Regeneration Vehicle Partitioning

Jeroen van Riel

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1 Conjecture

Let us first provide some words on notation. We use t_j^0 to denote the arrival time of some vehicle j. The *switch-over* time between vehicles crossing from different lanes is denoted by s. We use ϕ_W to denote *partial schedule* on the vehicles W, which is a mapping $\phi : W \to [0, \infty)$. We write σ_W whenever this schedule is (part of) and optimal schedule.

Definition 1.1 (Regeneration Vehicle)**.** Let V be a set of vehicles and ϕ_V some partial schedule. Vehicle j^* is a regeneration vehicle for ϕ_V whenever

$$\phi_V(l) \leq t_{j^*}^0 - s \tag{1}$$

for all $l \in V$ with $t_l^0 \leq t_{j^*}^0$.

Stated in the notation used here, the conjecture from [1] can be formulated as follows.

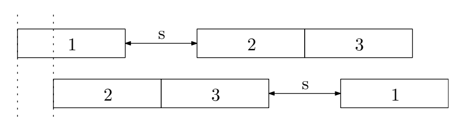
Proposition 1.1 (Regeneration Vehicle Partitioning)**.** *Let V be a set of vehicles. Let $W \subset V$ be a subset containing the k earliest arriving vehicles. Suppose that an optimal schedule σ_W contains a regeneration vehicle j^*, then we have*

$$\sigma_V(l) = \sigma_W(l), \tag{2}$$

for all vehicles $l \in A = \{l \in W : t_l^0 < t_{j^*}^0\}$ that arrived before j^*.

2 Counterexample

Let $r = r_2 - r_1$ and consider the schedule in the next figure. The first schedule



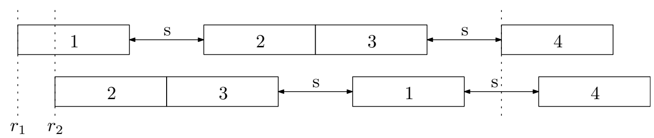
has $\sum C_j = 6p + 2s$ and the second schedule has $\sum C_j = 3r + 6p + s$. Therefore, the second schedule is optimal whenever

$$r < s/3. \tag{3}$$

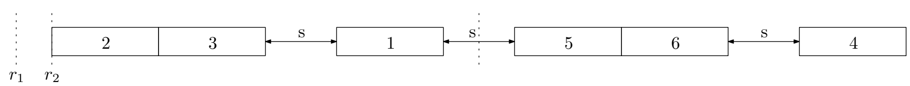
Now consider the situation in which we have an additional job 4 with release date $r_4 = 6p + 2s$. It is never optimal to schedule 4 before any of the other jobs, so the figure shows the two schedules to consider. The first schedule has $\sum C_j = 10p + 4s$ and the second schedule has $\sum C_j = 10p + 3s + 4r$. Therefore, the first schedule is better assuming

$$r > s/4. \tag{4}$$

For $W = \{1, 2, 3, 4\}$, let this schedule be denoted by σ_W. Observe that vehicle 4 is a regeneration vehicle for σ_W.



Now assume that (3) and (4) hold. Consider two additional jobs 5 and 6 with release date $r_5 = r_6 = r_4 + r$. Again, we see that vehicles 4, 5 and 6 should be ordered as $5, 6, 4$, so it is not hard to reason that the optimal schedule is given in the figure below and has $\sum C_j = 21p + 8s + 6r$. For $V = \{1, \ldots, 6\}$, this shows that $\sigma_V(l) \neq \sigma_W(l)$ for $A = \{1, 2, 3\}$.



References

[1] M. Limpens, “Online Platoon Forming Algorithms for automated vehicles: A more efficient approach,” Master’s thesis, Eindhoven University of Technology, Sept. 2023.