Macroeconometrics - Problem Set II

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(a) Create the spread as the difference between the long and the short rate. To transform the spread into a quarterly variable construct the within-quarter averages. Create the growth rate of real GDP.

For this task, we bring in data onto MATLAB concerning the 10-year government bond (GS10), the federal funds rate (FEDFUNDS), and the real GDP (GDPC1) sourced from the FRED database (https://fred.stlouisfed.org/), we take the full range of data for both variables and we join them into a unique dataset.

Next, we compute the monthrly spread as:

$$spread_t = GS10_t - FEDFUNDS_t \forall t \tag{1}$$

Next we want to consider a common process to have the data in the same time period. To do so, we find the earliest and latest common dates in the two tables of spread and real GDP . Then, the data in both tables is filtered to include only the rows with dates falling within the determined range, effectively synchronizing the datasets based on their date columns.

Since our task is to construct a VAR model, we need observations for the variable of interest covering the same frequency. Here, we notice that data on 10Y bonds and federal funds rate are monthly data, and so is the spread, while data on real GDP are quarterly. In order to obtain a measure for the quarterly spread, we re-sample the data aggregating it providing a new representation at a quarterly level with averaged values.

Finally, we manipulate GDP data annd the growth rate of GDP, denoted here as $G\hat{D}\_P$, is calculated using the formula:

$$G\hat{D}\_P_{t+1} = (\log(GDP_{t+1}) - \log(GDP_t)) \cdot 100 \quad \forall t \tag{2}$$

These variables are then employed to estimate a VAR(4) model for the growth rate of real GDP and the quarterly spread.

```
% Read data from the 'GS10.csv', 'FEDFUNDS.csv', and '
   GDPC1.csv' files
ten_gov_bond = readtable('GS10.csv');
fed_fund_rate = readtable('FEDFUNDS.csv');
real_GDP = readtable('GDPC1.csv');

% Specify the key variable for the join operation
keyVariable = 'DATE';

% Perform the join operation on 'DATE' between
   fed_fund_rate and ten_gov_bond
merged_data = join(fed_fund_rate, ten_gov_bond, 'Keys'
   , keyVariable);

% Calculate the 'spread' by subtracting 'FEDFUNDS'
   from 'GS10'
merged_data.spread = merged_data.GS10 - merged_data.
   FEDFUNDS;

% Create a new table with 'DATE' and 'spread'
spreadTable = table(merged_data.DATE, merged_data.
   spread, 'VariableNames', {'DATE', 'spread'});

% Display the result
disp(spreadTable);

% Find the earliest common date in both tables
minDate = max(min(spreadTable.DATE), min(real_GDP.DATE
   ));

% Find the latest common date in both tables
maxDate = min(max(spreadTable.DATE), max(real_GDP.DATE
   ));

% Select data between the earliest and latest common
   dates
spreadTable = spreadTable(spreadTable.DATE >= minDate
   & spreadTable.DATE <= maxDate, :);
real_GDP = real_GDP(real_GDP.DATE >= minDate &
   real_GDP.DATE <= maxDate, :);

% Convert the table to a timetable
spreadTable = table2timetable(spreadTable);

% Resample the data to quarterly frequency,
   calculating the mean
```

```
spreadTable = retime(spreadTable, 'quarterly', 'mean')
   ;

% Rename the 'spread' variable to 'quart_spread'
spreadTable = renamevars(spreadTable, 'spread', '
   quart_spread');

% Display the resampled and renamed data
disp(spreadTable);

% Calculate the percentage growth of GDP using the '
   diff' function
real_GDP.gdp_growth = [0; diff(real_GDP.GDPC1) ./
   real_GDP.GDPC1(1:end-1) * 100];
```

(b) Using the growth rates of real GDP and the spread, estimate a VAR(4).

The VAR(4) model allow for a dynamic analysis of the relationship between these variables over time (lagged terms let us capture potential delayed or persistent effects). Our estimation of this relationship can be expressed through the vector form:

$$Y = X\beta + \varepsilon$$

Where:

$$Y = \begin{bmatrix} \hat{G}DP_4 & spread_4 \\ \hat{G}DP_5 & spread_5 \\ \vdots & \vdots \\ \hat{G}DP_T & spread_T \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & \hat{G}DP_4 & spread_4 & \hat{G}DP_3 & spread_3 & \dots & \hat{G}DP_1 & \text{spread}_1 \\ 1 & \hat{G}DP_5 & spread_5 & \hat{G}DP_4 & spread_4 & \dots & \hat{G}DP_2 & spread_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \hat{G}DP_{T-1} & spread_{T-1} & \hat{G}DP_{T-2} & spread_{T-2} & \dots & \hat{G}DP_{T-4} & spread_{T-4} \end{bmatrix}$$

$$\beta = \begin{bmatrix} \mu_1 & \mu_2 \\ A_1^{1,1} & A_1^{1,2} \\ A_1^{2,1} & A_1^{2,2} \\ A_2^{1,1} & A_2^{1,2} \\ A_2^{2,1} & A_2^{2,2} \\ \vdots & \vdots \\ A_4^{1,1} & A_4^{1,2} \\ A_4^{2,1} & A_4^{2,2} \end{bmatrix}$$

In the codes below you can see how we solved this exercise. In particular, how we used the hand-created code of the VAR model.

```
% estimate VAR(4)
```

```
data = [real_GDP.gdp_growth, spreadTable.quart_spread
    ];
p = 4;
[B,Y,X]=VAREstim(data,p);
n=size(data,2);

%compute and plot the Wold representation

% 1) Companion form F
F=[B(2:end,:)'; eye(n*(p-1)) zeros(n*(p-1),n)];

% 2) Powers of F
for j=1:24
    FF=F^(j-1);
    C(:,:,j) = FF(1:n,1:n);
end
```

where the function of the VAR model is defined by:

```
function [phi,Y,X]=VAREstim(y,p)
Y=y(p+1:end,:);
T=size(Y,1);
X=ones(T,1);
for j=1:p
    X=[X y(p+1-j:end-j,:)];
end
phi=inv(X'*X)*X'*Y; %OLS estimation
```

Finally, we are interested in the Wold representation of the process. That is:

$$Y_t = \sum_{j=0}^{\infty} C_j \epsilon_t$$

with:

$$Y_t = \begin{bmatrix} \hat{G}DP_t \\ spread_t \end{bmatrix} \quad \text{and} \quad \epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}$$

To achieve such representation we transform the VAR(4) process as VAR(1) using the companion form:

$$Z_t = FZ_{t-1} + \varepsilon_t$$

with:

$$Z_t = \begin{bmatrix} \hat{G}DP_t \\ \text{spread}_t \\ \hat{G}DP_{t-1} \\ \text{spread}_{t-1} \\ \vdots \\ \hat{G}DP_{t-p+1} \\ \text{spread}_{t-p+1} \end{bmatrix}$$

$$F = \begin{bmatrix} A_1^{1,1} & A_1^{1,2} & \dots & A_4^{1,1} & A_4^{1,2} \\ A_1^{2,1} & A_1^{2,2} & \dots & A_4^{2,1} & A_4^{2,2} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & 0 \end{bmatrix}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

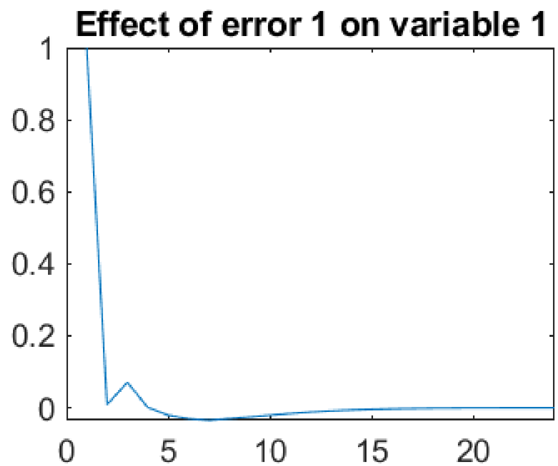
The Wold representation coefficients are derived by extracting the 2x2 submatrix from the top-left portion of each matrix F^j, where j takes values from 0 onwards. The theoretical description of the process is then implemented through the following Matlab process:

```
% 3) Take the appropriate submatrix from F to compute
    the imput response
% function

ii=0;
for j=1:n
     for i=1:n
          ii=ii+1;
          subplot(n,n,ii),plot(squeeze(C(j,i,:)));
          title ("Effect of error " + num2str(i) + " on
              variable " + num2str(j))
     end
end
```

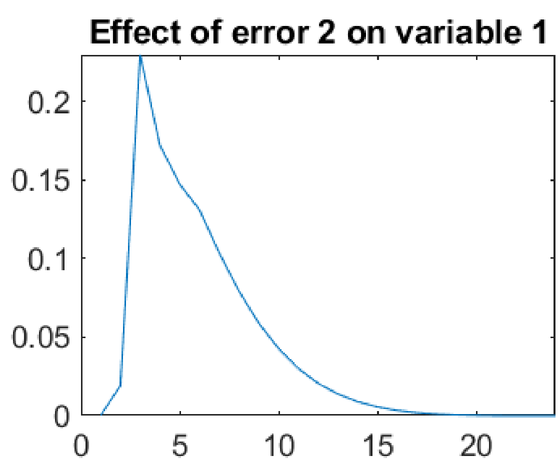
By examining the results, we can analyze the influence of shocks on the Impulse Response Functions (IRF) of real GDP growth rate and the spread level.

GDP Growth Rate - Error GDP Growth Rate
Concerning the first plot, we observe how the error term in the GDP growth rate influences itself. Initially, the magnitude is high, but this effect diminishes almost instantaneously, indicating a low persistence of this magnitude.



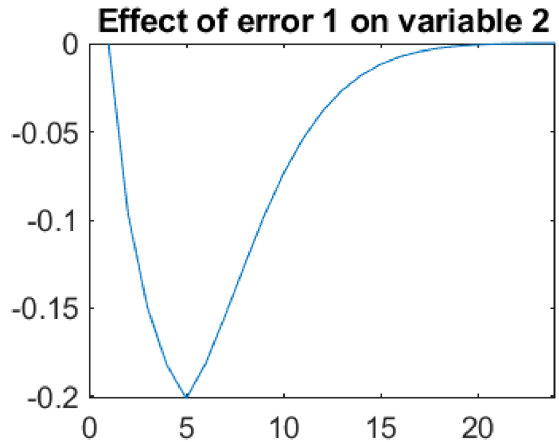
GDP Growth Rate - Error Spread

The impact of the spread on the GDP growth rate is intriguing. There seems to be no immediate impact; instead, the error term influences the GDP growth rate with a slight delay. Subsequently, the decline in the effect is gradual, demonstrating high persistence over time.



Spread - Error GDP Growth Rate

This scenario can be summarized as the reverse of "GDP Growth Rate - Error Spread." Specifically, we observe how the error in the GDP growth rate negatively influences the spread, and furthermore, this effect persists.



Spread - Error Spread

As depicted in the graph, the magnitude of the error term on the main variable is substantial, and it exhibits the most persistence among all scenarios, converging to the steady-state only after 15 periods.