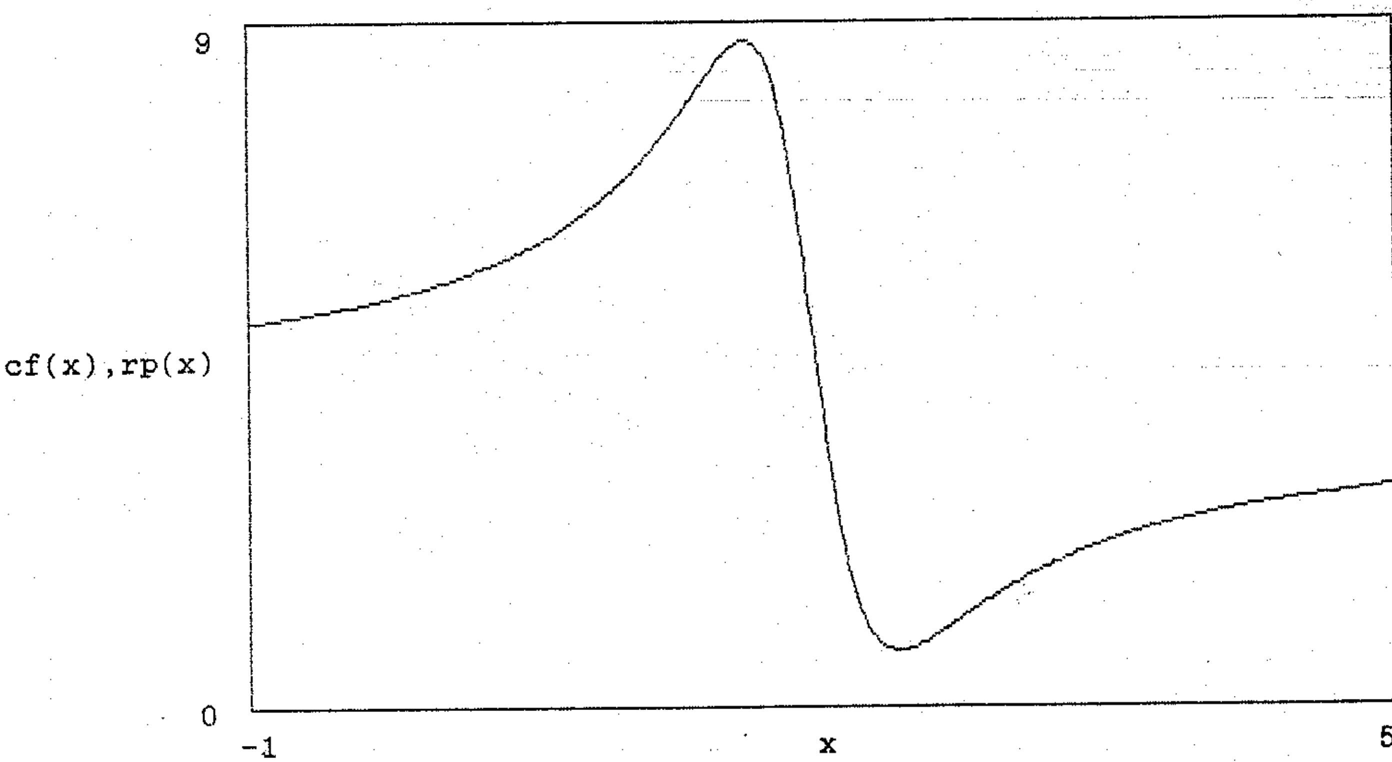
x := -1,-.98 ..5 FIGURE 1 from MathCAD v. 2.03

Here are two ways to express the same rational function:

$$cf(x) := 4 - \cfrac{3}{x - 2 - \cfrac{1}{x - 7 + \cfrac{10}{x - 2 - \cfrac{2}{x - 3}}}}$$

$$rp(x) := \frac{622 - x\cdot(751 - x\cdot(324 - x\cdot(59 - 4\cdot x)))}{112 - x\cdot(151 - x\cdot(72 - x\cdot(14 - x)))}$$



The coincidence of the graphs obtained by plotting both expressions confirms that they represent the same function, though they treat Roundoff, Overflow and Division-by-Zero differently.

For example, ...

cf(1) = ■	cf(2) = ■	cf(3) = ■	cf(4) = ■
singularity	singularity	singularity	singularity
rp(1) = 7	rp(2) = 4	rp(3) = 1.6	rp(4) = 2.5

Division-by-Zero cannot happen to rp(x) ; and it would be harmless in cf(x) too if the ∞ supplied by the hardware (it has an INTEL 80x87 that conforms to IEEE standard 754) were used as its designers intended. For instance, computing cf(3) would then produce correctly

$2/(x-3) = \infty$, $x-2-\infty = -\infty$, $10/\infty = 0$, $x - 7 - 0 = -4$, ...

eps := 0.5

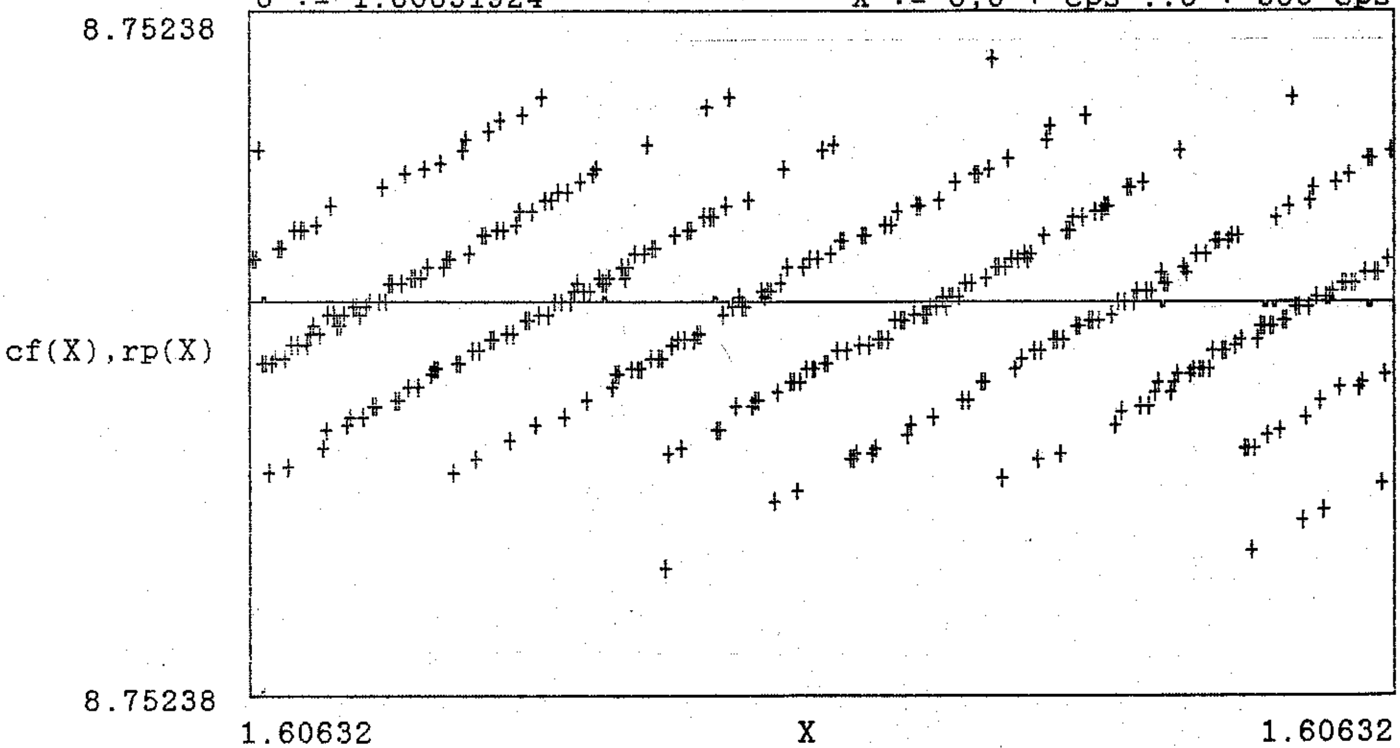
FIGURE 2

from MathCad v. 2.03

$$cf(x) := 4 - \cfrac{3}{x - 2 - \cfrac{1}{x - 7 + \cfrac{10}{x - 2 - \cfrac{2}{x - 3}}}}$$

$$rp(x) := \frac{622 - x\cdot(751 - x\cdot(324 - x\cdot(59 - 4\cdot x)))}{112 - x\cdot(151 - x\cdot(72 - x\cdot(14 - x)))}$$

U := 1.60631924 X := U,U + eps ..U + 360 eps



The nearly smooth graph ---- belongs to cf(x) ; the ragged graph of + 's belongs to rp(x) . Every point on each graph has been plotted to show not only how much worse roundoff affects rp(x) than cf(x) but also that roundoff is not nearly so random as some people think.

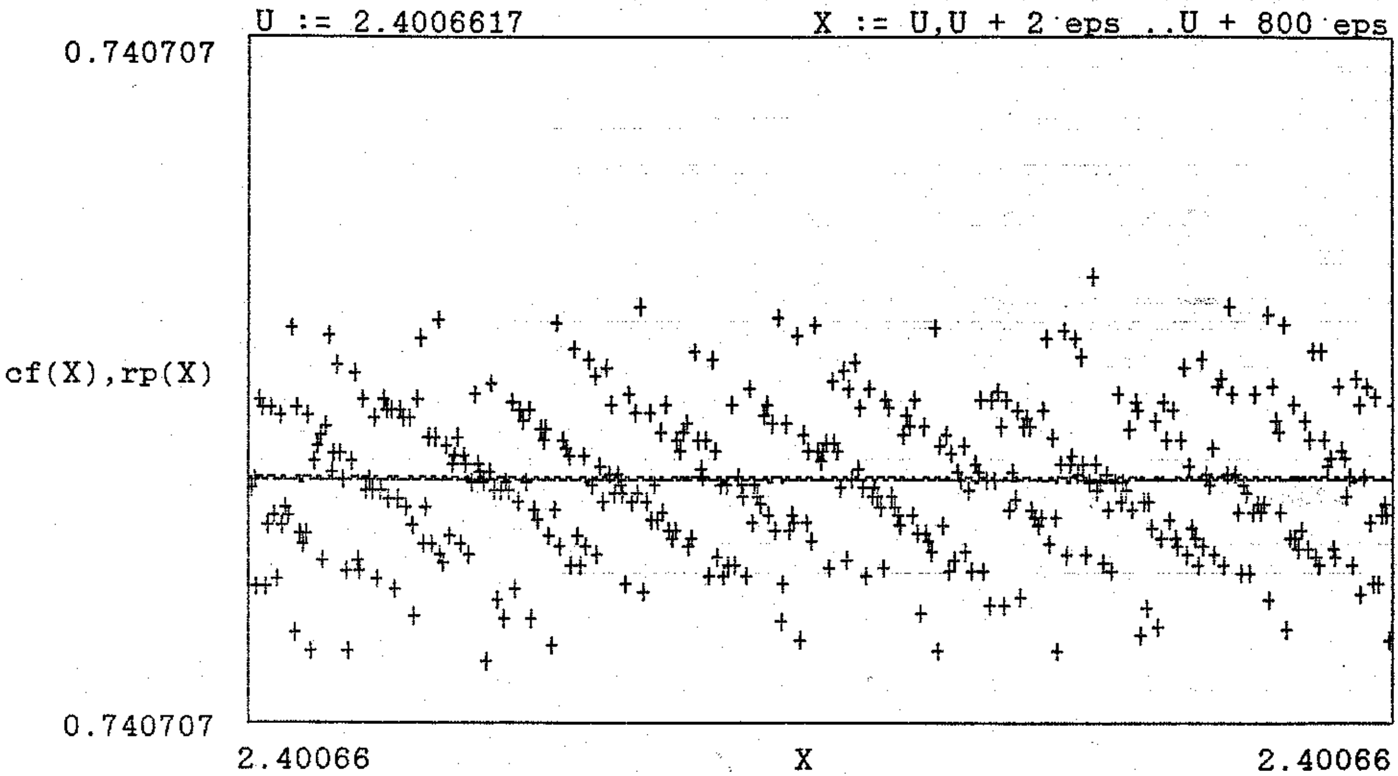
The next example illustrates that cf(x) is invulnerable to Overflow but rp(x) is not :

$cf\left[10^{77}\right] = 4$ correctly.

$rp\left[10^{77}\right] = \blacksquare\ \blacksquare$ overflow

FIGURE 3 from MathCad v. 2.03

The next graphs are included just to show that the previous one was not a fluke. They use different ranges of values for x .



V := 2 + 240·eps

e(x) := cf(x) - rp(x)

$$d(x) := \frac{cf(x) - cf(V)}{10}$$

The next graph shows how roundoff obscures rp(x) , but not cf(x) , by about 24 times as much as that function changes when x changes by one unit in its last place for values x slightly bigger than 2 ; for most other values of x roundoff in rp is much worse than this.