Learning to Control Traffic

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Learning problem

- set of problem instances $\mathcal{I}$
- distribution P over instances
- set of algorithms $\mathcal{A}$
- measure of optimality $m : \mathcal{I} \times \mathcal{A} \to \mathbb{R}$

based on [Bengio et al., 2020]

Learning problem

- general learning objective

$$\min_{a \in \mathcal{A}} \mathbb{E}_{i \sim P}\ m(i, a) \tag{1}$$

- no access to $\mathcal{I}$ or P, so use samples

$$\min_{a \in \mathcal{A}} \sum_{i \in D_{train}} \frac{1}{|D_{train}|} m(i, a) \tag{2}$$

Learning problem

- demonstration
- experience

Demonstration

- parameterization of algorithms, e.g., by using neural network with weights $\theta \in \mathbb{R}^p$

$$\min_{\theta \in R^p} \mathbb{E}_{i \sim P} m(i, a(\theta)) \tag{3}$$

Experience

- greedy TSP heuristic = picking next node

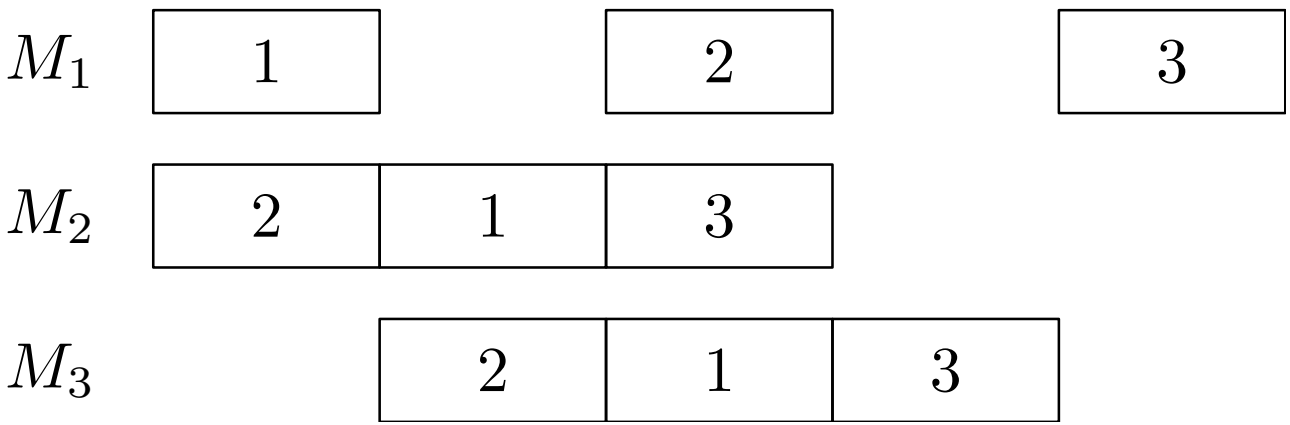
Learning to cut (example)

Job shop

- m machines
- n jobs
- fixed machine order for each job

Job shop

- example schedule



Job shop

- job j on machine i is operation (i, j)
- operations N
- order of operations for particular job j is fixed

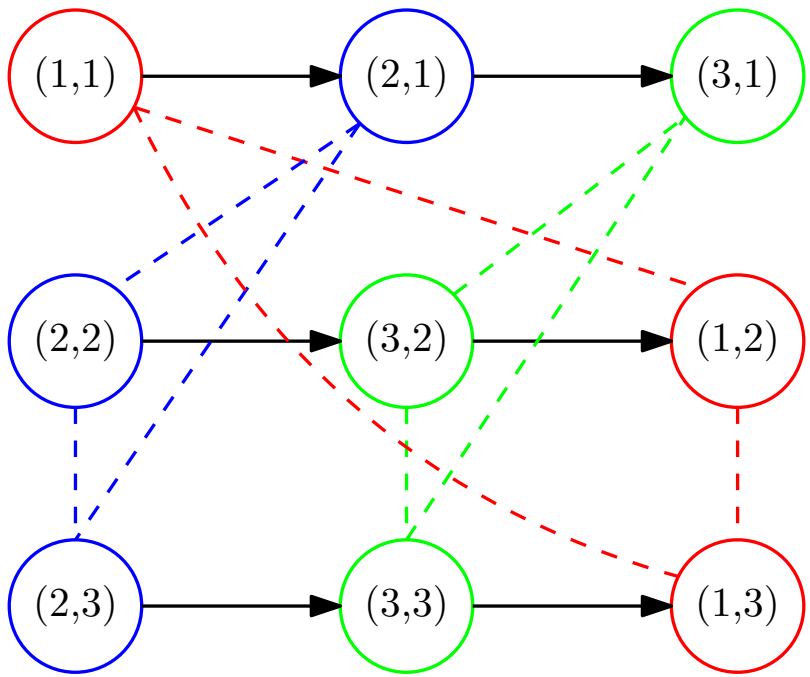
$$(i, j) \rightarrow (k, j) \in \mathcal{C}$$

- order among jobs j and l is optimization decision

$$(i, j) \rightarrow (k, l) \quad \text{or} \quad (i, l) \rightarrow (k, j)$$

Disjunctive graph

- directed graph $G = (N, \mathcal{C}, \mathcal{D})$
- conjunctive arcs
- disjunctive arcs



Job shop MILP

- makespan objective
- mixed-integer linear program

minimize $C_{\max}$

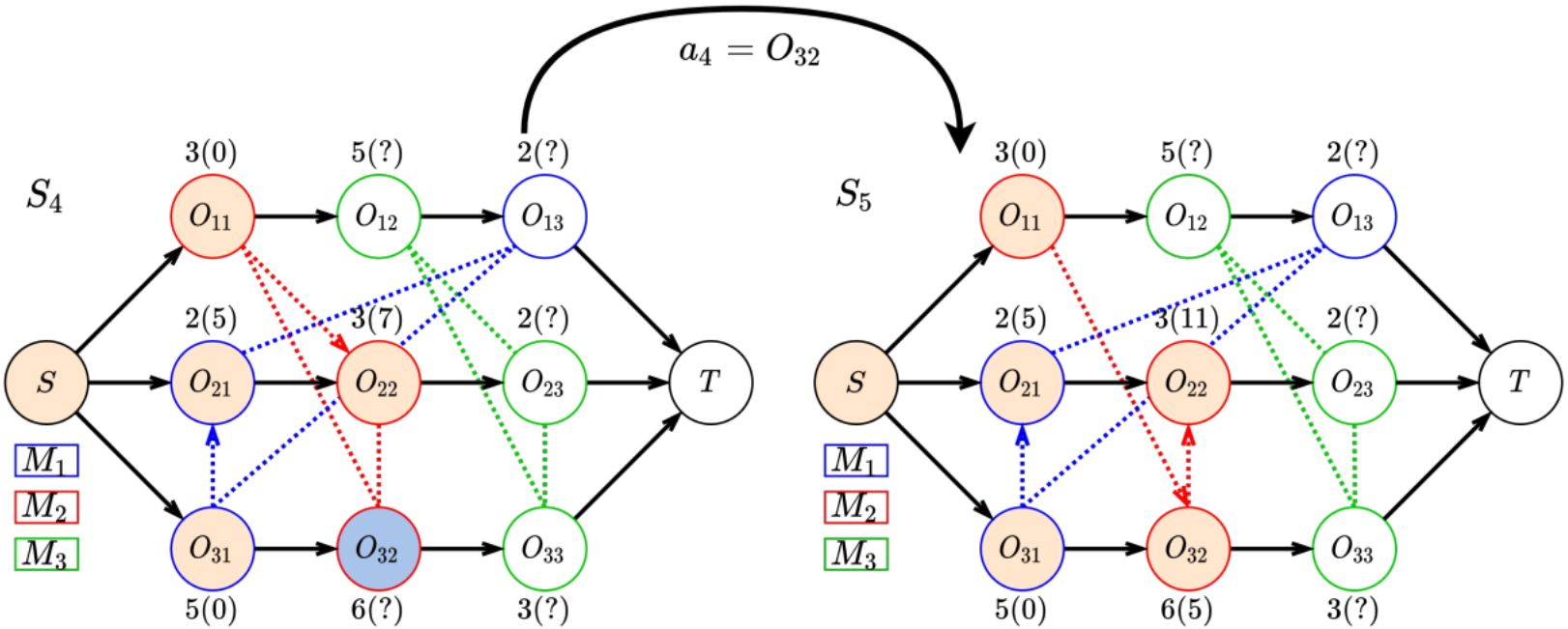
$$y_{ij} + p_{ij} \leq y_{kj} \qquad \text{for all } (i,j) \to (k,j) \in \mathcal{C}$$

$$y_{il} + p_{il} \leq y_{ij} \text{ or } y_{ij} + p_{ij} \leq y_{il} \qquad \text{for all } (i,l) \text{ and } (i,j), i = 1, \ldots, m$$

$$y_{ij} + p_{ij} \leq C_{\max} \qquad \text{for all } (i,j) \in N$$

$$y_{ij} \geq 0 \qquad \text{for all } (i,j) \in N$$

Zhang et al.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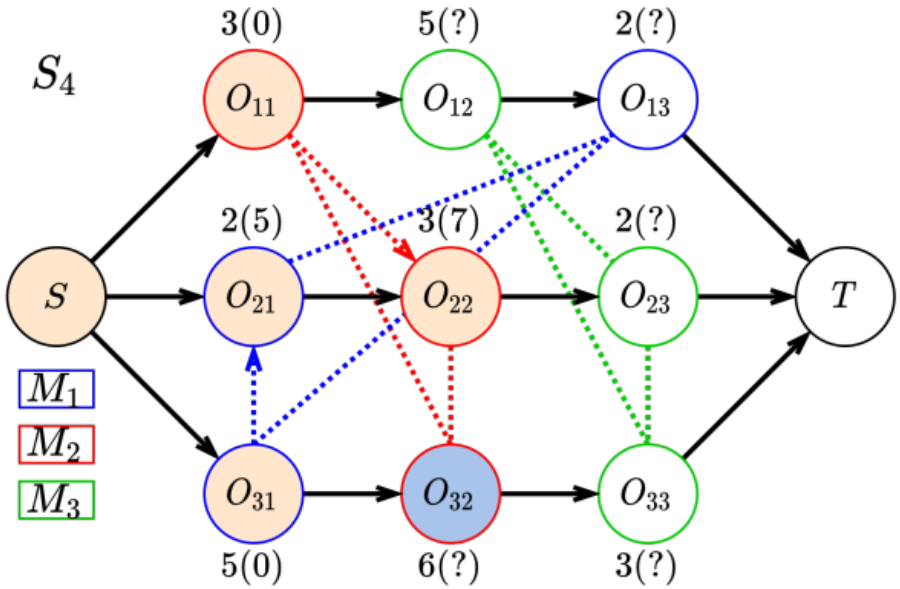
Zhang et al.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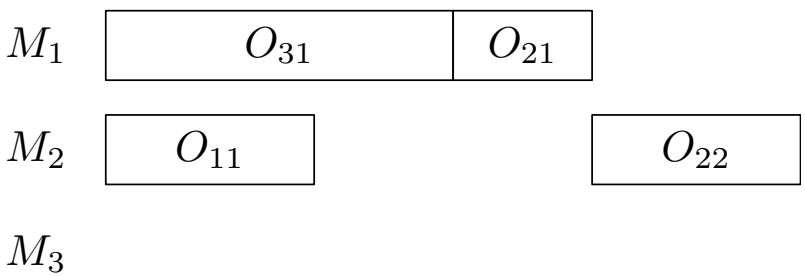
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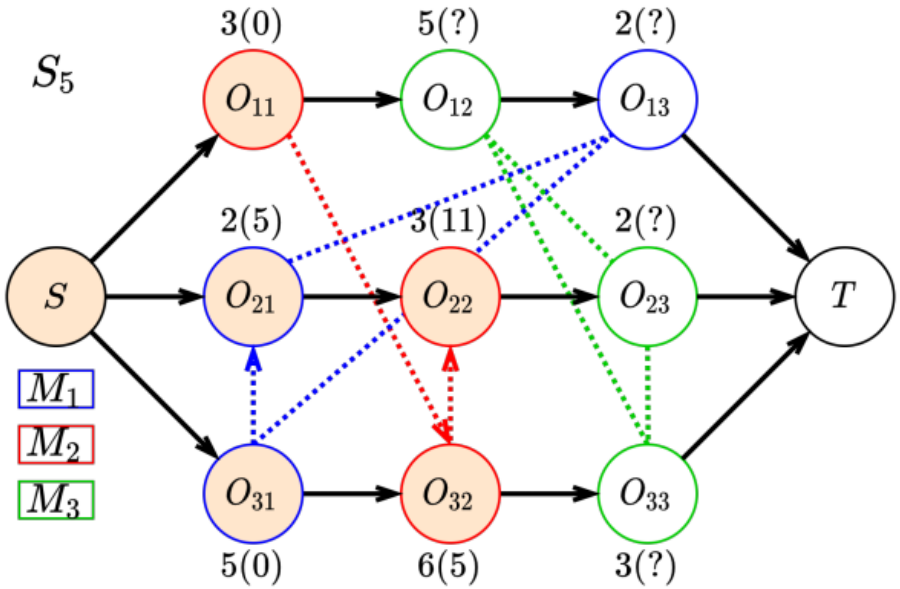
Zhang et al.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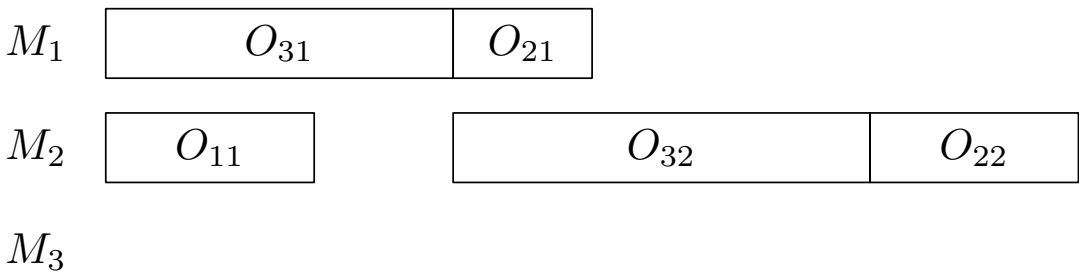
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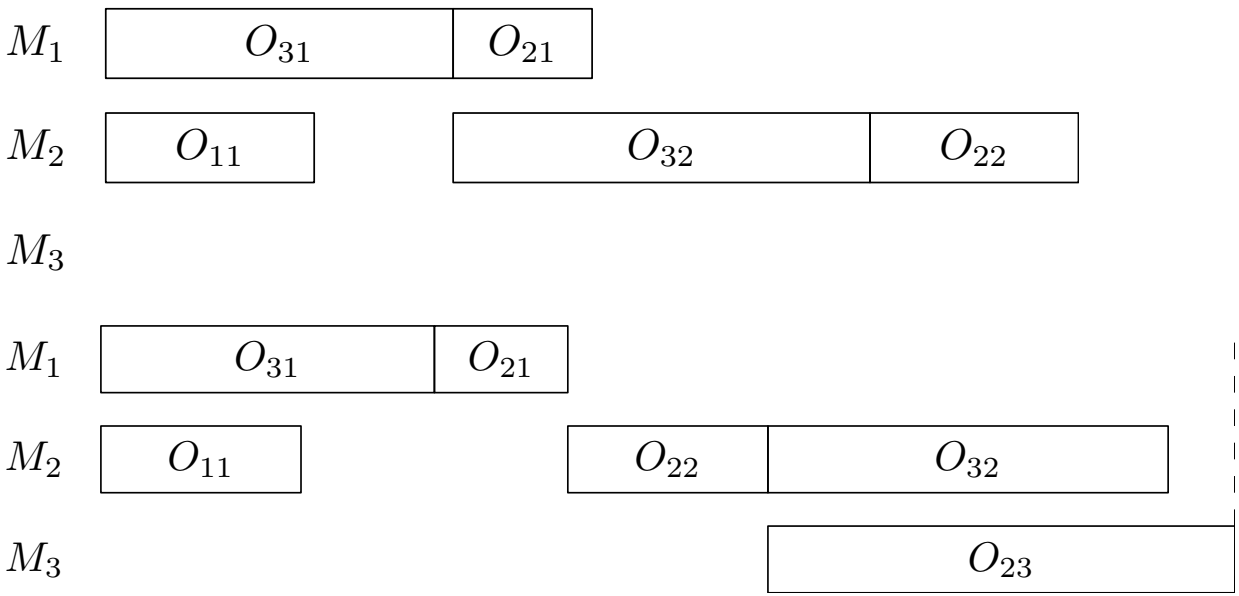

Tassel et al.

Traffic scheduling problem

- total completion time
- release dates
- chains
- setup times (switch-over)

References

Yoshua Bengio, Andrea Lodi, and Antoine Prouvost. Machine Learning for Combinatorial Optimization: A Methodological Tour d'Horizon, March 2020.