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Estimating Pearson's Correlation Coefficient With Bootstrap Confidence Interval From Serially Dependent Time Series[1]

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Pearson's correlation coefficient, r_{xy}*, is often used when measuring the influence of one time-dependent variable on another in bivariate climate time series. Thereby, positive serial dependence (persistence) and unknown data distributions impose a challenge for obtaining accurate confidence intervals for* r_{xy}*. This is met by the presented approach, employing the nonparametric stationary bootstrap with an average block length proportional to the maximum estimated persistence time of the data. A Monte Carlo experiment reveals that this method can produce accurate (in terms of coverage) confidence intervals (of type bias-corrected and accelerated). However, since persistence reduces the number of independent observations, substantially more data points are required for achieving an accuracy comparable to a situation without persistence. The experiment further shows that neglecting serial dependence may lead to serious coverage errors. The presented method proves robust with respect to distributional shape (lognormal/normal) and time spacing (uneven/even). The method is used to confirm that a previous finding of a correlation between solar activity and Indian Ocean monsoon strength in early Holocene is valid. A further result is that the correlation between sunspot number and cosmogenic* ^{10}*Be concentration vanishes after approximately 1870.*

KEY WORDS: BCa method, sun–climate relationship, irregular sampling interval, Monte Carlo simulation, persistence.

INTRODUCTION

One of the most often used statistical quantities, not only in geosciences, is Pearson's correlation coefficient, r_{xy}, which measures the degree of (linear) interrelation between two sampled (data size n) variables, x and y. Quite frequently x and y are measured over time, and a typical aim in correlation analysis of such bivariate time series is to value the evidence for an influence of one time-dependent variable on the other. For example, in one application shown here the influence of solar activity on monsoon rainfall during early Holocene is analyzed. Since

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geological interpretation of a detected correlation requires knowledge about the statistical accuracy, a confidence interval for r_{xy} or, at least, a test of the hypothesis "population correlation coefficient $\rho_{xy} = 0$" is required. Note that a confidence interval includes (but is not restricted to) a hypothesis test by looking whether or not the interval contains zero.

For typical geological or climatological time series, estimation of a confidence interval for r_{xy} is hindered by positive serial dependence, also termed *persistence* in the atmospheric sciences (Wilks, 1995). Persistence reduces the effective data size n_{eff} (von Storch and Zwiers, 1999) by an amount not known a priori. That means, the variance is approximately

$$\text{var}(r_{xy}) = \frac{\left(1 - \rho_{xy}^2\right)^2}{n_{\text{eff}} - 1} \left(1 + \frac{11\,\rho_{xy}^2}{2\,n_{\text{eff}}}\right) + \mathcal{O}\left(n_{\text{eff}}^{-3}\right),$$

with $n_{\text{eff}} = n$ for binormal data without serial dependence (Rodriguez, 1982) and $n_{\text{eff}} < n$ for data with positive serial dependence. Note that the problem of positive serial dependence reducing the effective data size applies in principle also to spatial data.

For binormally distributed data Fisher's (1921) transformation $z = \tanh^{-1}$ (r_{xy}) yields z being asymptotically distributed as N $(\tanh^{-1}$ (ρ_{xy}), $(n_{\text{eff}}$ $- 3)^{-1})$, providing an interval estimate for ρ_{xy} (see Rodriguez, 1982). The common problem, however, is nonnormal data distributions, frequently encountered not only in geosciences, which can make normal-theory-based intervals questionable.

The stationary bootstrap (Politis and Romano, 1994) is, besides the moving block bootstrap (Künsch, 1989), one obvious candidate to solve these problems. It is free of distributionary assumptions because it uses the data to mimic the distribution. Since it resamples data blocks (of variable length), persistence in the data is preserved.

The present paper advocates to use an average block length proportional to the estimated persistence time (Mudelsee, 2002) of the data. Monte Carlo simulations demonstrate that this approach yields acceptably accurate results over considerably large ranges of ρ_{xy} and persistence times, for various distributional shapes (normal, lognormal), for uneven as for even time spacing. The focus here is to evaluate necessary data sizes for obtaining accurate confidence intervals. This augments previous work on data size requirements for data without serial dependence (Bonett and Wright, 2000) and Spearman's correlation coefficient for data with serial dependence (Park and Lee, 2001). It turns out that the data size required may be substantially larger than for data without serial dependence. The accompanying Fortran 90 program PearsonT is described shortly.

BOOTSTRAP CONFIDENCE INTERVAL

The nonparametric stationary bootstrap (Politis and Romano, 1994) is used to construct a BCa (bias-corrected and accelerated) confidence interval (Efron and Tibshirani, 1993); see those references for a detailed statistical background.

Let $\{x(i), y(i)\}_{i=1}^{n}$ be a bivariate time series sampled at times $\{t(i)\}_{i=1}^{n}$ (which may be unevenly spaced) from an assumed weakly stationary process $\{X(i), Y(i)\}$. Calculate r_{xy}. The stationary bootstrap constructs a resampled time series $\{x^{*}(i), y^{*}(i)\}_{i=1}^{n}$ as follows. The first pair, $\{x^{*}(1), y^{*}(1)\}$, is randomly selected from the n pairs $\{x(i), y(i)\}_{i=1}^{n}$. The consecutive pairs are generated as follows: Let $\{x^{*}(i), y^{*}(i)\} = \{x(j), y(j)\}$, say, then with Prob $(1 - P)$, with P small, the successor of $\{x^{*}(i), y^{*}(i)\}$ is taken to be the successor of $\{x(j), y(j)\}$ in the original time series, that is, $\{x^{*}(i+1), y^{*}(i+1)\} = \{x(j+1), y(j+1)\}$, and with Prob P, $\{x^{*}(i+1), y^{*}(i+1)\}$ is a randomly selected pair of the original bivariate time series. Periodic boundary conditions are imposed to ensure that $\{x(n), y(n)\}$ has a successor, $\{x(1), y(1)\}$. Note that this resampling scheme is equivalent to a bootstrap in which blocks from the original time series are concatenated, with geometrically distributed block lengths (Politis and Romano, 1994). Intuitively, the bootstrap preserves serial dependence in the time series up to lags of order of the average block length, $\bar{d}/P$ (with the average time spacing, $\bar{d} = [t(n) - t(1)]/(n-1)$).

Assuming that the observed process, $\{X(i), Y(i)\}$, is (after standardization) a bivariate Gaussian autoregressive process (Appendix A) with estimated persistence times $\hat{\tau}_x$ and $\hat{\tau}_y$, it is reasonable to let the average block length be proportional to $\max(\hat{\tau}_x, \hat{\tau}_y)$. The Monte Carlo simulations (next Section) show that using

$$P = \bar{d}/[4 \cdot \max(\hat{\tau}_x, \hat{\tau}_y)]. \tag{1}$$

leads to confidence intervals with an acceptable accuracy. This choice should also allow for a certain degree of "model uncertainty" when the observed process has a different serial dependence structure than assumed. Next, from the resampled data, $\{x^{*}(i), y^{*}(i)\}_{i=1}^{n}$, the bootstrap replicate, r_{xy}^{*}, is calculated. The procedure resampling–calculation is repeated until B replicates exist. Efron and Tibshirani (1993) show that using $B \gtrsim 2000$ leads to sufficiently low "bootstrap noise," that is, the coefficient of variation of a confidence interval point in dependence of B approaches saturation. The bootstrap replications are taken to construct an equi-tailed $(1 - 2\alpha)$ confidence interval (i.e., with nominally Prob(r_{xy} < lower interval bound) = Prob(r_{xy} > upper interval bound) $= \alpha$) of type BCa as follows. The interval is given by

$$\left[r_{xy}^{*(\alpha 1)},\ r_{xy}^{*(\alpha 2)}\right],$$

the interval between the $(100 \cdot \alpha 1)$-th percentile point (calculated with interpolation) and the $(100 \cdot \alpha 2)$-th percentile point of the bootstrap distribution of r_{xy}^{*}. The percentile points are

$$\alpha 1 = \Phi\left[\hat{z}_0 + \frac{\hat{z}_0 + z^{(\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(\alpha)}\right)}\right],$$

$$\alpha 2 = \Phi\left[\hat{z}_0 + \frac{\hat{z}_0 + z^{(1-\alpha)}}{1 - \hat{a}\left(\hat{z}_0 + z^{(1-\alpha)}\right)}\right],$$

where Φ is the cumulative standard normal density, $z^{(\alpha)}$ is the $(100 \cdot \alpha)$-th percentile point of Φ, $\hat{z}_0$ (bias correction), is computed as

$$\hat{z}_0 = \Phi^{-1}\left(\frac{\text{number of replications where}\, r^*_{xy} < r_{xy}}{B}\right),$$

and $\hat{a}$ (acceleration), can (Efron and Tibshirani, 1993) be computed as

$$\hat{a} = \frac{\sum_{j=1}^{n}\left(\langle r_{xy|(j)}\rangle - r_{xy|(j)}\right)^3}{6\left[\sum_{j=1}^{n}\left(\langle r_{xy|(j)}\rangle - r_{xy|(j)}\right)^2\right]^{3/2}}$$

with $r_{xy|(j)}$ being the jackknife value of r_{xy}. That is, let $\{x(i), y(i)\}_{i=1|(j)}^{n}$ denote the original sample with pair $\{x(j), y(j)\}$ removed, then $r_{xy|(j)}$ is Pearson's r calculated from $\{x(i), y(i)\}_{i=1|(j)}^{n}$. The mean, $\langle r_{xy|(j)}\rangle$, is given by $(\sum_{j=1}^{n} r_{xy|(j)})/n$. Hall (1988) proved that BCa confidence intervals have advantageous second-order accuracy: let $r_{xy}(\alpha)$ be the single endpoint of confidence interval for ρ_{xy} with nominal one-sided coverage α, then Prob$[\rho_{xy} \le r_{xy}(\alpha)] = \alpha + C$ with the coverage error, C, being of order $(1/n)$. Obviously, small coverage error is a desirable property for a confidence interval. Simple percentile intervals, that is, neglecting estimation bias or a standard error of r_{xy} dependent on ρ_{xy}, for example, have only first-order accuracy (C of order $(1/\sqrt{n})$).

Previous Monte Carlo studies using the ordinary bootstrap (i.e., $P = 1$) for data without serial dependence revealed that the bad coverage performance of percentile intervals can be considerably improved with advanced methods such as bootstrap iteration (Hall, Martin, and Schucany, 1989). Our simulation study (next Section) suggests that also BCa intervals for r_{xy} perform accurate in such situations already for data sizes as low as 20 or 50.

MONTE CARLO EXPERIMENT

Since persistence reduces the number of independent observations the coverage performance of the method in dependence on the data size has to be evaluated.

To study from which value of n the second-order accuracy of BCa confidence interval produces empirical coverage points, γ, acceptably close to the nominal values, α and $1-\alpha$, Monte Carlo simulations are carried out. For each Monte Carlo design, that is, each predefined combination of

$$n \in \{10,\ 20,\ 50,\ 100,\ 200,\ 500,\ 1000,\ 2000\};$$

$$\rho_{xy} \in \{0.0,\ 0.3,\ 0.6,\ 0.9\};$$

$$\tau_x, \tau_y \in \{0.0,\ 2.0,\ 5.0\};$$

$$\alpha \in \{0.025,\ 0.05,\ 0.1\};$$

time spacing (equidistant 1.0, or a realization of a gamma distribution with three degrees of freedom (as used by Schulz and Stattegger, 1997) and $\bar{d}$ subsequently scaled to 1.0);

$\{X(i), Y(i)\}$ distribution (bivariate standard normal, or bivariate lognormal with $b_x = b_y = 5.976$ and $s_x = s_y = 0.164$ (i.e., variance unity and skewness 1/2), see Appendix A);

$n_{\text{sim}} = 10000$ bivariate time series are generated, r_{xy} with BCa interval calculated as in the preceding Section with $P = \bar{d}/[4 \cdot \max(\tau_x, \tau_y)]$ and the cases counted when the interval contains r_{xy}, or r_{xy} lies above the upper bound, or below the lower. ($P = 1$ is used for $\tau_x = \tau_y = 0.0$, and also in an additional run with $\tau_x = \tau_y = 2.0$ to study the influence of neglecting persistence.) The "simulation noise," σ, is negligible (nominally: $\sigma = \sqrt{\alpha(1-\alpha)/n_{\text{sim}}} \leq 0.003$). The selection of Monte Carlo designs aims to study coverage performance in dependence on parameters n, ρ_{xy}, τ_x, and τ_y under realistic conditions.

In the Monte Carlo experiment estimation of τ_x and τ_y is replaced by taking their known values to limit computing costs. This does not affect the implications of the experiment since the choice of P in Eq. (1) is to be seen as a rough guide where only gross estimation errors of $\hat{\tau}_x$ or $\hat{\tau}_y$ would influence the accuracy of the method.

Figures 1–5 show the results of the Monte Carlo experiment: empirical coverages against data size, for the various combinations of τ_x, τ_y, and P, in case of bivariate lognormal distributions with $\rho_{xy} = 0.60$ and uneven spacing. Figure 1 confirms that BCa confidence intervals for ρ_{xy} are rather accurate already for data sizes as low as 20 or 50 when no serial dependence is in the data ($\tau_x = \tau_y = 0$). Figure 5 reveals that not taking serial dependence into account when bootstrapping ($\tau_x \neq 0$, $\tau_y \neq 0$, $P = 1$) produces serious coverage errors that do not disappear when increasing the data size, n.

However, Figures 2–4 exhibit that although taking serial dependence correctly into account ($\tau_x = \tau_y = 2$ and $P = 1/4$; $\tau_x = 2$, $\tau_y = 5$, and $P = 1/20$; $\tau_x = \tau_y = 5$ and $P = 1/20$) considerably large data sizes ($n \gtrsim 200$ for $\max(\tau_x, \tau_y) = 2$ and $n \gtrsim 500$–1000 for $\max(\tau_x, \tau_y) = 5$) are required for an accuracy that is

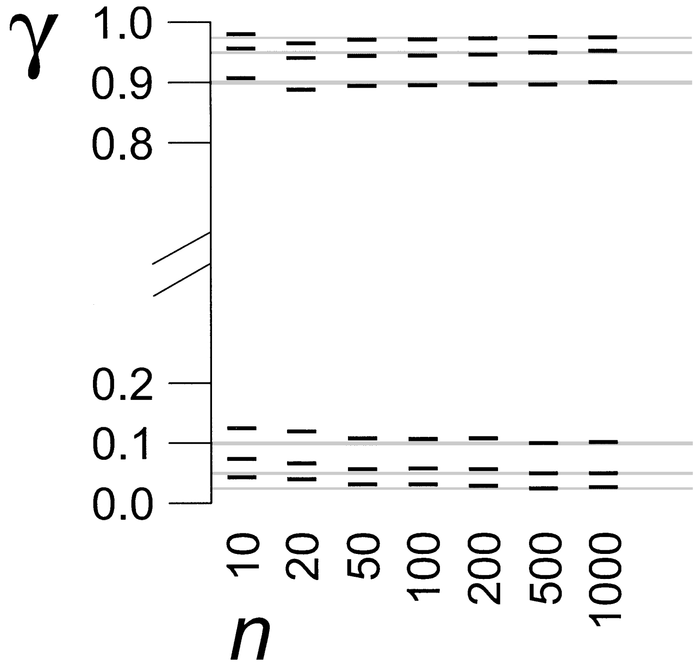

Figure 1. Monte Carlo experiment, result for uneven spacing ($\Gamma(3)$ realisation, scaled to $\bar{d} = 1$), bivariate lognormal autoregressive process, and $\rho_{xy} = 0.60$. Empirical coverage points, γ, are shown as bar symbols against data size, n. Three nominal α-levels used (0.025, 0.05, and 0.1), shown as horizontal lines with line thickness corresponding to standard error, σ. Persistence times: $\tau_x = 0.0$, $\tau_y = 0.0$; bootstrap probability: $P = 1$.

acceptable in geosciences (a nominal 95% interval should at least yield, say, 90% empirical coverage). Because the calculated BCa intervals for r_{xy} turn out liberal (i.e., they undercover) in the Monte Carlo experiment selection of a sufficiently small nominal value ($\alpha = 0.025$) is advised for applications.

The dependence of coverage accuracy on ρ_{xy} (results not shown) is rather weak and has negligible influence on interpretation of results obtained using the proposed method. Also dependence on the distributional shape of the data (log-normal/normal) is extremely weak (results not shown), attesting the method's robustness in this respect. Finally, whether the time spacing is equidistant or not is of marginal effect (results not shown).

Further Monte Carlo experiments (not shown) with $P = \bar{d}/[2 \cdot \max(\hat{\tau}_x, \hat{\tau}_y)]$ and $P = \bar{d}/t[\max(\hat{\tau}_x, \hat{\tau}_y)]$ produced larger coverage errors than using Eq. (1). The reason is presumably that in such cases the average bootstrap block length is too short to capture the serial dependence sufficiently.

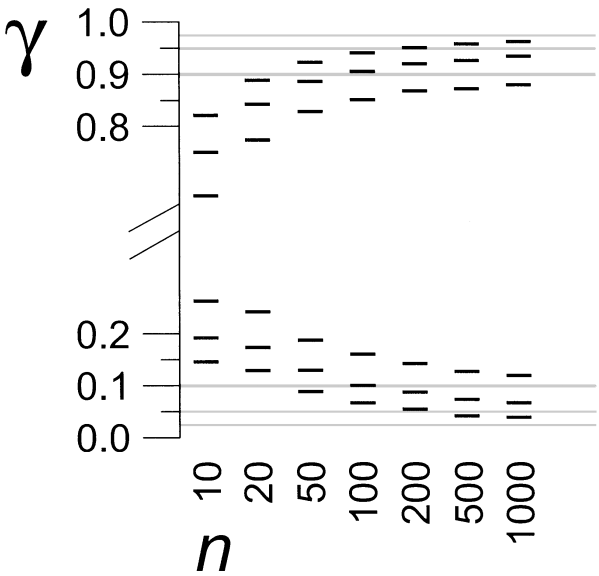

Figure 2. Monte Carlo experiment, result for $\tau_x = 2.0$, $\tau_y = 2.0$; bootstrap probability: $P = 1/8$ (other parameters as in Fig. 1).

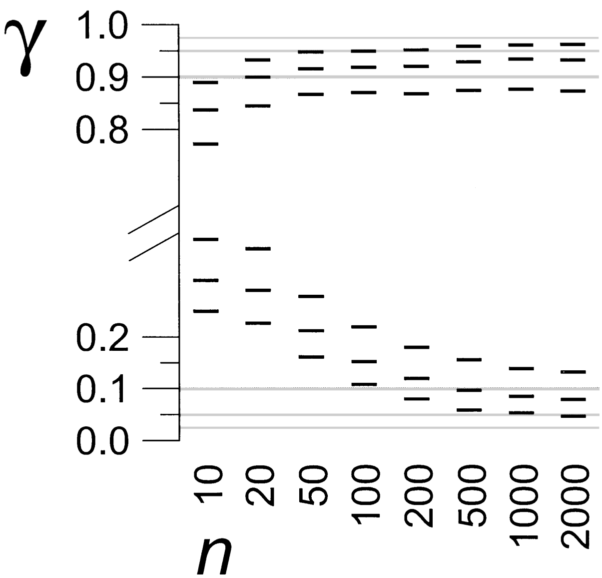

Figure 3. Monte Carlo experiment, result for $\tau_x = 2.0$, $\tau_y = 5.0$; bootstrap probability: $P = 1/20$ (other parameters as in Fig. 1).

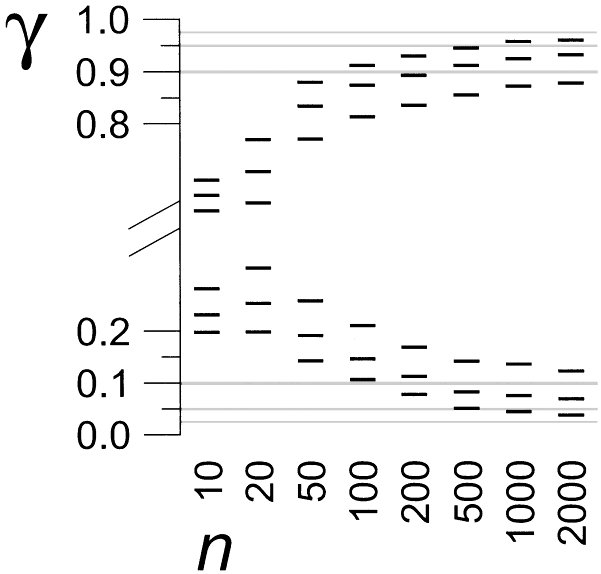

Figure 4. Monte Carlo experiment, result for $\tau_x = 5.0$, $\tau_y = 5.0$; bootstrap probability: $P = 1/20$ (other parameters as in Fig. 1).

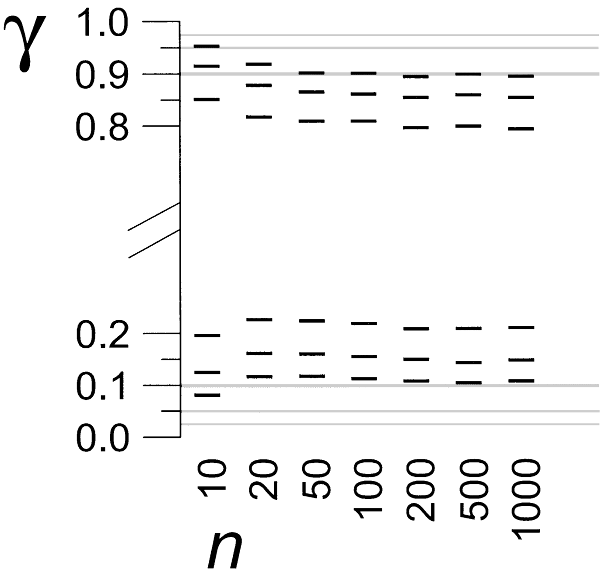

Figure 5. Monte Carlo experiment, result for $\tau_x = 2.0$, $\tau_y = 2.0$; bootstrap probability: $P = 1$ (other parameters as in Fig. 1).

APPLICATIONS

Solar Influence on Indian Ocean Monsoon

H5 is a stalagmite from Hoti cave (southern Oman) that grew in early Holocene, approximately from 9.6 to 6.1 ka (thousand years) before present (BP). The oxygen isotopic composition (δ^{18}O) of the speleothem is primarily determined by the δ^{18}O of drip water and, hence, the δ^{18}O of precipitation which is controlled by Indian Ocean monsoon intensity via an "amount effect" (see Neff and others, 2001). Thus, the H5 δ^{18}O record (Fig. 6(A)) (Neff and others, 2001) serves as indicator of Holocene monsoon rainfall (low δ^{18}O reflects strong rainfall). The timescale is based on 12 Th—U mass-spectrometric ages. The δ^{18}O record from H5 has been compared with Δ^{14}C in tree rings (Stuiver and others, 1998) which is a well-known indicator of solar activity (high Δ^{14}C reflects low activity) yielding an "excellent correlation" (Neff and others, 2001).

The original δ^{18}O record ($\bar{d} = 4.13$ a) is "downsampled" to the time points of the Δ^{14}C record ($n = 341$, $t(1) = 6.183$ ka BP, $t(n) = 9.643$ ka BP, $\bar{d} = 10.15$ a, equidistance with a few gaps) and not vice versa to avoid introducing artificial dependence due to the interpolation. PearsonT estimates $\hat{\tau}(\delta^{18}O) = 26.24$ a and $\hat{\tau}(\Delta^{14}\mathrm{C}) = 38.27$ a which means serial dependence of a strength ($\hat{\tau}/\bar{d} = 2.59$ and 3.77) comparable to those in the Monte Carlo experiment. The scatterplot (Fig. 6(B)) indicates positive correlation. The data size seems large enough for acceptably accurate confidence interval estimation. Pearson's correlation coefficient is estimated as $r_{xy} = 0.60$ with 95% BCa confidence interval [0.51; 0.66], confirming the conclusion of Neff and others (2001).

It is emphasized that this remarkably high correlation has been achieved by tuning the H5 δ^{18}O record when constructing the timescale (see Neff and others, 2001). The untuned H5 δ^{18}O record, correlated with Δ^{14}C, yields $r_{xy} = 0.11$ with 95% interval [-0.05; 0.27], a nonsignificant correlation. (Note that neglecting serial dependence would falsely indicate a significant correlation!) By tuning the monsoon indicator record, the nonsignificant correlation with solar activity has been turned into a remarkably high value. However, since the time shiftings in the tuning were always smaller than the dating error (Neff and others, 2001, Fig. 3 therein), there is good evidence for an influence of solar activity on Indian Ocean monsoon in early Holocene.

Solar Influence on Atmospheric ^{10}Be

The number of sunspots correlates positively with solar activity. Group sunspot number (Hoyt and Schatten, 1998) comprises more observations and is less noisy than Wolf's sunspot number. Production rate of cosmogenic nuclide ^{10}Be in the atmosphere is another indicator of solar irradiance (high ^{10}Be means

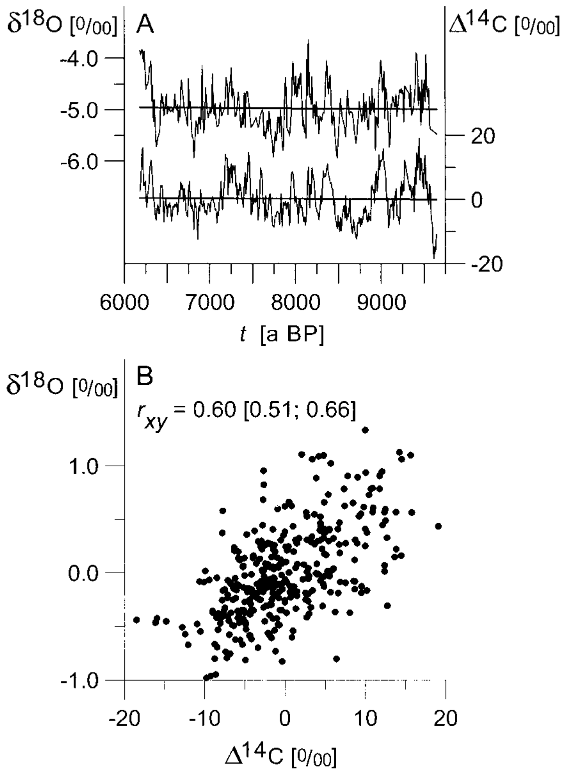

Figure 6. (A) H5δ^{18}O time series (Neff and others, 2001) as Indian Ocean monsoon indicator on tuned timescale, tree-ring Δ^{14}C time series (Stuiver and others, 1998) as solar activity indicator, linear trends. Time is in years before present (BP). (B) Scatterplot of detrended data, $n = 341$, $r_{xy} = 0.60$ with 95% confidence interval[0.51; 0.66].

low activity). However, ^{10}Be concentration depends additionally on atmospheric processes (Beer and others, 1994a). The ^{10}Be record from Greenland ice core Dye 3 (Beer and others, 1994b) allows a comparison with group sunspot number in yearly resolution (Fig. 7(A)).

A highly significant correlation, $r_{xy} = -0.45$ with 95% confidence interval $[-0.57; -0.31]$, is found for the interval from 1715 (end of Maunder Minimum of sunspot number) to 1870 (Fig. 7(B)). On the other hand, a nonsignificant correlation, $r_{xy} = +0.03$ with 95% confidence interval $[-0.15; +0.20]$, is found for the interval 1871–1985 (Fig. 7(C)). This vanishing correlation may be interpreted such that after approximately 1870 atmospheric processes have corrupted the quality of ^{10}Be to record solar activity variations (Hoyt and Schatten, 1997).

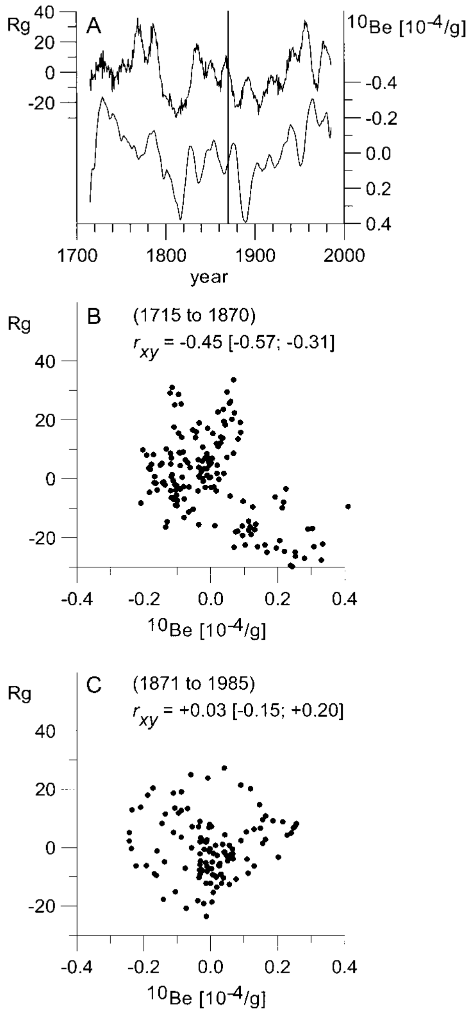

Figure 7. (A) Group sunspot number Rg (Hoyt and Schatten, 1998) as solar activity indicator, cosmogenic ^{10}Be in Dye 3 ice core (Beer and others, 1994b) as another proxy of solar activity. ^{10}Be time series was cubic-spline interpolated (Beer and others, 1994b). Both records have been detrended and low-pass (periods > 15 years) filtered prior to correlation analyses. Vertical line indicates year 1870. (B) Scatterplot for interval 1715–1870, $n = 156$, $r_{xy} = -0.45$ with 95% confidence interval $[-0.57; -0.31]$. (C) Scatterplot for interval 1871–1985, $n = 115$, $r_{xy} = +0.03$ with 95% confidence interval $[-0.15; +0.20]$.

CONCLUSIONS

The Monte Carlo simulations reveal that taking positive serial dependence (persistence) into account is essential for obtaining accurate confidence intervals (of type bootstrap BCa) for Pearson's correlation coefficient from bivariate climate time series data. The nonparametric stationary bootstrap (data blocks of variable length) performs as a method robust against the distributional shape of the data (bivariate lognormal/normal) and the time spacing (uneven/even). The presented approach, using an average block length proportional to the maximum estimated persistence time of the data, should provide further robustness against the exact form of serial dependence. Thus, the method presented should be widely applicable.

However, since persistence means a reduction of the effective data size, substantially more data are required ($n \gtrsim 200$ for $\max(\tau_x, \tau_y) = 2\bar{d}$, $n \gtrsim 500$–1000 for $\max(\tau_x, \tau_y) = 5\bar{d}$) than when no persistence is in the data for achieving an accuracy of the confidence interval that is acceptable in geosciences.

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APPENDIX A: BIVARIATE GAUSSIAN AND LOGNORMAL AUTOREGRESSIVE PROCESSES

Let $\{X(i)\}$ and $\{Y'(i)\}$ be strictly stationary Gaussian AR(1) processes at discrete, monotonically increasing times $t(i)$ with arbitrary spacing (Robinson, 1977):

$$X(1) = \epsilon_x(1) \sim N(0, 1),$$

$$Y'(1) = \epsilon_y(1) \sim N(0, 1),$$

and, for $i > 1$,

$$X(i) = \exp\left[-\frac{t(i) - t(i-1)}{\tau_x}\right] \cdot X(i-1) + \epsilon_x(i),$$
$$Y'(i) = \exp\left[-\frac{t(i) - t(i-1)}{\tau_y}\right] \cdot Y'(i-1) + \epsilon_y(i),$$
$$\epsilon_x(i) \sim N\left(0, 1 - \exp\left[-2\frac{t(i) - t(i-1)}{\tau_x}\right]\right),$$
$$\epsilon_y(i) \sim N\left(0, 1 - \exp\left[-2\frac{t(i) - t(i-1)}{\tau_y}\right]\right),$$

with the $\epsilon_x(i)$ and $\epsilon_y(i)$, for $i \geq 1$, both being independent sequences. Let further $\epsilon_x(i)$ and $\epsilon_y(i)$ be also mutually independent and define

$$Y(i) = \rho_{xy} \cdot X(i) + \sqrt{1 - \rho_{xy}^2} \cdot Y'(i)$$

for $i \geq 1$. Then the process $\{X(i), Y(i)\}$ is a bivariate Gaussian autoregressive process with zero means, unit variances, and further second-order properties:

$$E[X(i) \cdot Y(i)] = \rho_{xy},$$
$$E[X(i+h) \cdot X(i)] = \exp\left[-\frac{t(i+h) - t(i)}{\tau_x}\right],$$
$$E[Y(i+h) \cdot Y(i)] = \rho_{xy}^2 \cdot \exp\left[-\frac{t(i+h) - t(i)}{\tau_x}\right] + \left(1 - \rho_{xy}^2\right) \cdot \exp\left[-\frac{t(i+h) - t(i)}{\tau_y}\right],$$
$$E[X(i+h) \cdot Y(i)] = \rho_{xy} \cdot \exp\left[-\frac{t(i+h) - t(i)}{\tau_x}\right],$$

lag $h > 0$, making it strict stationary for persistence times τ_x, $\tau_y > 0$ (see Priestley, 1981, section 9.1 therein). Formulating the autoregressive process by means of persistence time has the advantage that this quantity corresponds directly to the relevant physical timescale (independence of time spacing). The persistence time measures the "memory" of the (climatic) signal which has been observed by the sampled time series.

In the Monte Carlo experiment, time series were generated either from this bivariate Gaussian autoregressive process or from a bivariate lognormal autoregressive process. The latter was obtained by transforming the bivariate Gaussian autoregressive process, say $\{X^{\mathrm{N}}(i), Y^{\mathrm{N}}(i)\}$ with correlation ρ_{xy}^{N}, as follows:

$$X(i) = \exp\left[s_x \cdot X^{\mathrm{N}}(i) + \log(b_x)\right],$$
$$Y(i) = \exp\left[s_y \cdot Y^{\mathrm{N}}(i) + \log(b_y)\right],$$
$$\rho_{xy} = \left[\exp\left(\rho_{xy}^{\mathrm{N}} \cdot s_x \cdot s_y\right) - 1\right] \Big/ \sqrt{\left[\exp\left(s_x^2\right) - 1\right] \cdot \left[\exp\left(s_y^2\right) - 1\right]},$$

with scale parameters b_x, b_y and shape parameters s_x, s_y (see Johnson, Kotz, and Balakrishnan, 1994).

Since ρ_{xy} and the persistence times appear together in Eq. (2), in principle, an iterated estimation method for $\hat{\tau}_x$ and $\hat{\tau}_y$ would be indicated. However, PearsonT performs only one iteration loop: estimation of τ_x and τ_y from their respective time series using the method of Mudelsee (2002). This algorithm takes $\hat{\tau}_x = \operatorname{argmin}(S(\cdot))$ with least-squares sum

$$S(\tau) = \sum_{i=2}^{n} (x(i) - x(i-1) \exp\{[t(i) - t(i-1)]/\tau\})^2.$$

In case of uneven spacing, minimization has to be done numerically. Using only one iteration loop seems to be justified since in the present context only a rough estimate is needed in Eq. (1). Only if $\hat{\tau}_x$ and $\hat{\tau}_y$ show very large differences (a factor of, say, ≥ 10), further analysis of the dependence structure seems to be necessary.

APPENDIX B: COMPUTER PROGRAM

PearsonT is a Fortran 90 program which has been tested under Windows 98 and invokes Gnuplot program (version 3.6 or higher) for visualization. After starting PearsonT, supply path and data file name. The time series are then linearly detrended (ordinary least squares) to satisfy weak stationarity assumption. More complicated trend functions should be subtracted or variance stabilizing transformations carried out prior to PearsonT analysis. Persistence times are estimated using the algorithm of Mudelsee (2002) with automatic bias correction. Next, time series and their trends are plotted on screen, followed by an $x - y$ scatterplot, and the result window which informs about (1) data file name, (2) current time interval, (3) n, (4) $\hat{\tau}_x$ and $\hat{\tau}_y$, and (5) r_{xy} with BCa confidence interval ($B = 2000$, $\alpha = 0.025$). You can now select a new time interval and repeat the analysis for the data inside, or select the original interval, or exit and output the result, data, trends, and detrended data to file "PearsonT.dat." File "readme.txt" provides further details. Executables of PearsonT are available from the author's homepage, http://www.uni-leipzig.de/~meteo/MUDELSEE/.