

## Penalty Matrix

The penalty matrix  $\Omega$  in ?? is a combination of three sub matrix  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$ , which are in the following form

$$\begin{aligned}
\Omega_{k,k}^{(1)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{00}^{(k)}(t)}{dt^2} \frac{d^2 h_{00}^{(k)}(t)}{dt^2} dt = \frac{12}{\Delta_k^3} \\
\Omega_{k,k+1}^{(1)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{00}^{(k)}(t)}{dt^2} \frac{d^2 h_{01}^{(k+1)}(t)}{dt^2} dt = \frac{-12}{\Delta_k^3} \\
\Omega_{k+1,k+1}^{(1)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{01}^{(k+1)}(t)}{dt^2} \frac{d^2 h_{01}^{(k+1)}(t)}{dt^2} dt = \frac{12}{\Delta_k^3} \\
\Omega_{k,k}^{(2)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{10}^{(k)}(t)}{dt^2} \frac{d^2 h_{10}^{(k)}(t)}{dt^2} dt = \frac{4}{\Delta_k^2} \\
\Omega_{k,k+1}^{(2)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{10}^{(k)}(t)}{dt^2} \frac{d^2 h_{11}^{(k+1)}(t)}{dt^2} dt = \frac{2}{\Delta_k} \\
\Omega_{k+1,k+1}^{(2)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{11}^{(k+1)}(t)}{dt^2} \frac{d^2 h_{11}^{(k+1)}(t)}{dt^2} dt = \frac{4}{\Delta_k} \\
\Omega_{k,k}^{(3)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{00}^{(k)}(t)}{dt^2} \frac{d^2 h_{10}^{(k)}(t)}{dt^2} dt = \frac{6}{\Delta_k^2} \\
\Omega_{k,k+1}^{(3)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{00}^{(k)}(t)}{dt^2} \frac{d^2 h_{11}^{(k+1)}(t)}{dt^2} dt = \frac{6}{\Delta_k^2} \\
\Omega_{k+1,k}^{(3)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{10}^{(k)}(t)}{dt^2} \frac{d^2 h_{01}^{(k+1)}(t)}{dt^2} dt = \frac{-6}{\Delta_k^2} \\
\Omega_{k+1,k+1}^{(3)} &= \int_{t_k}^{t_{k+1}} \frac{d^2 h_{01}^{(k+1)}(t)}{dt^2} \frac{d^2 h_{11}^{(k+1)}(t)}{dt^2} dt = \frac{-6}{\Delta_k^2}
\end{aligned}$$