You are currently working with physicists that are using a particle collider for some experiments. They have been testing the monitoring equipment, by passing a single particle and monitoring its motion on a plane, where the particle is at position $(0,0)$ at the starting time $t = 0$. (The units have been omitted, as the measurements are on such a small scale, and in order to anonymize the computations, somewhat.) They want to model the measured motion from the data, and predict the position of the particle at a given point within the duration of the measurements.

For this project, data is provided in a text file named `partmove.txt`, which has numbers at each line, separated by spaces within lines, and lines separated by carriage returns, and no other characters are included. The first line contains N, which is an integer from 1 to 16. The next N lines contains pairs of floating point numbers corresponding to the position (x_i, y_i) at the ith measurement, where $0 < t_1 < t_2 < \cdots < t_N$ is the time at which the measurements are taken. The next line will contain $t\ x\ y$, where t time at which the position is to be predicted, and (x, y) the actual position at which the particle is found at time t.

1. The measurements are taken at evenly-spaced intervals, with $t_N = 2$. Model the position of the particle using polynomials in each direction, and output the discrepancy between the interpolated position at time t with the given (x, y). **(3)**

2. The measurements are taken at evenly-spaced intervals, with $t_N = 2$. Model the position of the particle using natural cubic splines in each direction, and output the discrepancy between the interpolated position at time t with the given (x, y). **(3)**

3. The measurements are taken $1 + t_i$, where t_i at the roots of $T_N(t)$, the Chebyshev polynomial of degree N. Chebyshev polynomials, with degree $n \geq 0$, can be given in the following form

$$T_n(t) = \cos(n \arccos t).$$

   Model the position of the particle using polynomials, and output the discrepancy between the interpolated position at time t with the given (x, y). **(3)**

4. The measurements are taken at $1 + t_i$, where t_i the roots of $P_N(t)$, the Legendre polynomial of degree N. Legendre polynomials, with degree $n \geq 0$, are given recursively as

$$P_0(t) \equiv 1; \quad P_1(t) = t; \quad (n+1)P_{n+1}(t) + nP_{n-1}(t) = (2n+1)tP_n(t), n > 0.$$

   It is known that the roots of the Legendre polynomial P_n are also simple and lie in the interval $[-1, 1]$. It is also known that

$$P_n'(t) = \sum_{\substack{k=0 \\ n+k \text{ even}}}^{n-1} (2k+1)P_k(t),$$

   and iterative methods for determining the roots of the Legendre polynomial P_n often start from the roots of the Chebyshev polynomial T_n.

   Model the position of the particle using polynomials, and output the discrepancy between the interpolated position at time t with the given (x, y). **(3)**

5. The measurements are taken at evenly-spaced intervals, with $t_N = 2$. It is presumed that the model of the position of the particle is dependent only on the initial velocity and an unknown constant force—that is, the position is quadratic in both directions. Output the discrepancy between the interpolated position at time t with the given (x, y). **(3)**