Computer System Support for Scientific and Engineering Computation

Lecture 23 - July 19, 1988 (notes revised June 14, 1990)

1 Floating-Point Exceptions (Continued)

1.1 Cray

The Cray has 64 bit words, and its single precision floating point format has 1 sign bit, 15 exponent bits and 48 bits of significand. The exponent bias is 2^{14}. Cray manuals use octal notation, so that they write the bias as 40000_8. Whenever the biased exponent is less than 20000_8, the result is flushed to 0, and whenever the biased exponent is greater than 60000_8, the number becomes an "indefinite". So there are effectively only 14 exponent bits.

By default, a program halts execution when an indefinite (a number of the form $6xxxx_8$) is created. The indefinite has encoded in it the operation that caused the exception. It is possible to change the default so that computation continues, with the indefinite being propagated like NaN's. However, unlike IEEE arithmetic, which has three kinds of indefinites ($-\infty$, $+\infty$, and NaN) with a careful set of rules for how they propagate thru arithmetic operations, the Cray's indefinites do not combine in a useful way.

Cray expects exceptions to be handled by not getting any! For this purpose, Cray lets you provide a bit vector that specifies elements of a vector on which no operation should be performed. The intended use of this facility is to inhibit division by zero, so that exceptions can be avoided.

1.2 Apple

Apple has a very complete set of primitive functions, language extensions and library routines called SANE that work across their entire product line (except UNIX on the Mac II). SANE has the three precisions single, double, extended, and always computes intermediate results in extended precision (assignments implement rounding to narrower precision). Thus the primary reason for providing three types is to control the amount of storage. SANE includes routines for setting and saving rounding modes, rounding precision, exception flags, and trap enables. Users can install their own trap handler(s). When a trap handler is called, it is given the operation and the addresses of the operands and result, so that it can modify them. Up to this point, SANE has the same flavor (although more carefully done) as exception handling on the VAX and IBM/370. The new element that SANE provides is a pair of routines called `ProcEntry` and `ProcExit`. ProcEntry saves the current floating point environment, and resets the environment to the IEEE default. ProcExit restores the

environment, adding to it any flags that were set at the time of the ProcExit call (and causing corresponding halts if they are enabled). An example of its use is

```
myprog()
    ProcEntry(myCallerEnvironment)
    ...
    y = 2*arctan(sqrt((x+1)/(x-1)))
    SetException(DivByZero, false)
    ProcExit(myCallerEnvironment)
```

If x is 1, then there will be a divide by zero exception, but y will get the correct value of π, so the divide-by-zero flag is cleared. Any other exception that occurs (for example an invalid exception if x is -3) will be passed on to the calling program. Although `ProcEntry` and `ProcExit` are very useful, they could become inefficient if in a long chain of procedure calls, each procedure invoked `ProcEntry` and `ProcExit`.

On machines with a Motorola 68881, exception handling is the same, but programmers must use the 68881's trap handling mechanism rather than the software trap mechanism. By default, transcendental functions are computed in software. Users desiring more speed can ask for the on-chip transcendentals, which are faster but less accurate.

SANE is designed to require the minimum number of changes to the host language, but some are unavoidable. For example, standard Pascal has only one floating point type, which isn't sufficient to exploit SANE's three precisions. Compilers must also be altered to cope with IEEE comparison involving NaNs and with I/O of NaNs and infinities.

In summary, SANE provides a portable floating point environment with little overhead, but it appears that its exception handling and trapping features are not heavily used by customers, perhaps because most are disinclined to change their habits developed elsewhere.

1.3 APL

A slightly different strategy from the `ProcEntry` approach is used in STSC APL for the IBM PC, to handle a similar problem. APL has a global variable called `CT` which controls the tolerance for comparisons. When comparing two numbers, the comparison will be true if it holds within this tolerance. This has strange consequences. For example, you can have x=y and y=z but not x=z!

In STSC APL, you can declare `CT` to be local to a procedure, and it will have a "copy-on-write" behavior. It initially has the same value as the global `CT`, but if it is modified in the subroutine, it will affect the compare tolerance only in the procedure, and not change the global value of `CT`. Thus if you write to the local `CT`, you will get a local copy. This is a nicer model for the programmer than `ProcEntry`, but it does require changing the compiler. There is a fine point connected to associating complex operations with an assignment. Suppose you have the code fragment

```
C = A*B;
RndMode = Up
D = A*B;
```

Suppose the assignment to `RndMode` has the side effect of changing the rounding mode. An over-optimizing compiler might see the common subexpression `A*B` and avoid the second multiply, not realizing that it may have a different value because the rounding mode has changed.

2 Precise Interrupts

When discussing exceptions, it is useful to have the concept of a *precise interrupt.* Consider the instruction sequence

```
100    E = E + 1
200    A = B / C
300    B = D + F
```

Suppose that the division operation causes an overflow exception. We would like the trap handler to see the state of the machine as if all the instructions preceeding the division had completed, and none of the instructions following the diviison had initiated. Thus if the trap handler accesses E, it should have its value incremented by statement 100, and if it accesses B, it should not have had its value destroyed by statement 300.

There are two main obstacles to precise interrupts. The first is the compiler. An optimizer might have decided to move statement 100 after statement 200. The second is the hardware. The hardware might perform statements 200 and 300 in parallel, and by the time the division operation overflows, statement 300 could have completed and overwritten B. To some extent this is similar to what happens with virtual memory systems, where an instruction can fault because the data it is accessing has been paged out. However, in this case the fault handler is specified by the system and is going to do a very predictable thing, namely bring the data into memory. User-specified trap handlers for floating point show much more diversity in their behavior.

The DEC VAX family has a further distinction between *traps* and *faults.* A fault occurs (logically) before the offending instruction executes, and so returning from the exception handler will execute the instruction that caused the fault. A trap occurs after the offending instruction executes, so returning from the exception handler will begin with the instruction after the offending instruction.

Machines that have overlapped instruction execution (due to pipelining or multiple floating point units) often simulate precise interrupts with extra hardware. One approach is to take checkpoints, and when an exception occurs roll back to the last checkpoint, proceeding from the checkpoint one instruction at a time. Another approach is to have shadow registers to save the state of registers that might be needed later by the trap handler. The approach taken by the HP spectrum is to ensure that the hardware never overwrites the arguments to a floating point operation until it is sure that operations will not trap. In each of these schemes, extra hardware is required to guarantee precise interrupts.

SANE

Standard Apple Numeric Environment

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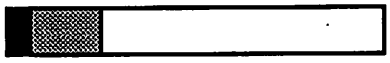
Clayton Lewis

Apple Numerics Group

SANE Data Types

- **Single**

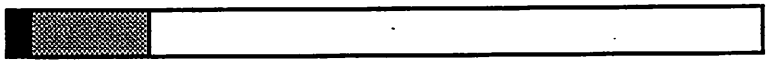
  Precision 7 - 8 Range 10^{-45} to 10^{38}



  1 8 23 = 32 bits

- **Double**

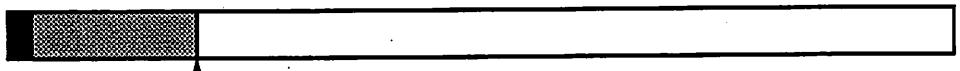
  Precision 15 - 16 Range 10^{-324} to 10^{308}



  1 11 52 = 64 bits

- **Extended**

  Precision 19 - 20 Range 10^{-4951} to 10^{4932}



  1 15 64 = 80/96 bits

  68881 format stores 16-bits of junk here

- **Comp(utational)**

  Precision 18 - 19 Range -10^{19} to 10^{19} integers

  1 63 = 64 bits

* Range and Precision are decimal approximations

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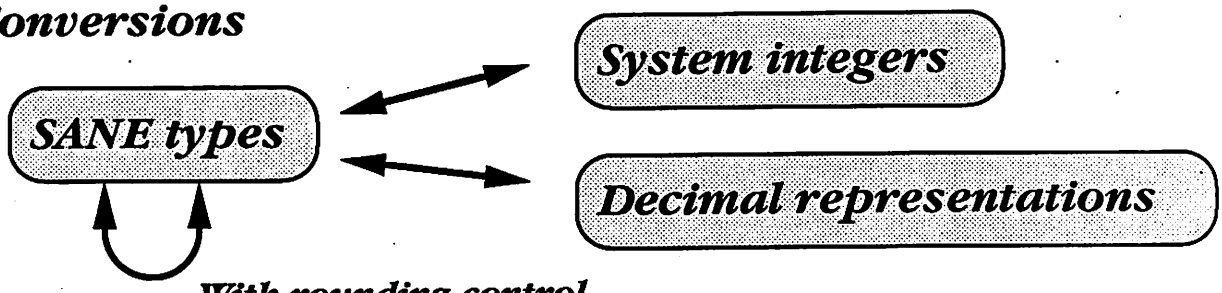
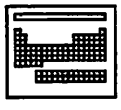
SANE Operations

- +, -, *, ÷, √, *Remainder, round-to-integer*
  - *With rounding control*
  - *Correct to the last bit*

- *Comparisons*
- *Conversions*



- *IEEE auxiliary operations*



absolute value *negate* *nextafter* *scaleb*
logb *classify* *copysign*

- *Elementary functions*

annuity
compound

finance

e^x $e^x - 1$
2^x x^i x^y

exponentials

ln x
ln(1+x)
$\log_2 x$

logarithms

sin *cos*
tan *arctan*

trigonometric

random

random numbers

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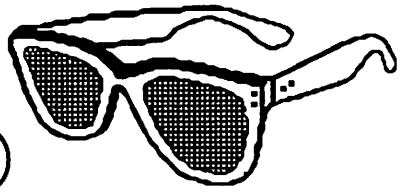
SANE in High Level Languages

- *SANE languages support the abstract environment:*

  Data types

  Operations

  Exceptions



- *SANE languages offer:*

  - *Minimum impact on language standards*
  - *SANE features not expressible in the minimally-modified language are placed in a library*
  - *Old sources run (more accurately)*
  - *Expressions evaluated in extended precision*
  - *Data types, operations, and environment under full user control*
  - *I/O supports NaNs and Infinities*
  - *Source constants held in extended precision*

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SANE Availability

- *SANE is available on all Apple machines*

Macintosh II	Apple //
Macintosh	Apple //GS

- *SANE is available in software and in hardware*

Software on all machines without FPU
Hybrid on machines with FPU
Direct 68881 on machines with FPU

- *SANE is available in high-level languages*

C	FORTRAN
Pascal	BASIC

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SANE Library Services

➪ Conversions between binary formats

Num2Integer	Num2Single
Num2Longint	Num2Double
Num2Extended	Num2Comp

Num2Dec
Dec2Num
Num2Str
Str2Num

Str2Dec
CStr2Dec
Dec2Str

➪ IEEE and IEEE auxiliary functions

Remainder	CopySign	ClassSingle	SignNum
Rint	NextSingle	ClassDouble	NaN
Scalb	NextDouble	ClassComp	
Logb	NextExtended	ClassExtended	

➪ Environmental access routines

SetEnvironment	ProcEntry	SetPrecision	
GetEnvironment	ProcExit	GetPrecision	
SetException	SetHalt	SetRound	SetHaltVector
TestException	TestHalt	GetRound	GetHaltVector

➪ Elementary functions

➪ Relation

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SANE Exceptions

- *What happens when an exception occurs*

  Flags are raised

  

  Corresponding halts are checked

  

  - *Invalid*
  - *Overflow*
  - *Underflow*
  - *Division-by-zero*
  - *Inexact*

- *A reasonable value is returned*

  

  Closest machine representation
  NaN (Not a Number)
  Infinity

- *Examples*

$\sqrt{-3}$	→	*NaN*	*(invalid)*
8/0	→	*Inf*	*(division-by-zero)*
0/0	→	*NaN*	*(invalid)*
*2*maxvalue*	→	*Inf*	*(overflow, inexact)*
minvalue/2	→	*0*	*(underflow, inexact)*
1/3	→	*.33333333333333333*	*(inexact)*

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Control after Exceptions

* Software SANE (software-based)

User sets and queries current halt vector

Information is placed on the stack

Operation

Arguments (with result already delivered)

Exceptions from this operation

* Hardware SANE (68881-based)

User sets and queries current halt vectors

The 68881 trapping model is followed
(programs run in supervisor mode)

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