University of Rwanda - College of Science and Technology
School of ICT - Computer Science - Year 2
Analysis of Algorithms - CAT 1
October 19, 2022

1. For each of the following pairs of functions, either $f(n)$ is in $O(g(n))$, $f(n)$ is in $\Omega(g(n))$, or $f(n) = \Theta(g(n))$. Determine which relationship is correct and briefly explain why.

   (a) $f(n) = log(n^2)$; $g(n) = log(n) + 5$ [2 marks]

   (b) $f(n) = \sqrt{n}$; $g(n) = log(n^2)$ [2 marks]

2. Place the following functions into increasing asymptotic order. $f_1(n) = n^2 log_2(n)$, $f_2(n) = n(log_2 n)^2$, $f_3(n) = \sum_{i=0}^{n} 2^i$, $f_4(n) = log_2(\sum_{i=0}^{n} 2^i)$ [4 marks]

3. A common problem for compilers and text editors is determining whether the parentheses in a string are balanced and properly nested. For example, the string $((())()) ()$ contains properly nested pairs of parentheses, which the strings $)()($ and $())$ do not. [4 marks]

   (a) Give an algorithm that returns true if a string contains properly nested and balanced parentheses, and false if otherwise. [4 marks]

   (b) What is the time complexity of the algorithm used in (a)? [2 marks]

4. Is the array with values $(23; 17; 14; 6; 13; 10; 1; 5; 7; 12)$ is a max-heap? [2 marks]

5. Give an efficient algorithm to rearrange an array of n keys so that all the negative keys precede all the nonnegative keys. Your algorithm must be in-place, meaning you cannot allocate another array to temporarily hold the items. How fast is your algorithm? [5 marks]