# Why algorithms?



- The practical view: for **profit**.
- **Build better systems/applications** in terms of reduced latency to use the service.  
→ Make your users happy so that they will keep using your service (and you will keep earning)!

# Why algorithms?



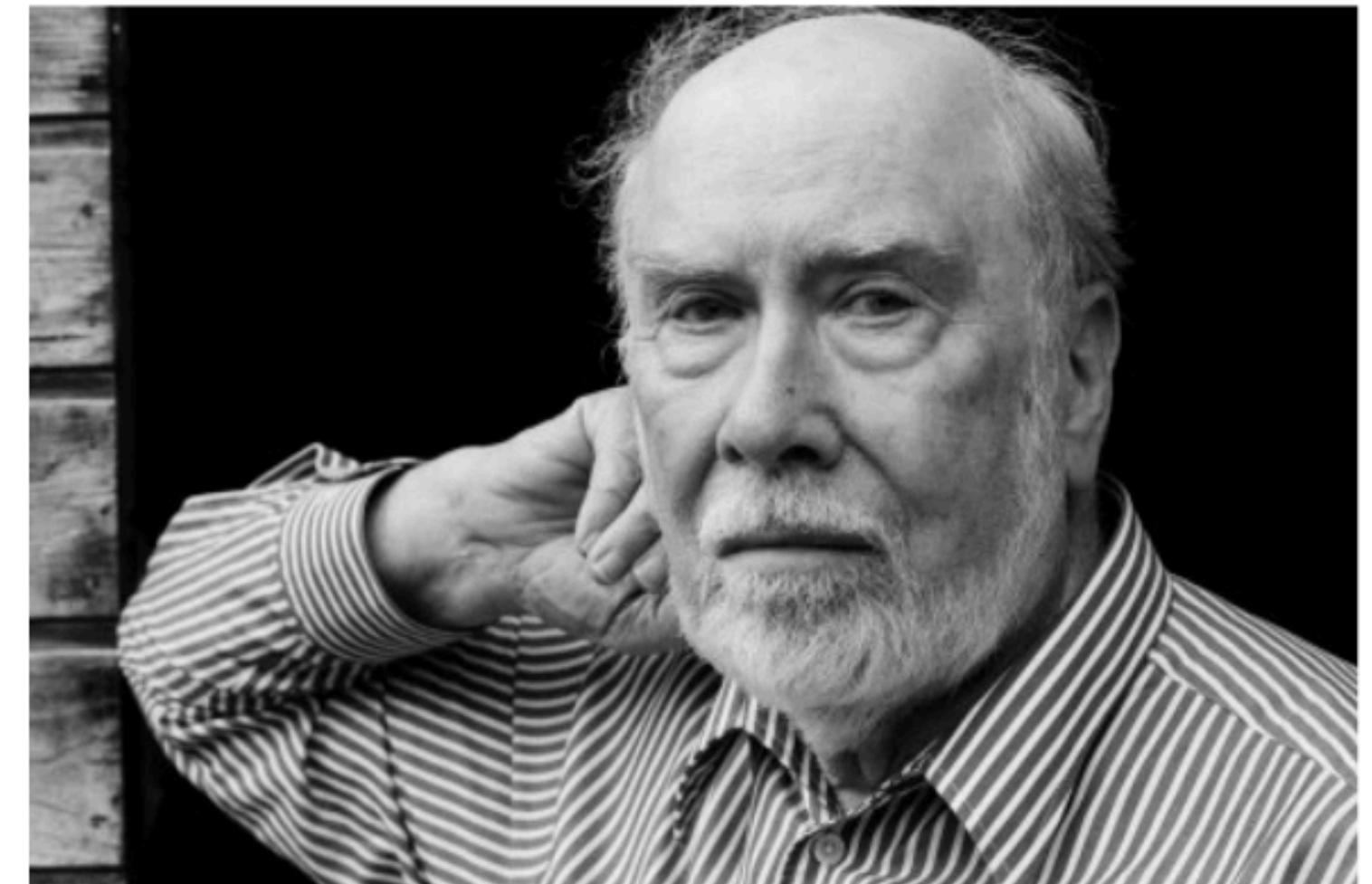
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- **Build better systems/applications** in terms of reduced latency to use the service.  
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- **Save computer resources** (power and storage machines).

# The increase of data does not scale with technology

- These considerations are **even more relevant today** than in the past.
- Today we are facing a **data explosion** phenomenon.

*“Software is getting slower  
more rapidly than hardware  
becomes faster.”*

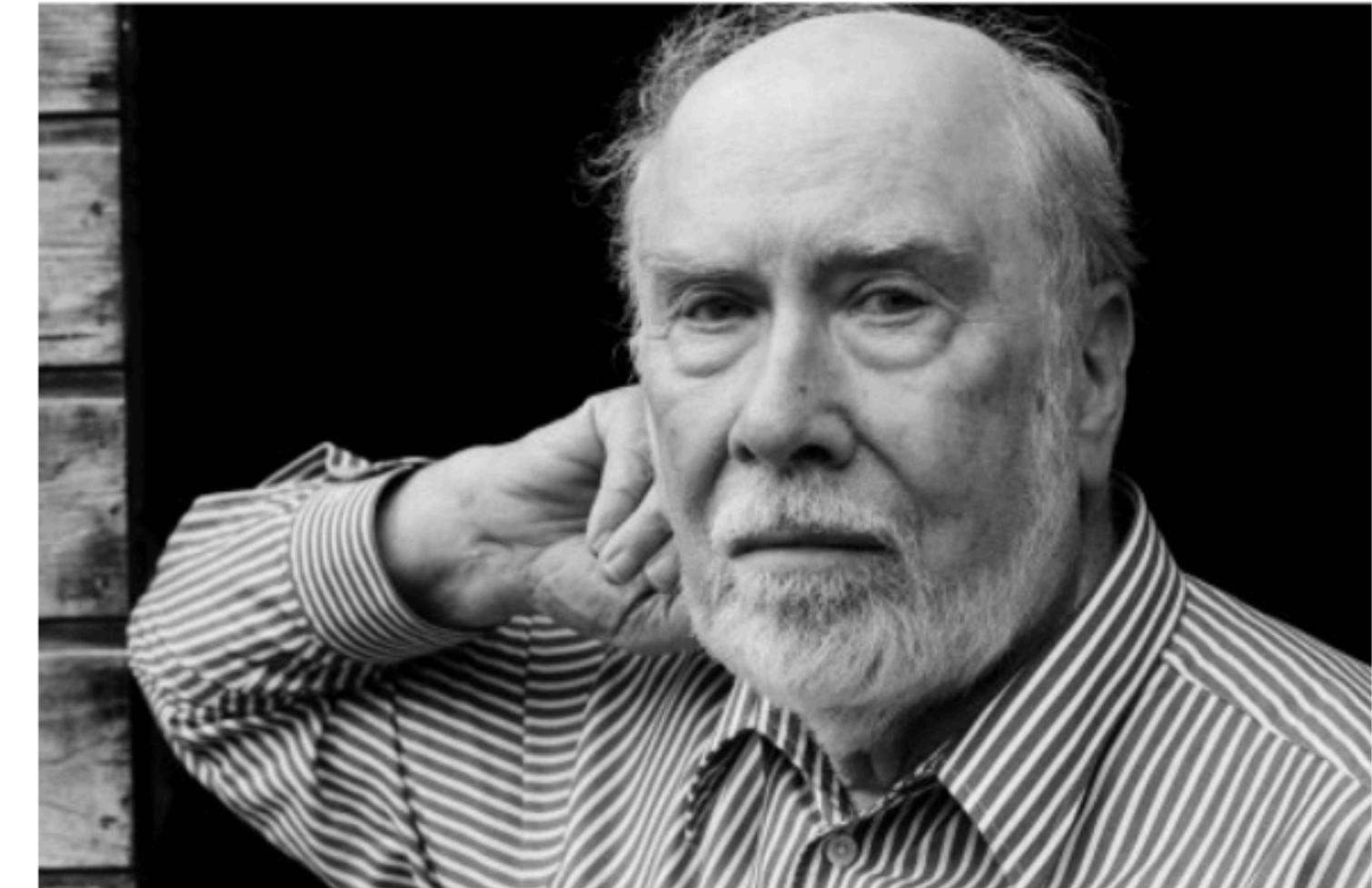


Niklaus Wirth

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→ Lesson learnt: **a better algorithm is always better than a better computer!**

Niklaus Wirth

# Data explosion

- More data...



# Data centers

- More computers...



# Applications are more data intensive than ever

- More electricity spent → more money spent!
- **The more efficient an algorithm is, the less electricity it requires to run.**



# Why algorithms?

- We need **good programmers to implement efficient algorithms.**

*"Bad programmers worry about the code.  
Good programmers worry about data  
structures and their relationships."*



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Google

YAHOO!

bing

facebook



IBM

Dropbox

ORACLE

eBay

amazon

# Why algorithms? – Recap

- To **better understand** what we can do with computers.
- To **solve** real-world problems that could be otherwise impossible to solve.
- To get a **well-paid job**.

# Why algorithms? – Recap

- To **better understand** what we can do with computers.
- To **solve** real-world problems that could be otherwise impossible to solve.
- To get a **well-paid job**.

→ **No reason not to study Computer Science and algorithms!**

# Analysis of algorithms

- When developing a solution to a problem with an algorithm, we are concerned about two things:

- the **running time** of the algorithm;



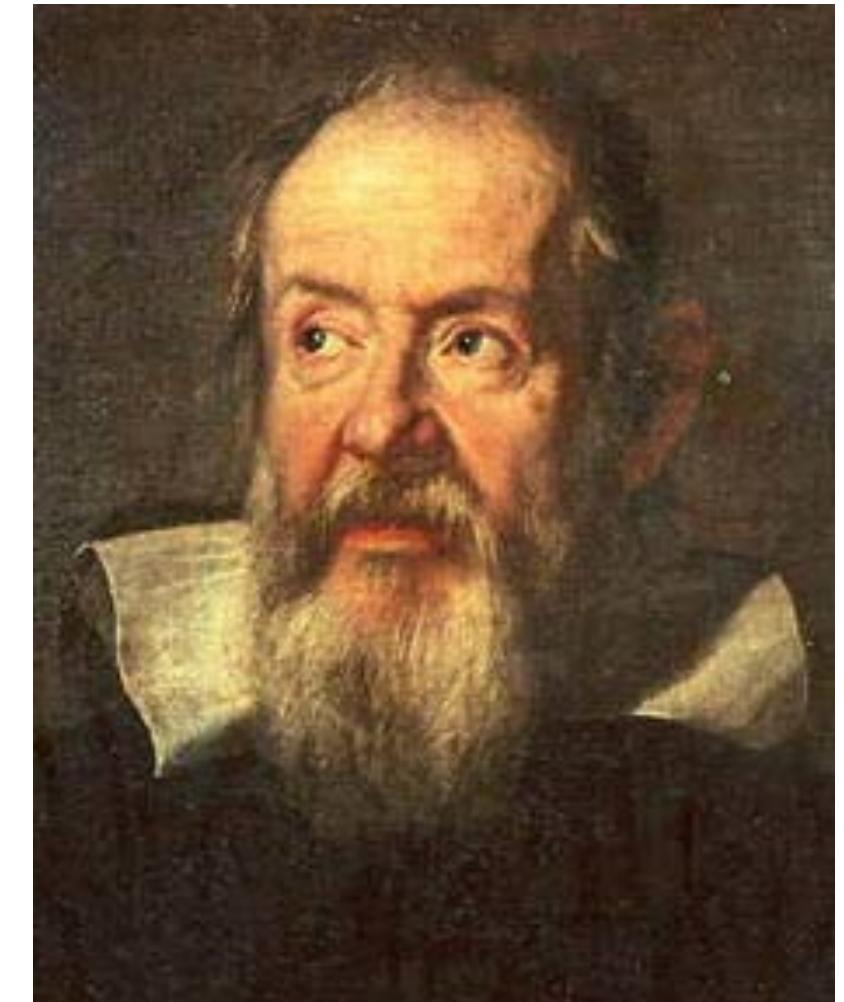
- the **space** taken by the data structure(s) it uses.



- **The less, the better.**
- **Trade-off between time and space** of a solution.

# The running time – The scientific method

- Scientific method:
  1. **Observe.**
  2. Formulate an **hypothesis.**
  3. Make a **prediction.**
  4. **Validate:** if prediction is valid, then stop; repeat otherwise.



Galileo Galilei

# The running time – The scientific method

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???

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all_occ_count_v1(S):  
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v1

```
all_occ_count_v2(S):    v2  
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5. for i = 1..|C|:  
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```

S	v1	v2
0.5M	118 ms	3 ms
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M = 1 million; 1 ms = 1/1000 sec

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- First observation: as the **input doubles in size, also the running time of both v1 and v2 doubles.**
- First hypothesis: the running time has a **linear** dependency from the input size.

M = 1 million; 1 ms = 1/1000 sec

# The running time – The scientific method

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1. C[1..256] = [0,0,...,0]
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4.     C[j] += 1
5. for i = 1..|C|:
6.     print(char(i),C[i])
```

S	v1	v2
0.5M	118 ms	3 ms ≈39
1M	201 ms ≈1.70	6 ms ≈33 ≈2.00
2M	372 ms ≈1.85	13 ms ≈29 ≈2.17
4M	721 ms ≈1.94	26 ms ≈27 ≈2.00

- First observation: as the **input doubles in size, also the running time of both v1 and v2 doubles.**
- First hypothesis: the running time has a **linear** dependency from the input size.

M = 1 million; 1 ms = 1/1000 sec

# The running time – The scientific method

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all_occ_count_v2(S):      v2
1. C[1..256] = [0,0,...,0]
2. for i = 1..|S|:
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4.     C[j] += 1
5. for i = 1..|C|:
6.     print(char(i),C[i])
```

S	v1	v2
0.5M	118 ms	3 ms ≈39
1M	201 ms ≈1.70	6 ms ≈33 ≈2.00
2M	372 ms ≈1.85	13 ms ≈29 ≈2.17
4M	721 ms ≈1.94	26 ms ≈27 ≈2.00

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- First observation: as the **input doubles in size, also the running time of both v1 and v2 doubles.**
- First hypothesis: the running time has a **linear** dependency from the input size.
- Second observation: v1 tends to be  $\approx 27\text{-}30\times$  **slower** than v2 for large inputs.

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- The scientific method is great to validate our hypotheses.
- But one should come up with an hypothesis first. We derived our hypothesis via **direct observation** of the running time.
- However, looking at the running time alone does not explain **what** the algorithm is doing.
- We would like to have a **model** to **predict the running time**.

# The running time – Deriving a model

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- **Q.** What is an "operation" ?
- By "operation" we mean some **elementary operation** that a computer can execute, like: assignments, addition/subtraction, multiplication/division, read a cell of an array, comparing two integers/characters, etc.
- **Simplification:** such elementary operations take a (usually, very small) unit of time, say  $c$ .

Example 1:

```
x = 1  
y = 2  
z = x + y  
  
4 ops
```

Example 2:

```
S[3] = 5  
z = S[3] * 4  
  
5 ops
```

Example 3:

```
for i = 1..|S|:  
  x = i + 3  
  
~2|S| ops
```

# Counting occurrences – Analysis

- Let's **count the number of operations** our two algorithms perform. Let  $n = |S|$ .

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occ_count(S,x):  
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all_occ_count_v2(S):
```

```
1. C[1..256] = [0,0,...,0] ← 256 ops
2. for i = 1..|S|:
3.   j = int(S[i])
4.   C[j] += 1
5. for i = 1..|C|:
6.   print(char(i),C[i]) ← 256 ops
```

~  $3n$  ops

a total of ~  $3n + 256 \times 2$  ops ≈  $3n$   
when  $n$  is large (e.g.,  $n = 1$  million)

# Counting occurrences – Analysis

- To sum up.

	v1	v2
num. operations	$\sim 65n$	$\sim 3n$

- We can conclude that:
  - Both v1 and v2 have running time that grows **linearly** in  $n$  (the length of the input string).
  - But v2 executes way fewer operations, hence it is **much** faster ( $\approx 20\text{-}30\times$  faster).

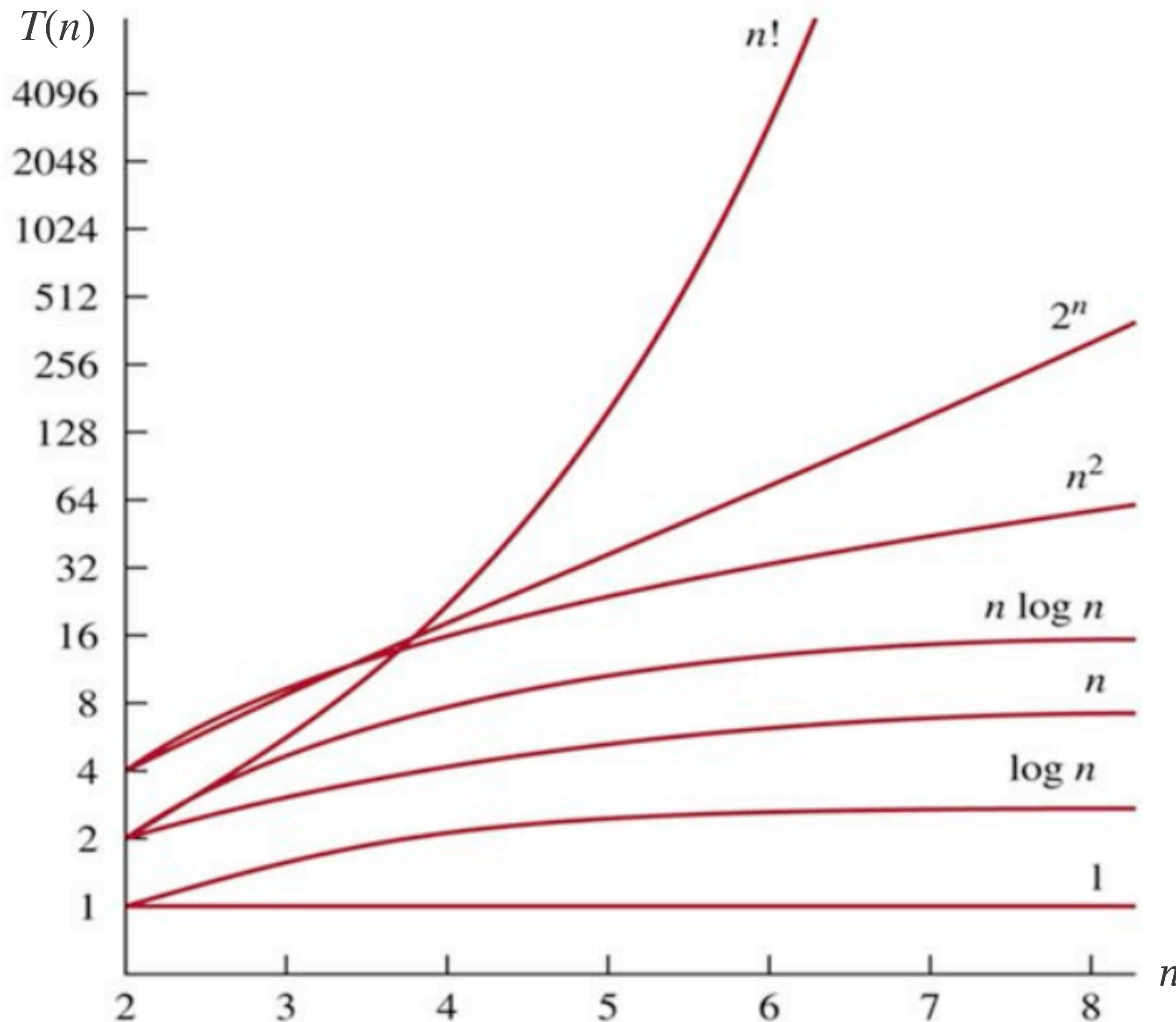
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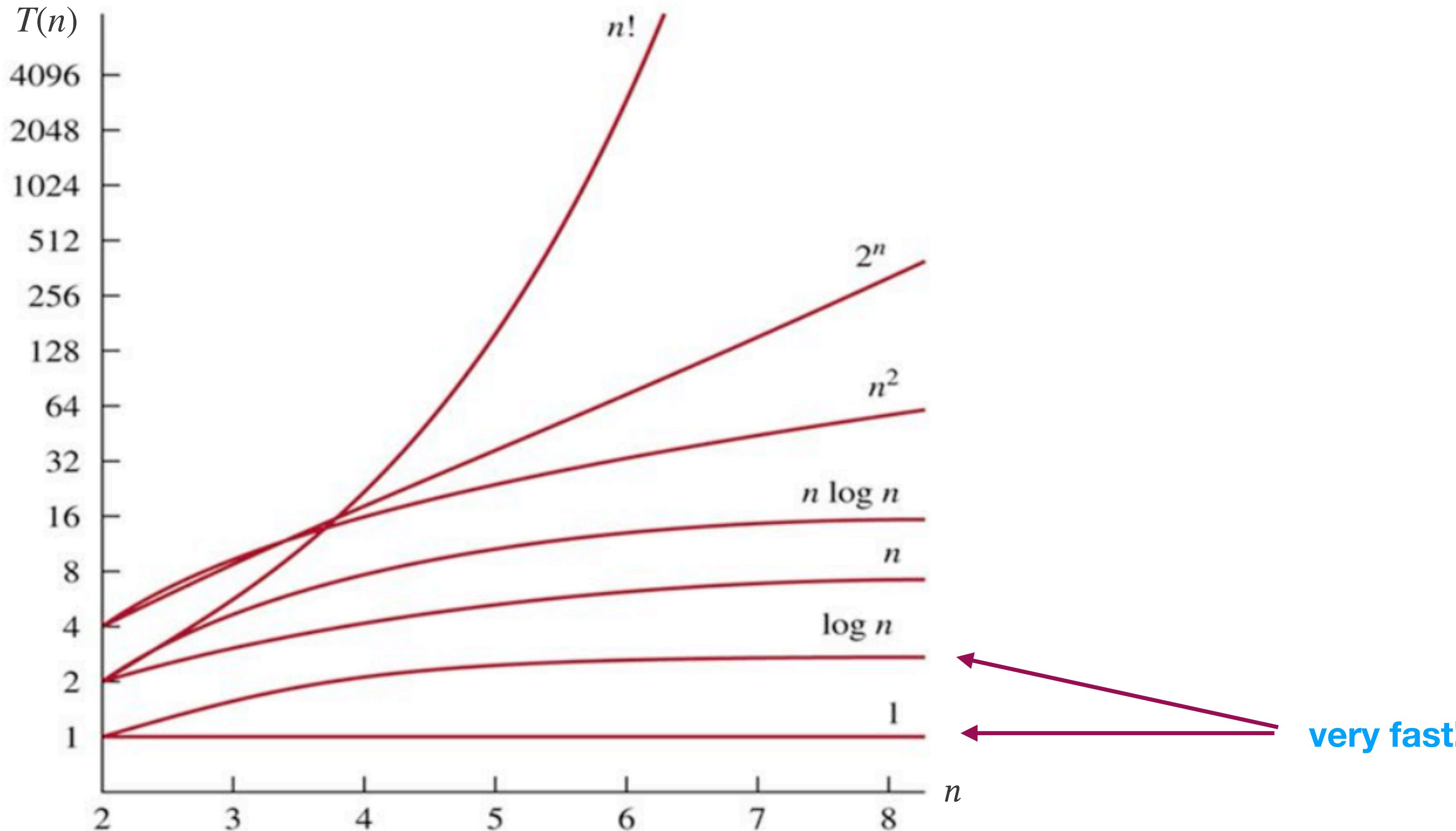
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  - But v2 executes way fewer operations, hence it is **much** faster ( $\approx 20\text{-}30\times$  faster).
- **Linear running time is not the only possibility!**

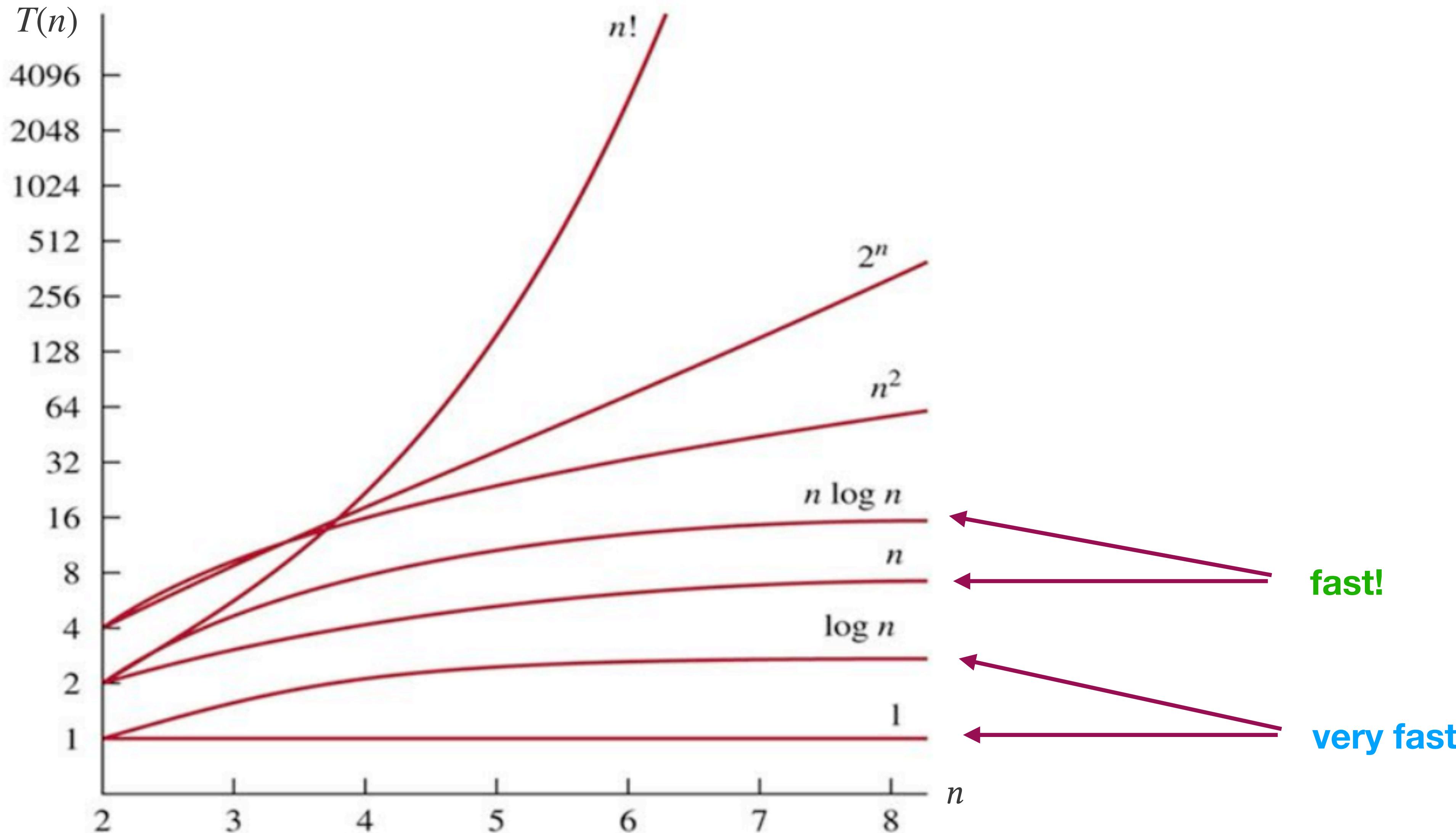
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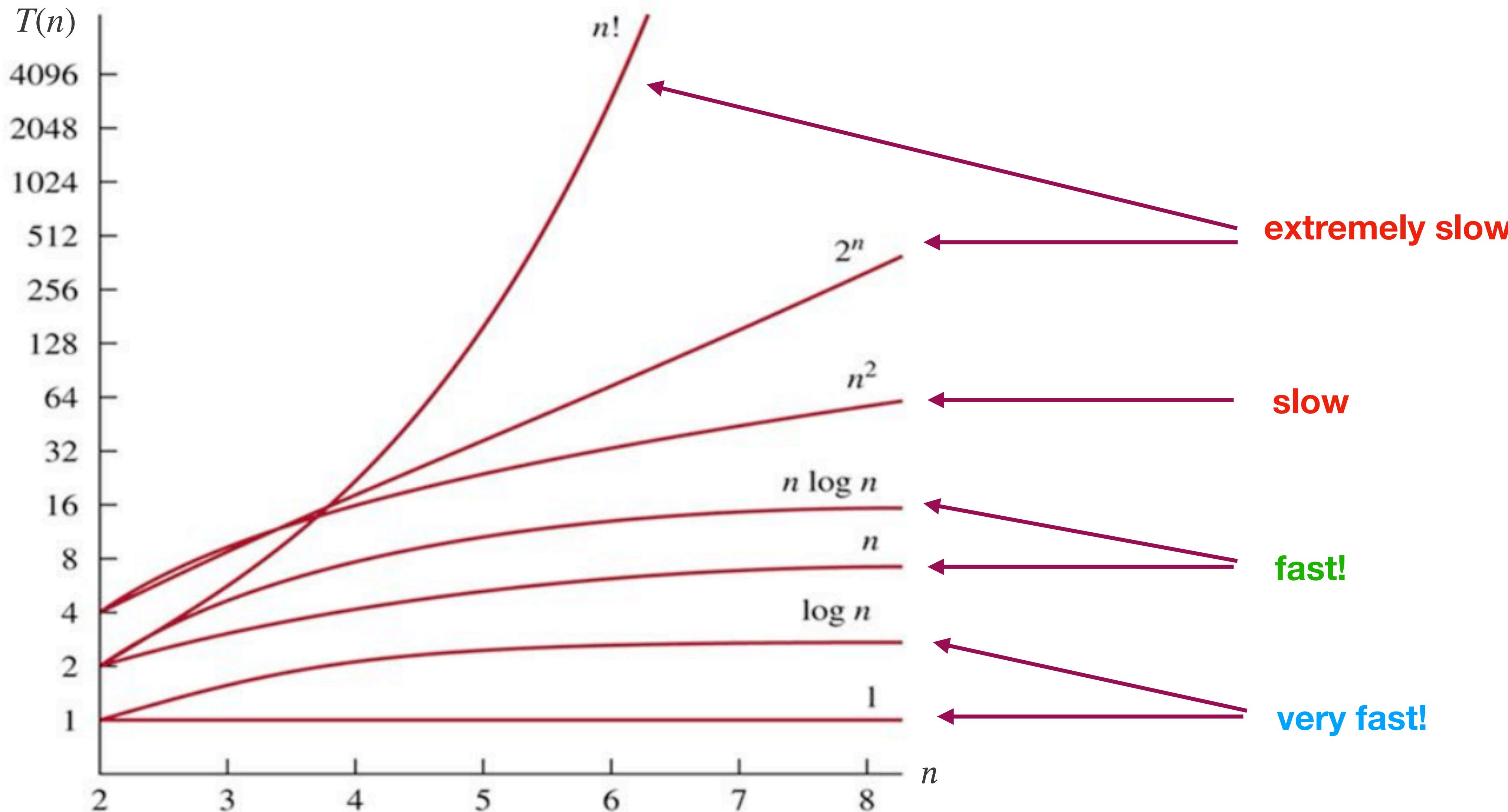
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# The space

- Intuitively: all the bytes that are maintained/manipulated by the algorithm during its execution.

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4-byte integer

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4 bytes x 256 = 1024 bytes = 1 KiB

# Part 2 – Summary

- Three good reasons to study algorithms:
  - understand; solve; earn.
- Analysis of algorithms:
  - scientific method is good to confirm/reject hypotheses;
  - we need a model to predict the running time and space consumed by an algorithm.
- Model: count the number of operations performed by an algorithm.
- Alg. v2 is 30X faster than algorithm v1 but also consumes 1KiB of extra memory.

## **Part 3 – Some example problems: integer search and sub-string search**

# Integer search

- **Problem.** We are given a **sorted** integer array A, say of length  $n$ , and an integer x. We want to determine whether x is in A and, if so, return its **position** in A.

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A = [3, 5, 7, 13, 14, 15, 34, 45, 66, 78, 123, 443, 601]  
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- We will see **two algorithms** to solve this problem, with **radically different** running times.

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- **Idea 1.** For each integer  $A[i]$ ,  $i = 1..n$ , check if it is equal to  $x$ . If so, return  $i$ . If no integer is equal to  $x$ , then return -1.
- Pseudo code.

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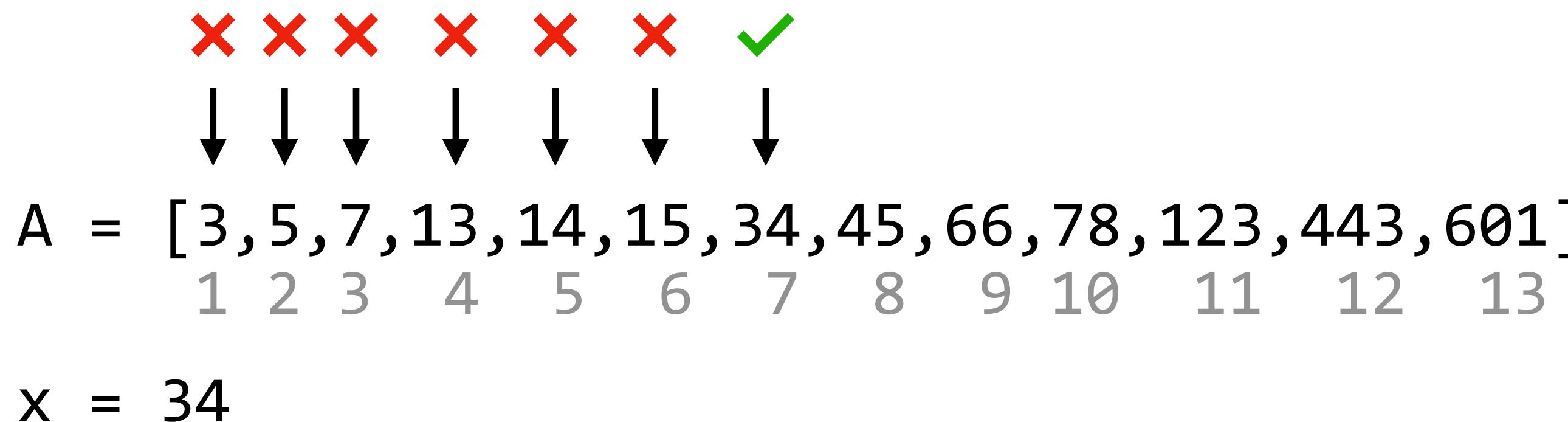
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  - **Average case:**  $\sim 1/2 \cdot 2n = n$  ops.
- So **the running time is linear** in the length of the array.

# A better search strategy

- **Idea 2.** Exploit that fact that the array A is **sorted**.
- **Intuition:** Suppose you have  $x=34$  and you look at a random position in A, say at position 11. What can you say about the position of  $x$ ?

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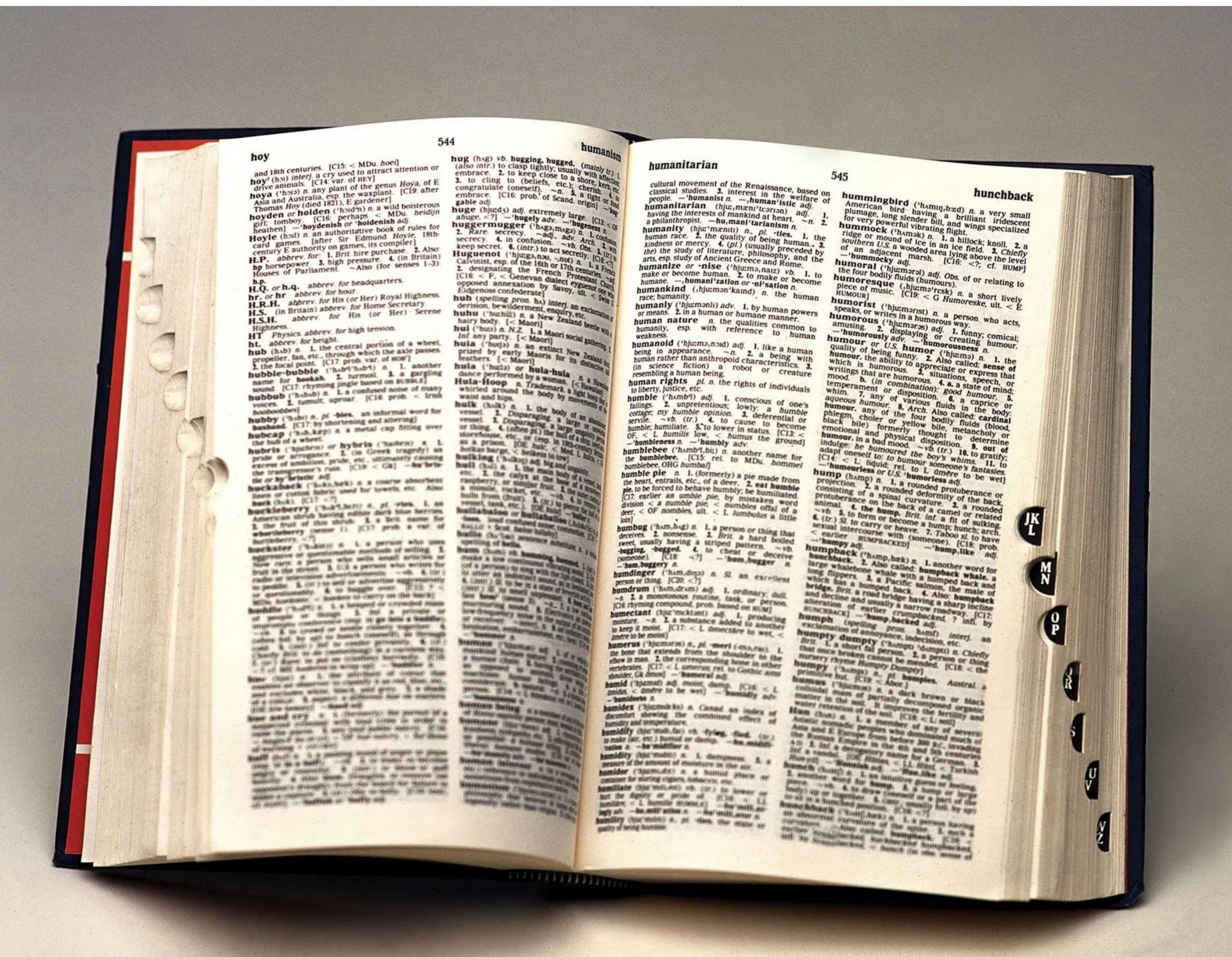
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# A better search strategy

- If you think, this is exactly the way we search for a word in a dictionary!
  - If we are searching for the word "*freshness*" we do not start from the beginning of the dictionary...but probably look for words that start with *f*.
  - In fact, words in a vocabulary are **sorted** lexicographically...

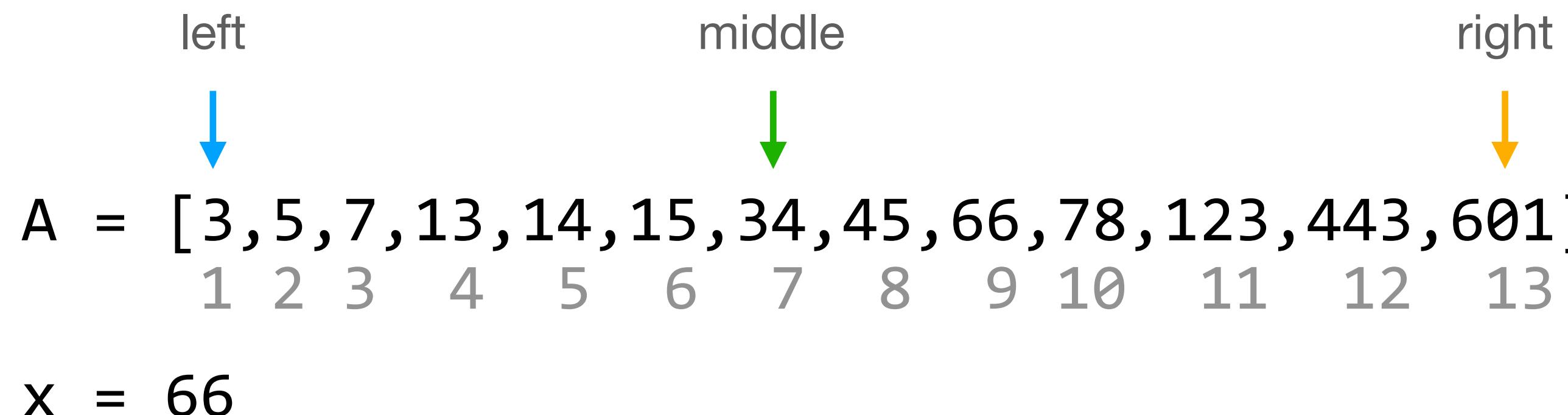


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- **Our refined strategy.** Look at the element in **middle** position,  $y=A[n/2]$ :  
if  $x = y$ , then we are done;  
if  $x < y$ , then continue searching in the **left half** (i.e.,  $A[1..n/2-1]$ );  
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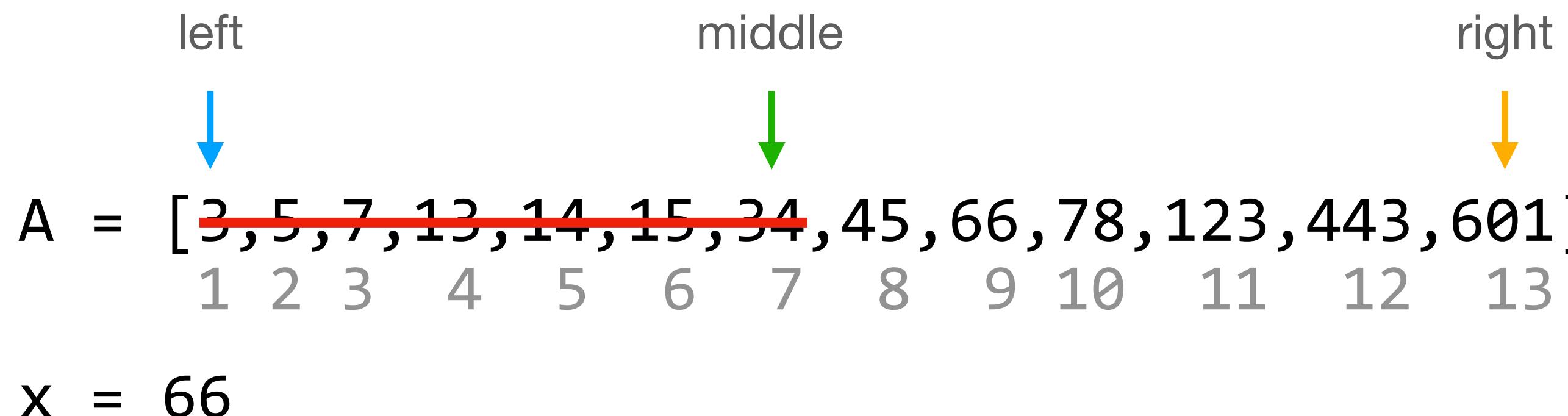
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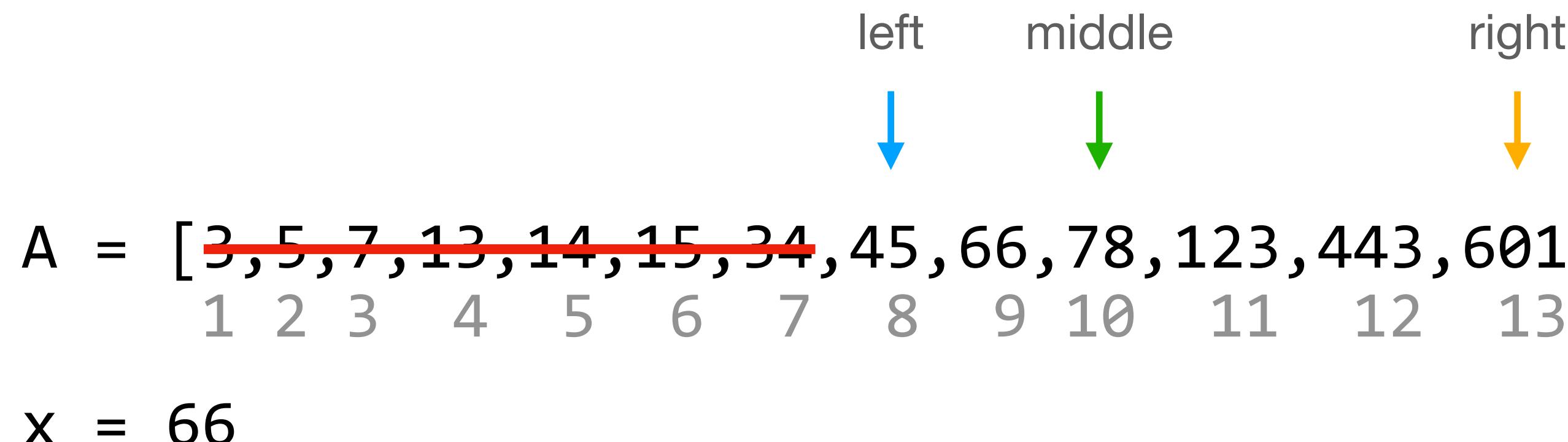
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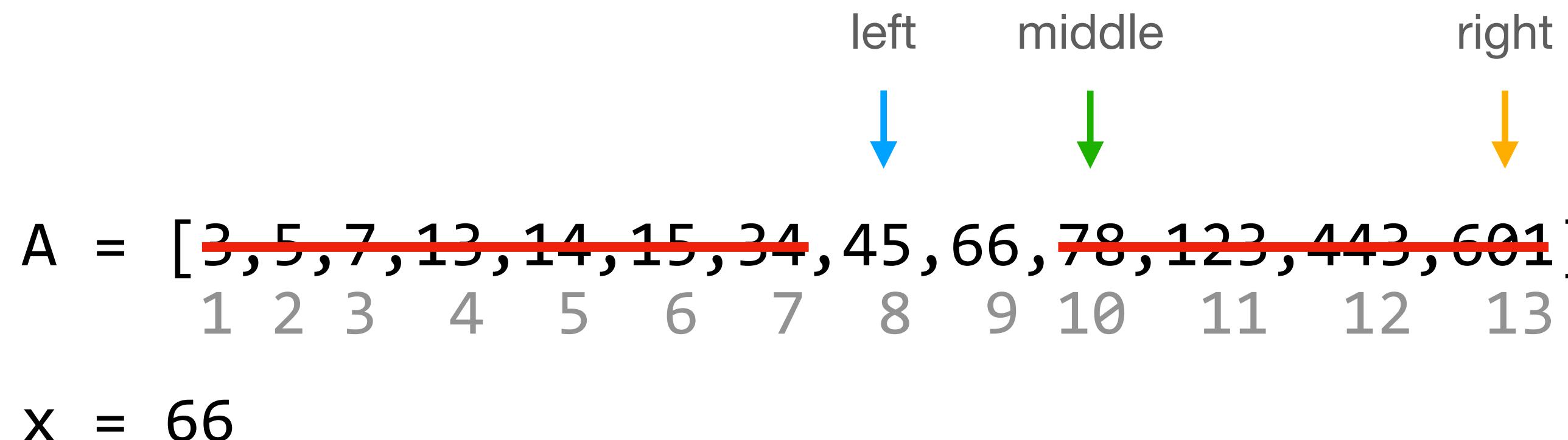
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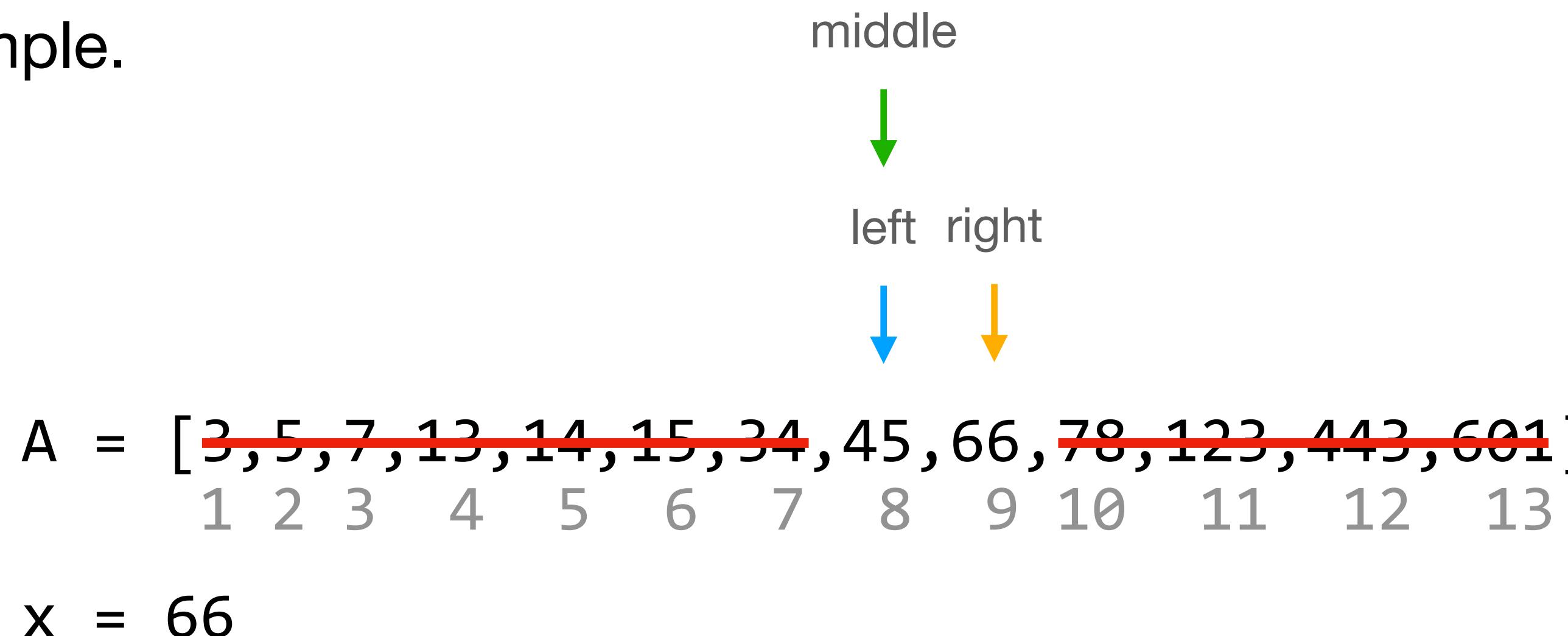
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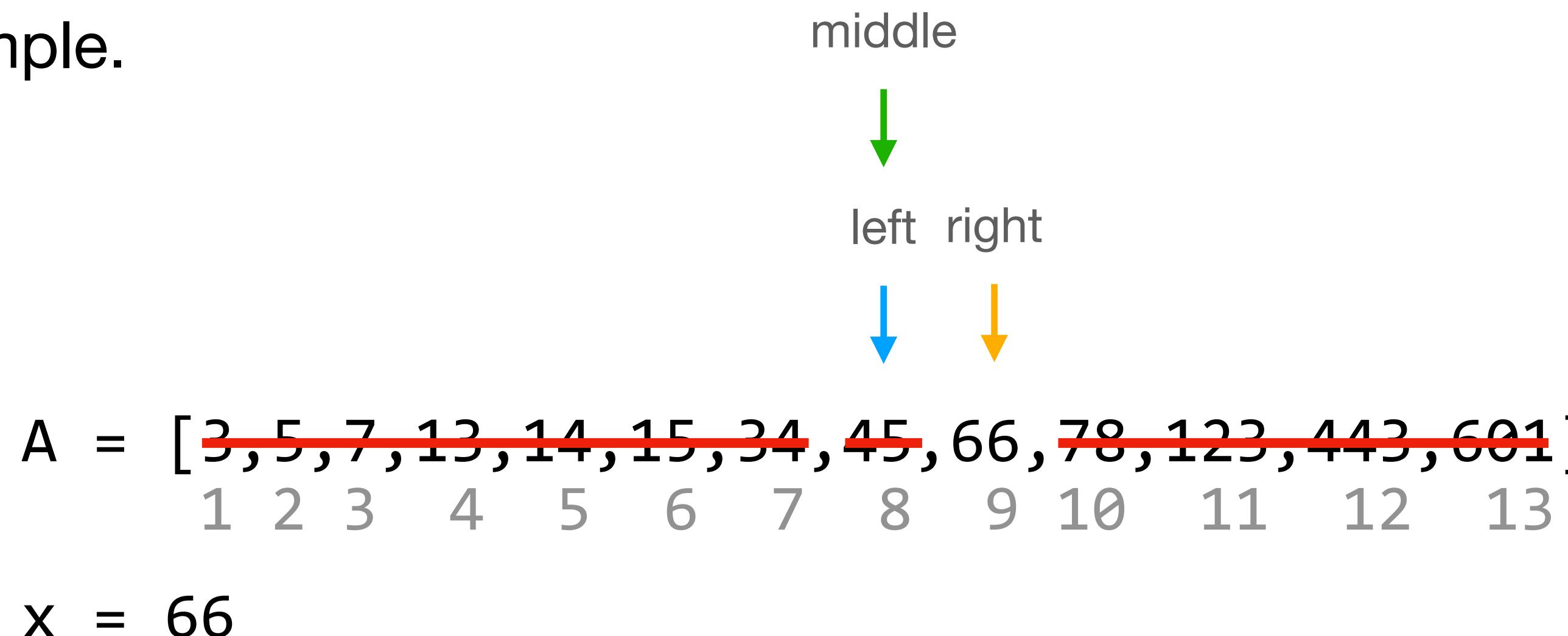
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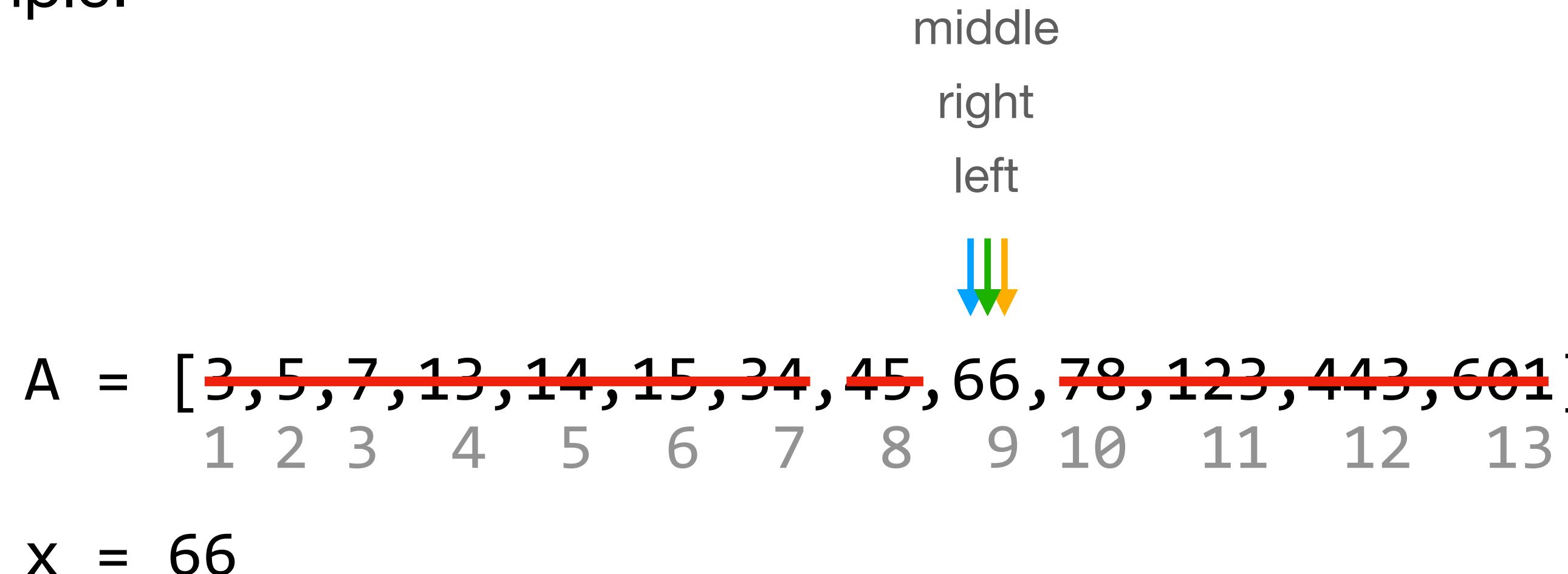
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# Binary search – Analysis

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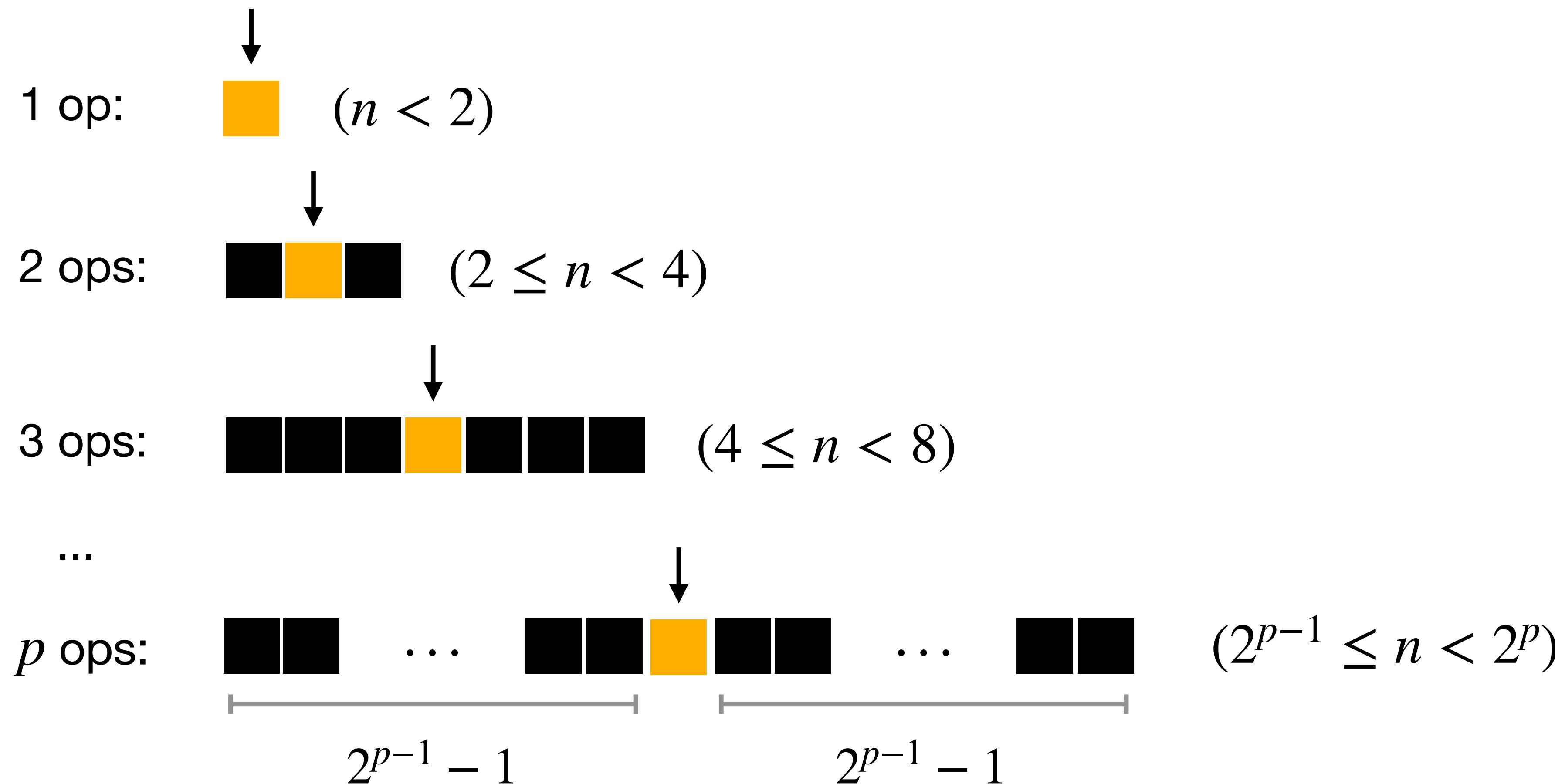
1 op:   $(n < 2)$

2 ops:   $(2 \leq n < 4)$

3 ops:   $(4 \leq n < 8)$

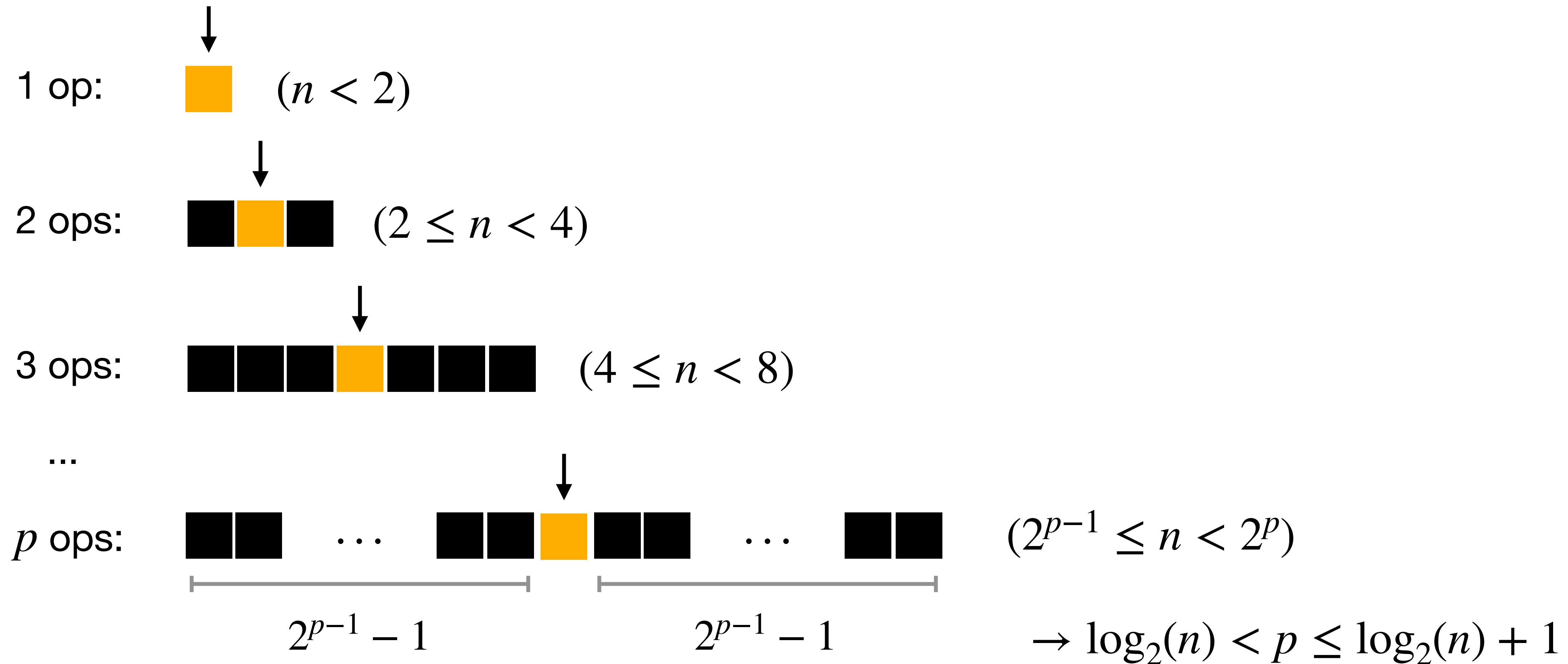
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# Linear search vs. binary search

	num. operations	$n = 100,000$	$n = 1,000,000$	$n = 10,000,000$
<b>Linear search</b>	$\sim n$	305 ms	3,400 ms	36,000 ms
<b>Binary search</b>	$\sim \log_2(n)$	0 ms	1 ms	3 ms

Running time to search for 10,000 integers.

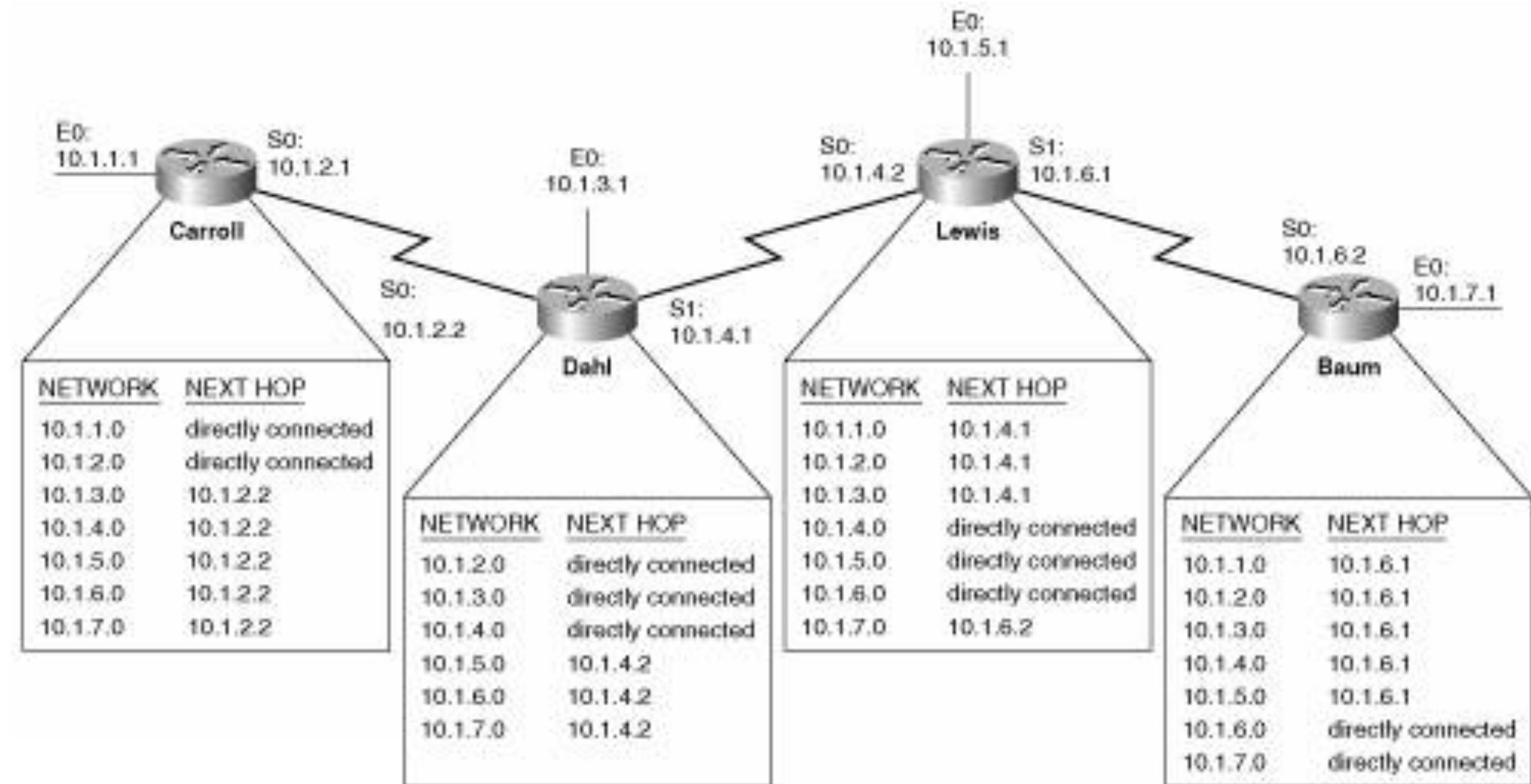
# IP address lookup

- Each packet has an IP destination address which is a big integer number.

- This number is searched, at each hop, in a sorted table of destinations IP addresses.

- Search is done via binary search.**

Hence binary search is probably **the most run algorithm in the world!**



# Sub-string search

- **Problem.** We are given two strings,  $T$  and  $P$ , respectively of length  $n$  and  $m$ , with usually  $n \gg m$ , and we are asked to find all the occurrences of  $P$  in  $T$ .
- $T$  is also called the *text* and  $P$  is called the *pattern*.
- Example.

$P = S I P$

$T = M I S S I S S I P P I L I P P I S I P$

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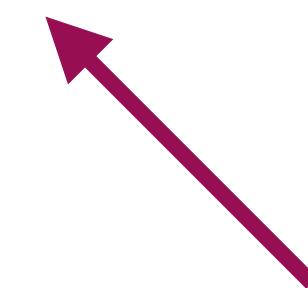
$P = S I P$

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# The Linux utility grep

```
[giulio@xor:~$ grep --help
Usage: grep [OPTION]... PATTERNS [FILE]...
Search for PATTERNS in each FILE.
Example: grep -i 'hello world' menu.h main.c
PATTERNS can contain multiple patterns separated by newlines.
```

```
giulio@xor:~$ grep flower GoogleBooks.2-grams
```



search for all occurrences of "**flower**"  
in the file "**GoogleBooks.2-grams**"

# Brute-force algorithm

- **Idea 1.** Compare every sub-string of  $T$  of length  $m$ ,  $T[i \dots i + m - 1]$ , for  $1 \leq i \leq n - m + 1$ , with  $P$  and check if they are equal.

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$S \ I \ P$

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$S \color{green} I \color{red} P$   
 $\color{red} S I P$

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...

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- **Q.** How many operations?

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 $S I P$

⋮

- **Q.** How many operations?

- We compare two sub-strings of length  $m$  spending  $\sim m$  operations.
- We have a total of  $n - m + 1$  total sub-string comparisons, which is  $\approx n$  when  $n \gg m$ .
- Hence, a total of  $\sim mn$  operations.

# Brute-force algorithm

- **Summary.** Compare  $P$  to  $T[i .. i + m - 1]$ , from **left to right**, for every  $1 \leq i \leq n - m + 1$ .
- Very easy to implement; analysis is straightforward.
- Usually sufficiently fast if  $m$  is small.

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- **Summary.** Compare  $P$  to  $T[i \dots i + m - 1]$ , from **left to right**, for every  $1 \leq i \leq n - m + 1$ .
- Very easy to implement; analysis is straightforward.
- Usually sufficiently fast if  $m$  is small.
- Could be **slow** if  $m$  is sufficiently long.
- **Q.** How to make it faster?

# Boyer-Moore algorithm

- **Intuition.** Compare **from right to left**. If the last character does **not** match, then stop comparing and jump ahead.

P = S I P

T = M I S S I S S I P P I L I P P I S I P

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$S \ I \ P$

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S I P

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## P = S I P

T = M I S S I S S I P P I L I P P I S I P

# SIR

S I A

S I

S I P

S I F

S I P

A long horizontal red arrow pointing to the left, indicating a previous page or section.

'L' does not belong to the pattern:  
jump  $m$  characters ahead!

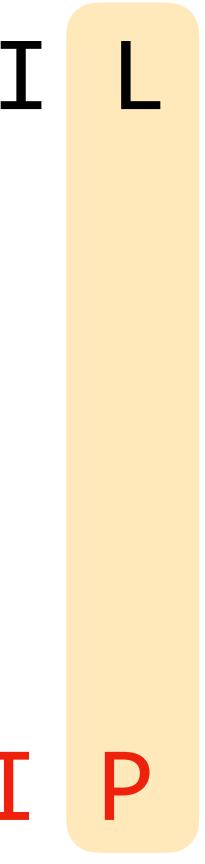
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$S I P$



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$$P = S \cdot I \cdot P$$

T = M T S S T S S T P P T I T P P T S T P

# SIA

# SIR

S I

SIR

S I P

S T P

S T P

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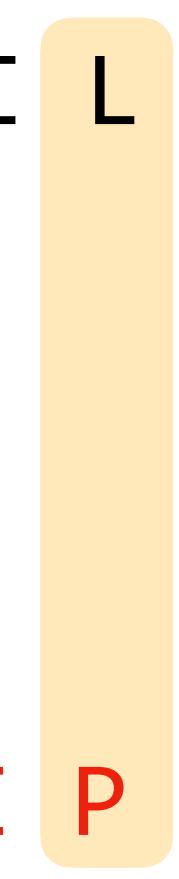
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$S I P$



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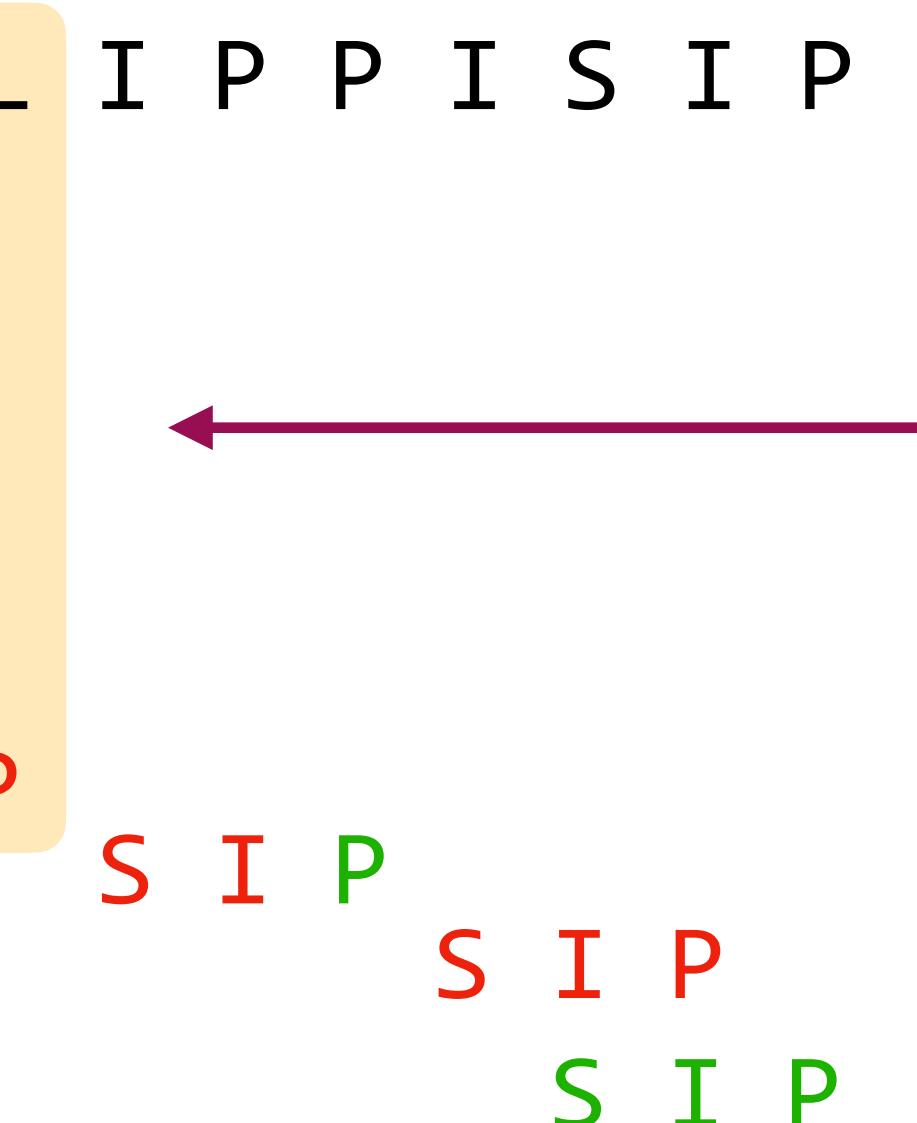
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$S I P$



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jump  $m$  characters ahead!

- If the above case is frequent (as it usually **is** in practice), then we perform  $\sim n/m$  operations!

# Karp-Rabin algorithm

- **Idea.** Calculate a function  $h(P)$  that returns an **integer number** and compare this number to  $h(T[i \dots i + m - 1])$ . If the two numbers are equal, then we have found a match.
- Two integers can be compared with 1 operation, which is much faster than doing a string comparison ( $\sim m$  operations).

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- Two integers can be compared with 1 operation, which is much faster than doing a string comparison ( $\sim m$  operations).
- **Key.** Calculate the function  $h$  efficiently for every sub-string  $T[i \dots i + m - 1]$ , using a constant number of operations, and not  $m$  operations.
- **Note.** Function  $h$  is called a *hash* function.

# Karp-Rabin algorithm – Rolling hash function

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- How to compute  $h$ , i.e., obtain an integer number from a string?
- Remember the **ASCII** table (e.g., of size 127), mapping characters to integers.
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P = S I P

ASCII 83 73 80

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- Remember the **ASCII** table (e.g., of size 127), mapping characters to integers.
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$$\begin{array}{ccccccc} P & = & S & I & P \\ \text{ASCII} & & 83 & 73 & 80 \end{array} \rightarrow h(P) = 83 \times b^2 + 73 \times b + 80 = 1,348,058 \text{ for } b = 127.$$

# Karp-Rabin algorithm – Rolling hash function

- **Key.** Calculate the function  $h$  efficiently for every sub-string  $T[i \dots i + m - 1]$ , using a constant number of operations, and not  $m$  operations.
- **Problem.** How to calculate

$$h(T[i + 1 \dots i + m]) = T[i + 1] \cdot b^{m-1} + T[i + 2] \cdot b^{m-2} + T[i + 3] \cdot b^{m-3} + \dots + T[i + m]$$

from

$$h(T[i \dots i + m - 1]) = T[i] \cdot b^{m-1} + T[i + 1] \cdot b^{m-2} + T[i + 2] \cdot b^{m-3} + \dots + T[i + m - 1]$$

using a **constant number of operations** ?

# Karp-Rabin algorithm – Rolling hash function

- Let's consider an example.

$T = M I S S I S S I P P I L I P P I S I P$

$$T[1..3] = M I S \rightarrow h(T[1..3]) = T[1] \cdot b^2 + T[2] \cdot b + T[3]$$

$$T[2..4] = I S S \rightarrow h(T[2..4]) = T[2] \cdot b^2 + T[3] \cdot b + T[4]$$

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subtract

add

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subtract  
add

- Hence, it is easy to derive that

$$h(T[i+1..i+m]) = (h(T[i..i+m-1]) - T[i] \cdot b^{m-1}) \cdot b + T[i+m].$$

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- Hence, it is easy to derive that

$$h(T[i+1..i+m]) = (h(T[i..i+m-1]) - T[i] \cdot b^{m-1}) \cdot b + T[i+m].$$

- Just 4 operations (not  $m$ ) !

$b^{m-1}$  can be pre-computed

# Karp-Rabin algorithm

- The function  $h$  is computed using a constant number of operations for each sub-string: this leads to a **simple linear-time algorithm**  $\rightarrow \sim n$  operations.

$$P = S I P \rightarrow h(P) = 1348058$$

T = M I S S I S S I P P I L I P P I S I P

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$$\text{M I S } h(\text{MIS}) = 1251287$$

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T = M I S S I S S I P P I L I P P I S I P

M I S  $h(MIS) = 1251287$

I S S  $h(ISS) = 1188041$

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T = M I S S I S S I P P I L I P P I S I P

M I S  $h(MIS) = 1251287$

I S S  $h(ISS) = 1188041$

S S I  $h(SSI) = 1349321$

S I S  $h(SIS) = 1348061$

I S S  $h(ISS) = 1188041$

S S I  $h(SSI) = 1349321$

S I P  $h(SIP) = 1348058$

...

# Karp-Rabin algorithm

- The function  $h$  is computed using a constant number of operations for each sub-string: this leads to a **simple linear-time algorithm**  $\rightarrow \sim n$  operations.

$$P = S I P \rightarrow h(P) = 1348058$$

T = M I S S I S S I P P I L I P P I S I P

M I S  $h(MIS) = 1251287$

I S S  $h(ISS) = 1188041$

S S I  $h(SSI) = 1349321$

S I S  $h(SIS) = 1348061$

I S S  $h(ISS) = 1188041$

S S I  $h(SSI) = 1349321$

S I P  $h(SIP) = 1348058$

...

- Caveat.** When  $m$  increases, the integers output by  $h$  increase as well. Thus we take the  $h \bmod p$ , where  $p$  is a *big prime* number.

# Summary of sub-string search

	num. operations	space	Moby Dick (1.3 MB)	Sherlock Holmes (6.5 MB)
<b>Brute force</b>	$\sim mn$	constant	3.5 ms	15.1 ms
<b>Boyer-Moore</b>	$\sim n/m$	$\sim k$	0.9 ms	4.5 ms
<b>Karp-Rabin</b>	$\sim 4n$	constant	1.3 ms	6.3 ms

$k$  is the  
alphabet size

time to search all occurrences of the  
pattern  $P = "not\ only\ all\ that"$

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- There are **many more** string search algorithms!
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- There are **many more** string search algorithms!
- So far, we have considered solutions to the sub-string search problem that do **not** use a data structure built from the text.
- **Intuition:** if we pre-process the text  $T$  into a data structure, we can find the occurrences of the pattern  $P$  faster.
- Clear **trade-off between space and time** of the solution.
- These trade-offs are at the heart of all problems in Computer Science.

# The Suffix Array data structure

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1	2	3	4	5	6	7	8	9	10	11	12	
$T =$	m	i	s	s	i	s	s	i	p	p	i	\$

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$T = m$	i	s	s	i	s	s	i	p	p	i	\$ 1
	i	s	s	i	s	s	i	p	p	i	\$ 2
	s	s	i	s	s	i	p	p	i	\$ 3	
	s	i	s	s	i	p	p	i	\$ 4		
	i	s	s	i	p	p	i	\$ 5			
	s	s	i	p	p	i	\$ 6				
	s	i	p	p	i	\$ 7					
	i	p	p	i	\$ 8						
	p	p	i	\$ 9							
	p	i	\$ 10								
	i	\$ 11									
	\$ 12										

Step 1: we take all the suffixes of  $T$ .  
('\$' is the smallest character.)

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	i	s	s	i	s	s	i	p	p	i	\$	2	
	s	s	i	s	s	i	p	p	i	\$	3		
	s	i	s	s	i	p	p	i	p	i	\$	4	
	i	s	s	s	i	p	p	i	p	i	\$	5	
	s	s	s	i	p	p	i	p	p	i	\$	6	
	s	s	s	i	p	p	i	i	i	i	\$	7	
	s	s	s	i	p	p	i	i	i	i	\$	8	
	i						i	i	i	i	\$	9	
							i	i	i	i	\$	10	
							i	i	i	i	\$	11	
							i	i	i	i	\$	12	

→

12	\$											
11	i	\$										
8	i	p	p	i	s	i	\$					
5	i	s	s	i	p	i	s					
2	i	s	s	i	p	i	s					
1	m	i	t	s	i	t	s					
10	p	p	s	s	i	p	s					
9	p	p	s	s	i	p	s					
7	p	p	s	s	i	p	s					
4	p	p	s	s	i	p	s					
6	p	p	s	s	i	p	s					
3	p	p	s	s	i	p	s					
2	p	p	s	s	i	p	s					
1	p	p	s	s	i	p	s					

We take all the suffixes of  $T$ .  
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	s	s	i	s	s	i	p	p	i	\$	3	
	s	i	s	s	i	p	p	i	\$	4		
	i	s	s	i	p	p	i	\$	5			
	s	s	i	p	p	i	\$	6				
	s	i	p	p	i	\$	7					
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	i	\$	11									
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Step 1: we take all the suffixes of  $T$ . ('\$' is the smallest character.)

Step 2: we sort them lexicographically.



# The Suffix Array data structure

- The SA of  $T$  looks like this.
- Examples.

$$SA[3] = 8$$

means that the 3-rd smallest suffix of  $T$  begins at position 8;

$$SA[6] = 1$$

means that the 6-th smallest suffix of  $T$  begins at position 1.

- Let's now see how, with SA and  $T$ , we can **search** for a pattern  $P$ .

	1	2	3	4	5	6	7	8	9	10	11	12
$T =$	m	i	s	s	i	s	s	i	p	p	i	\$
$SA =$	[12,	11,	8,	5,	2,	1,	10,	9,	7,	4,	6,	3]
	1	2	3	4	5	6	7	8	9	10	11	12
	\$	i	i	i	i	m	p	p	s	s	s	s
		\$	p	s	s	i	i	i	i	i	i	i
			i	p	i	s	s	s	s	s	s	s
				i	p	i	p	i	p	i	p	i
					i	s	s	i	s	i	s	s
						p	i	p	i	p	i	\$
							i	p	i	p	i	\$

# Searching with the Suffix Array

- With  $T$  and  $SA$  we can search for  $P$  by **binary search**:
  1. compare  $P$  with the string starting at  $T[SA[\lfloor n/2 \rfloor]]$
  2. if **equal**, then a match if found in  $T$  at  $SA[\lfloor n/2 \rfloor]$
  3. if **smaller**, recurse on  $SA[1.. \lfloor n/2 \rfloor - 1]$
  4. **otherwise**, recurse on  $SA[\lfloor n/2 \rfloor + 1..n]$

- Example.

$P = \text{ssi}$

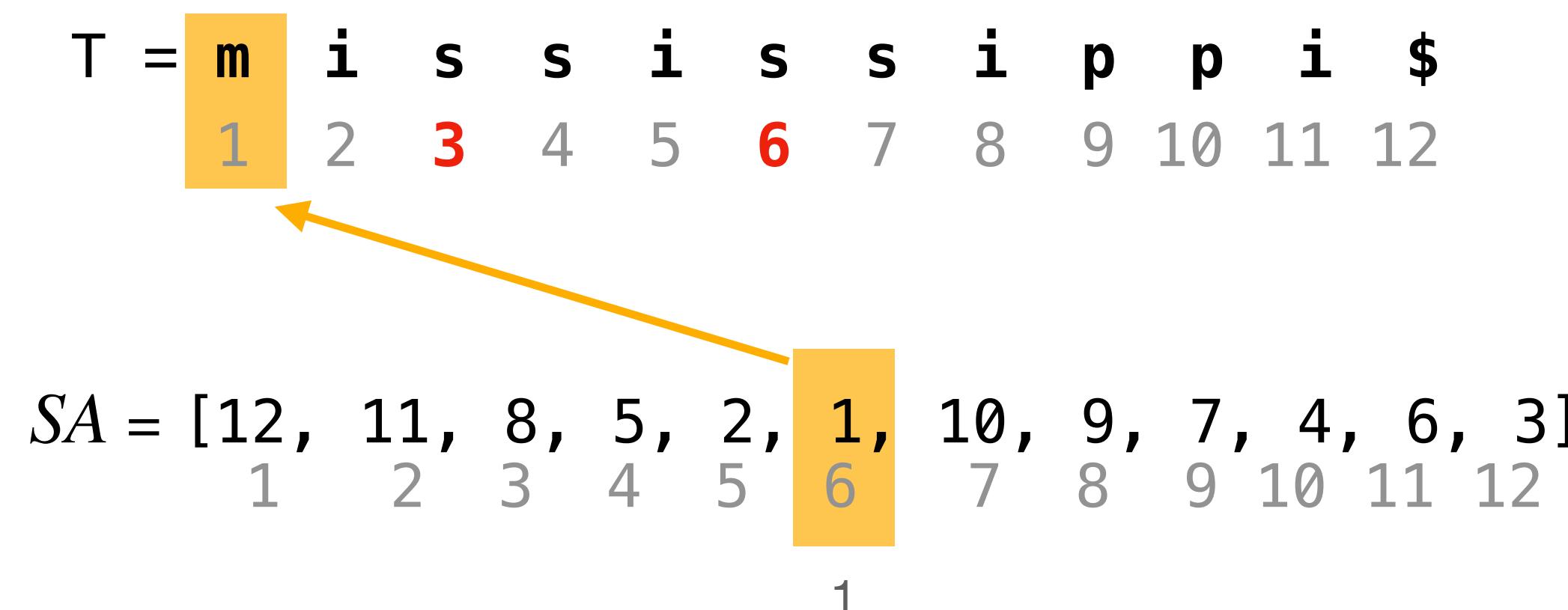
$T = m \ i \ s \ s \ i \ s \ s \ i \ p \ p \ i \ \$$   
1 2 3 4 5 6 7 8 9 10 11 12

$SA = [12, 11, 8, 5, 2, 1, 10, 9, 7, 4, 6, 3]$   
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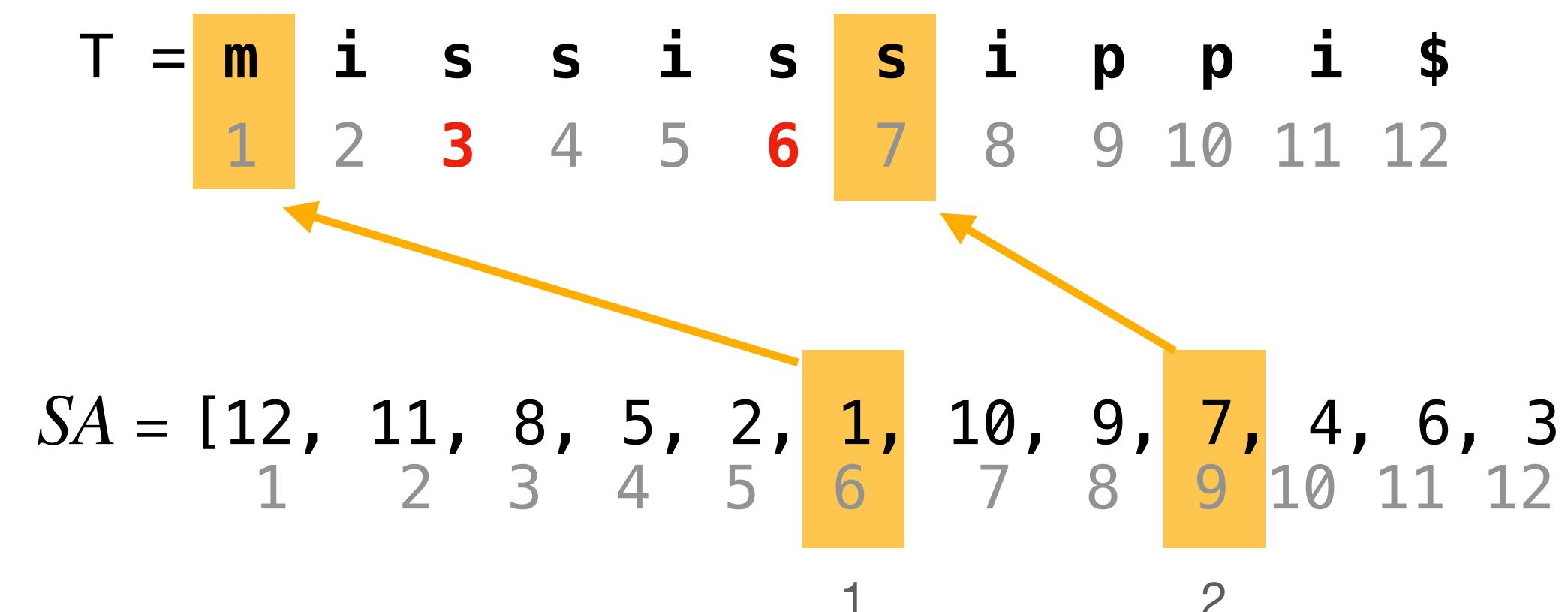
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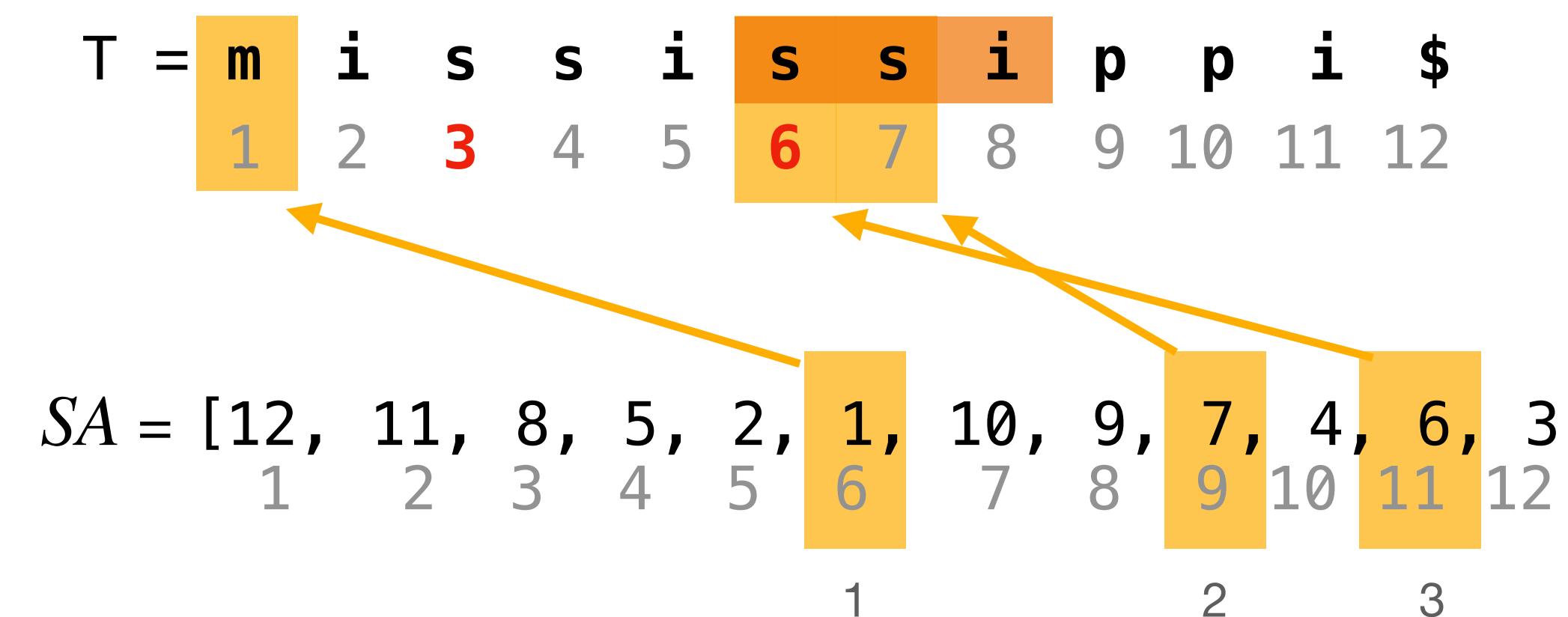
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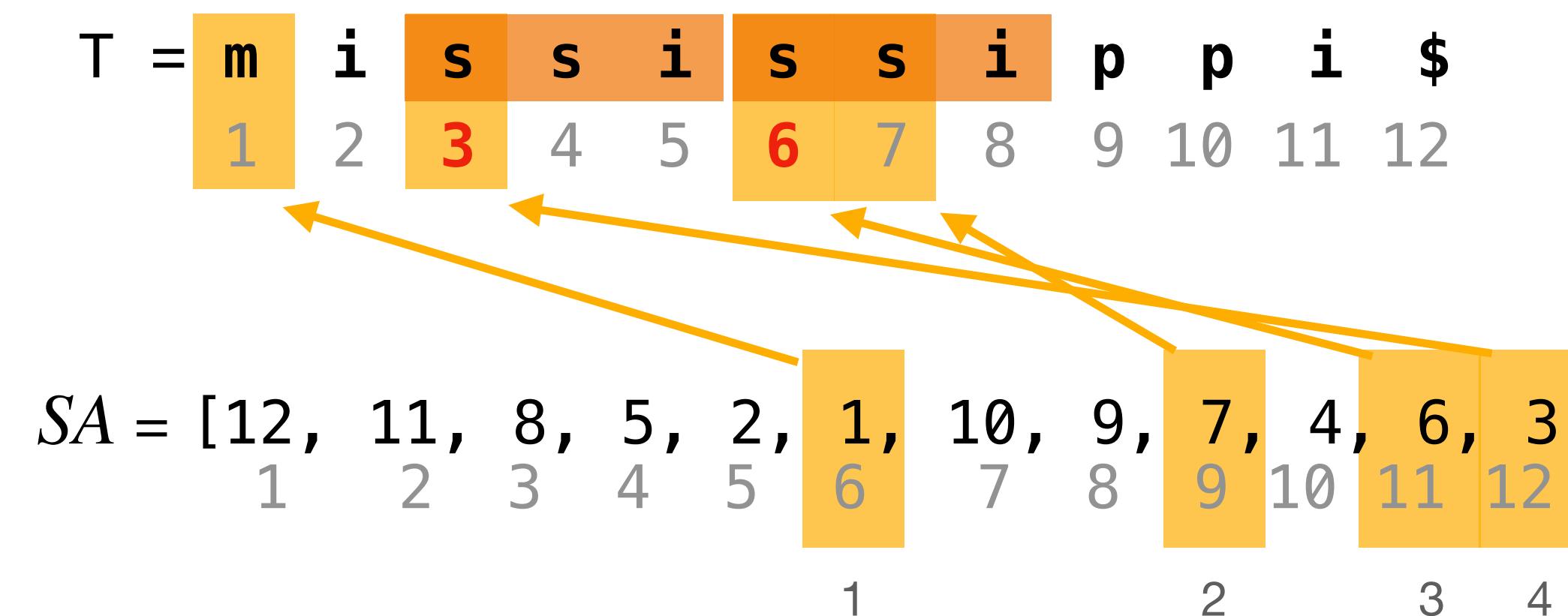
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  - Hence  $P$  can be searched in  $\sim m \log_2(n)$  operations.
- Space?
  - The SA is an integer array; each integer takes a value in the range  $[1..n]$  and therefore requires  $\lceil \log_2(n) \rceil$  bits to be represented.
  - Hence, the SA takes a total of  $n \lceil \log_2(n) \rceil$  bits. (More than the text itself!)

# Summary of sub-string search – Update

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<b>Suffix Array</b>	$\sim m \log_2(n)$	$n \log_2(n)$	<b>0.001 ms</b>	<b>0.001 ms</b>

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time to search all occurrences of the  
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**Thank you!**

**Questions?**