$$\alpha_1 \, x_1 + \alpha_2 \, x_2 + \alpha_3 \, x_3$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \leftarrow$$

$f: V \to W \quad V, W$ vector spaces

"linear": $f(x+y) = f(x) + f(y) \quad$ (I)

$$f(\alpha x) = \alpha \, f(x) \qquad (II)$$

Example: $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \sin(x)$$

• (I). $f(x+y) = \sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y) \overset{?}{\neq} \sin(x) + \sin(y)$

• $f(0) \dots ?$

$V: \quad v_1, v_2, v_3 \dots, v_m \in V \qquad f: V \to W \text{ linear}$

$W: \quad v_1, w_2, w_3, \dots w_n \in W$

$x = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \cdots + \alpha_m v_m$

$f(x) = f(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \cdots + \alpha_m v_m)$

$\overset{(I)}{=} f(\alpha_1 v_1) + f(\alpha_2 v_2) + f(\alpha_3 v_3) + \cdots + f(\alpha_m v_m)$

$\overset{(II)}{=} \alpha_1 f(v_1) + \alpha_2 f(v_2) + \alpha_3 f(v_3) + \cdots + \alpha_m f(v_m)$

$\underbrace{f(v_1) \in W}_{\begin{pmatrix} \\ \\ \end{pmatrix}} \quad \underbrace{f(v_2) \in W}_{\begin{pmatrix} \\ \\ \end{pmatrix}} \quad \underbrace{f(v_3) \in W}_{\begin{pmatrix} \\ \\ \end{pmatrix}} \quad \underbrace{f(v_m) \in W}_{\begin{pmatrix} \\ \\ \end{pmatrix}}$