Single Intersection Scheduling with Deep Q-Learning

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1 Introduction

Recall our definition of the Single Interesection Scheduling Problem. We saw that it can be concisely formulated as a Mixed-Integer Linear Program. Therefore, one direct solution approach is to apply standard software for solving this class of problems. We will use the commercial Gurobi solver. Alternatively, we will show how the problem can be formulated in terms of *dispatching* jobs. We will argue that this formulation allows us to tackle the problem with reinforcement learning techniques and compare the performance of these methods to that of the exact procedure.

2 Environment

We are interested in whether we can learn a procedure to generate good schedules. To start with, we propose to focus on dispatching procedures, which means that the schedule is constructed by inserting jobs one by one. Therefore, we will define this procedure in terms of a Markov Decision Process (MDP).

Assume we have a single intersection with K lanes. Because this corresponds with a single machine in the machine scheduling context, we will use the first index to refer to the lane, so we use r_{ij} and p_{ij} to denote the release date and processing time, respectively, of the jth platoon of vehicles on lane i. Note that it is not necessary to specify the number of vehicles of a platoon, because of the Platoon Preservation Theorem. Instead, we only consider the total processing time.

Let $i \in \{0, \dots, K-1\}$ be an arbitrary lane and let n_i denote the (possibly infinite) number of total arrivals to this lane. The release dates and processing times can be considered part of the specification of the MDP, which means that $(r_{ij})_{j=1}^{n_i}$ and $(p_{ij})_{j=1}^{n_i}$ are fixed. In the following experiments, we assume that the arrival process is as follows. We assume the processing times are integers distributed as $p_{ij} \sim \text{Uni}[L, H]$. The processing times are distributed as $p_{ij} \sim \text{Geom}(\theta)$ such that $P(p_{ij} = n) = (1-\theta)^{n-1}\theta$, so it is the required number of Bernoulli trials before the first success, where θ is the success probability. We define interarrival times $X_{ij} \sim \text{Exp}(\lambda)$, parameterized such that $\mathbb{E}[X_{ij}] = \lambda$. The release dates are given by

$$r_{ij} = \sum_{l=1}^{j-1} p_{il} + \sum_{l=1}^{j} X_{il}. \tag{1}$$

In a dispatching approach, platoons are assigned to a timeslot one by one. Because of the precedence constraints between platoons on the same lane, this is equivalent to deciding in which order to serve the lanes. Therefore, the action a of the scheduler is to decide whether to keep serving the current lane ($a = 0$), or to serve the next lane ($a = 1$). At step t, let $c^{(t)}$ denote the lane that was last served and let $m_i^{(t)}$ denote the number of scheduled platoons from lane i. Let $T^{(t)}$ denote the completion time of the last scheduled platoon at step t, so before any platoons are scheduled, we have $T^{(0)} = 0$. The state at step t can then simply be written as

$$s^{(t)} = (c^{(t)}, T^{(t)}, m_1^{(t)}, \dots, m_K^{(t)}) \in \{1, \dots, K\} \times \mathbb{R} \times \mathbb{N}^K. \tag{2}$$

Note that we do not consider the starting times y_{ij} to be explicitly part of the state, because only completion time $T^{(t)}$ of the last scheduled platoon is necessary for the state to be Markovian.

Furthermore, the state transitions are very simple. Whenever the same lane is served, $a^{(t)} = 0$, we have $c^{(t+1)} = c^{(t)}$, when serving the next lane, $a^{(t)} = 1$, we have $c^{(t+1)} = c^{(t)} + 1 \mod K$. The completion time $T^{(t+1)}$ is calculated as follows. To simplify notation, let $i^{(t)} = c^{(t+1)}$ and $j^{(t)} = m_{i^{(t)}}^{(t)} + 1$, then

$$T^{(t+1)} = T^{(t)} + sa^{(t)} + p_{i^{(t)}j^{(t)}}, \tag{3}$$

where s is the switch-over time. Observe that the pair $(i^{(t)}, j^{(t)})$ identifies the platoon that is scheduled in the transition from t to $t + 1$ with starting time given by

$$y_{i^{(t)}j^{(t)}} = T^{(t)} + sa^{(t)}. \tag{4}$$

For the remaining part of the state, with $i \in \{0, \dots, K\}$, we always have

$$m_i^{(t+1)} = m_i^{(t)} + \mathbb{1}\{i = i^{(t)}\}. \tag{5}$$

Recall that we defined the objective for optimal schedules as minimizing the total delay for platoons. When assuming $n_i < \infty$ for all lanes i, we could decide to only define episodic reward according to total delay in the complete schedule. However, to deal with possibly infinite episodes and to provide the scheduler with earlier reward signals, we define the reward to be

$$r^{(t)} = (y_{i^{(t)}j^{(t)}} - r_{i^{(t)}j^{(t)}}) \cdot p_{i^{(t)}j^{(t)}}. \tag{6}$$

As we indicated above, the exact arrivals $\{(p_{ij}, l_{ij})_{j=1}^{\infty}\}_{i=1}^{K}$ can be considered a fixed part of the specification of a particular instance of the MDP. Therefore, we regard it as a constant part of the state. When the scheduler has no access to these arrivals in any way, it is not very interesting to solve the MDP. Therefore, we should further specify what information is available to the scheduler. By defining the exact *observations* that become available to the scheduler, we obtain a Partially Observable Markov Decision Process (POMDP).

We assume that the scheduler does not know about all arrivals upfront, but sees a certain number $h_i^{(t)}$ of next arrivals at lane i. Thefore, the visible *horizon* of lane i at step t consists of the release dates and processing times

$$\mathcal{H}_i^{(t)} = \{(p_{ij}, l_{ij}) : j \in \{m_i^{(t)}, m_i^{(t)} + 1, \dots, m_i^{(t)} + h_i^{(t)}\}\}. \tag{7}$$

Therefore, the observation at step t is deterministic and given by

$$o^{(t)} = \{\mathcal{H}_i^{(t)}\}_{i=1}^{K}. \tag{8}$$

In general, the length $h_i^{(t)}$ of the horizon may depend on the arrival times. This would be the case when we want to model a fixed look-ahead time t_a. However, in what follows, we will assume that $h_i^{(t)} = h$ for some fixed h. This makes the implementation simpler, because the vector that encodes the horizon has fixed dimensions in this case, so we do not have to implement some sort of *masking*.

3 Deep Q-Learning

Let the *total discounted return* from step t be defined as

$$G_t = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}, \tag{9}$$

where γ is the *discount factor*. The state-action value $q_\pi(s, a)$ is defined as the expected total discounted return by taking action a in state s and following π afterwards, so we have

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a], \tag{10}$$

for all $s \in \mathcal{S}$ and $a \in \mathcal{A}$. These values are also refered to as *q-values*. It turns out that all optimal policies share the same q-values, so the optimal state-action value function can be defined as

$$q_*(s, a) = \max_\pi q_\pi(s, a). \tag{11}$$

It can be shown that

$$q_*(s, a) = \mathbb{E}\left[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1}, a')\right], \tag{12}$$

which is known as the *Bellman equation*.

λ	exact ($g = 0.1$)	dqn ($h = 10$)
2	-705	-1012
3	-445	-375
4	-258	-251
5	-185	-201

Table 1: Episodic reward (rounded to integers) for each of the four scenarios for both methods. The results for the exact approach are averaged over 100 randomly generated instances. The results for the DQN method are obtained from a single run (consisting of multiple episodes).

The main goal of Q-learning is to estimate the optimal q-values. This is done using the following *temporal difference update*

$$\hat{q}(s_t, a_t) \leftarrow \hat{q}(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a) - \hat{q}(s_t, a_t)]. \tag{13}$$

This method is an *off-policy* method, because it can be shown that $\hat{q}$ will converge to q_* regardless of the policy π that is being followed, as long as all states are visited infidelity often.

3.1 Function approximation

3.2 Experience replay

4 Experiments

Assume we have two lanes, with fixed switch-over time $s = 2$ and a fixed number of arrivals $n = 30$ for each lane. We assume that $p_{ij} \sim \text{Uni}[1, 3]$. Consider four scenarios with $\lambda_1 = 2, \lambda_2 = 3, \lambda_3 = 4, \lambda_4 = 5$.

We ue the commercial Gurobi solver to obtain an estimate for the optimal expected objective. More precisely, we estimate

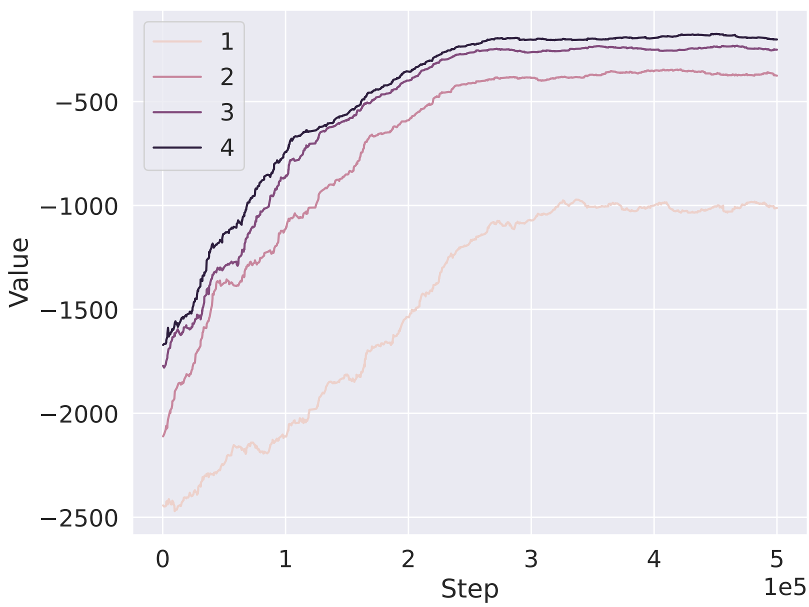
$$\mathbb{E}\left[\sum_j C_j\right], \tag{14}$$

where the expectation is given over the distribution of the arrival process

$$\{(p_{ij}, l_{ij})_{j=1}^{\infty}\}_{i=1}^{K} \tag{15}$$

as defined above. To estimate this quantity, we solve **100** randomly generated instances with an optimality gap of $g = 0.1$.

For the DQN, we use a fixed horizon of $h = 10$. The smoothed return obtained by a **single run** of the DQN algorithm for each scenario is shown in Figure 4.


1
2
3
4
−500
−1000
−1500
−2000
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Value
0
1
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1e5