

# Fingerprints of a Non-Inflationary Universe from Massive Fields



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and

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Majorana-Raychaudhuri Seminar  
November 8, 2024

Mostly based on joint work with

Xingang Chen



and

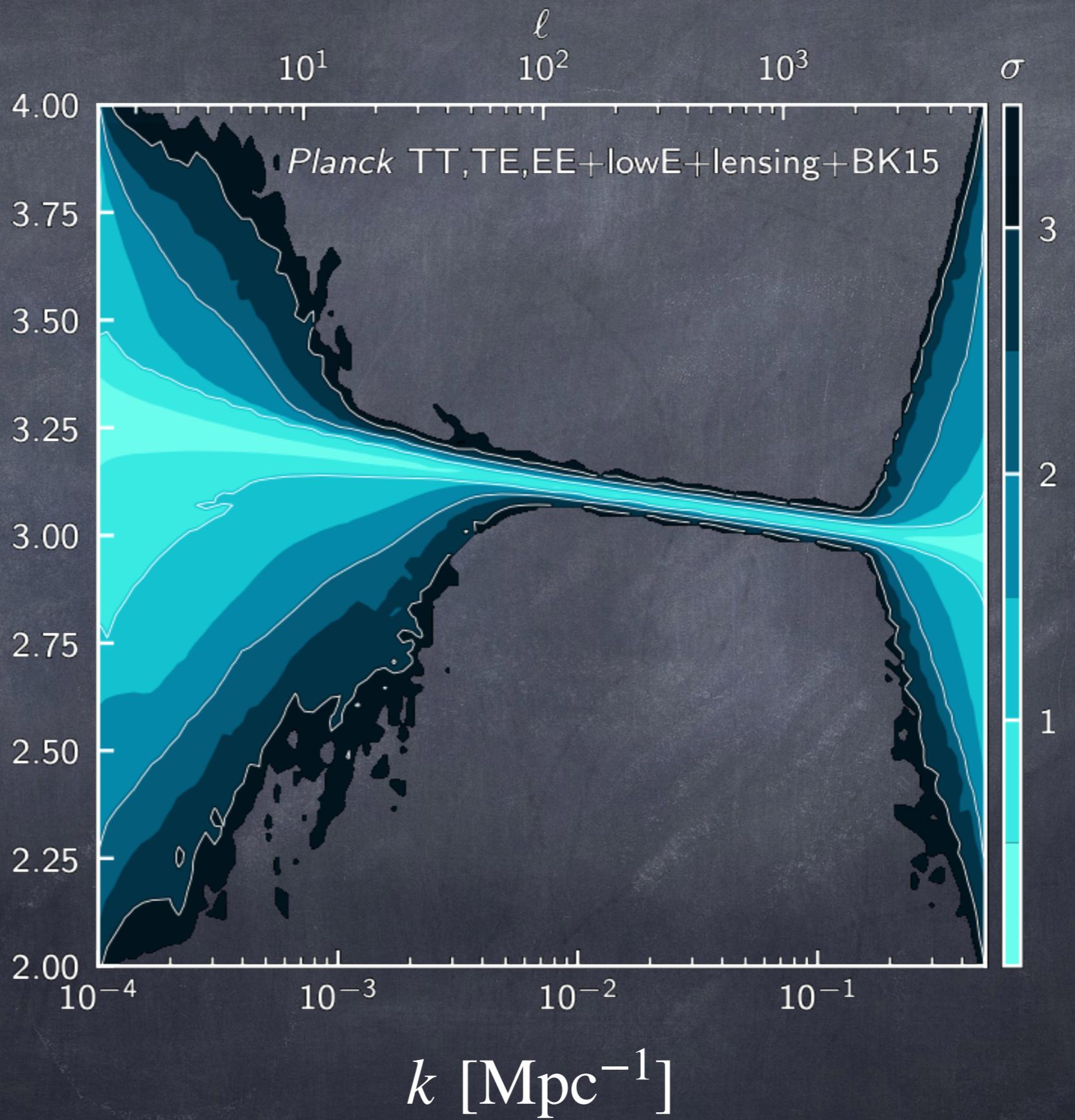
Reza Ebadi



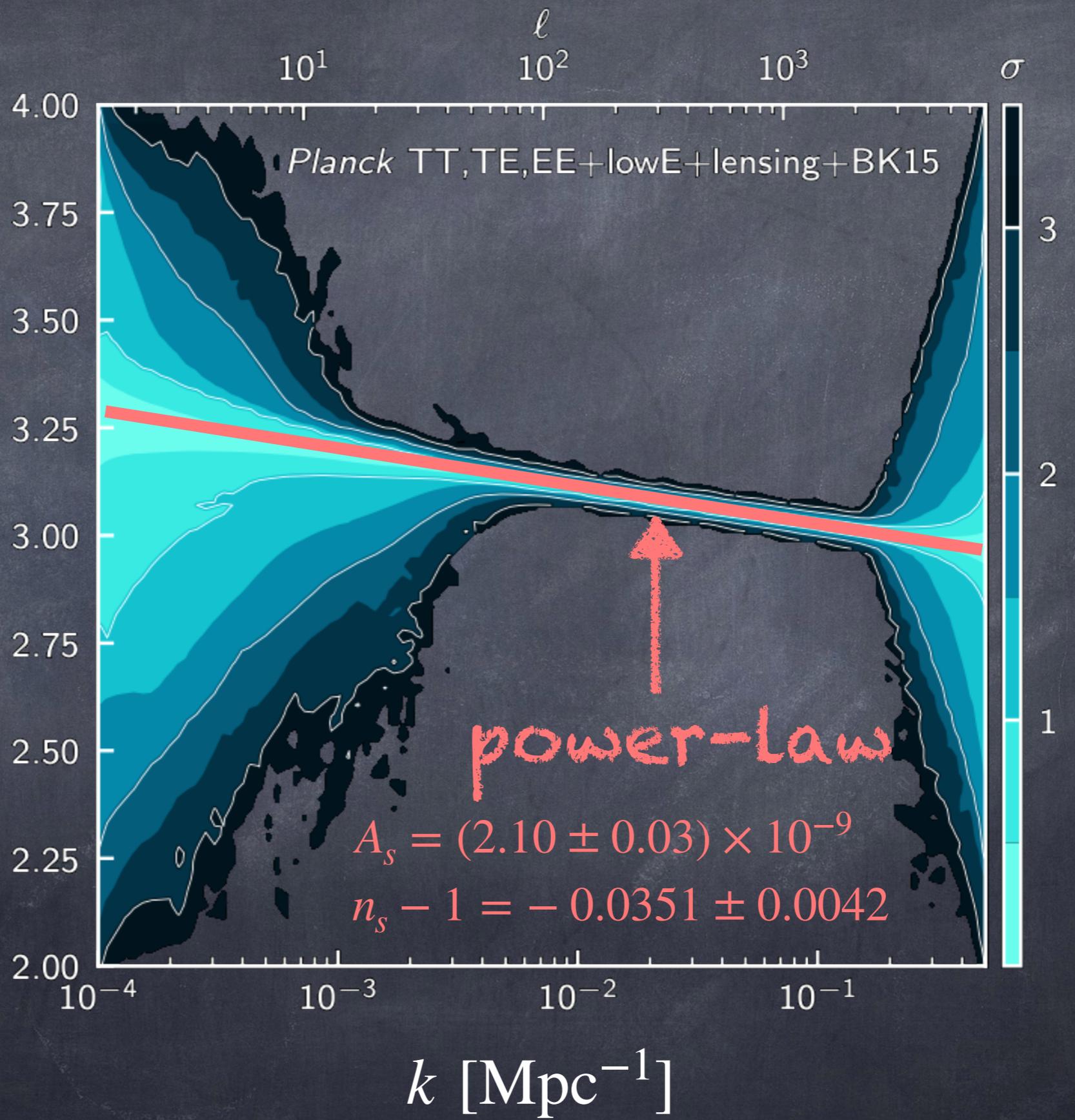
JCAP09(2024)026, arXiv:2405.11016

Part 1  
What we know of  
the very early universe

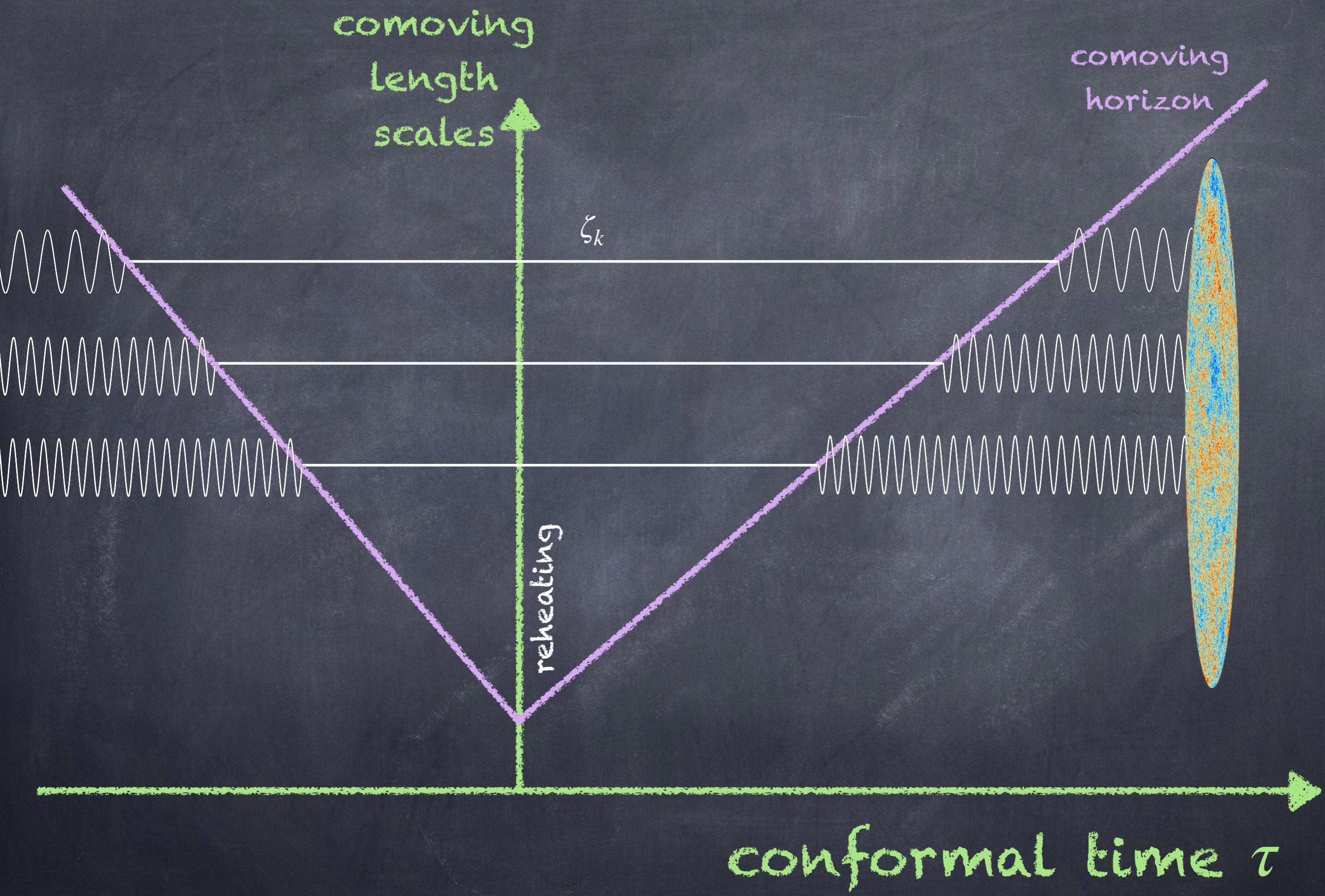
$$\ln \left( 10^{10} \mathcal{P}_\zeta \right)$$



$$\ln \left( 10^{10} \mathcal{P}_\zeta \right)$$



Part 2a  
How do we explain this?  
(scenarios for the very early universe)



$$\text{Comoving horizon} = \frac{1}{a|H|}, \quad H \equiv -\frac{\dot{a}}{a}$$

Shrinking comoving horizon:  $\frac{d}{d\tau} \left( \frac{1}{a|H|} \right) < 0$

$$\frac{d}{d\tau} \left( \frac{1}{a|H|} \right) = \begin{cases} -(1-\epsilon) & (\text{if } H > 0) \\ 1-\epsilon & (\text{if } H < 0) \end{cases} \stackrel{!}{<} 0 \iff \begin{cases} w < -\frac{1}{3} & \text{and } H > 0 \\ w > -\frac{1}{3} & \text{and } H < 0 \end{cases}$$

$$1-\epsilon = 1 + \frac{\dot{H}}{H^2} \stackrel{\text{Friedmann}}{=} -\frac{1}{2}(1+3w), \quad w \equiv \frac{P}{\rho}$$

$$\text{Comoving horizon} = \frac{1}{a|H|}, \quad H \equiv -\frac{\dot{a}}{a}$$

$$\text{Shrinking comoving horizon: } \frac{d}{d\tau} \left( \frac{1}{a|H|} \right) < 0$$

$$\iff \begin{cases} w < -\frac{1}{3} \text{ and } H > 0 \\ w > -\frac{1}{3} \text{ and } H < 0 \end{cases} \iff \begin{cases} \ddot{a} > 0 \text{ and } \dot{a} > 0 \\ \ddot{a} < 0 \text{ and } \dot{a} < 0 \end{cases}$$

$\iff$   $\begin{cases} \text{expanding and inflating} \\ \text{contracting and decelerating} \end{cases}$

$$\text{Comoving horizon} = \frac{1}{a|H|}, \quad H \equiv -\frac{\dot{a}}{a}$$

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$\iff$   $\begin{cases} \text{expanding and inflating} \\ \text{contracting and decelerating} \end{cases}$



inflationary  
cosmology

$$\text{Comoving horizon} = \frac{1}{a|H|}, \quad H \equiv -\frac{\dot{a}}{a}$$

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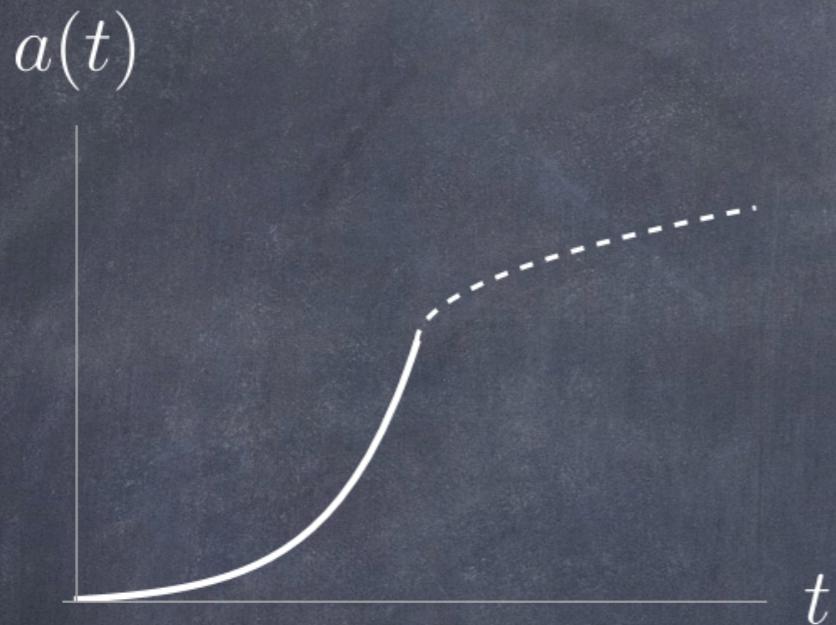
$$\text{Shrinking comoving horizon: } \frac{d}{d\tau} \left( \frac{1}{a|H|} \right) < 0$$

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$\iff$   $\begin{cases} \text{expanding and inflating} \\ \text{contracting and decelerating} \end{cases}$

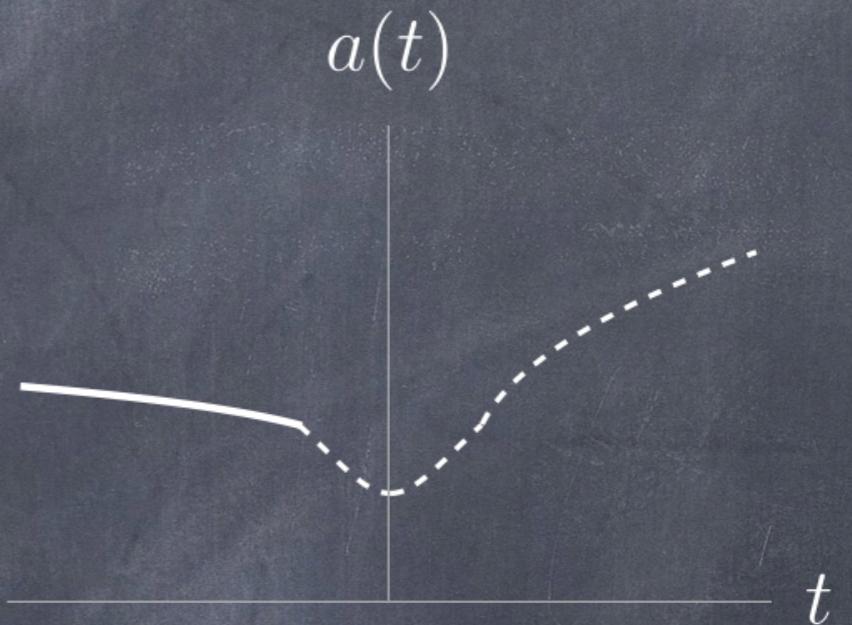
(typically the same requirement for the flatness problem)

## inflationary cosmology



Guth (1980),  
...  
...

## contracting cosmology (before a bounce)



Gasperini-Veneziano (1993),  
Khoury-Ovrut-Steinhardt-Turok (2001),  
JQ+ (2014, ...),  
...

Part 2b  
What about the perturbations?

GR/modified gravity + single/multifield  
canonical scalar/k-essence/Horndeski

$$\implies S_s^{(2)} = \frac{1}{2} \int d^3x d\tau \left( (v'_s)^2 + \frac{z''}{z} v_s^2 - (\partial_i v_s)^2 \right)$$

$$\frac{z''}{z} = \frac{\nu^2 - 1/4}{\tau^2}, \quad \nu \approx \frac{3}{2}$$

$$v_s = -\frac{\sqrt{-\pi\tau}}{2} H_\nu^{(1)}(-k\tau) \stackrel{-k\tau \nearrow 1}{\simeq} \frac{e^{-ik\tau}}{\sqrt{2k}}$$

$$\mathcal{P}_s(k) = \frac{k^3}{2\pi^2} \frac{|v_s|^2}{z^2} \stackrel{-k\tau \searrow 1}{\sim} k^{3-2\nu}$$

Couple examples

(GR + BD + single minimally coupled scalar field of constant  $\epsilon$  and constant  $c_s$ )

$$\frac{z''}{z} = \frac{a''}{a} \stackrel{!}{\approx} \frac{2}{\tau^2} \quad \Leftrightarrow \quad a(\tau) \sim \tau^{-1} \quad \text{or} \quad \tau^2$$

# Couple examples

(GR + BD + single minimally coupled scalar field of constant  $\epsilon$  and constant  $c_s$ )

de Sitter exponential expansion  
(fast expansion)

$$\frac{z''}{z} = \frac{a''}{a} \stackrel{!}{\approx} \frac{2}{\tau^2} \quad \Leftrightarrow \quad a(\tau) \sim \tau^{-1} \quad \text{or} \quad \tau^2$$



# Couple examples

(GR + BD + single minimally coupled scalar field of constant  $\epsilon$  and constant  $c_s$ )

de Sitter exponential expansion  
(fast expansion)

$$\frac{z''}{z} = \frac{a''}{a} \stackrel{!}{\approx} \frac{2}{\tau^2} \quad \Leftrightarrow \quad a(\tau) \sim \tau^{-1} \quad \text{or} \quad \tau^2$$



matter-dominated contraction  
(fast contraction)



Part 2c  
What scenarios are consistent?

inflation

matter  
bounce

genesis

ekpyrosis

scale factor

expands

contracts

expands

contracts

"speed"

fast

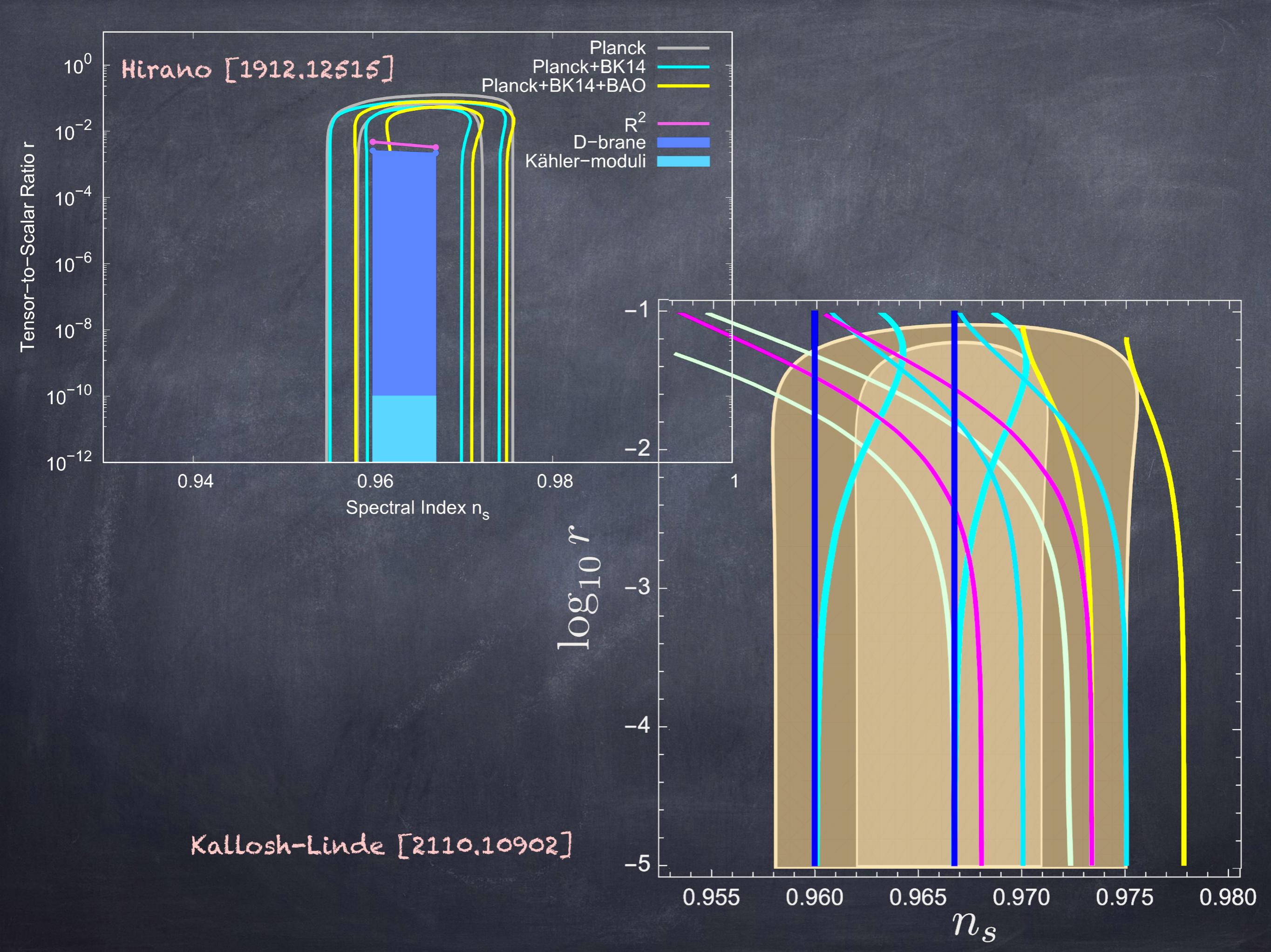
fast

slow

slow

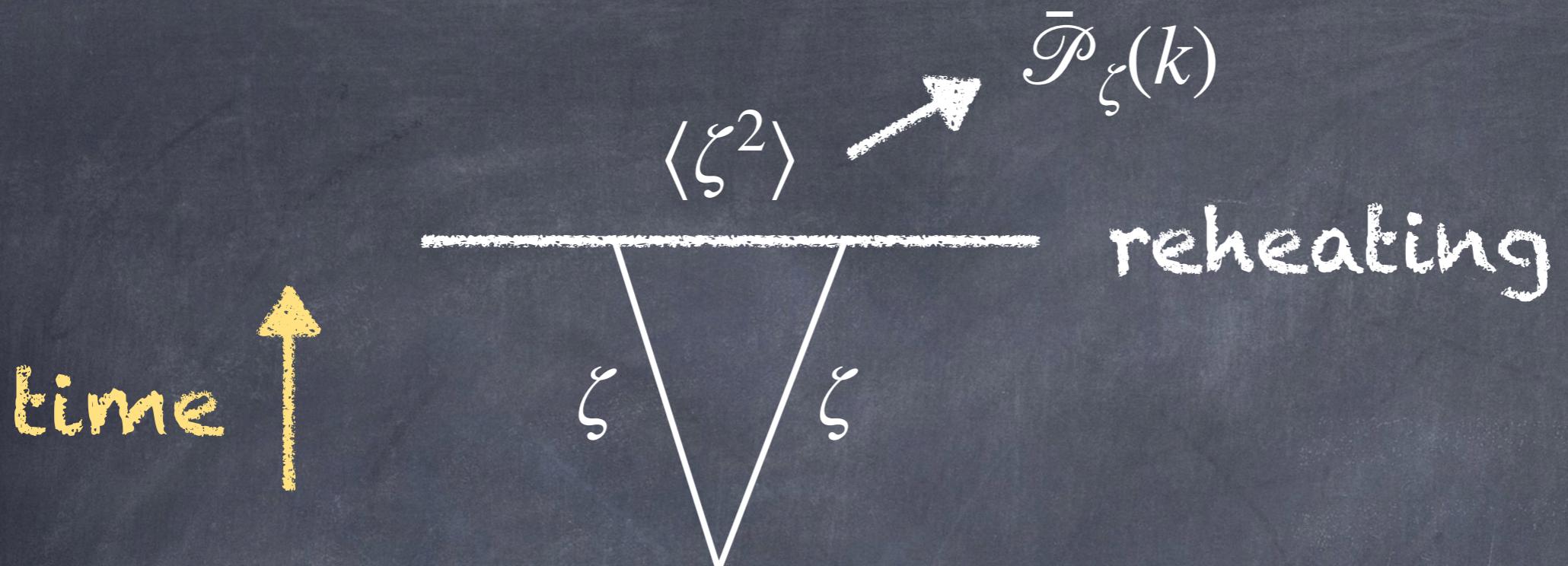
More in the supplementary material

$$\left. \begin{array}{l}
\langle \zeta^2 \rangle \quad \left\{ \begin{array}{l} A_s = (2.10 \pm 0.03) \times 10^{-9} \\ n_s - 1 = -0.0351 \pm 0.0042 \end{array} \right. \\
\langle h_{ij}^2 \rangle \quad r = \frac{A_t}{A_s} < 0.036 \text{ (95 \% CL)} \\
\langle \zeta^3 \rangle \quad \left\{ \begin{array}{l} f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \\ f_{\text{NL}}^{\text{equil}} = -26 \pm 47 \\ f_{\text{NL}}^{\text{ortho}} = -38 \pm 24 \end{array} \right. \\
\langle \zeta^4 \rangle \quad g_{\text{NL}}^{\text{local}} = (-5.8 \pm 6.5) \times 10^4 \\
\quad \quad \quad \dots
\end{array} \right\} \quad \text{all consistent with zero}$$

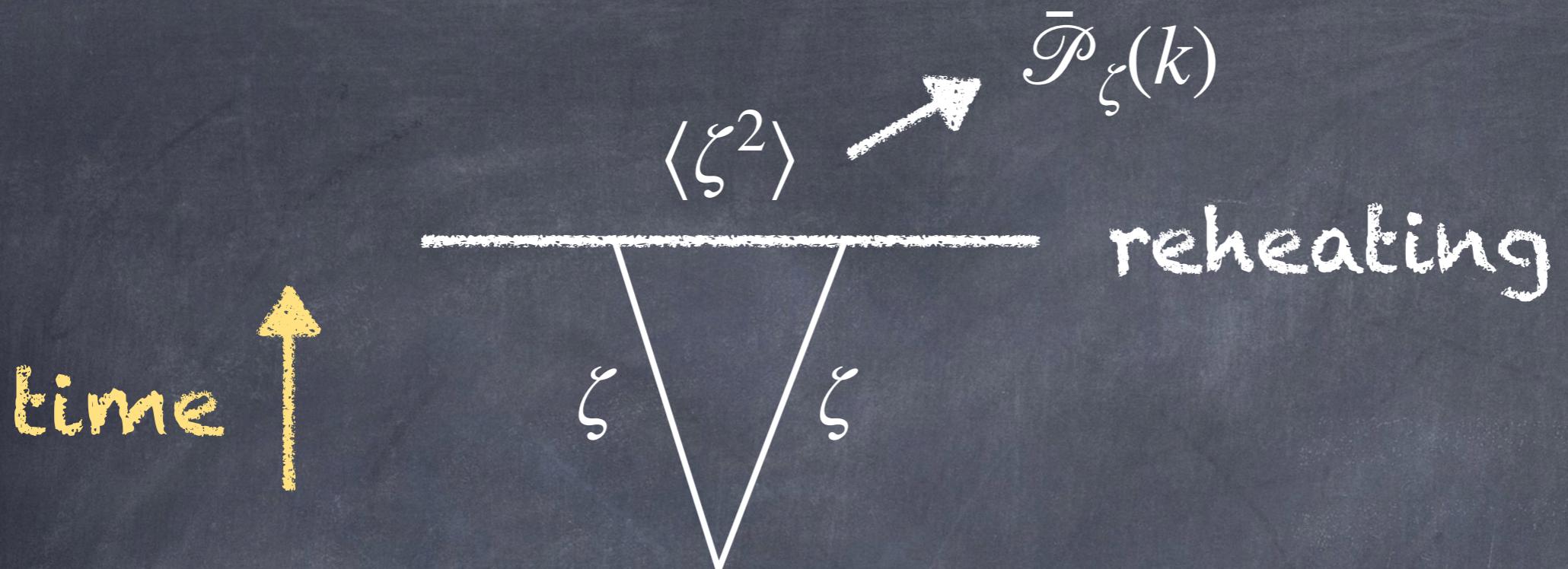


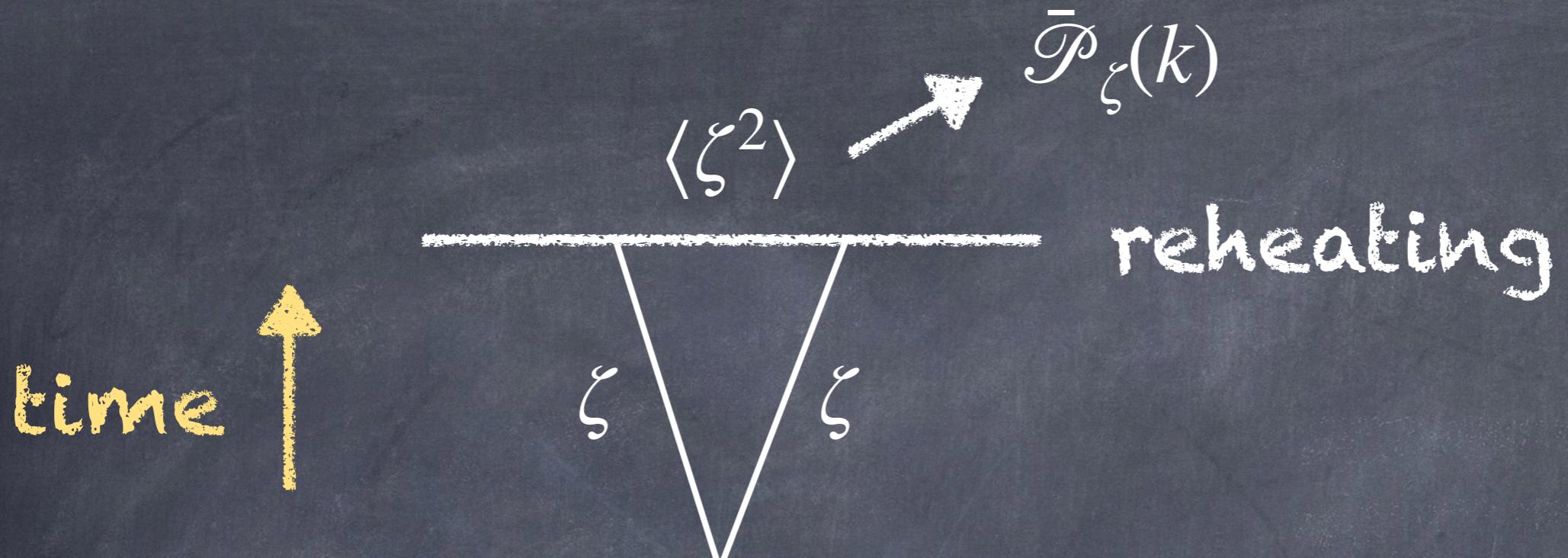
## Part 3

We want to do better and  
have more to look after!  
(add massive fields)

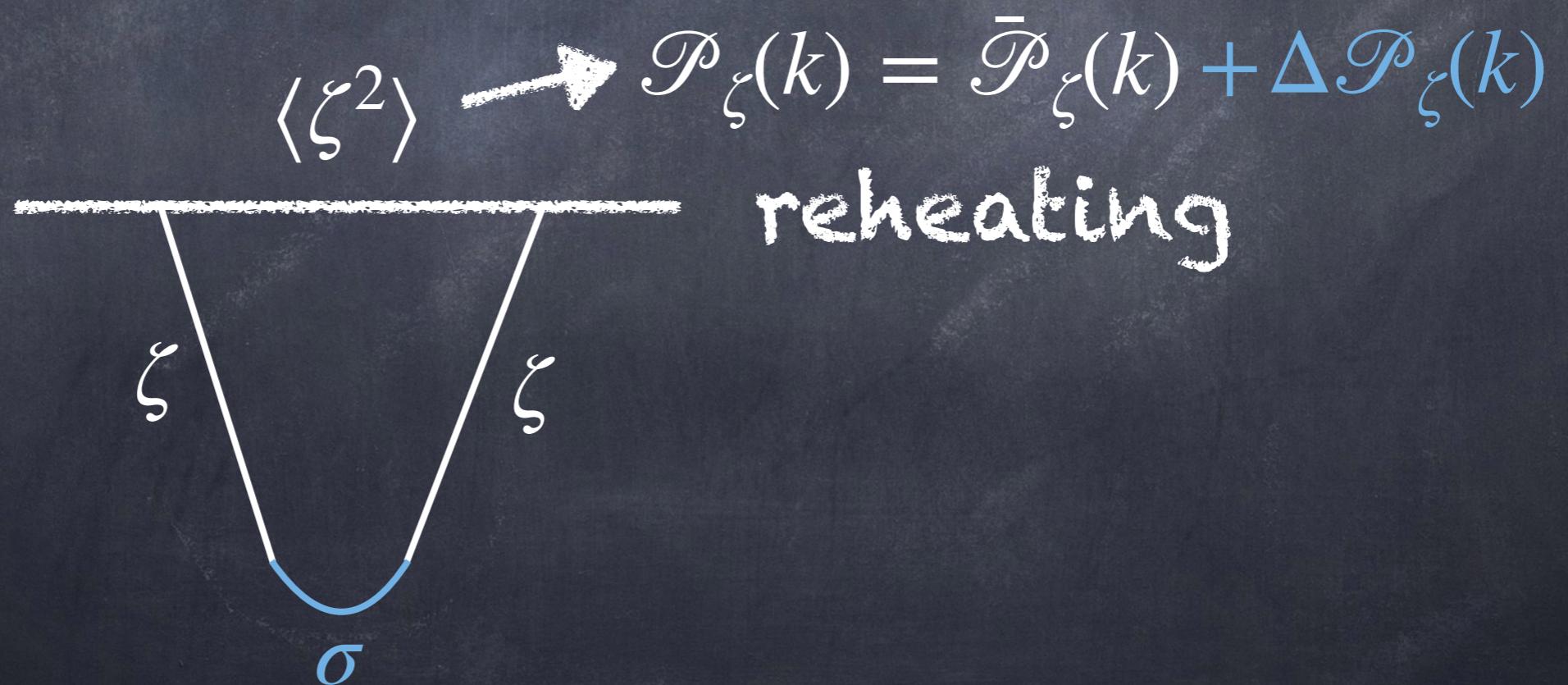


$$\langle \zeta \zeta \rangle \longrightarrow \langle \text{in} | \text{in} \rangle$$





Add interaction with massive field  $\sigma$



## Quantum vacuum:

$$ds^2 = a(\tau)^2(-d\tau^2 + d\vec{x}^2)$$

large  $k$   
early time  $\Rightarrow \partial_\tau^2 \zeta_k + k^2 \zeta_k = 0 \Rightarrow \zeta_k(\tau) \sim e^{-ik\tau}$

## Massive spectator fields:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

large  $m_\sigma$   
early time  $\Rightarrow \partial_t^2 \sigma + m_\sigma^2 \sigma = 0 \Rightarrow \sigma(t) \sim e^{\pm im_\sigma t}$

$$a d\tau = dt$$

in-in:

free theory vacuum

$$\langle \Omega | \hat{\zeta}^2 | \Omega \rangle = \langle 0 | (\bar{T} e^{i \int d\tau \mathcal{H}_{\text{int}}}) \hat{\zeta}^2 (T e^{-i \int d\tau \mathcal{H}_{\text{int}}}) | 0 \rangle$$



interaction  
vacuum

$$= \underbrace{\langle 0 | \hat{\zeta}^2 | 0 \rangle}_{\bar{\mathcal{P}}_{\zeta}(k)} + 2\text{Im} \underbrace{\langle 0 | \hat{\zeta}^2 \int d\tau \mathcal{H}_{\text{int}} | 0 \rangle}_{\Delta \mathcal{P}_{\zeta}(k)} + \dots$$

$$\Rightarrow \frac{\Delta \mathcal{P}_{\zeta}}{\bar{\mathcal{P}}_{\zeta}} \sim 2\text{Im} \int d\tau \sigma(\partial \zeta)^2$$

$$\int d\tau e^{\pm i m_{\sigma} t} e^{-2ik\tau}$$

$$\int d\tau e^{\pm im_\sigma t} e^{-2ik\tau} \longrightarrow$$

highly oscillatory  
integrals average  
to zero

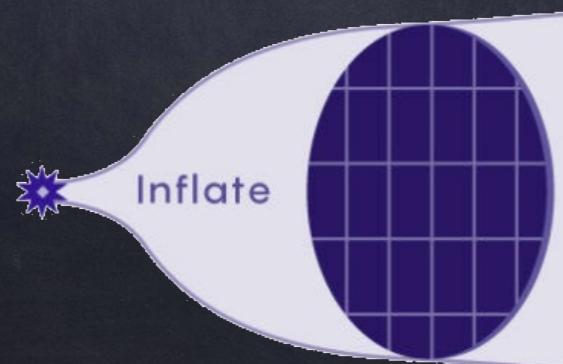
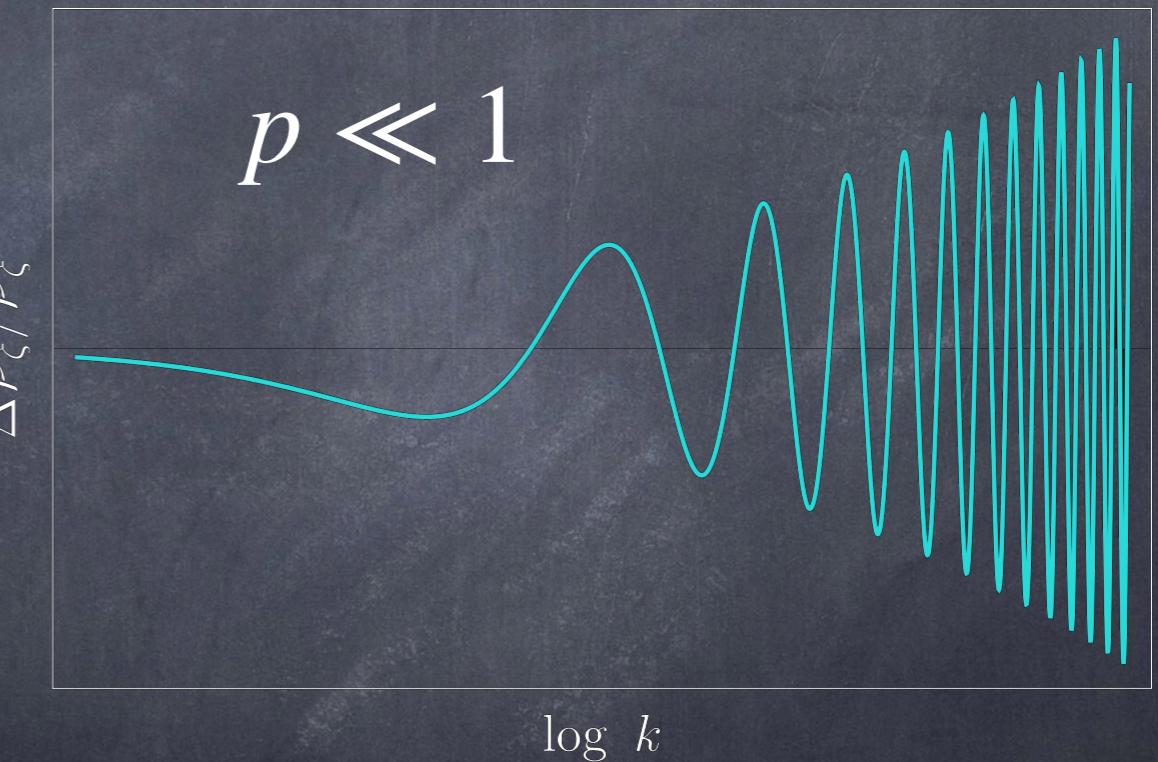
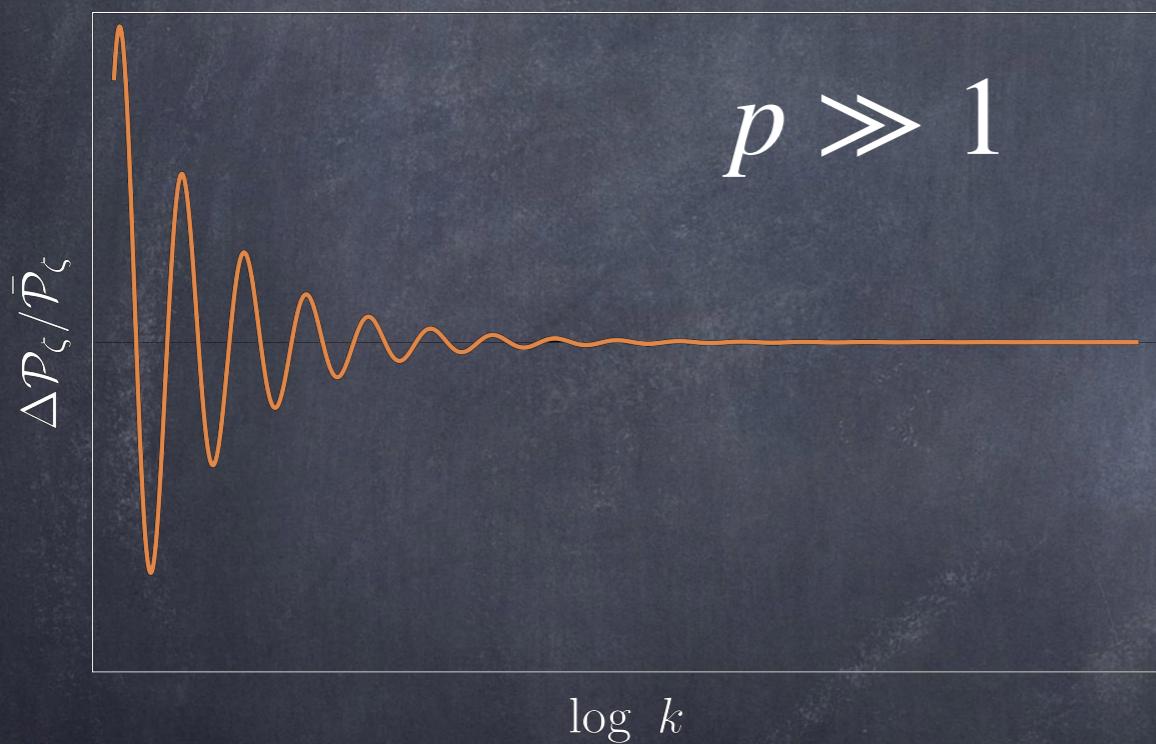
heavy fields can usually be  
integrated out in low-energy  
effective field theories

Except for resonance  
(saddle point) when

$$\frac{d}{d\tau} (\pm im_\sigma t - 2ik\tau) \Bigg|_{\tau=\tau_{\text{res}}} = 0 \Rightarrow m_\sigma = \frac{2k}{a(\tau_{\text{res}})}$$

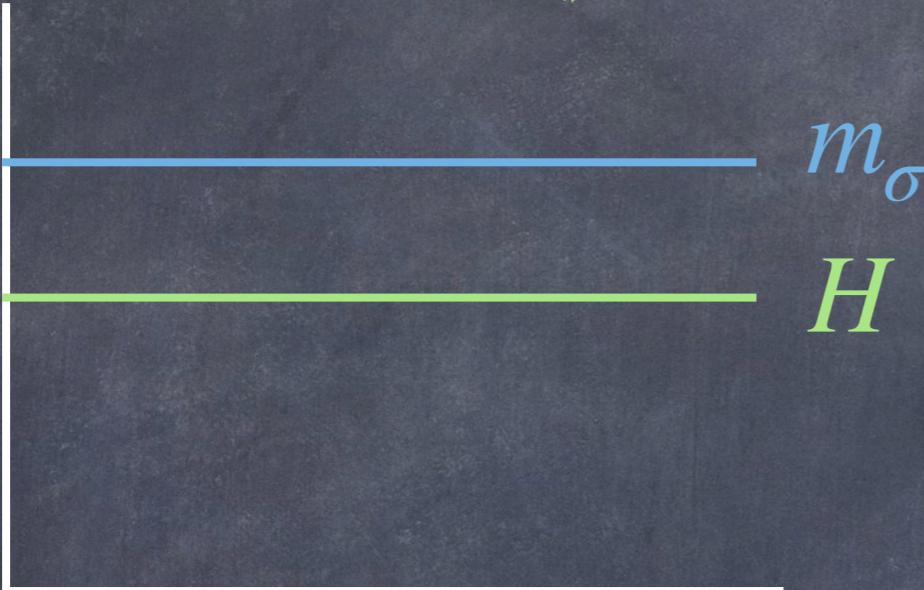
$$a(t) \propto |t|^p$$

$$\frac{\Delta \mathcal{P}_\zeta}{\bar{\mathcal{P}}_\zeta} \sim \text{Im} \int d\tau e^{\pm im_\sigma t} e^{-2ik\tau} \sim k^{-\frac{3}{2} + \frac{1}{2p}} \sin(k^{1/p})$$



inflationary  
cosmology

evergreen



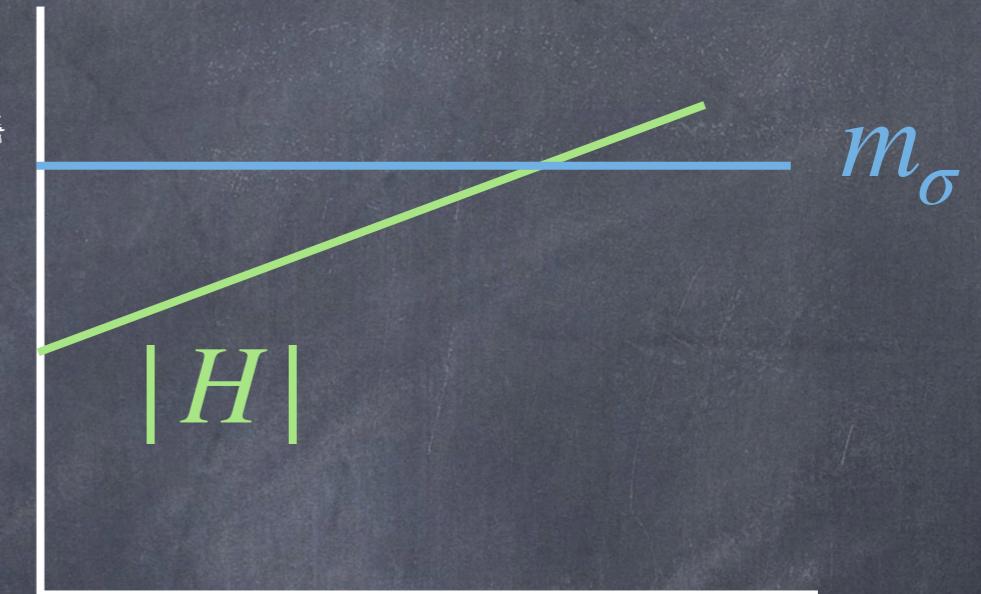
time

cosmological  
collider

Chen and Wang (2009),  
Arkani-Hamed and Maldacena (2015),  
...  
...

contracting  
cosmology  
(before a bounce)

evergreen

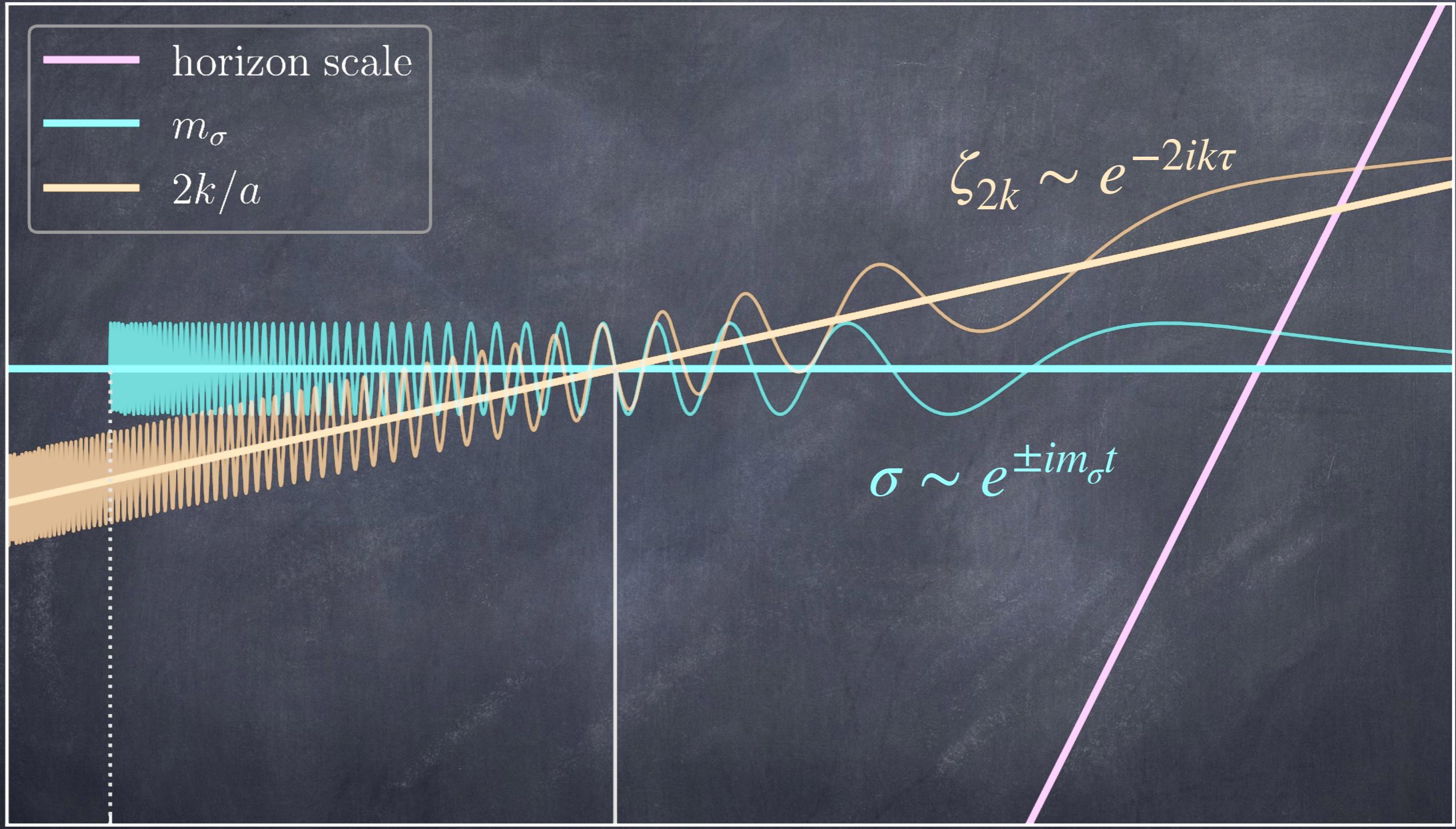


time

particle  
scanner

Chen-Loeb-Xianyu (2019)

physical energy scales



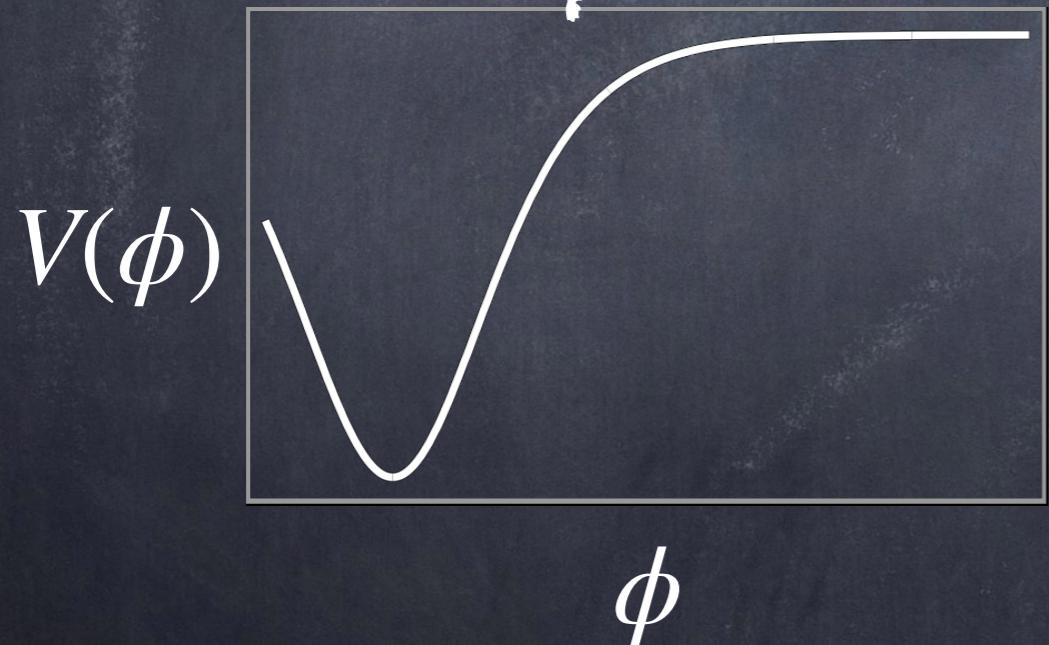
# Part 4a

## Modelling massive fields

We do not know what degrees of freedom there are at high energies...

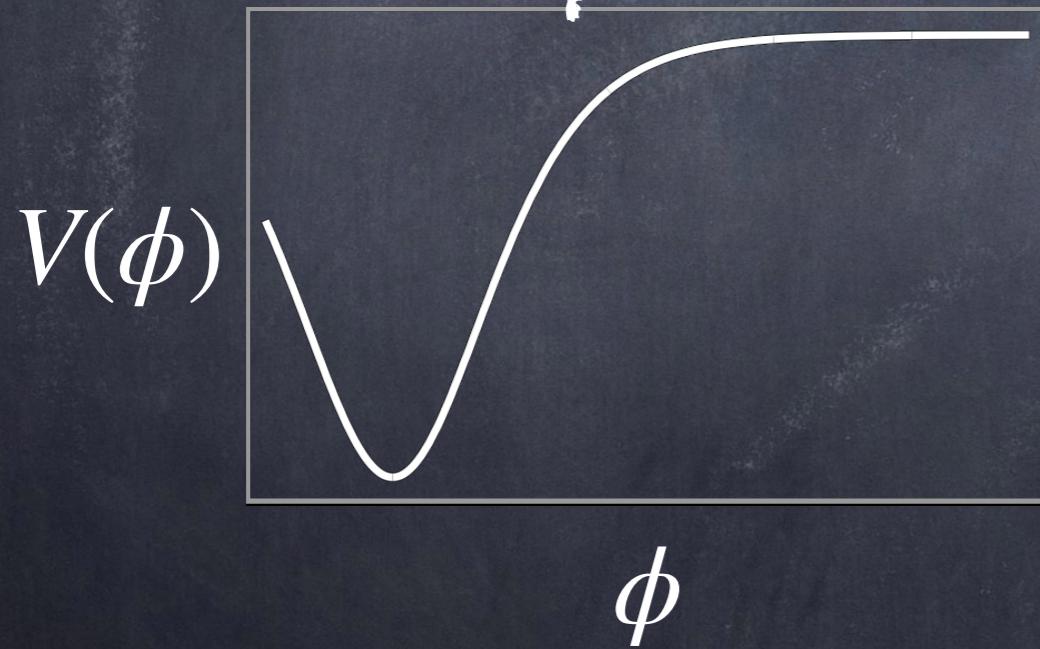
We do not know what degrees of freedom there are at high energies...

We hope for...

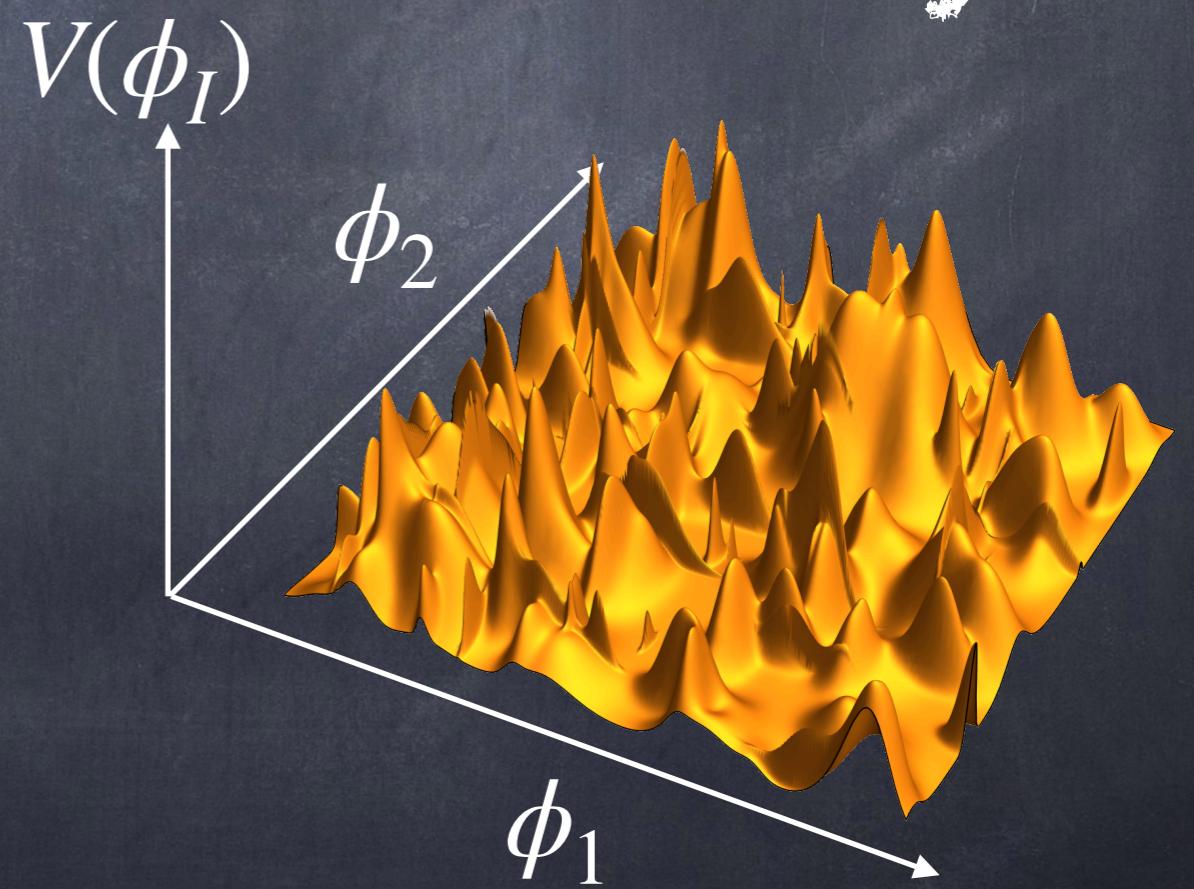


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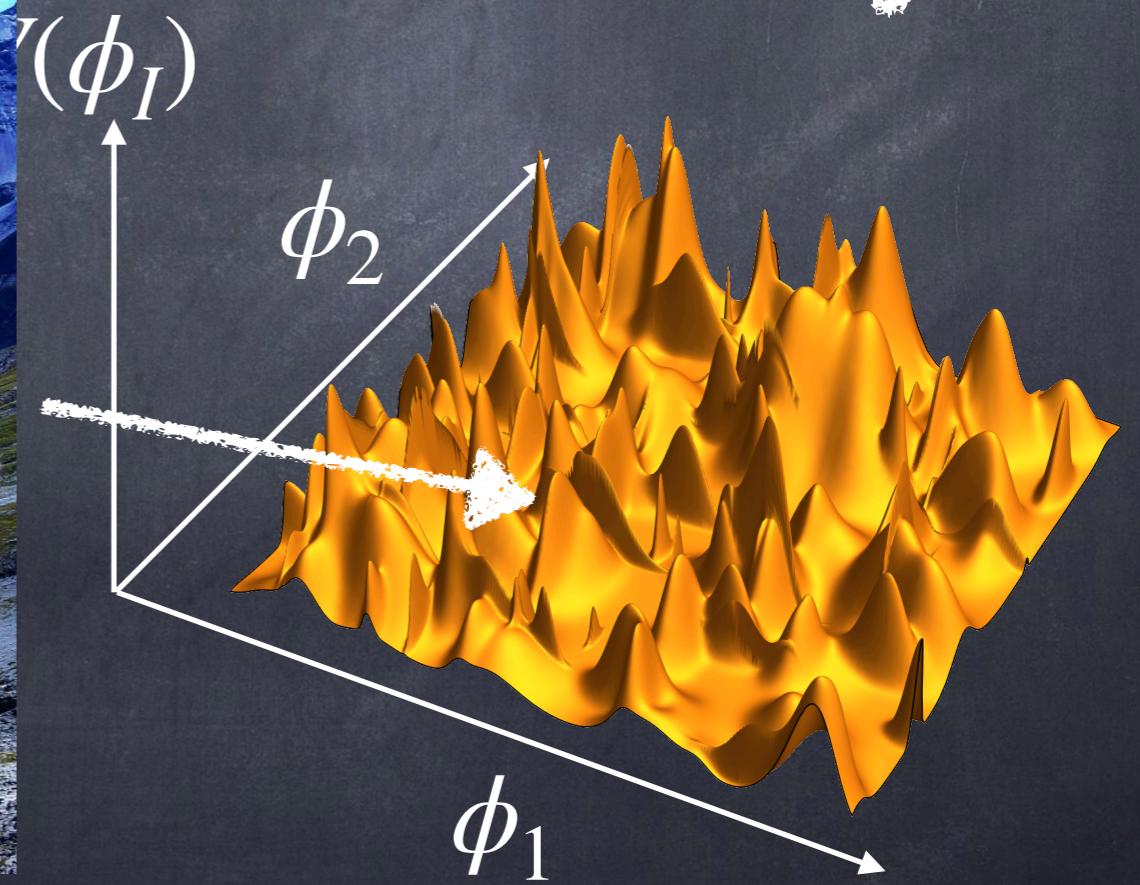
But the reality...



We do not know what degrees of freedom there are at high energies...

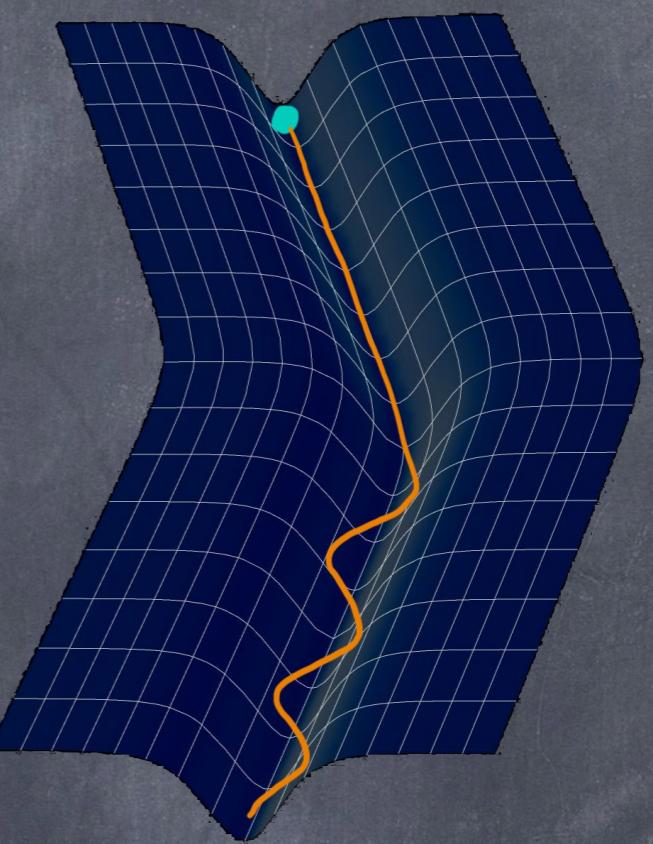
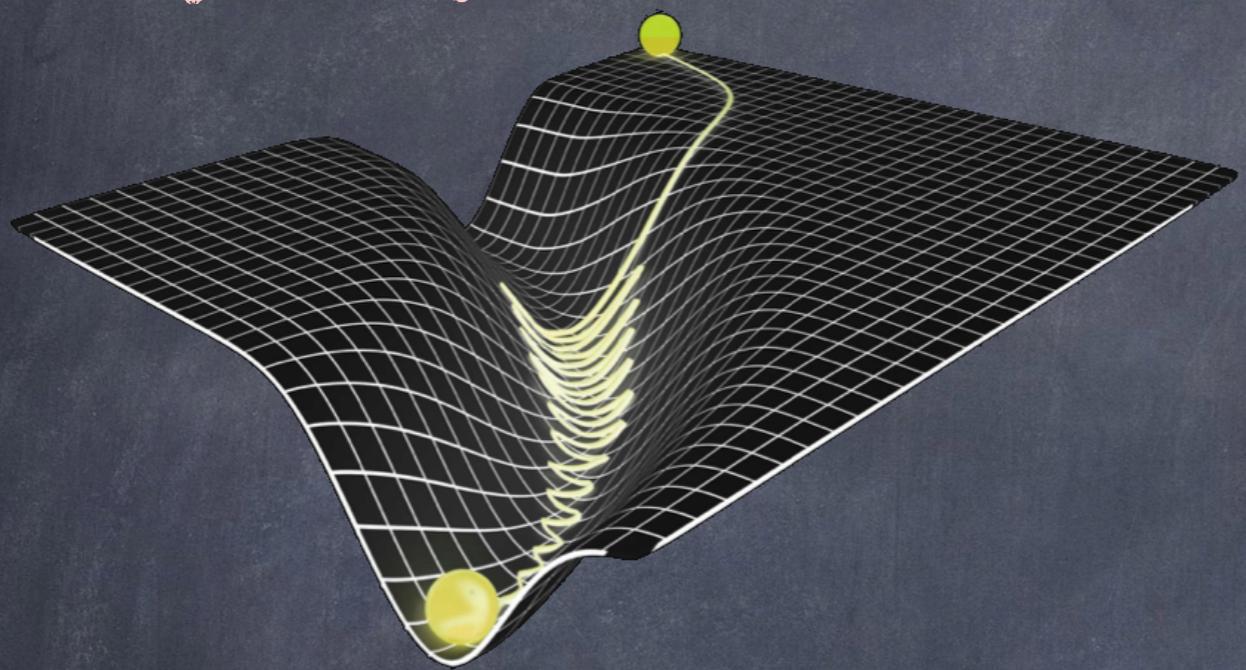


But the reality...



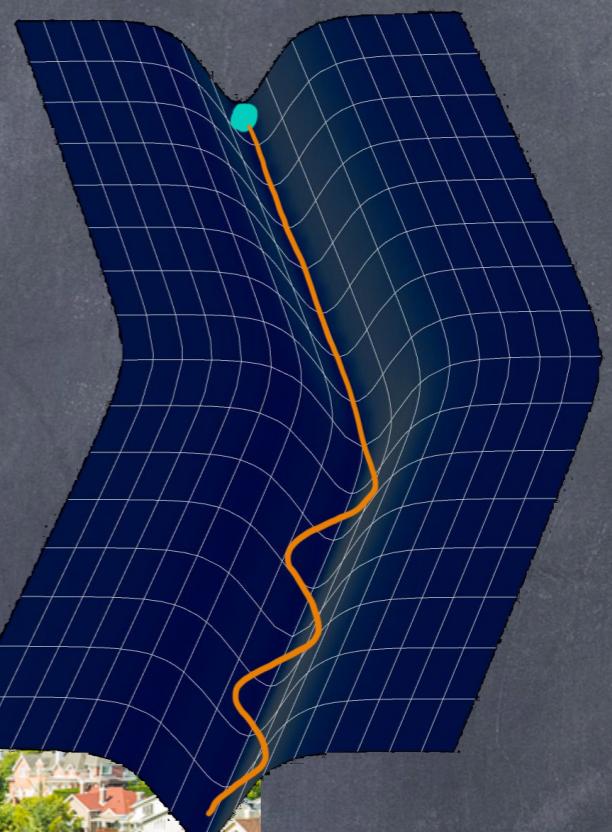
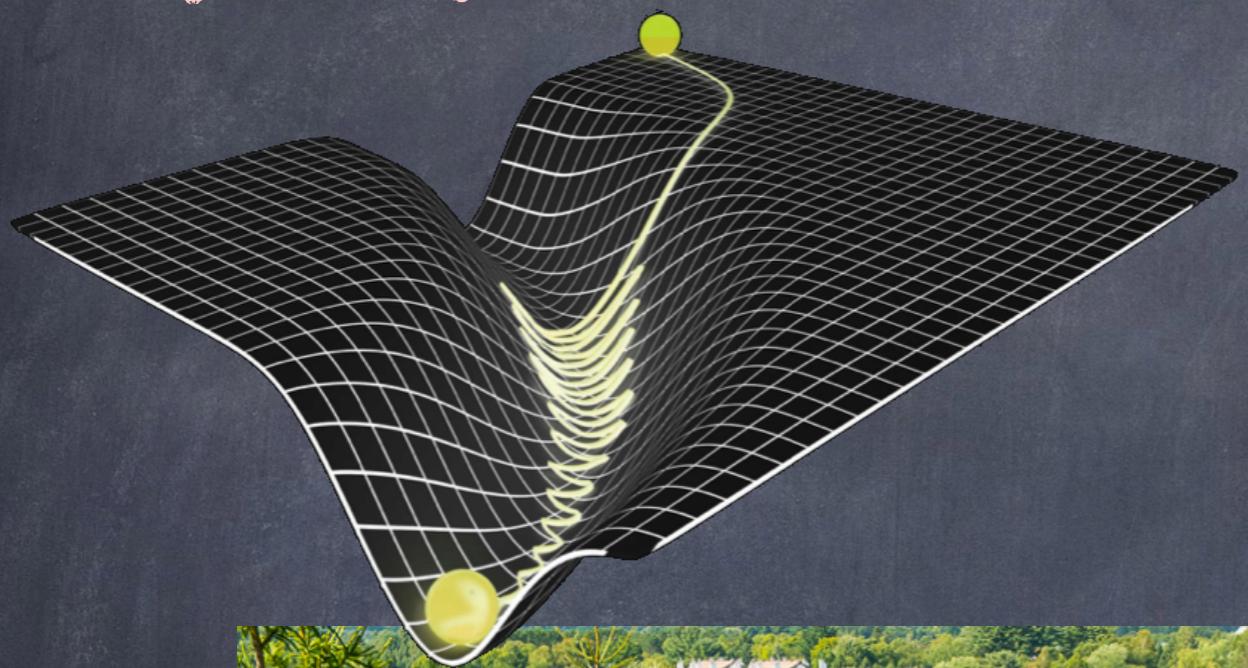
JQ-Chen-Ebadi (2024)

Chen-Namjoo-Wang (2015)

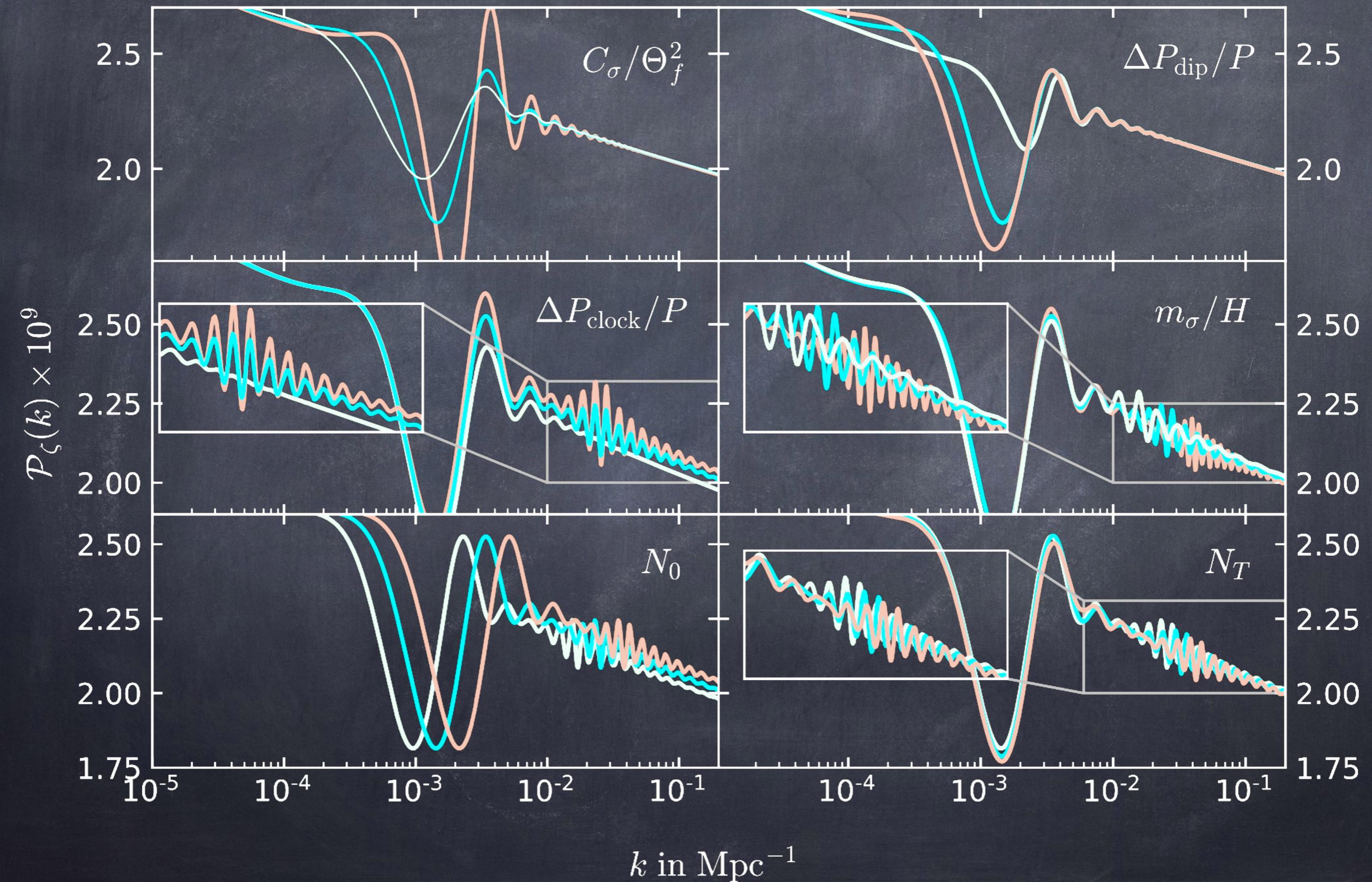


JQ-Chen-Ebadi (2024)

Chen-Namjoo-Wang (2015)



# In inflation...



# Part 4b

## First realization in a non-inflationary alternative: ekpyrotic scenario (slow contraction)

first 2 slides can be skipped in the interest of time

# Vanilla ekpyrosis

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}(\partial\phi)^2 + V_0 e^{-\sqrt{2/p}\phi} - \frac{1}{2}e^{-\sqrt{2/b}\phi}(\partial\chi)^2$$

$$(V_0 > 0, \quad 0 < p < 1/3, \quad b \approx p)$$

$$\delta_\phi \mathcal{L} = 0 \quad \Rightarrow \quad a(t) \propto (-t)^p \quad (t < 0)$$

$$\delta_\chi \mathcal{L} = 0 \quad \Rightarrow \quad n_s - 1 \approx 2 \left( 1 - \sqrt{\frac{p}{b}} \right)$$

isocurvature to curvature perturbation conversion

$$\mathcal{P}_\chi(k) \xrightarrow{\text{bounce}} \mathcal{P}_\zeta(k)$$

Battarra+ (2014)  
Fertig+ (2016)  
...

# Implementing a classically excited massive scalar field

Add to  $\mathcal{L}$ :

$$-\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2$$

And change:  $(\partial\phi)^2 \longrightarrow \left(1 + \Xi(\phi)\frac{\sigma}{\varrho}\right)(\partial\phi)^2$

$$(\partial\chi)^2 \longrightarrow \left(1 + \frac{\sigma}{\Lambda}\right)(\partial\chi)^2$$

$\Xi(\phi)$

Step  $\frac{1}{2} \left( 1 - \tanh \left[ \frac{\phi - \phi_0}{\delta} \right] \right)$

Plateau

$$\frac{1}{2} \left( \tanh \left[ \frac{\phi - \phi_e}{\delta} \right] - \tanh \left[ \frac{\phi - \phi_0}{\delta} \right] \right)$$

$$\Delta\phi_\Xi \equiv \phi_0 - \phi_e$$

Bump

$$e^{-(\phi-\phi_0)^2/\delta^2}$$

$\phi_e$

$\phi_0$



$\phi$  evolution

Bump

Plateau

Step

## Part 5 Results (background)

Focus on bump. Other couplings in the extra material.

# Bump

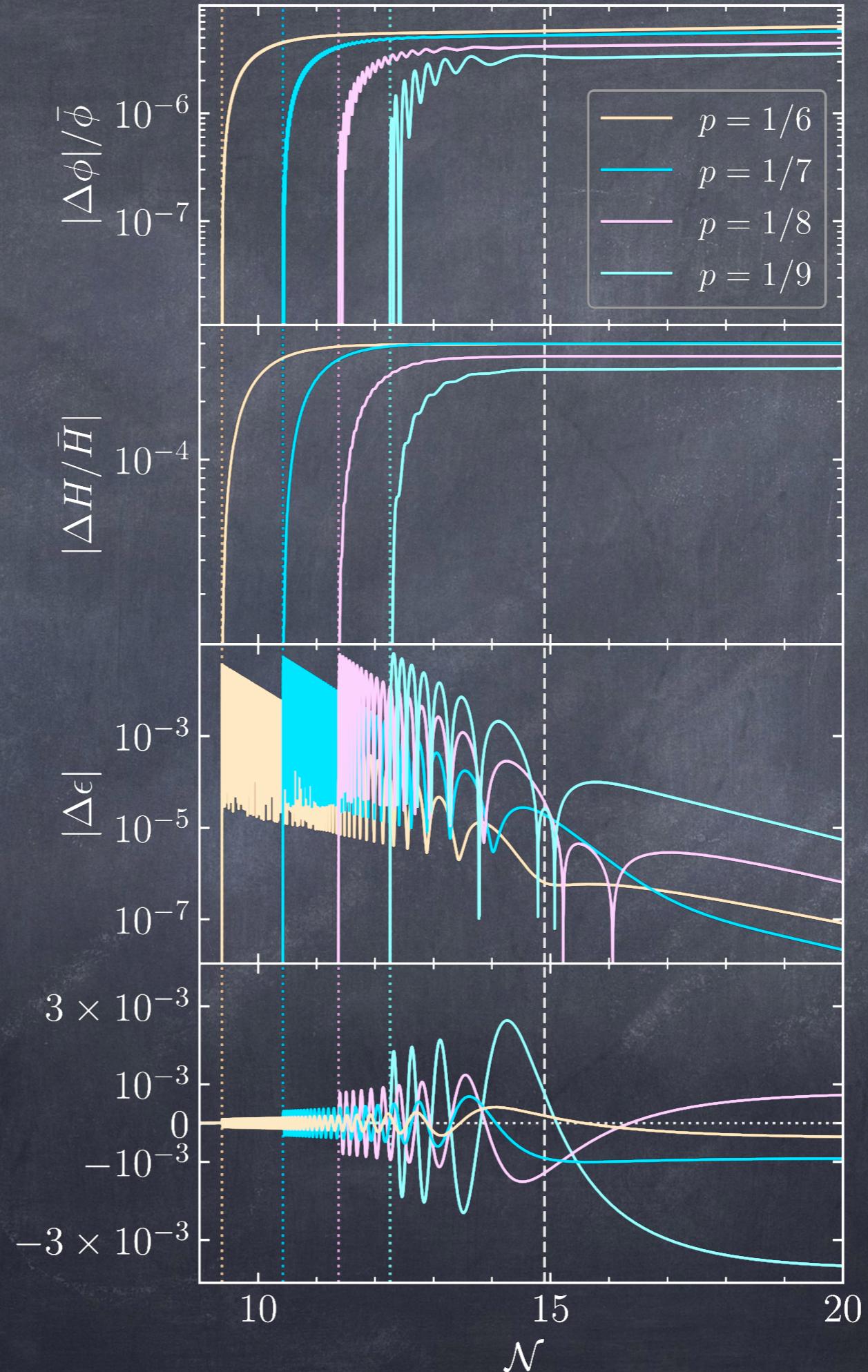
$$\ddot{\sigma} + 3\bar{H}\dot{\sigma} + m_\sigma^2 \sigma = 0$$

$$\ddot{\sigma} + \frac{3p}{t}\dot{\sigma} + m_\sigma^2 \sigma = 0$$

$$\begin{aligned} \sigma(t) &= (-t)^\alpha (c_1 J_\alpha(-m_\sigma t) \\ &\quad + c_2 Y_\alpha(-m_\sigma t)) \end{aligned}$$

$$\alpha = \frac{1 - 3p}{2}$$

$$\begin{aligned} \sigma(t) &\sim \frac{\sin(-m_\sigma t)}{(-m_\sigma t)^{3p/2}} \\ (-m_\sigma t &\gg 1) \end{aligned}$$



$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

$$d\mathcal{N} \equiv d\ln(a|H|)$$

# Part 6a

## Revisiting perturbations

### Analytically

can be skipped in the interest of time

# Perturbations

$$S \supset -\frac{1}{2} \int d^4x \sqrt{-g} \left( 1 + \frac{\sigma}{\Lambda} \right) \Omega^2(\phi) (\partial\chi)^2, \quad \Omega^2(\phi) = e^{-\sqrt{2/b}\phi}$$

$$= \frac{1}{2} \int d^3x d\tau \tilde{z}^2 \left( (\chi')^2 - (\partial_i \chi)^2 \right), \quad \tilde{z}^2 = \left( 1 + \frac{\sigma}{\Lambda} \right) z^2, \quad z^2 = a^2 \Omega^2(\phi)$$

$$\Rightarrow \chi_k'' + \left( \frac{(\bar{z}^2)'}{\bar{z}^2} + 2\Delta \left( \frac{a'}{a} \right) - \sqrt{\frac{2}{b}} \Delta \phi' + \frac{\sigma'/\Lambda}{1 + \sigma/\Lambda} \right) \chi_k' + k^2 \chi_k = 0$$

A diagram illustrating the components of the differential equation. A blue arrow points from the term  $(\bar{z}^2)'/\bar{z}^2$  to the label  $\bar{\mathcal{P}}_s(k)$ . A green bracket underlines the terms  $2\Delta(a'/a)$  and  $\sqrt{2/b} \Delta \phi'$ , which are grouped together and labeled "massive field induced oscillations". A yellow arrow points from the term  $\sigma'/\Lambda / (1 + \sigma/\Lambda)$  to the label "direct coupling to massive field".

$\bar{\mathcal{P}}_s(k)$       massive field induced oscillations      direct coupling to massive field

# Perturbations (interaction picture)

$$\mathcal{L}_0 = \frac{1}{2}z^2 \left( (\chi')^2 - (\partial_i \chi)^2 \right) , \quad \delta \mathcal{L} = \frac{\sigma}{\Lambda} \mathcal{L}_0$$

$$\mathcal{H}_0 = \frac{1}{2}z^2 \left( (\chi'_{\text{int}})^2 + (\partial_i \chi_{\text{int}})^2 \right) , \quad \mathcal{H}_{\text{int}} = -\frac{1}{2} \frac{\sigma}{\Lambda} z^2 \left( (\chi'_{\text{int}})^2 - (\partial_i \chi_{\text{int}})^2 \right)$$

$$\begin{aligned} \frac{\Delta \mathcal{P}_s}{\bar{\mathcal{P}}_s} &= \text{Im} \int_{-\infty}^{\tau_{\text{end}}} d\tau \underbrace{\frac{\sigma}{\Lambda} \bar{z}^2 \left( (\bar{\chi}'_k)^2 - k^2 \bar{\chi}_k^2 \right)}^{*} \\ &= \frac{\pi}{4} k^2 \tau \left( H_\nu^{(1)}(-k\tau)^2 - H_{\nu-1}^{(1)}(-k\tau)^2 \right)^* \end{aligned}$$

# Sharp feature signal

$$\sigma(t) = \sigma_0 \Theta(\tau - \tau_0)$$

$$\frac{\Delta \bar{\mathcal{P}}_s}{\bar{\mathcal{P}}_s} = -\frac{\pi \sigma_0}{4\Lambda} \operatorname{Im} \left[ ik\tau \left[ (\cot(\pi\nu) - i) J_{\nu-1}(-k\tau) + \csc(\pi\nu) J_{1-\nu}(-k\tau) \right] H_\nu^{(1)}(-k\tau) \right]_{\tau_0}^{\tau_{\text{end}}}$$

$$\Rightarrow \frac{\Delta \bar{\mathcal{P}}_s}{\bar{\mathcal{P}}_s} \simeq -\frac{\sigma_0}{2\Lambda} (1 + \cos(-2k\tau_0)) = -\frac{\sigma_0}{2\Lambda} \left( 1 + \cos\left(\frac{2k}{k_0}\right) \right)$$

$$(\nu \approx 3/2, \quad -k\tau_0 \gg 1, \quad -k\tau_{\text{end}} \ll 1, \quad k_0 \equiv -1/\tau_0)$$

# Clock signal

$$\sigma(t) = \Theta(t - t_0)\sigma_0 \left(\frac{t_0}{t}\right)^{3p/2} \sin[-m_\sigma(t - t_0)]$$

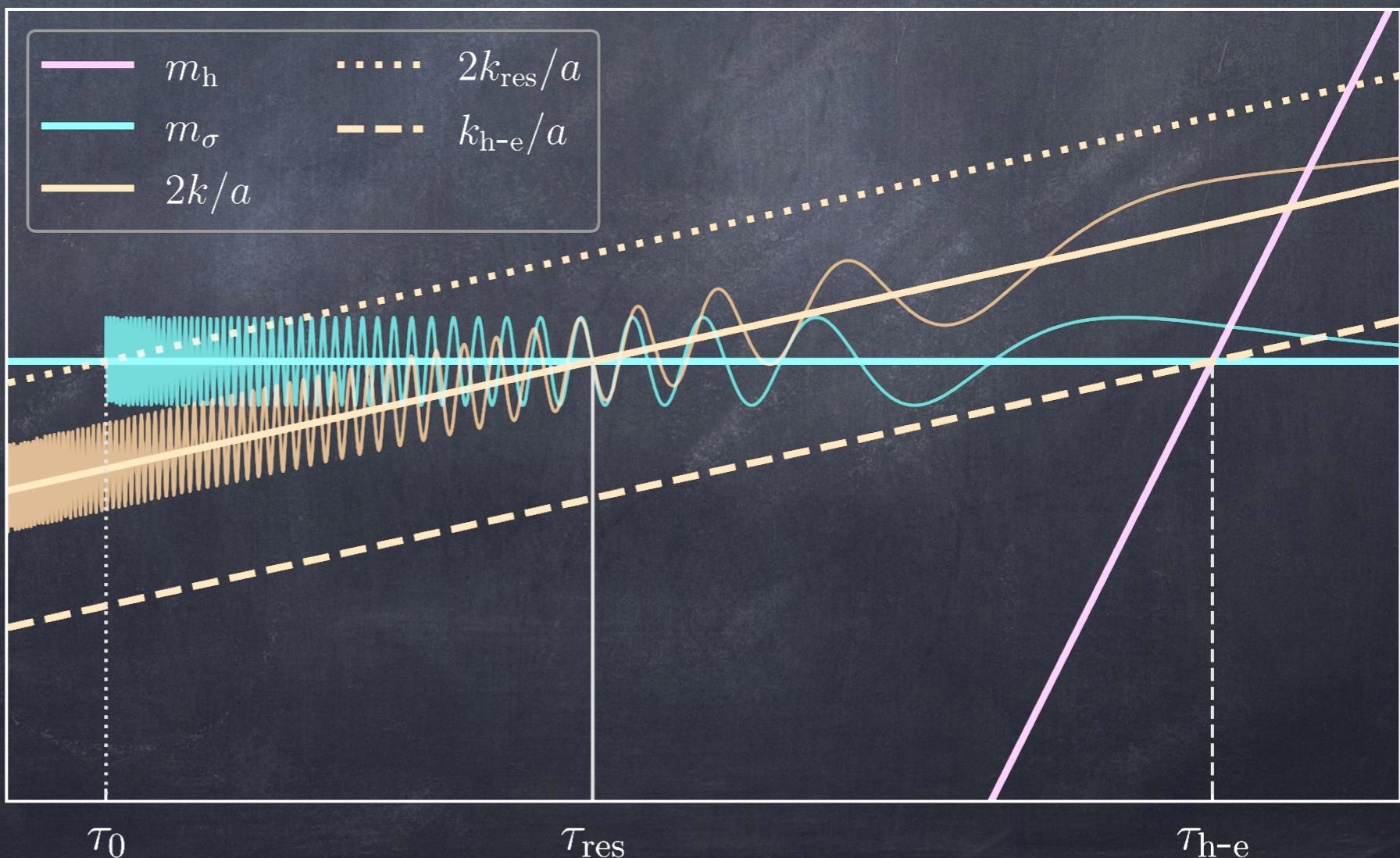
$$(-m_\sigma t \gg 1)$$

$$\frac{\Delta \mathcal{P}_s}{\bar{\mathcal{P}}_s} \simeq \Theta(k_{\text{res}} - k) \Theta(k - k_{\text{h-e}}) \frac{\sigma_0}{2\Lambda} \sqrt{\frac{\pi m_\sigma}{2|H_0|}} \left(\frac{k}{k_{\text{res}}}\right)^{\frac{1-3p}{2p}} \sin \left( p \frac{m_\sigma}{m_{\text{h0}}} \left(\frac{k}{k_{\text{res}}}\right)^{1/p} + \varphi \right)$$

$$k_{\text{res}} = \frac{m_\sigma}{m_{\text{h0}}} k_0$$

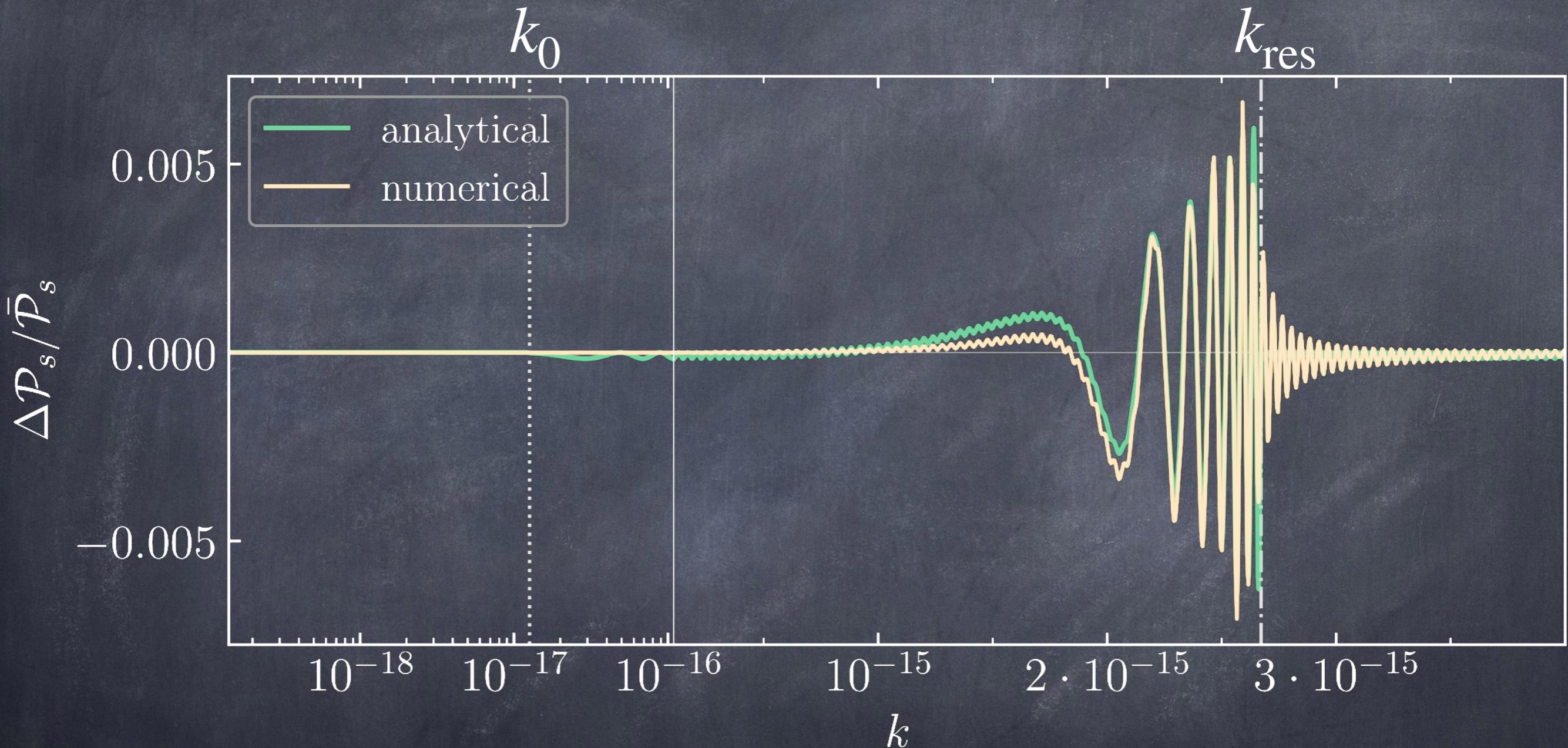
$$m_{\text{h0}} = \frac{1-p}{p} |H_0|$$

$$k_{\text{h-e}} = \left(\frac{m_\sigma}{m_{\text{h0}}}\right)^{1-p} k_0$$



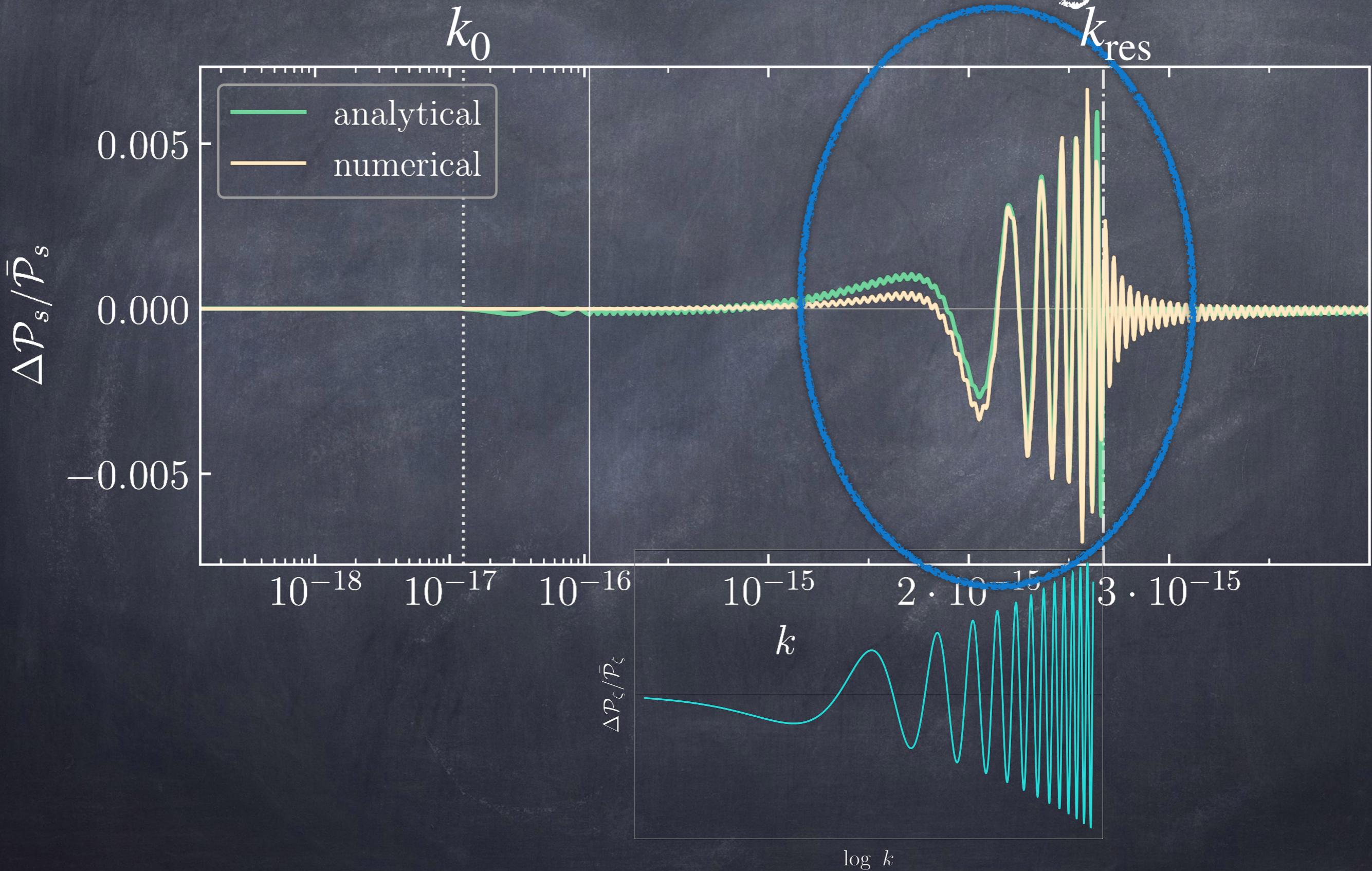
Part 6b  
Revisiting perturbations  
Numerically

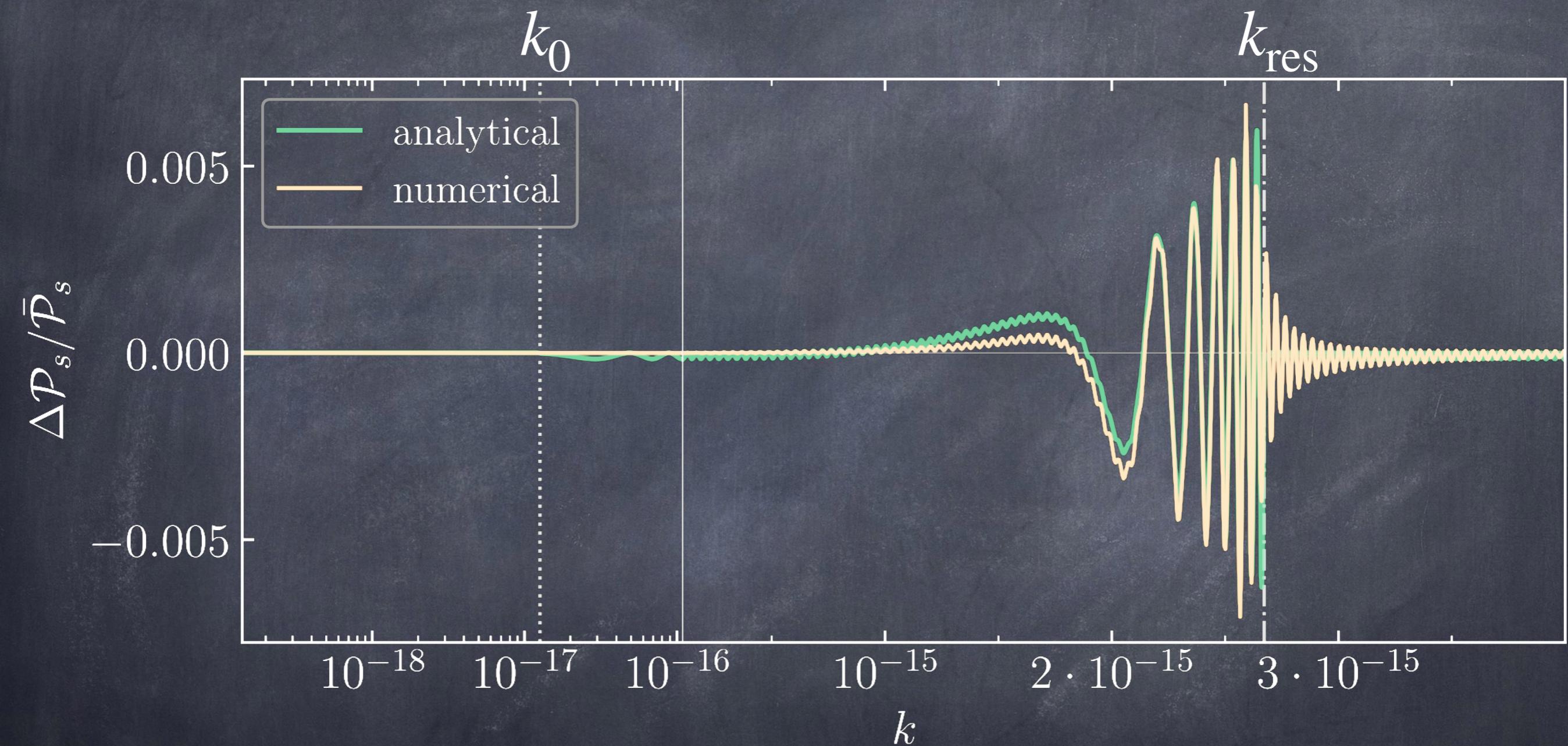
# Bump

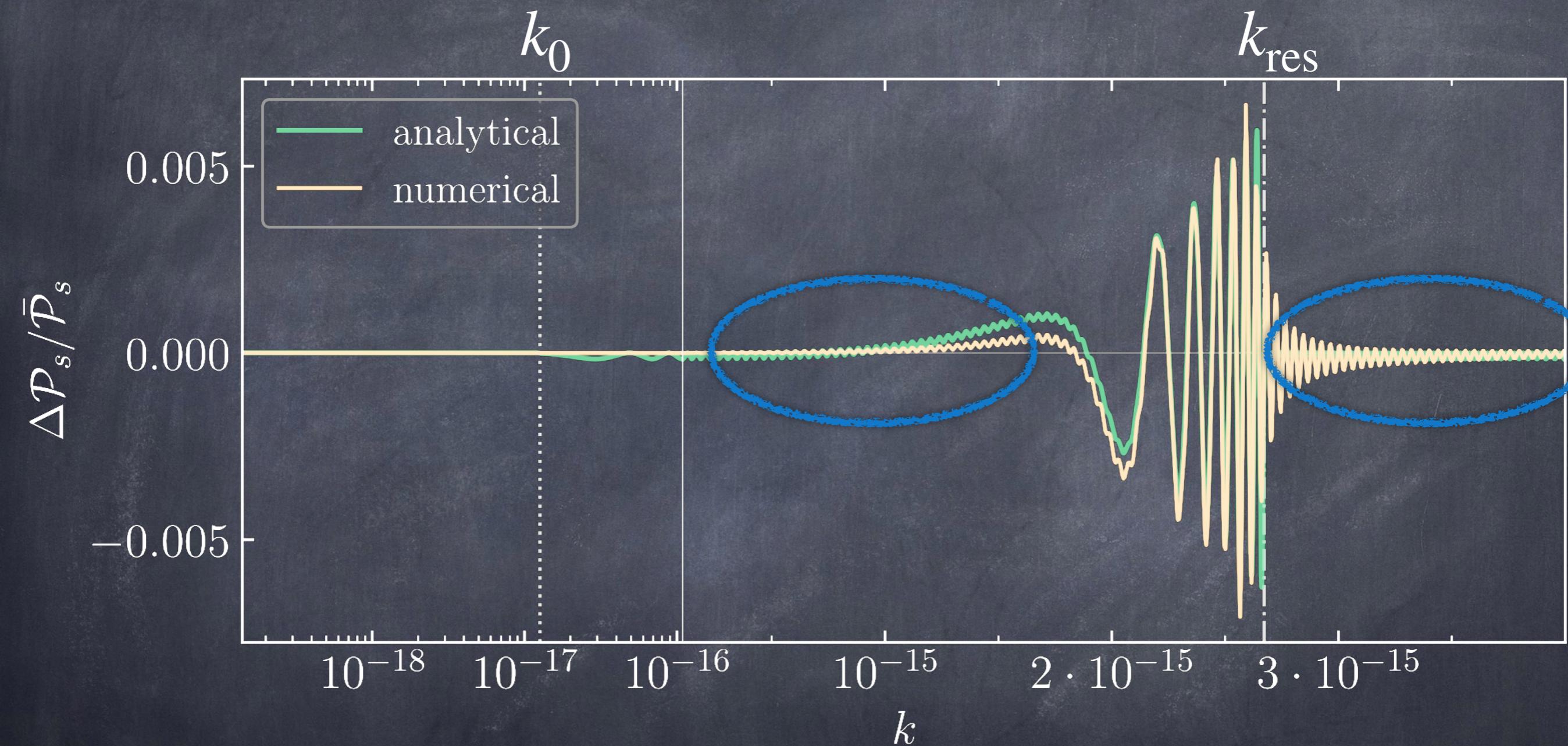


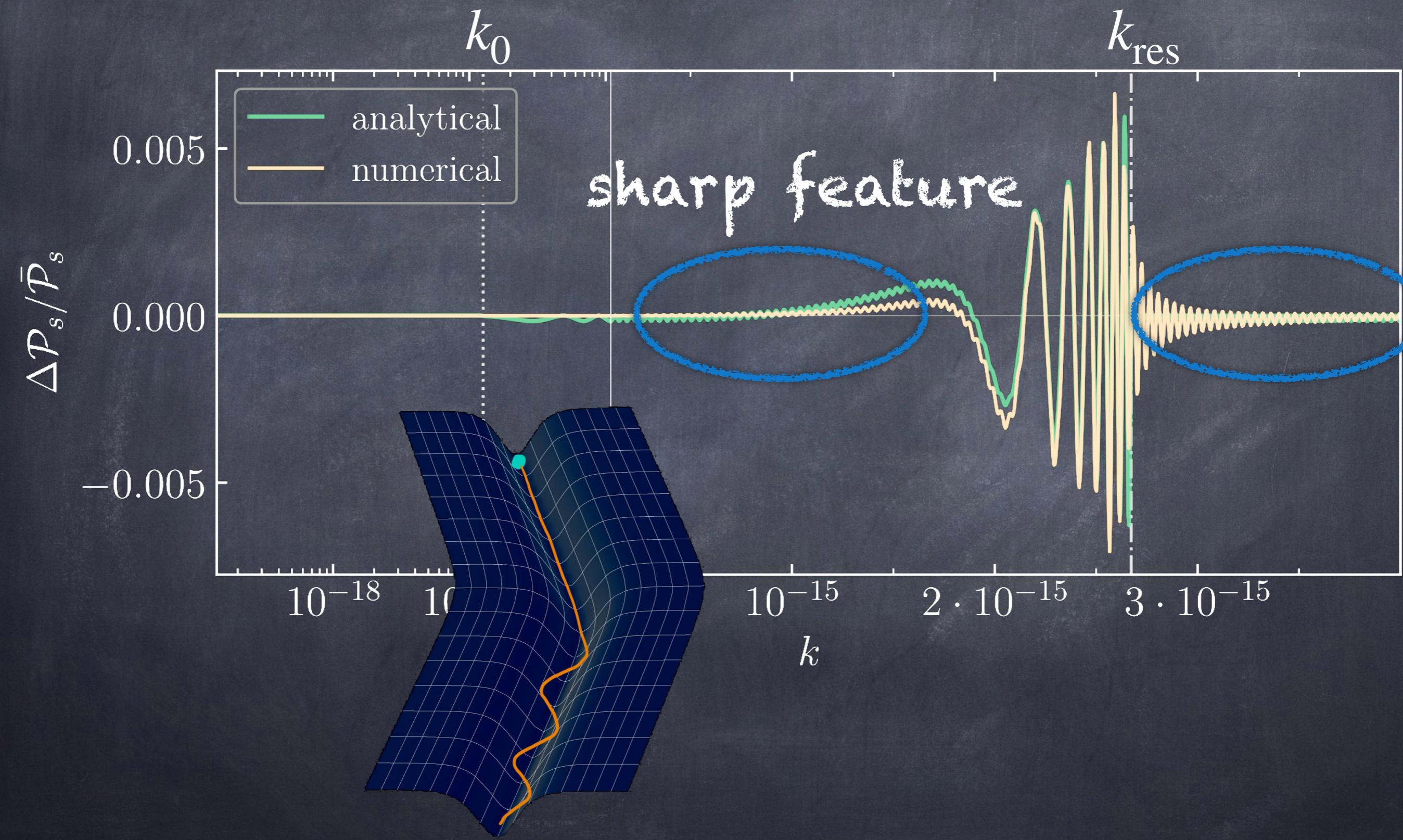
# Bump

# clock signal

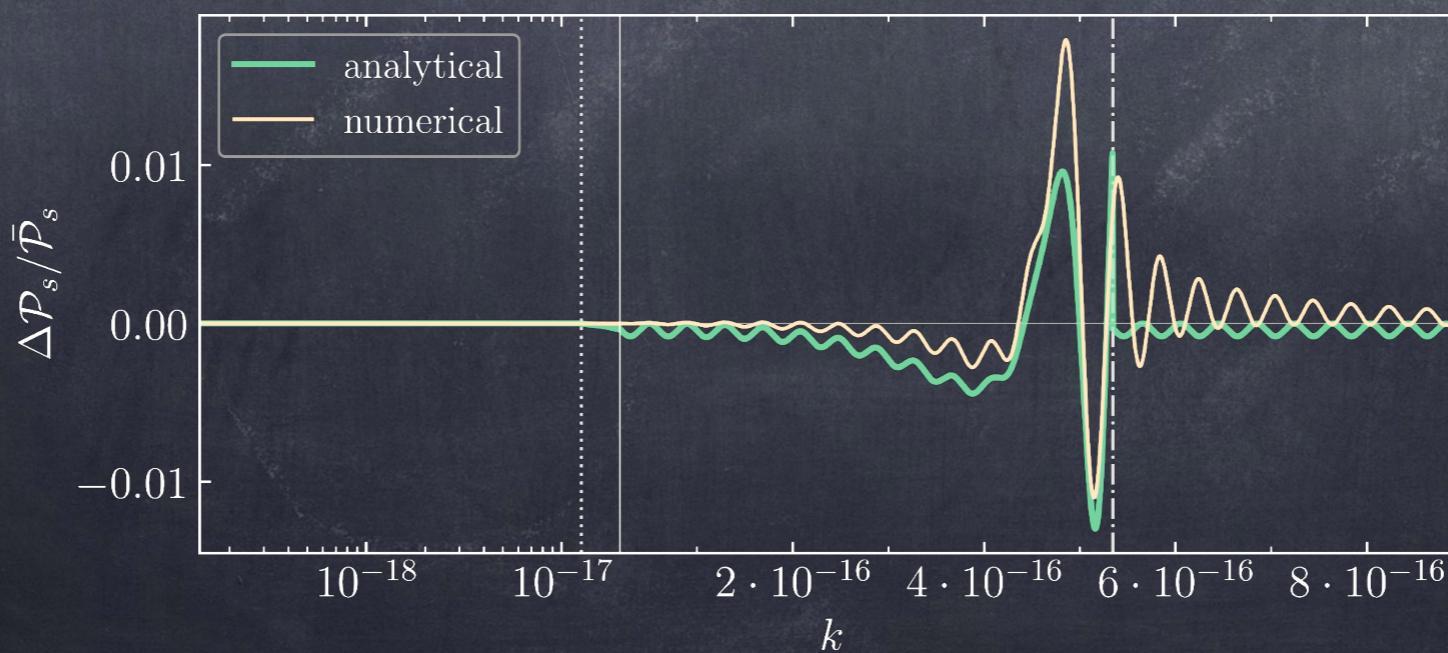
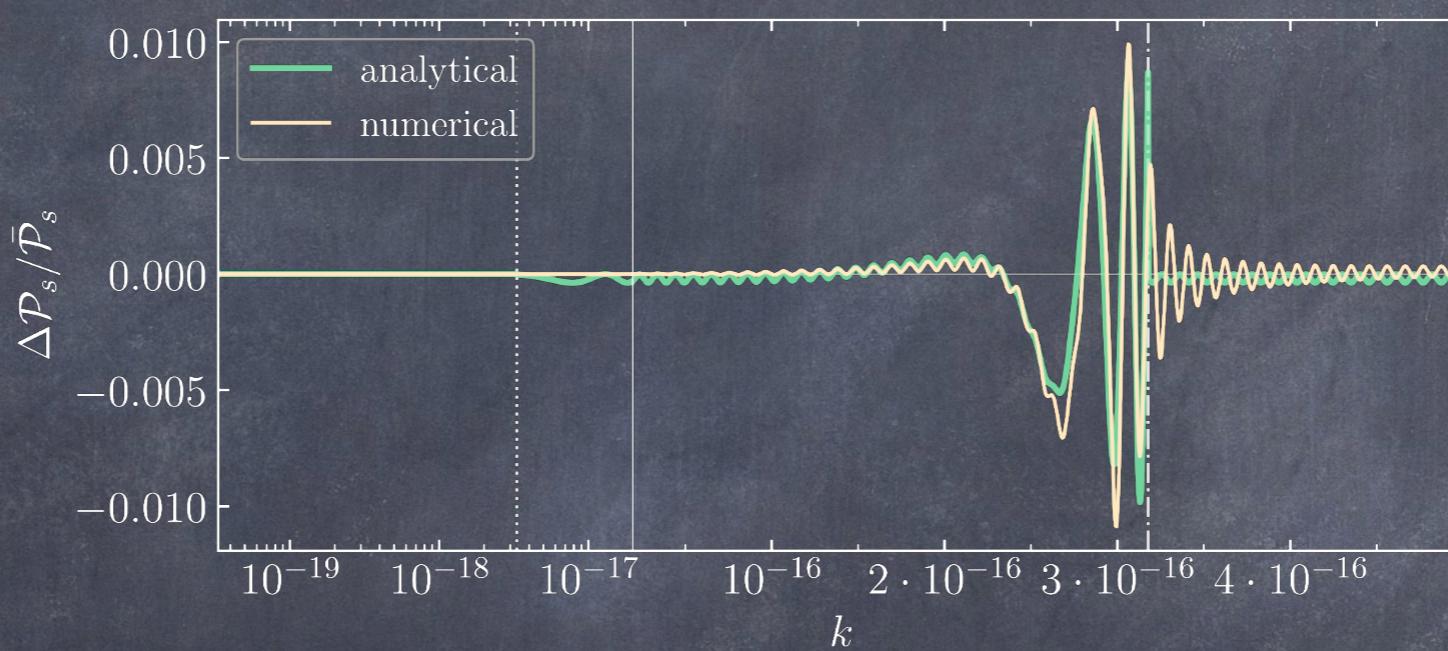
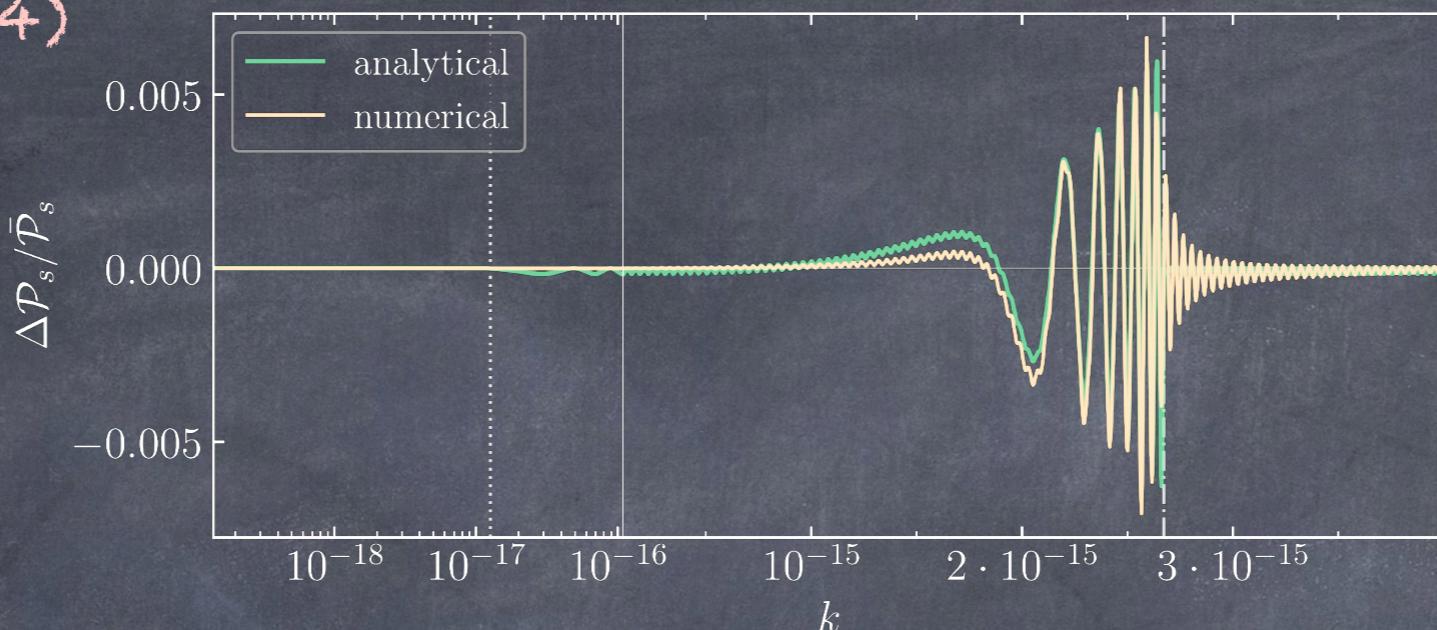


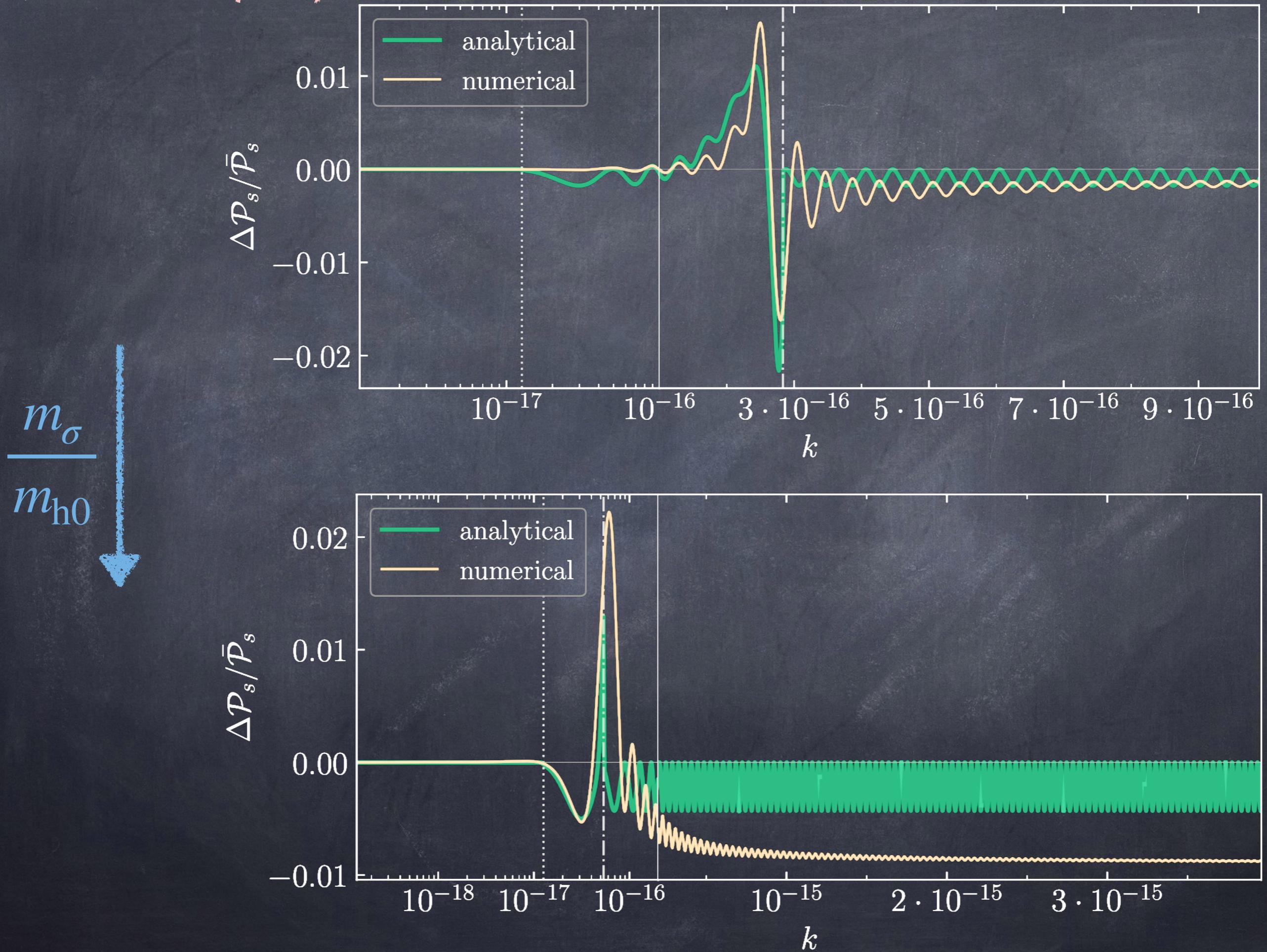


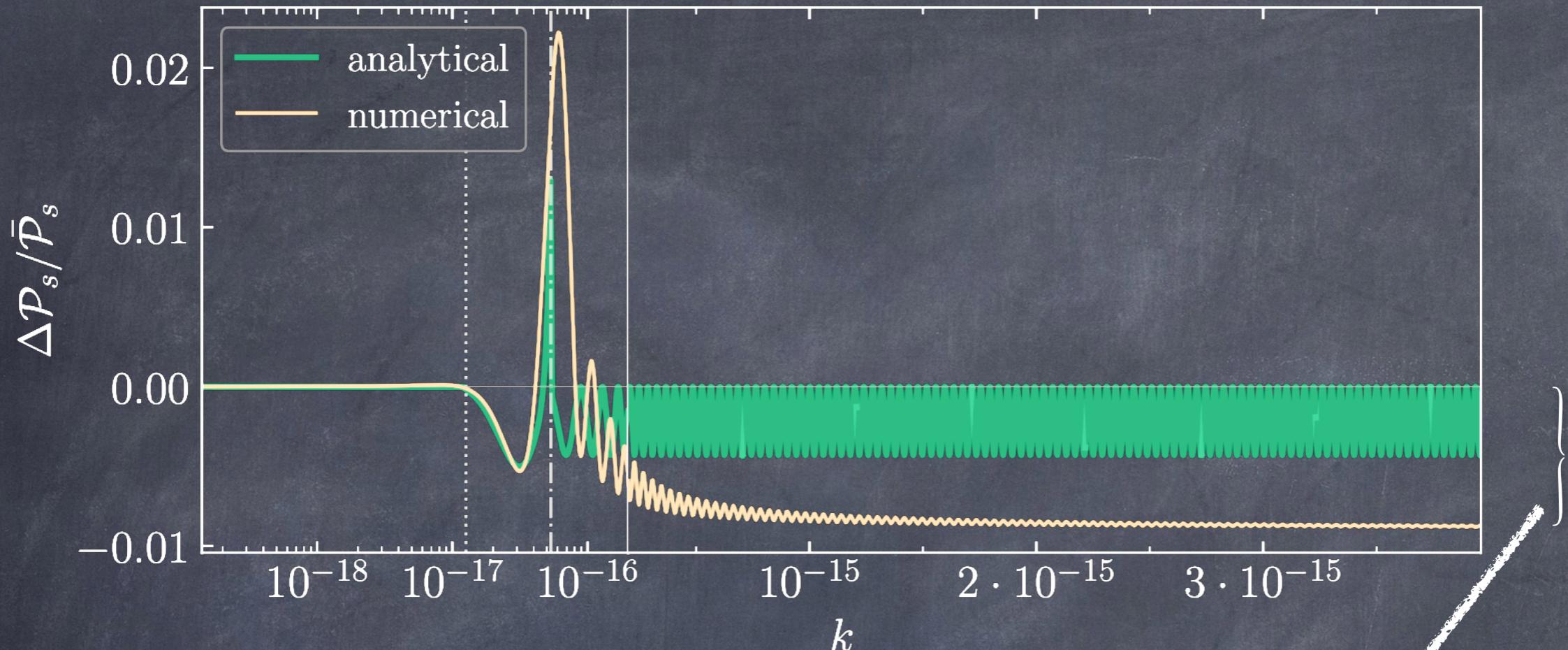




$$\frac{m_\sigma}{m_{h0}} \downarrow$$



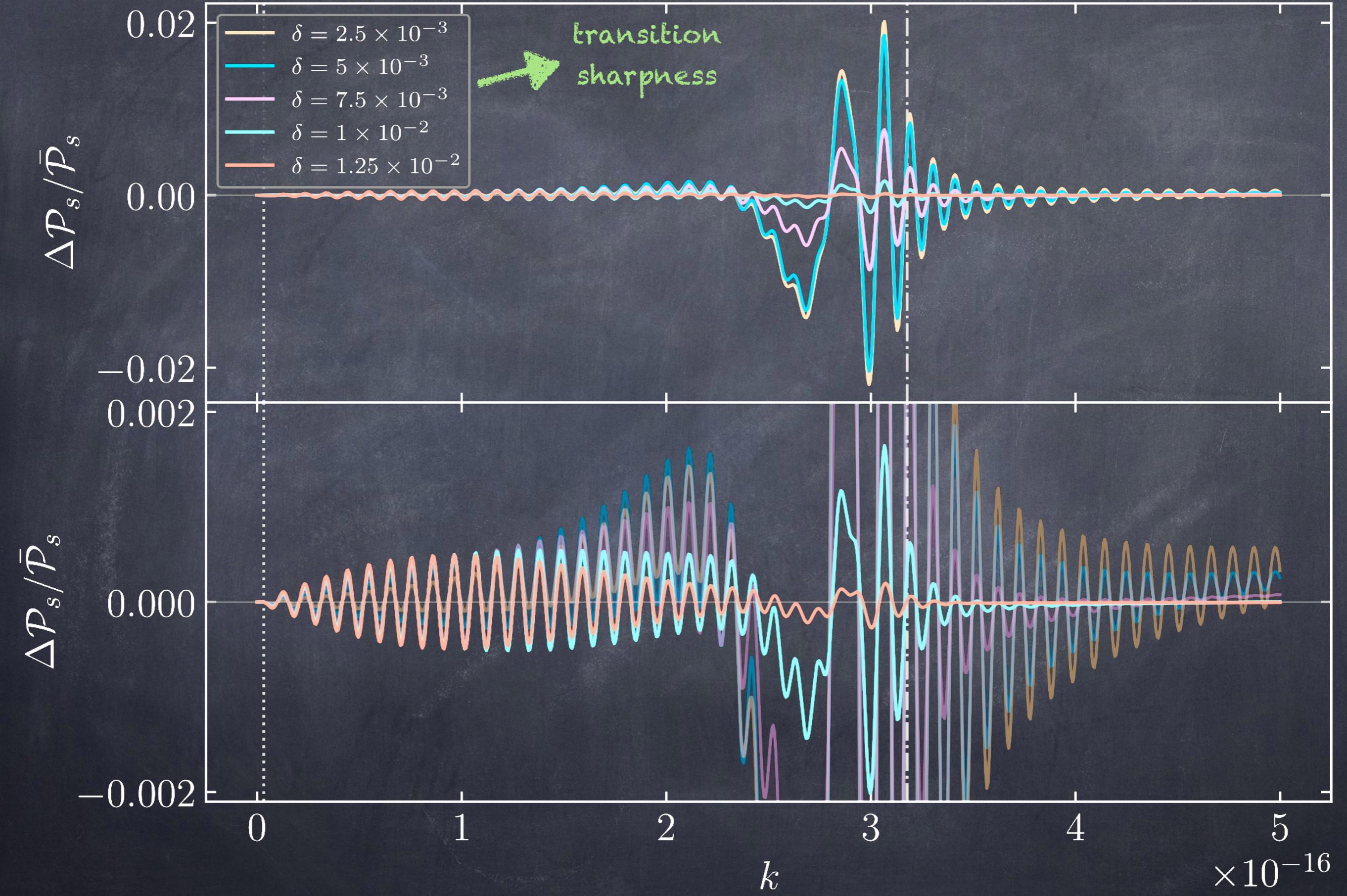


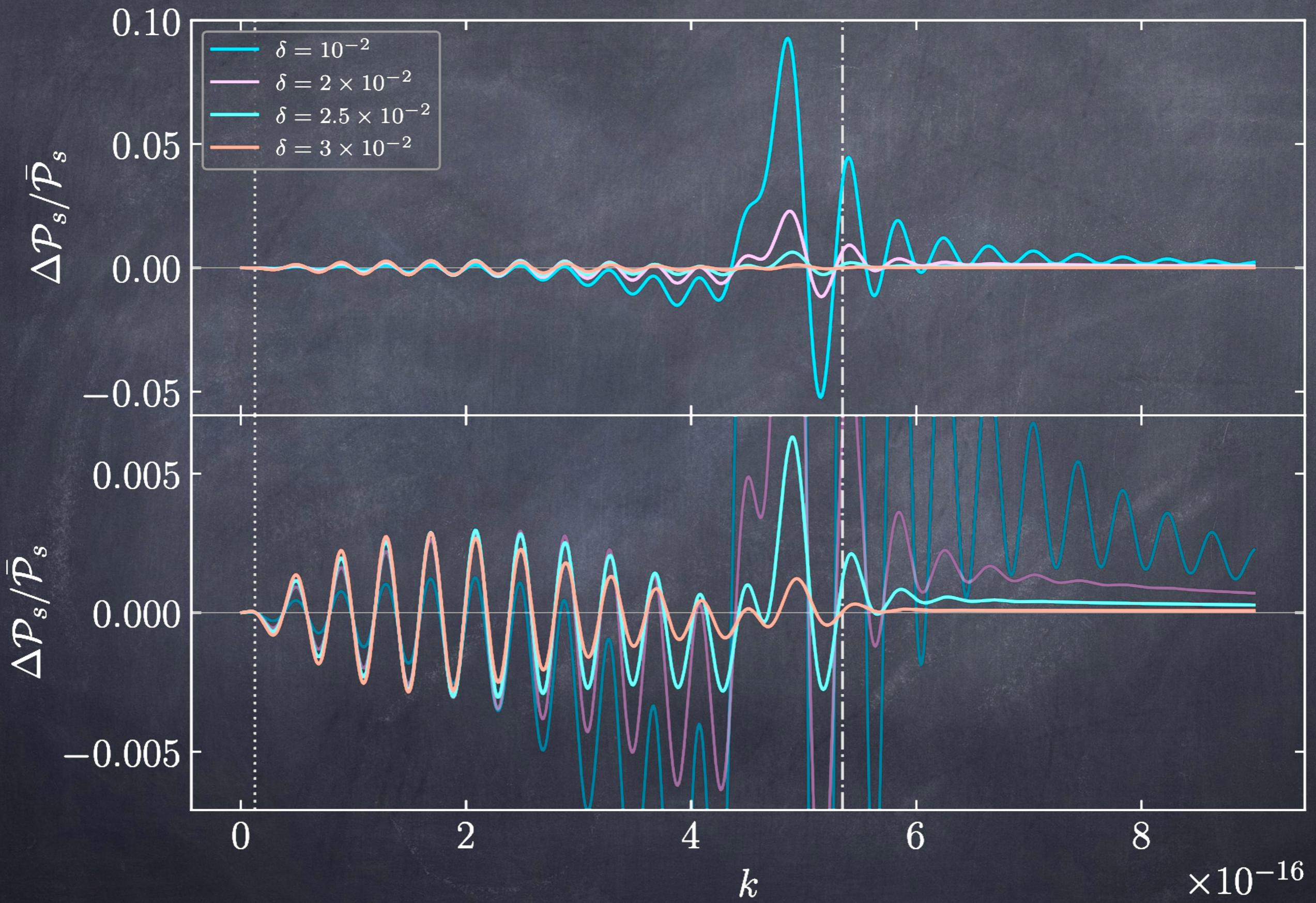


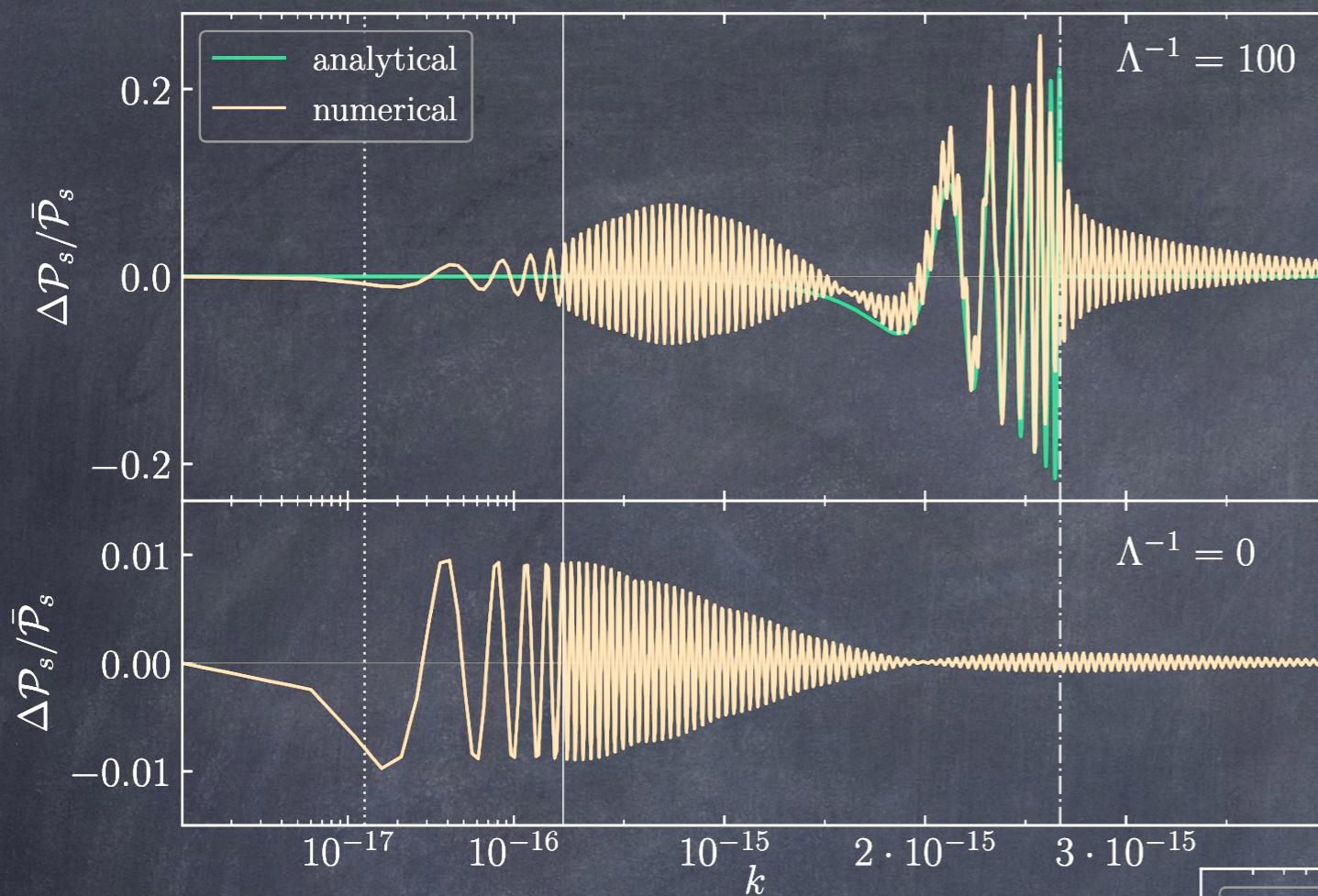
$$\frac{\Delta \mathcal{P}_s}{\bar{\mathcal{P}}_s} \xrightarrow{k \gg k_{\text{res}}}$$

$$-\frac{\sigma_{\text{late}}}{\Lambda}$$

Late-time freezing  
of massive field  
as it becomes light  
after horizon crossing

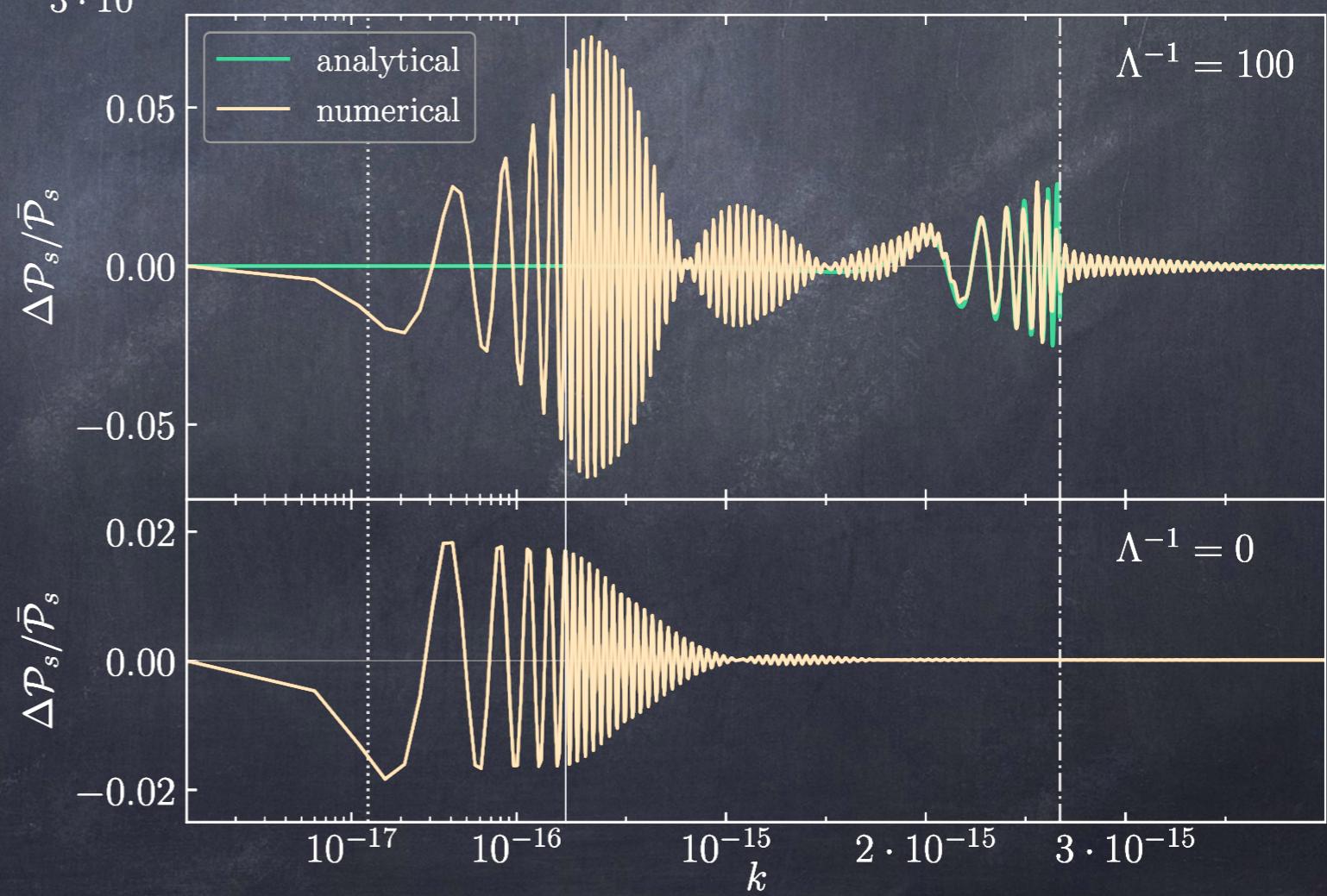






$$\mathcal{L}^{(2)} \supset -\frac{1}{2} \left(1 + \frac{\sigma}{\Lambda}\right) (\bar{z} + \Delta z)^2 (\partial \chi)^2$$

$$\mathcal{L}^{(2)} \supset -\frac{1}{2} (\bar{z} + \Delta z)^2 (\partial \chi)^2$$



# Part 7

## Summary and outlook

# What we achieved

- Slow contraction (a.k.a. ekpyrotic cosmology) can support spectator massive fields
- Better if  $\sigma(\partial\phi)^2$  coupling goes to zero
- Confirmed clock signal and how it connects to sharp feature signal

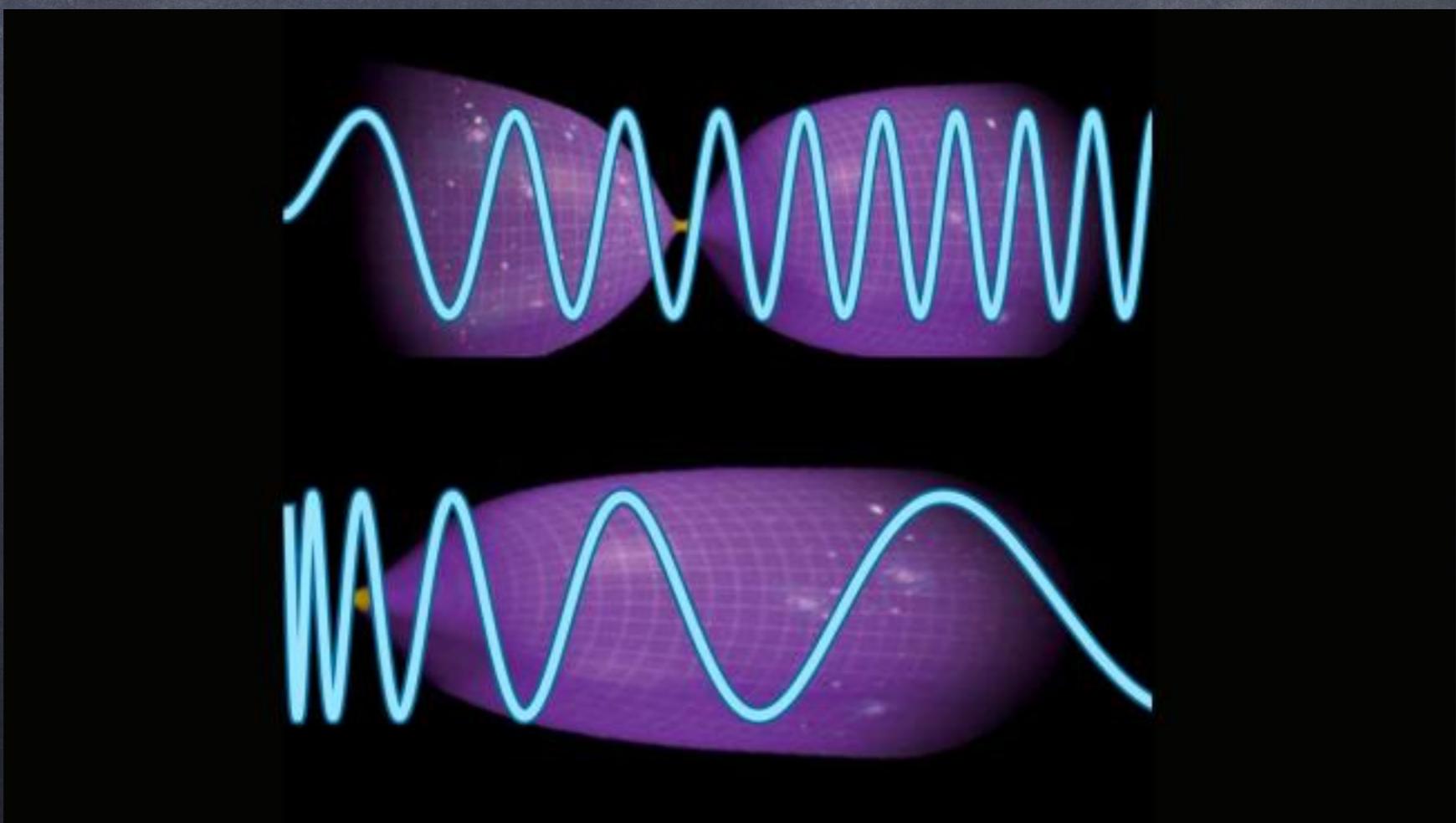
Signal could really tell apart  
→ inflationary cosmology  
from ekpyrotic cosmology

# Still many things to explore!

- Further implementations
- Fit to data
- Full bispectrum calculation
- Full quantum standard clocks models  
and more cosmological collider business
- Non-standard clocks (time-dependent masses)
- Other alternatives
- ...

Thank you for your attention!

Questions?



Extra

	inflation	matter bounce	genesis	ekpyrosis
scale factor	expands	contracts	expands	contracts
"speed"	fast	fast	slow	slow
number of fields	1 or more	1 or more	2 or more	2 or more
content	positive, flat potential	pressureless matter	CFT	negative, steep potential
property	$ds \text{ } SO(4,1)$ $\rightarrow ISO(3)$	needs to be stabilized	$Mink \text{ } SO(4,2)$ $\rightarrow SO(4,1)$	smoothing attractor

inflation

$$n_s - 1 = -2\epsilon - \eta$$

$$\alpha_s = 2\epsilon_{,\mathcal{N}} - \frac{\epsilon_{,\mathcal{N}\mathcal{N}}}{\epsilon} + \left(\frac{\epsilon_{,\mathcal{N}}}{\epsilon}\right)^2$$

$$r = 16\epsilon$$

$$f_{\text{NL}} \sim \mathcal{O}(\epsilon, \eta, c_s^{-2})$$

matter  
bounce

$$n_s - 1 = 12w - 9w_{,\mathcal{N}}$$

$$\alpha_s = -12w_{,\mathcal{N}}$$

$$r = 24c_s$$

$$f_{\text{NL}} \sim \mathcal{O}(1, c_s^{-2})$$

ekpyrosis

$$n_s - 1 = 2\left(1 - \sqrt{\frac{p}{b}}\right) - \frac{7\epsilon_{,\mathcal{N}}}{3\epsilon}$$

$$\alpha_s = -\sqrt{\frac{p}{b}}\frac{\epsilon_{,\mathcal{N}}}{\epsilon} + \frac{7\epsilon_{,\mathcal{N}\mathcal{N}}}{3\epsilon} - \frac{7}{3}\left(\frac{\epsilon_{,\mathcal{N}}}{\epsilon}\right)^2$$

$$r \lll 1 \qquad (n_t = 2)$$

$$f_{\text{NL}} \sim \mathcal{O}(1)$$

inflation

$$n_s - 1 = -2\epsilon - \eta$$

$$\alpha_s = 2\epsilon_{,\mathcal{N}} - \frac{\epsilon_{,\mathcal{N}\mathcal{N}}}{\epsilon} + \left(\frac{\epsilon_{,\mathcal{N}}}{\epsilon}\right)^2$$

$$r = 16\epsilon$$

$$f_{\text{NL}} \sim \mathcal{O}(\epsilon, \eta, c_s^{-2})$$

Lehners and Wilson-Ewing (2015) matter  
Li-JQ-Wang-Cai (2017)  
Akama-Hirano-Kobayashi (2020) bounce  
...

$$n_s - 1 = 12w - 9w_{,\mathcal{N}}$$

$$\alpha_s = -12w_{,\mathcal{N}}$$

$$r = 24c_s$$

$$f_{\text{NL}} \sim \mathcal{O}(1, c_s^{-2})$$

ekpyrosis

$$n_s - 1 = 2 \left( 1 - \sqrt{\frac{p}{b}} \right) - \frac{7\epsilon_{,\mathcal{N}}}{3\epsilon}$$

$$\alpha_s = -\sqrt{\frac{p}{b}} \frac{\epsilon_{,\mathcal{N}}}{\epsilon} + \frac{7\epsilon_{,\mathcal{N}\mathcal{N}}}{3\epsilon} - \frac{7}{3} \left( \frac{\epsilon_{,\mathcal{N}}}{\epsilon} \right)^2$$

$$r \lll 1 \quad (n_t = 2)$$

Fertig-Lehners-Mallwitz (2014)

Lehners and Wilson-Ewing (2015)

$$f_{\text{NL}} \sim \mathcal{O}(1)$$

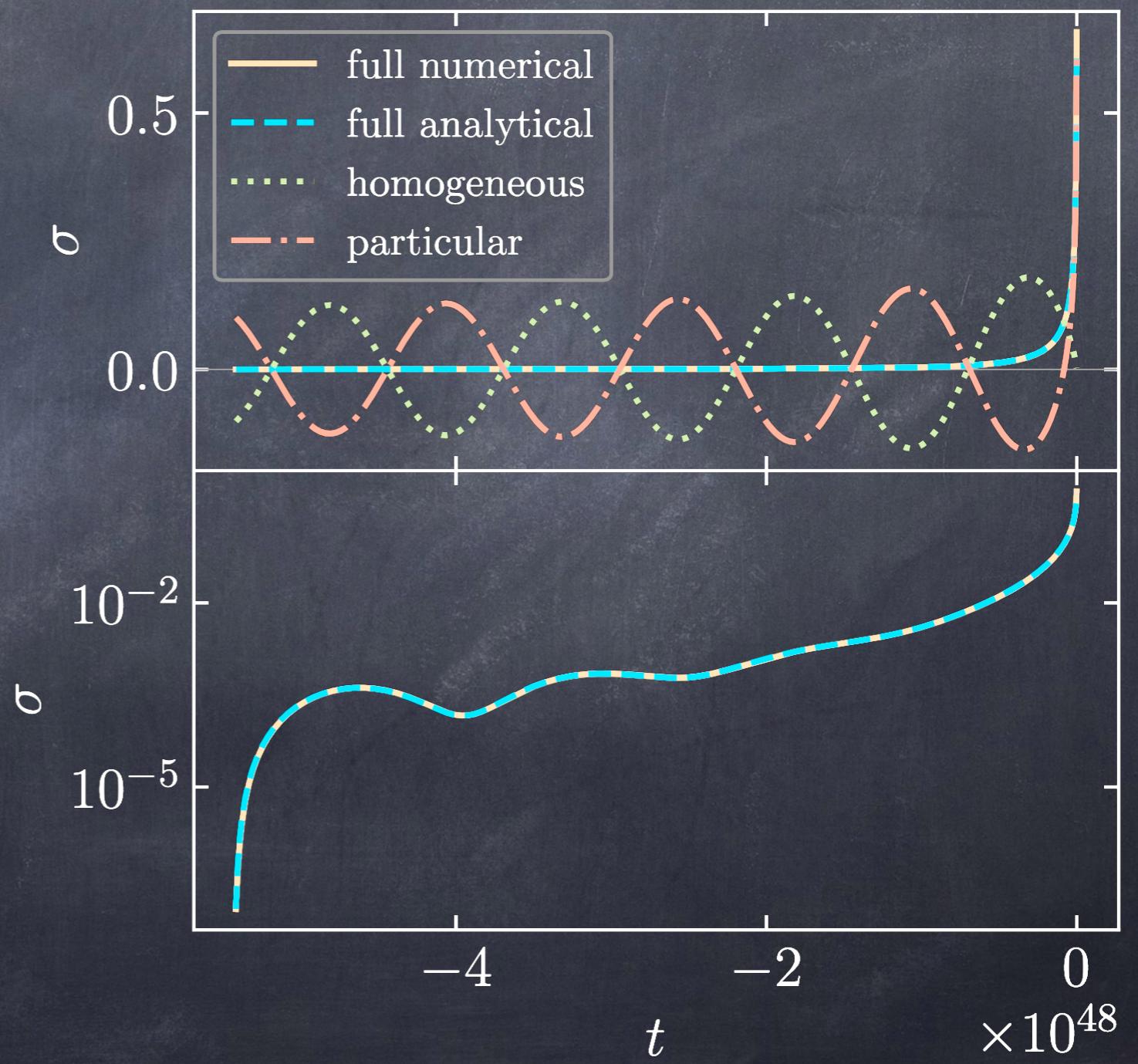
...

# Plateau Step

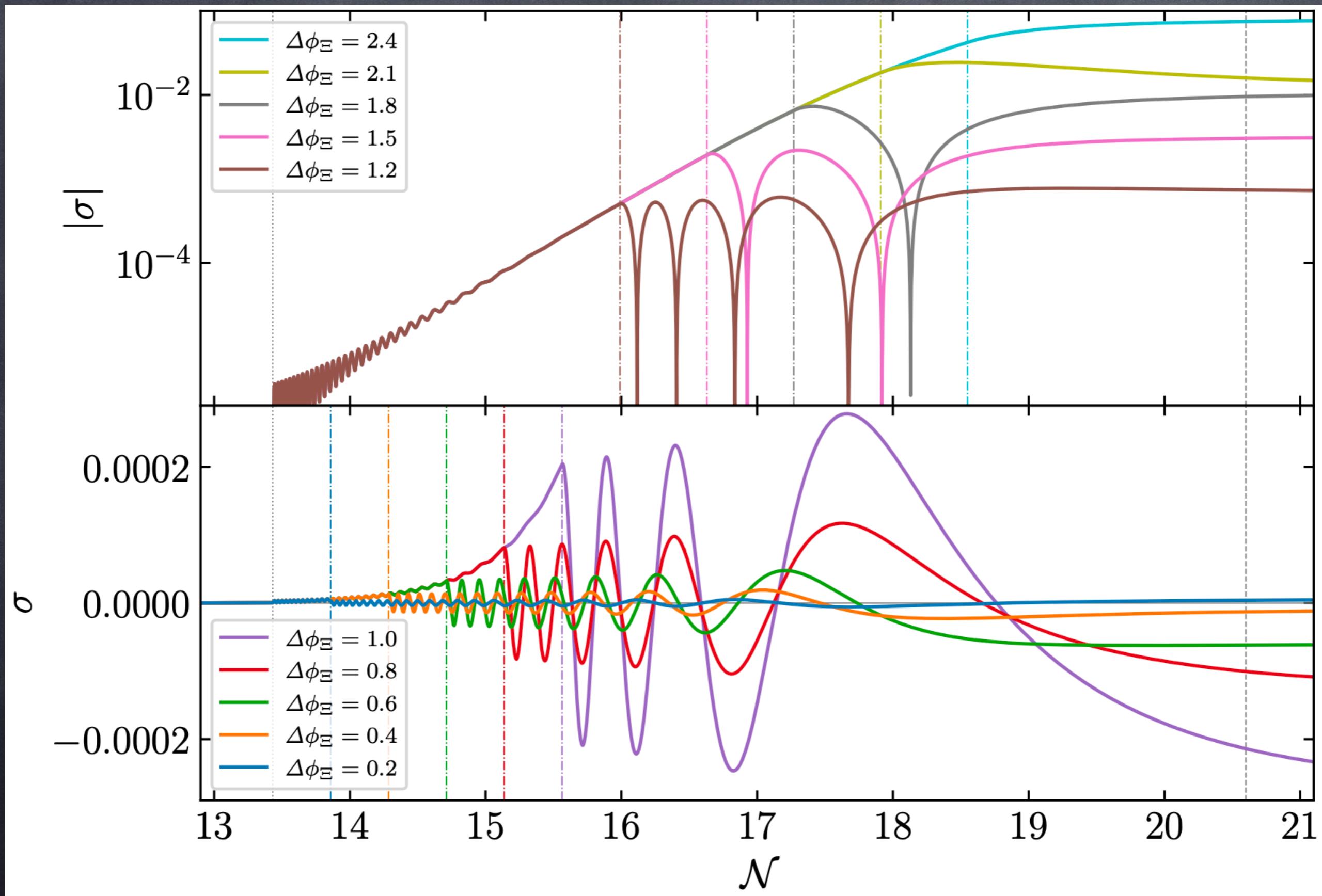
$$\ddot{\sigma} + 3\bar{H}\dot{\sigma} + m_\sigma^2 \sigma = \frac{\dot{\phi}^2}{2\varrho} \Rightarrow \ddot{\sigma} + \frac{3p}{t}\dot{\sigma} + m_\sigma^2 \sigma = \frac{p}{\varrho t^2}$$

$$\Rightarrow \sigma_p(t) = \frac{\pi p}{4Q \sin(\pi\alpha)} \left( \frac{J_\alpha(-m_\sigma t) {}_1F_2(-\alpha; 1-\alpha, 1-\alpha; -y^2)}{\alpha^2 \Gamma(-\alpha) y^\alpha} + e^{-i\pi\alpha} y^\alpha J_{-\alpha}(m_\sigma t) G_{1,3}^{2,0} \left( y^2 \middle| 0,0,-\alpha \right) \right)$$

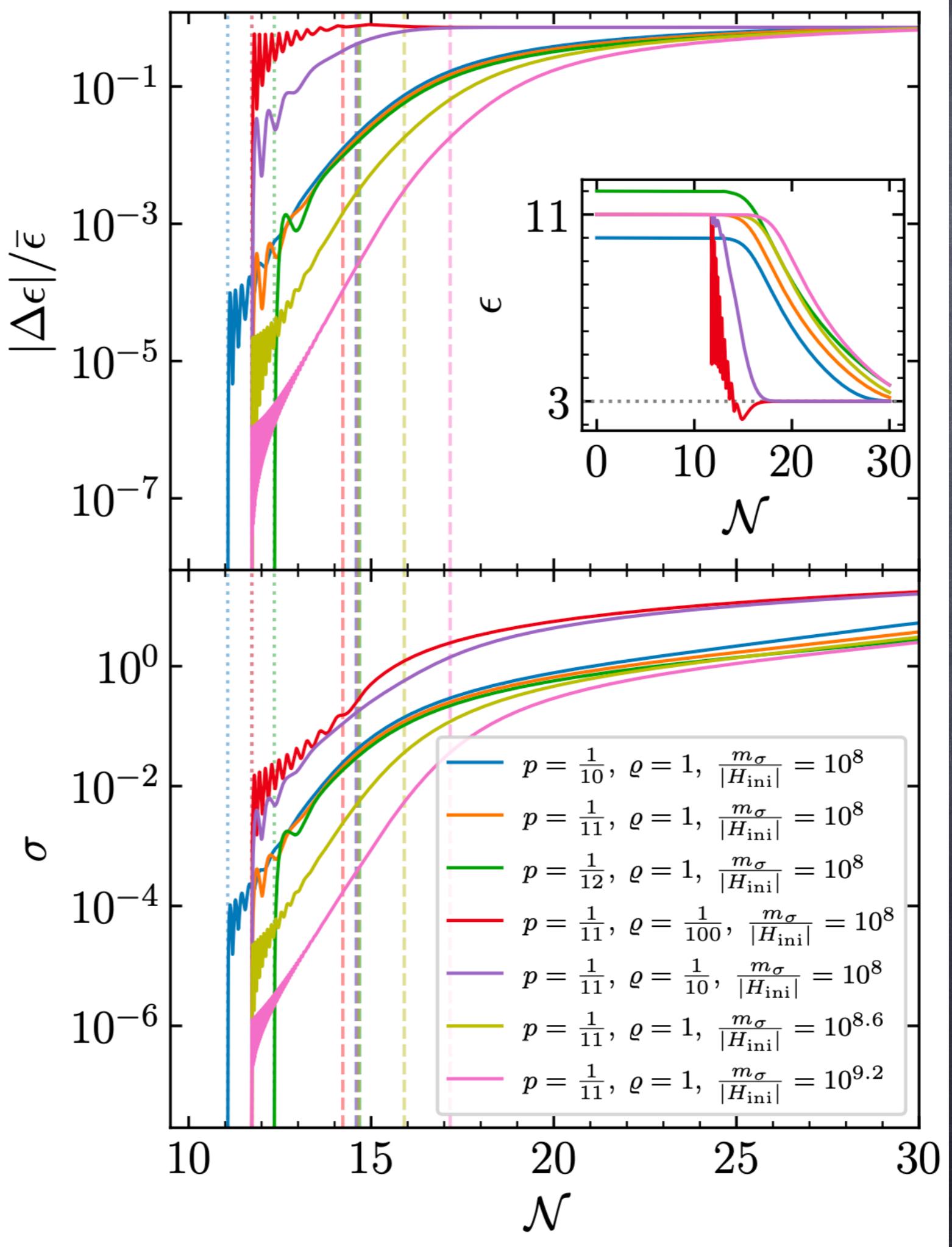
$$\Rightarrow \sigma_p \sim \frac{\sin(-m_\sigma t)}{(-m_\sigma t)^{3p/2}}, \quad (-m_\sigma t \gg 1)$$

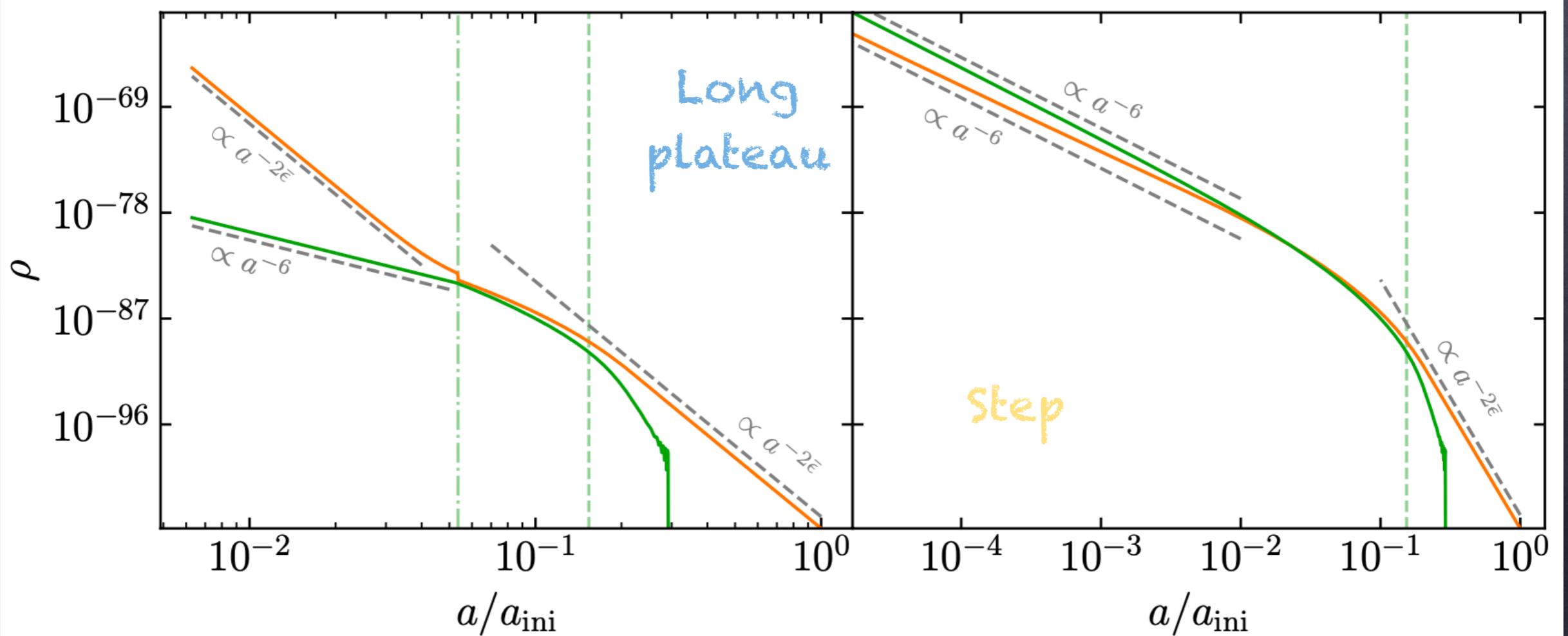
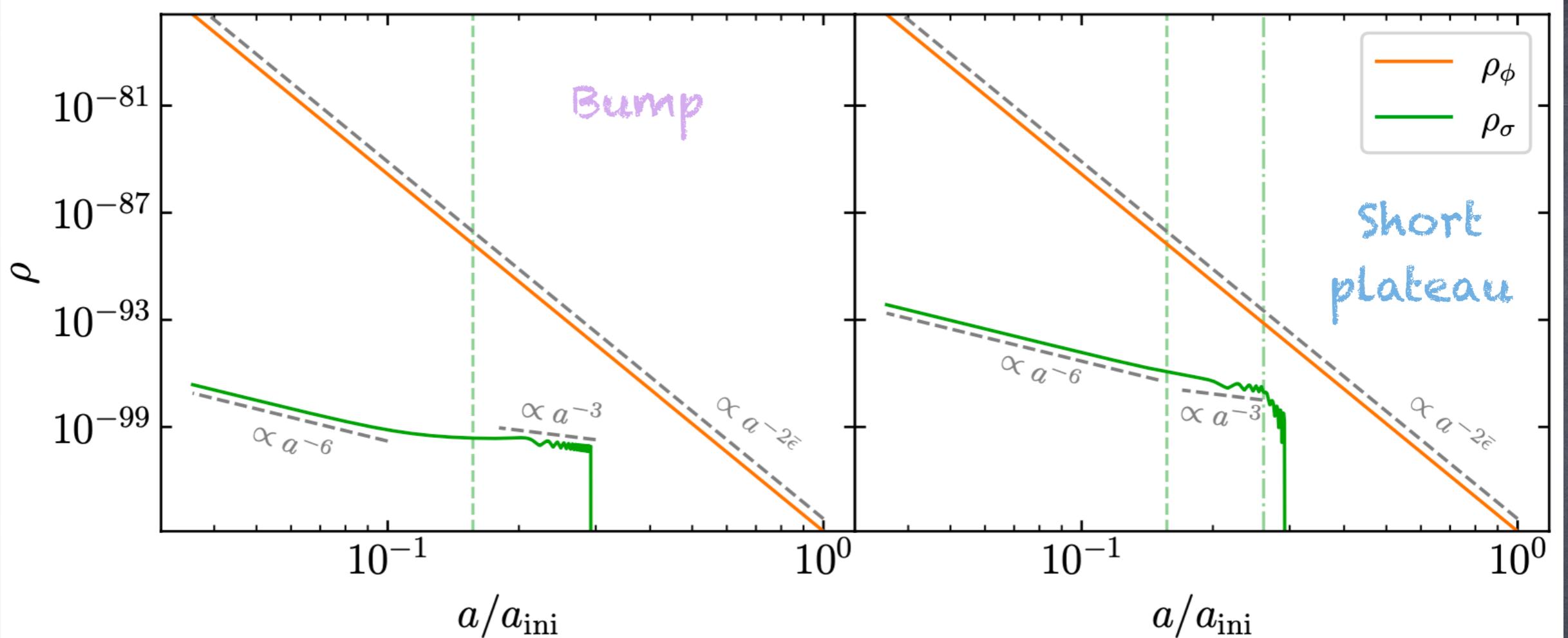


# Plateau



Step





Plateau

