APPC
TP 6 SVD & NNMF

Thomas Robert

SVD

First, let's try to factorize X matrix using SVD method.

We get a K rank factorization, however, how to interpret negative values ?

```
% compute 16 first SVD
K = 16;
[U, S, V] = svds(X, K);

% estimate
Xhat = U*S*V';

% plot
i = 1;
for k = randperm(n, 8)
    if (i <= 4) subplot(4,4,i);
    else        subplot(4,4,4 + i); end
    imshow(reshape(X(k,:), 50, 50), [])
    title(['Original n°' int2str(k)])
    if (i <= 4) subplot(4,4,4 + i);
    else        subplot(4,4,8 + i); end
    imshow(reshape(Xhat(k,:), 50, 50), [])
    title(['Estimé n°' int2str(k)])
    i = i + 1;
end

figure
for i = 1:K
    subplot(4,ceil(K/4),i)
    imshow(reshape(V(:,i), 50, 50), [])
    title(['Composante V_{' int2str(i) '}'])
end
```

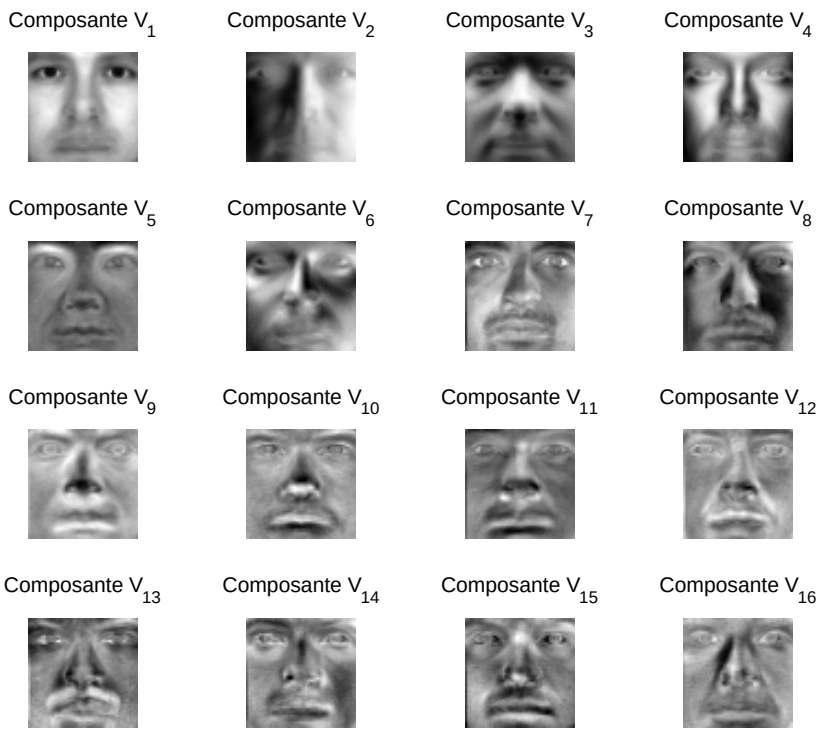

Figure 1 : First components of the SVD.

Figure 2 : Results of the estimates.

NNMF

To solve this problem of negative values, let's use Non-negative Matrix Factorization of the same data.

We will use proximal gradient method or this.

Because of this constrain of positiveness, we can no longer find components with values that cancels each other. Therefore, the new components contains more null values on some zones of the picture, so that components add themselves and represent differents parts of the face.

```
% Params
K = 16;
eps = 1e-1;
kmax = 1000;
rhoU = 1e-5;
rhoV = 1e-5;

% Init
U = zeros(n,K);
V = rand(p,K);
J = Inf;
Js = [];

% Iterate
k = 1;
while (J > eps && k < kmax)
    VOld = V;

    for i = 1:n
        U(i,:) = max(0, U(i,:)' - rhoU * V'*(V*U(i,:)' - X(i,:)'));
    end

    for j = 1:p
        V(j,:) = ( V(j,:)' - rhoV * (U'*(U*V(j,:)' - X(:,j))) )';
    end

    k = k + 1;
    J = norm(VOld - V);
    Js(end+1) = J;
    %
end

% estimate
Xhat = U*V';

% plot
i = 1;
for k = randperm(n, 5)
    subplot(2,5,i)
    imshow(reshape(X(k,:), 50, 50), [])
    title(['Original n°' int2str(k)])
    subplot(2,5,5+i)
    imshow(reshape(Xhat(k,:), 50, 50), [])
    title(['Estimé n°' int2str(k) ' (K = ' int2str(K) ')'])
    i = i + 1;
end

figure
for i = 1:K
    subplot(4,ceil(K/4),i)
    imshow(reshape(V(:,i), 50, 50), [])
    title(['Composante V_{' int2str(i) '}'])
end

figure
plot(Js)
title('Evolution of V matrix (norm of the variation)');
xlabel('Iteration')
ylabel('Variation norm')
```

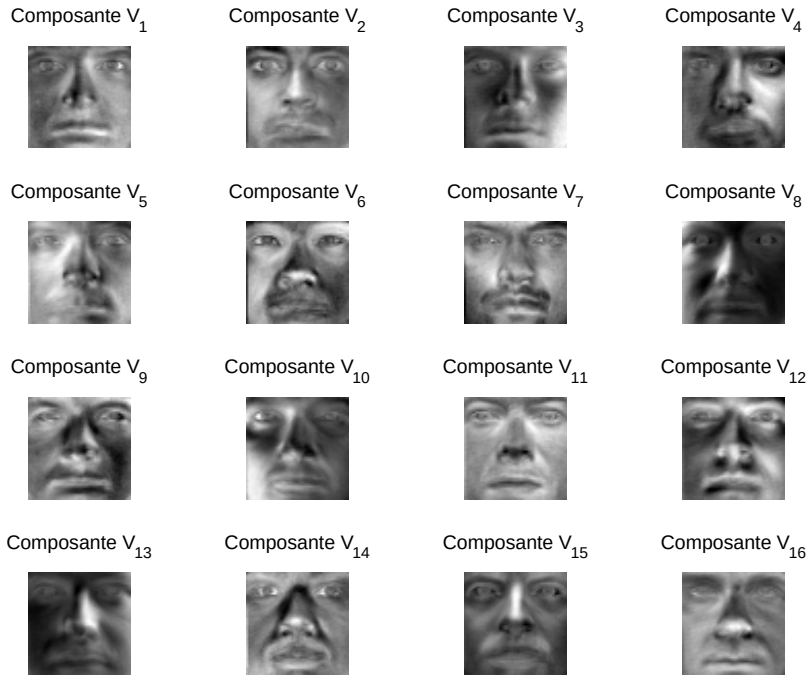

Figure 3 : V matrix of the NNMF.

Figure 4 : Results of the estimate.

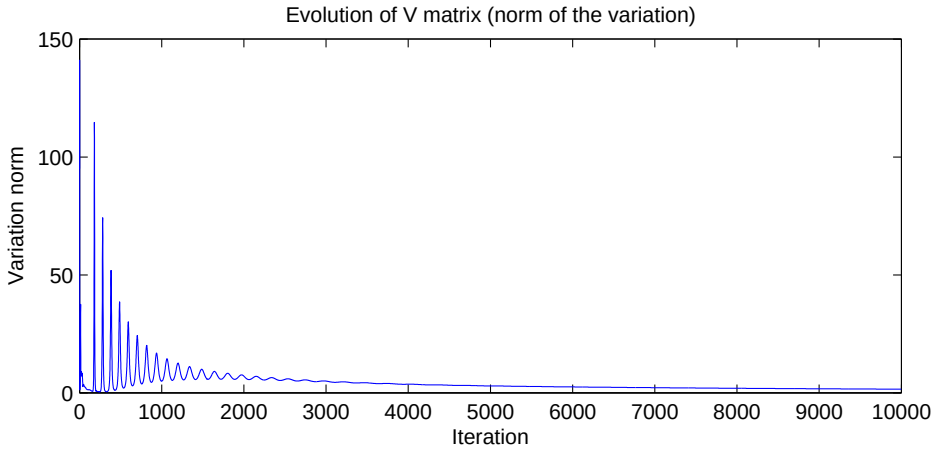

Figure 5 : Evolution of the V matrix.