Numerical Methods (CS 357)

Worksheet

Problem 1. Reduced and full QR

Given a matrix A of size $m \times n$ with $m \geq n$, what is the matrix size of reduced QR?

(A) Q: $n \times m$ – R: $m \times m$

(B) Q: $n \times n$ – R: $n \times m$

(C) Q: $m \times m$ – R: $m \times n$

(D) Q: $m \times n$ – R: $n \times n$

Problem 2. Least-squares residual

Given a QR factorization $A = QR$ with $Q = I$, what is the square of the 2-norm of the residual of solving the least-squares problem $Ax \approx b$ going to be if

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \qquad R = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}?$$

Problem 3. Norms and Matrices

Given

$$Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 4 \end{bmatrix},$$

what is $\|Q^T b\|_2^2$?