Question 1 (10 marks)

Once you know how to run simulations, the obvious thing to do is see what would happen in a zombie apocalypse.

Conceptually, the zombie apocalypse works as follows. We begin with a 2-dimensional grid of squares (the "world"). A number of healthy people are randomly distributed throughout the world, as well as one zombie. Each day, each person either stays still or moves one square up, down, right, or left, or one square diagonally. If a zombie spends a day on the same square as a healthy person, the healthy person becomes infected (i.e., becomes a zombie).

Your implementation should have four parts. First, specify the parameters of the simulation. Second, initialise the people who will live (*and die*) in the simulation. Third, run the simulation (i.e., at each timestep, update the locations of each person and whether or not they are a zombie). Finally, plot and output the results.

Parameters

The simulation should be initialized with four parameters: the size of the 2-dimensional world people inhabit (`sidelength`, default 40 squares per side), the maximum time to run the simulation for (`maxtime`, default 1000 days), the number of people (`npeople`, default 100), and the rate at which zombies spontaneously enter remission and return to being healthy humans (`remission`, default 0).

People

Each person will be associated with a number of pieces of information: (i) their x coordinate (an integer between 1 and the size of the world), (ii) their y coordinate (same), and (iii) whether or not they're a zombie.

Simulation

The "simulation" consists of the calculations that occur inside the loop over time.

First, we need to figure out where people move to. For this, we'll need an inner loop over people. At each timestep there are nine options, corresponding to the eight immediate neighbours of a person, and their current position (meaning they don't move). If people move outside the world ($x, y < 0$ or $x, y >$ `sidelength-1`), they come through the other side of the world (i.e. we shall assume we have periodic boundary conditions). So, a person who moves to $x = -1$ will instead move to $x =$`sidelength-1`.

Second, we need to figure out which people are zombies and which are not zombies.

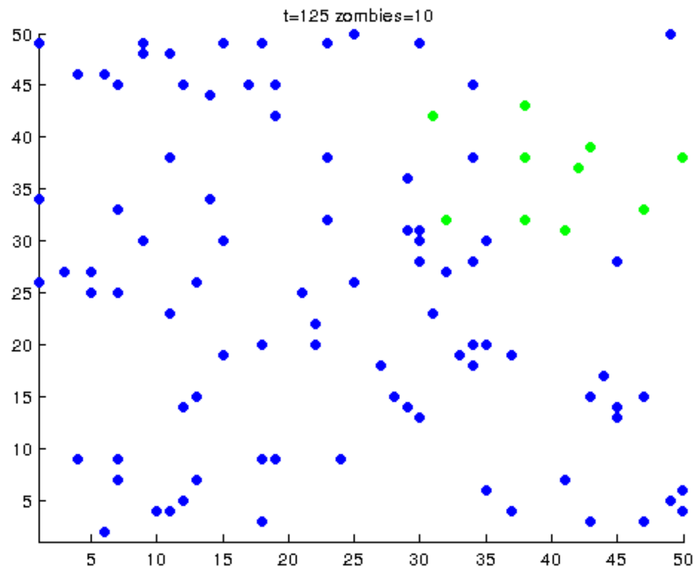
Third comes the tricky part: we need to handle infections. To do this, we can loop over just the zombies, then loop over all the non-zombies. For each zombie/non-zombie pair, we need to check whether the non-zombie is occupying the same square as the zombie (i..e., their x and y coordinates are identical), and if so, infect the non-zombie (i.e. turn the non-zombie into a zombie).

Finally, for each zombie, we need to check if they spontaneously return to being a human: i.e., with probability `remission` at each time step the zombie may turn back into a non-zombie.

Plotting

Construct a scatter plot, where a dot shows the position of a zombie or a non-zombie on the xy plane. Plot healthy people in blue and zombies in green.

If you've done everything correctly, the simulation should take about 1 second of actual time for every 10 days of simulated time, and you should get an output like this:



Questions

(1.1) What is the average length of time until the last human gets infected? Provide also an estimate of the uncertainty in your value.

(1.2) Averaging over many runs: Plot the number of zombies as a function of time.

(1.3) Averaging over many runs: Plot the number of infections per day (the infection rate) as a function of the fraction of zombies in the population.

(1.4) Given your results in (1.3), postulate a discrete map for the zombie fraction (Z), i.e. $Z_{n+1} = f(Z_n, t_n)$.

(1.5) If you halve the size of the world (i.e., set `sidelength=20`), how does this change the time it takes until the last human is infected? Briefly discuss the real-world implications of this result, e.g. in terms of how diseases such as tuberculosis spread in crowded hospitals and prisons.

(1.6) Using the original size of the world (`sidelength=40`), change the spontaneous zombie remission rate from 0 to 0.01 (i.e., every 100 days on average a given zombie will spontaneously return to being human). Discuss the new dynamics of the model, with reference to steady states and/or equilibrium points (if you wish, you can also include a graph of the number of zombies over time to illustrate your point). Be careful to identify all possible types of behaviour (a "type" being a qualitatively distinct behaviour).