PROBABILITY

Probability is a numerical measure of the likelihood that an event will occur. Probability values are always assigned on a scale from 0 to 1. A probability near zero indicates an event is unlikely to occur; a probability near 1 indicates an event is almost certain to occur

Event - An individual outcome of a sample space is called a simple event; it is a collection or set of one or more simple events in a sample space.

Exhaustive Event - All possible outcomes must be included.

Mutually exclusive Event - which means that no two outcomes can occur at the same time i.e. if the events have no sample points in common.

Sample Space - A sample space of a random experiment is a list of all possible outcomes of the experiment. The outcomes must be exhaustive and mutually exclusive and is denoted by S.

Requirements of Probabilities

Given a sample space , the probabilities assigned to the outcomes must satisfy two requirements.

1. The probability of any outcome must lie between 0 and 1; that is,

for each *i*

*[Note: is the notation we use to represent the probability of outcome i]*

1. The sum of the probabilities of all the outcomes in a sample space must be 1. That is,

**Three Approaches to Assigning Probabilities**

1. The classical approach - when all the experimental outcomes are equally likely.
2. The relative frequency approach - when data are available to estimate the proportion.
3. The subjective approach - An excellent example is derived from the field of investment. An investor would like to know the probability that a particular stock will increase in value. Using the subjective approach, the investor would analyze a number of factors associated with the stock and the stock market in general and, using his or her judgment, assign a probability to the outcomes of interest.

# Counting Rules

* Multiple-step experiments - The first counting rule applies to multiple-step experiments. Consider the experiment of tossing two coins. If an experiment can be described as a sequence of k steps with n1 possible outcomes on the first step, n2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by (n1).(n2)...(nk).
* Combinations A second useful counting rule allows one to count the number of experimental outcomes when the experiment involves selecting ***r*** objects from a (usually larger) set of ***n*** objects. It is called the counting rule for combinations.

The notation ***!*** means factorial

= FACT(number)

= COMBIN(n, r)

= COMBINA(n, r)

[<https://corporatefinanceinstitute.com/resources/excel/functions/combin-function/>]

* Permutation - A third counting rule that is sometimes useful is the counting rule for permutations. It allows one to compute the number of experimental outcomes when n objects are to be selected from a set of N objects where the order of selection is important. The same ***r*** objects selected in a different order are considered a different experimental outcome. The counting rule for permutations closely relates to the one for combinations; however, an experiment results in more permutations than combinations for the same number of objects because every selection of n objects can be ordered in n! different ways.

= PERMUT(n, r)

= PERMUTATIONA(n, r)

[<https://corporatefinanceinstitute.com/resources/excel/functions/permutationa-function/>]

# Basic Relationships of Probability

## Complement of an Event

Given an event A, the complement of A is defined to be the event consisting of all sample points that are not in A. The complement of A is denoted by Ac.

**P(A) + P(Ac) = 1**

**P(Ac) = 1 - P(A)**

## Addition Law

The addition law is helpful when we are interested in knowing the probability of two or more events occurring. we need to discuss two concepts related to the combination of events: the ***union*** of events and the ***intersection*** of events. Given two events A and B, the union of A and B is defined as follows.

* Union of Two Events - The union of A and B is the event containing all sample points belonging to A or B or both. The union is denoted by . The addition law of probability is used to compute the probability of a union of two events.
* Intersection of Two Events - Given two events A and B, the intersection of A and B is the event containing the sample points belonging to both A and B. The intersection is denoted by . the multiplication law is used to compute the probability of the intersection of two events.

### Non-Mutually Exclusive Events

### Mutually Exclusive Events

# Conditional Probability

We use the notation | to indicate that we are considering the probability of event A given the condition that event B has occurred. Hence, the notation P(A|B) reads “the probability of A given B.”

### Non-Independent Events

***Joint Probability*** - The intersection of events C and D is the event that occurs when both C and D occur. It is denoted as C and D. The probability of the intersection is called the joint probability. This type of combination is called a union of two events.

***Marginal Probability*** - marginal probabilities are found by summing the joint probabilities in the corresponding row or column of the joint probability table.

Multiplication Law

### Independent Events

OR

Multiplication Law

# Bayes’ Theorem

In the discussion of conditional probability, we indicated that revising probabilities when new information is obtained is an important phase of probability analysis. Often, we begin the analysis with initial or prior probability estimates for specific events of interest. Then, from sources such as a sample, a special report, or a product test, we obtain additional information about the events. Given this new information, we update the prior probability values by calculating revised probabilities, referred to as posterior probabilities. Bayes’ theorem provides a means for making these probability calculations.

# APPLICATIONS

1. APPLICATIONS in FINANCE [Mutual Funds] - A mutual fund is a pool of investments made on behalf of people who share similar objectives. In most cases, a professional manager who has been educated in finance and statistics manages the fund. He or she makes decisions to buy and sell individual stocks and bonds in accordance with a specified investment philosophy. For example, there are funds that concentrate on other publicly traded mutual fund companies. Other mutual funds specialize in Internet stocks (so-called dot-coms), whereas others buy stocks of biotechnology firms. Surprisingly, most mutual funds do not outperform the market; that is, the increase in the net asset value (NAV) of the mutual fund is often less than the increase in the value of stock indexes that represent their stock markets. One reason for this is the management expense ratio (MER) which is a measure of the costs charged to the fund by the manager to cover expenses, including the salary and bonus of the managers. The MERs for most funds range from .5% to more than 4%. The ultimate success of the fund depends on the skill and knowledge of the fund manager. This raises the question, which managers do best? Determinants of Success among Mutual Fund Managers
2. Reminder