



Bell Laboratories

Victor A. Vyssotsky
Executive Director
Circuit Provisioning
Systems Division

6 Corporate Place
Piscataway, N.J. 08854
Phone (201) 981-6622

February 12, 1979

Professor W. Kahan
Electronics Research Laboratory
College of Engineering
University of California
Berkeley, California 94720

Dear Professor Kahan:

Stu Feldman passed some of your reports on pocket calculators along to me; you had left them off at Bell Labs when you gave a talk here, and Feldman knew I'd enjoy them, because of all the grumbling I do on the subject. Hooray! Right on!

I own and use a TI SR-52 and a TI Programmable 59, and have developed a fine mixture of fondness and loathing for them, because they perform so well until they sandbag me in the ways you describe.

For example, although I knew about many of the anomalies you describe, I was perplexed for some time recently by a program of mine which insisted on producing seriously wrong answers. I finally realized that on the TI Programmable 59 it can happen that if $a > b > 0$ and $c > 0$, $ac < bc$; the inequality strictly reverses for certain values. Well, of course, I had a test in my program which was based on the assumption that if $a > b > 0$ and $c > 0$ then $ac > bc$, and so in certain cases the program took the wrong path.

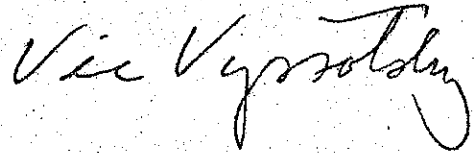
This discovery, and the time it took to make it, annoyed me enough so that I wrote to TI and asked: a) what algorithm is used for hardware multiplication on the Programmable 59, and b) what bounds on relative accuracy I can depend on for the product of two numbers. After a long delay, TI wrote me back, without answering either of my questions; their letter said that the accuracy of arithmetic on the Programmable 59 is adequate for engineering calculation because quantities used in engineering are almost never more accurate than one part in 10^8 . The letter went on to say that they didn't see why I should be concerned about what happens to the last few digits.

Professor W. Kahan (Continued)

Now, to be fair, this is much the same response I got from UNIVAC when I forwarded to them a test case I ran on one of their UNIVAC 1110 double precision library routines, a test case which showed the last 21 bits of the result in error, compared to the "to within two bits" accuracy claimed by UNIVAC. UNIVAC's response was that I shouldn't have tried that particular case, because they couldn't see any possible use for the answer; that led to a long and acrimonious exchange between me and UNIVAC. (However, they did concede that I had a point on another test case I sent them at the same time, which showed that their integer binary-to-decimal conversion converted 2^{34} to an incorrect integer value.)

To conclude by getting back to my reason for writing, I have spent more hours in the last twenty years than I care to recall puzzling over the anomalies of machine arithmetic and library routines on machines from the IBM-650 and 704 on. I have been forced to, by divide instructions which don't divide, sine routines which don't compute the sine, random number generators with sequential correlations, and all of the other unnecessary annoyances that spring from the viewpoint that "it doesn't matter how it handles the last few bits." I applaud your superb exposition, and I hope you will keep at it, ever louder, until your view prevails everywhere.

Sincerely yours,

A handwritten signature in cursive script, reading "Vic Vyssotsky". The signature is written in dark ink and is positioned to the right of the typed name "Vic Vyssotsky".