① Linear Algebra Basics

0 What's a vector? $\begin{pmatrix} 5 \\ 7 \\ 12 \\ 4 \end{pmatrix} \leftarrow$ that, duh.

True. But actual definition much broader.

To define, need concept of number.

0 Examples of numbers? of classes of numbers?

So the integers work, right?

No. Will not be concerned with them in this class.

Why not? Need usable division in our numbers.

math-speak: • numbers must form a "field" ← Wikipedia that on a rainy day

$\Leftrightarrow$ satisfy the field "axioms" (= rules)

• integers form only a "ring"

But: possible to define a field with some finite sets of integers. example: "GF(2)"

Another examples: complex numbers ⇝ Python

More info: Ch.1 of book

0 [So... what is a vector?

0 [Tied for the worst-bombing Q on the pre-quiz.

Definition: A set V is called a vector space (= a bag full of vectors) iff:

- $v + w \in V$ for $v, w \in V$
- $\alpha v \in V$ for α a number (from a field) and $v \in V$.

(with sane rules for arithmetic ⇝ Wikipedia)
("axioms")

[Sounds awfully abstract? Think interfaces in object-oriented programming.

```
interface Vector {
    Vector add(Vector x, Vector y);
    ...
}
```

0 [Is a number a vector?

lec2

<u>admin bits</u>

hw0 → online code (please be kind)

discuss pre-quiz

numpy demo

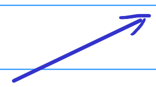
lec3

recap vector space def.

discuss pre-lecture quiz vectors

0 [So, a number is a vector. Is a vector also a number?

<u>Other examples:</u>

- $\begin{pmatrix} 5 \\ 4 \\ 7 \\ 12 \end{pmatrix}$ ← n-tuples of numbers are a form of vector
-  arrows ... how?
- images Demo
- shapes Demo
- text Demo
- video
- you name it

0 [Computational representation: sparse vs. dense ?

0 [Why does every non-empty vector space contain a null vector?

<u>Convention:</u> Restrict "vector space" to being non-empty.

0 [What's the smallest possible vector space?

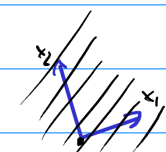
Prelecture combin.

What can you do with a vector space?

"coefficient"

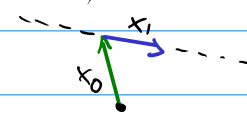
scalar vector

Linear combinations: $\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$



Affine combinations: $\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$

and $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$



Convex combinations: $\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$

with $\alpha_1, \alpha_2, \ldots, \alpha_n \geq 0$

and $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$

Demo

lec 4

hw0 due tonight

discuss pre-lec quizzes: vectors, combinations, matrices

recap conv. comb demo

bit.ly/357-comb

What do you get out of all convex combinations...

... of 4 vectors in 2D?

... of 4 vectors in 3D?

Write an einsum for regular old matrix multiplication.

Is a dot product a feature of a vanilla vector space as defined here?

What's einsum-speak for dot products?

$\Sigma a_i b_i$

lec5

hw1 out

Qs: Assume coding background? (IE etc.)

Dimension of generator?

Fav. color?

$ signs

answers for hw0 → Wed

read up?

einsum

<u>Span</u>: all linear combinations of a set of vectors $x_1, \dots, x_n$

$$\mathrm{span}\{x_1, \dots, x_n\} = \{ \alpha_1 x_1 + \dots + \alpha_n x_n : \alpha_i \text{ is any scalar} \}$$

"generators"

O [A vector space? What do we need to check?

Span and linear combinations are a really neat idea. Why?
We can deal with many vector spaces in terms of coordinates.

Vectors could be really awkward to write down. (images? shapes?)
Coordinates are easy to write down.

If we fix some generators, then we can write down every vector in the span of the generators just by writing down coordinates.

⚠ I know you want to, but don't call them "basis" just yet.

$0.2 \cdot$  $+\ 0.5 \cdot$  $+\ (-2) \cdot$  $=$ 

work with these: $\begin{pmatrix} 0.2 \\ 0.5 \\ -2 \end{pmatrix}$ sweep these under the rug:   

"coordinates" "generators"

But: need generators to reconstruct meaning!

Can also write down coordinates of vectors consisting of numbers. Uh-oh.

$$0.2 \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + 0.5 \cdot \begin{pmatrix} 8 \\ 0 \\ 2 \end{pmatrix} + (-2) \cdot \begin{pmatrix} 5 \\ 0 \\ 8 \end{pmatrix}$$

work with these: $\begin{pmatrix} 0.2 \\ 0.5 \\ -2 \end{pmatrix}$ sweep these under the rug: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 8 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ 8 \end{pmatrix}$

"coordinates" "generators"

And never confuse them! They look similar, but they're not the same thing. A coordinate must always be a coefficient to a generator.

$0.2 \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ✓

$\begin{pmatrix} 0.2 \\ 0.5 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ✗ meaningless!

0 There's only one situation where coordinate vectors and the "underlying" vectors coincide. When?

<Demo: Klein book → Vec class>

So, given a coordinate vector $(2, 6)^T$ with respect to the generators

$$g_1 = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \qquad g_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

what's (a) the underlying vector? (b) the coordinates of that with respect to

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} ?$$

Transforming between different coordinates: one example of a linear function.