Rupture detection (CUSUM, filtered derivative)

April 14, 2015

1 Useful imports

In [1]:
```
import matplotlib
import matplotlib.pyplot as plt
import pickle
import numpy as np
import pylab
import scipy.stats
%matplotlib inline
pylab.rcParams['figure.figsize'] = (14.0, 8.0)
```

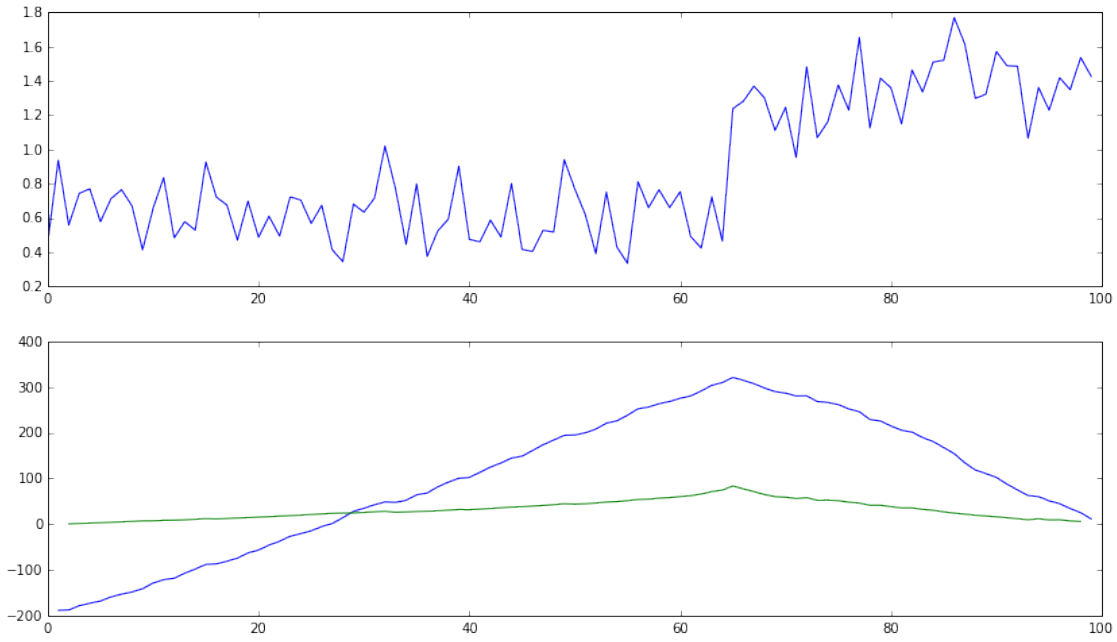
2 CUSUM

2.1 Offline

In [2]:
```
data = pickle.load(open('cusum.pick','rb'))

data = data[1]

signal = data['signal']
mu0 = data['mu0']
mu1 = data['mu1']
sigma0 = data['sigma0']
sigma1 = data['sigma1']

n = len(signal)

def likelihood(mu, sigma, x):
    x = np.atleast_1d(x)
    xl = -np.log(sigma) - 1. / 2 * np.log(2. * np.pi) - (x - mu)**2 / (2 * sigma**2)
    return np.sum(xl)

def vraissemblanceParamsConnus (signal, mu0, mu1, muTilde, sigma0, sigma1, sigmaTilde):
    n = len(signal)
    vraissemblance = np.zeros((n-1,))

    for t0 in range(1, n):
        l1 = likelihood(mu1, sigma1, signal[t0:])
        l0 = likelihood(mu0, sigma0, signal[:t0])
        l00 = likelihood(muTilde, sigmaTilde, signal)

        vraissemblance[t0 - 1] = l0 + l1 - l00
```

```
        return vraissemblance


    def vraissemblanceParamsInconnus (signal):
        n = len(signal)
        vraissemblance = np.zeros((n-3,))

        for t0 in range(2, n-1):

            mu0 = np.mean(signal[:t0])
            mu1 = np.mean(signal[t0:])
            muTilde = np.mean(signal)

            sigma0 = np.std(signal[:t0])
            sigma1 = np.std(signal[t0:])
            sigmaTilde = np.std(signal)

            l1 = likelihood(mu1, sigma1, signal[t0:])
            l0 = likelihood(mu0, sigma0, signal[:t0])
            l00 = likelihood(muTilde, sigmaTilde, signal)

            vraissemblance[t0 - 2] = l0 + l1 - l00

        return vraissemblance

    plt.subplot(2,1,1)
    plt.plot(signal)
    plt.subplot(2,1,2)
    plt.plot(range(1,n), vraissemblanceParamsConnus(signal, mu0, mu1, mu0, sigma0, sigma1, sigma0))
    plt.plot(range(2, n-1), vraissemblanceParamsInconnus(signal))
```

Out[2]: [<matplotlib.lines.Line2D at 0x10d0da890>]

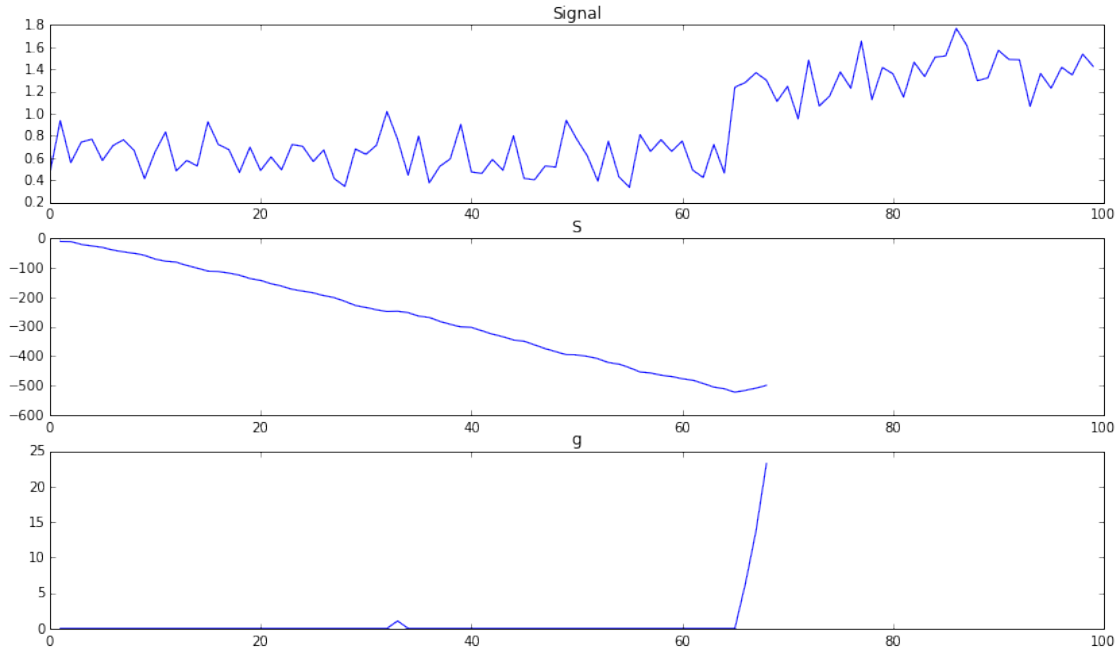


2.2 Online

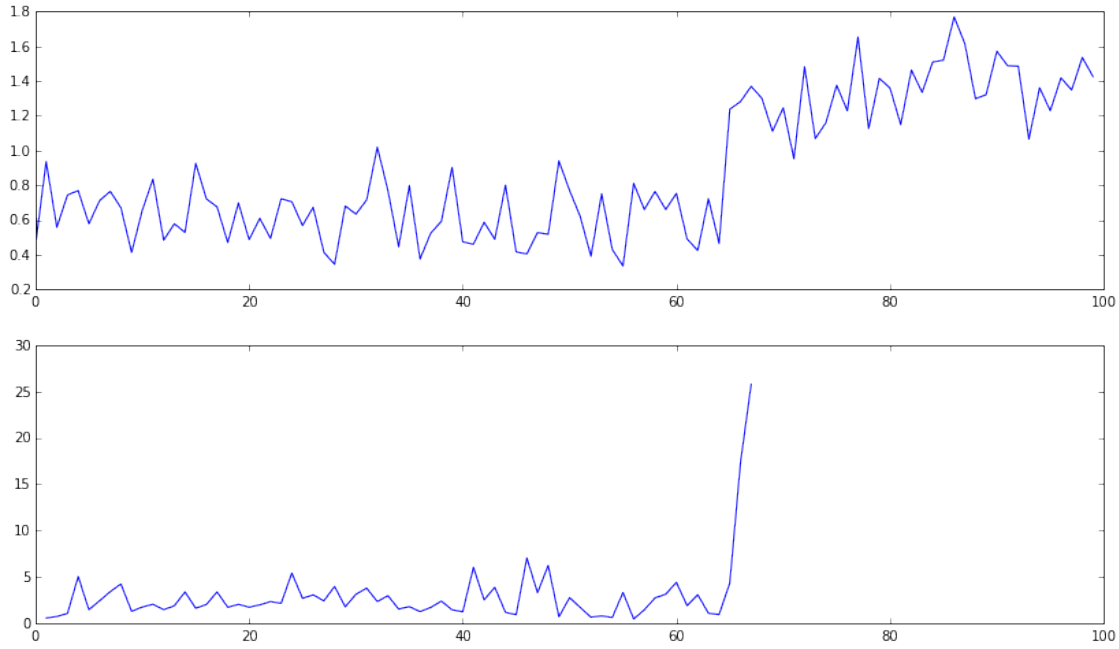
In [3]:
```
data = pickle.load(open('cusum.pick','rb'))

data = data[1]

signal = data['signal']
mu0 = data['mu0']
mu1 = data['mu1']
sigma0 = data['sigma0']
sigma1 = data['sigma1']

def likelihood(mu, sigma, x):
    x = np.atleast_1d(x)
    xl = -np.log(sigma) - 1. / 2 * np.log(2. * np.pi) - (x - mu)**2 / (2 * sigma**2)
    return np.sum(xl)

def vraissemblanceParamsConnus (signal, mu0, mu1, sigma0, sigma1):

    l0 = likelihood(mu0, sigma0, signal)
    l1 = likelihood(mu1, sigma1, signal)

    return l1 - l0

def vraissemblanceParamsInconnus (signal, mu0, sigma0):

    k = len(signal) - 1
    Sjs = np.zeros((k,))

    for j in range(k):

        mu1 = np.mean(signal[j:])
        sigma1 = np.std(signal[j:])

        l0 = likelihood(mu0, sigma0, signal[j:])
        l1 = likelihood(mu1, sigma1, signal[j:])

        Sjs[j] = l1 - l0

    return Sjs

def ruptureOnlineParamsConnus (signal, mu0, mu1, sigma0, sigma1):

    n = len(signal)
    S = np.zeros((n-1,))
    g = np.zeros((n-1,))
    h = 20

    for i in range(1,n):
        Sk = vraissemblanceParamsConnus(signal[:i], mu0, mu1, sigma0, sigma1)
        S[i-1] = Sk;
```

```
        mk = np.min(S[:i])
        gk = Sk - mk
        g[i-1] = gk

        if (gk > h):
            t0 = np.argmin(S[:i])
            break

    ta = i
    print("Rupture at " + str(t0) + " found at " + str(ta))

    plt.subplot(3,1,1)
    plt.plot(signal)
    plt.xlim([0,n])
    plt.title('Signal')
    plt.subplot(3,1,2)
    plt.plot(range(1,ta+1), S[:ta])
    plt.xlim([0,n])
    plt.title('S')
    plt.subplot(3,1,3)
    plt.plot(range(1,ta+1), g[:ta])
    plt.xlim([0,n])
    plt.title('g')
    plt.show()

def ruptureOnlineParamsInconnus (signal, mu0, sigma0):

    n = len(signal)
    g = np.zeros((n-2,))
    h = 20

    for i in range(2,n):
        Sjs = vraissemblanceParamsInconnus(signal[:i], mu0, sigma0)
        gk = np.max(Sjs);
        g[i-2] = gk;

        if (gk > h):
            t0 = np.argmax(Sjs)
            break

    ta = i
    print("Rupture at " + str(t0) + " found at " + str(ta))

    plt.subplot(2,1,1)
    plt.plot(signal)
    plt.xlim([0,n])
    plt.subplot(2,1,2)
    plt.plot(range(1,ta), g[:ta-1])
    plt.xlim([0,n])
    plt.show()

ruptureOnlineParamsConnus (signal, mu0, mu1, sigma0, sigma1)
ruptureOnlineParamsInconnus (signal, mu0, sigma0)
```

Rupture at 64 found at 68



Rupture at 65 found at 68



3 Filtered derivative

3.1 Seuil fixé

In [4]:
```
C = 0.5
```

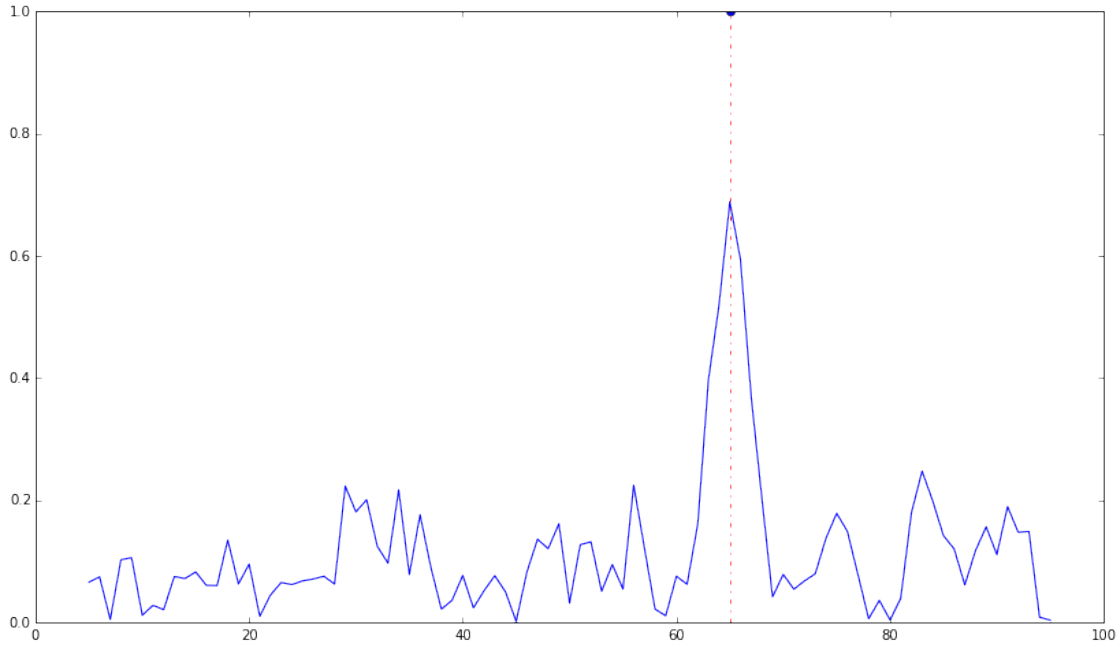
In [5]:
```
data = pickle.load(open('cusum.pick','rb'))

        data = data[1]

        signal = data['signal']
        mu0 = data['mu0']
        mu1 = data['mu1']
        sigma0 = data['sigma0']
        sigma1 = data['sigma1']
```

In [6]:
```
def calculDifference(A1, A2, signal):
            n = len(signal)

            D = np.zeros((n,))
            D[:] = np.NAN

            # for i, k in enumerate(range(A1, n - A2 + 1)):
            for k in range(A1, n - A2 + 1):
                D[k] = np.abs(np.mean(signal[k:k+A2]) - np.mean(signal[k-A1:k]))

            return D

        D = calculDifference(5, 5, signal)
```

In [7]:
```
def detectionRupture(D, C):
            zonesRupture = np.greater(D, C)
            pointsRupture = []

            ruptStart = 0;
            for i in range(1, len(zonesRupture)):
                if zonesRupture[i] == 1 and zonesRupture[i-1] == 0:
                    ruptStart = i
                elif zonesRupture[i] == 0 and zonesRupture[i-1] == 1:
                    pointsRupture.append(ruptStart + np.argmax(D[ruptStart:i+1]))

            return (zonesRupture, pointsRupture)

        (zonesRupture, pointsRupture) = detectionRupture(D, C)

        plt.plot(D)
        plt.stem(pointsRupture, np.ones((len(pointsRupture),)), '-.r')
```

```
/usr/local/lib/python2.7/site-packages/IPython/kernel/__main__.py:2: RuntimeWarning: invalid value encour
  from IPython.kernel.zmq import kernelapp as app
```

Out[7]:
```
<Container object of 3 artists>
```



3.2 Recherche seuil C

```
In [8]: def calculMsRandperm(A1, A2, signal, n):
            Ms = np.zeros((n,))

            for i in range(n):
                signalPerm = np.random.permutation(signal)
                Ms[i] = np.nanmax(calculDifference(A1, A2, signalPerm))

            return Ms

        Ms = calculMsRandperm(5, 5, signal, 1000)
```

```
In [9]: def findC(Ms, p):
            return scipy.stats.norm.ppf(1 - p,loc=np.mean(Ms), scale=np.std(Ms))
```

```
In [13]: C = findC(Ms, 0.1)

         D = calculDifference(10, 10, signal)
         (zonesRupture, pointsRupture) = detectionRupture(D, C)
         C
```

```
/usr/local/lib/python2.7/site-packages/IPython/kernel/__main__.py:2: RuntimeWarning: invalid value encoun
  from IPython.kernel.zmq import kernelapp as app
```

```
Out[13]: 0.76301602488289277
```