Numerical Methods (CS 357)

Worksheet

Problem 1. Back-substitution

Consider an $n \times n$ upper triangular matrix. How many operations are carried out in back-substitution?

(A) Independent of n

(B) Proportional to n

(C) Proportional to n^2

(D) Proportional to n^3

Problem 2. Rank and Nullspace

A matrix of size 10×7 has a row space of dimension 5. What is the dimension of its nullspace?

(A) 2

(B) 7

(C) 10

(D) 0

(E) 5

Problem 3. Elimination matrices

Consider the elimination matrices

$$A = \begin{pmatrix} 1 & & \\ & 1 & \\ & 2 & 1 \\ & -4 & & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & & \\ & 1 & \\ & -2 & 1 \\ & 4 & & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \\ & & 5 & 1 \end{pmatrix}.$$

What is ABC?

(A) $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 5 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & -4 & 1 & \\ & 8 & & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & 2 & 1 & \\ & 2 & & 1 \end{pmatrix}$

Problem 4. Invariants of LU

Suppose you have an LU factorization $PA = LU$.
Is rowspace(U) = rowspace(PA)?

(A) Yes

(B) No

Problem 5. Invariants of LU

Suppose you have an LU factorization $PA = LU$.
Is $N(U) = N(PA)$?
(The original version of this question specified got the answer wrong.)

(A) Yes

(B) No