

# Worksheet

## Problem 1. Value of the condition number

Consider the matrix

$$A = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

What's the value of the 2-norm-based condition number of  $A$ ?

## Problem 2. Problems with Rank Finding

Which of the following is a problem with finding the rank of a number of vectors computationally using pivoted Gaussian elimination?

- (A) Infeasibly expensive (in terms of computational work)
- (B) May break down if near-zeros occur in input
- (C) Answer brittle/poorly defined due to rounding errors
- (D) Only works for 'square' sets of vectors, i.e. sets of  $n$  vectors of length  $n$

## Problem 3. Computational expense of solving many linear systems

Suppose you have both the inverse  $A^{-1}$  and a  $PLU$  factorization of an  $n \times n$  matrix. What is true about the computational expense of finding the solution of  $k$  linear systems  $Ax_i = b_i$  ( $i = 1, \dots, k$ ) using both of these methods?

- (A) Using the inverse is asymptotically cheaper ( $n^2$  vs  $n^3$ )
- (B) Asymptotically, the two methods have the same computational cost ( $n^2$  and  $n^2$ )
- (C) Asymptotically, the two methods have the same computational cost ( $n^3$  and  $n^3$ )
- (D) Using LU is asymptotically cheaper ( $n^3$  vs  $n^2$ )

#### Problem 4. Nullspace Finding

Given a LU factorization  $PA = LU$  of a matrix  $A$ , we know that the nullspace is preserved by this factorization as  $N(PA) = N(U)$ . Which of the following are true statements?

- (A) The nullspace of  $A$  can be “read off” from  $U$  with little (at most linear in  $n$ ) computational work.
- (B) Having an LU factorization of  $A$  does not help significantly with computing the nullspace of  $A$ .
- (C) Computing the nullspace is inherently brittle because of rounding error.
- (D) Matrices in echelon form do not have a nullspace.