The Real Business Cycle Mode: Working Through A DSGE Model

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1 Introduction

I have developed a dynamic stochastic general equilibrium (DSGE) model to analyze the influence of a technological shock on the business cycle. The underpinning of this model draws inspiration from the work of Finn E. Kydland and Edward C. Prescott in their paperTime to Build and Aggregate Fluctuations [KP82]. My model is particularly based on the version utilized by Professor Karl Whelan in his presentation on, MA Advanced Macroeconomics - The Real Business Cycle Model [Whe16]. The primary focus of my research is to automate a specific facet of DSGE models, specifically the Real Business Cycle (RBC) model. The RBC model, rooted in neoclassical economics, seeks to elucidate business cycles through technological shocks and shifts in productivity. At its core, the model assumes that economic fluctuations are fundamentally propelled by real factors, as opposed to monetary ones. This research encompasses two main objectives. Firstly, I will apply Klein's Method [Pau00], a technique I acquired during the Numerical Methods for Economics course, to solve DSGE models. To accomplish this, I will implement the numerical algorithm delineated in DeJong and Dave 2011 (Chapter 2) [DD11], employing Python for execution. This implementation will enable me to present a dynamic solution for the model's economic dynamics. Secondly, I will scrutinize the model's response to technological disruption and analyze its ramifications on the business cycle.

2 The Model Economy

The core components of our RBC model include:

Consumption and Saving: In the RBC paradigm, households steer their consumption and saving choices in alignment with their intertemporal preferences. Their objective centres on judiciously apportioning resources across time to optimize their cumulative utility over a lifetime. Confronted with the ramifications of technology shocks, households possess the propensity to recalibrate their consumption and saving trajectories, thereby engendering a smoother trajectory of overall well-being. Favourable technology shocks hold the potential to engender augmented income, thus yielding elevated consumption thresholds. Conversely, adverse shocks may elicit a response wherein households pare back immediate consumption and amplify savings, bolstering their readiness for the vagaries of an uncertain future.

Labour Market: Within the RBC framework, the labour market wields substantial influence over the contours of economic outcomes. This labour market is characterized by its malleability, where workers evince a readiness to modulate their labour provision in response to shifts in the real wage rate. The real wage rate factors in adjustments for inflation, thereby endowing it with pivotal significance in appraising the appeal of labour supply. In light of technology shocks that reverberate through firms' labour demands, fluctuations in the real wage rate assume a pivotal function in upholding equilibrium within the labour market. Notably, wage adjustments emerge as a critical linchpin in reestablishing equilibrium amidst the undulations of the economy during periods of turbulence.

Production: Firms hold a pivotal role within the framework of the RBC model, as they are central to the production process. Leveraging inputs like labour and capital, alongside the prevailing technological conditions, firms generate goods and services. The production function encapsulates the technological interplay between inputs and outputs. The firm's objective lies in the maximization of profits, accomplished by adeptly harmonizing these inputs to satiate the demand for their offerings.

Implicit in the production process is the reflection of the economy's productivity level; alterations in technology can exert substantial influence over the aggregate output and the broader economic performance.

Investment: Firms within the RBC construct formulate investment determinations predicated on their foresight regarding forthcoming profits and the extant interest rate environment. Positive technology shocks possess the capacity to augment firms' anticipations concerning future profitability, thereby propelling escalated investments in novel capital and technology. Investments geared toward augmenting productivity and broadening production capabilities serve to magnify the favourable impacts of technology shocks on the tapestry of economic expansion. Conversely, unfavourable technology shocks may impel firms to adopt a circumspect stance vis-à-vis their future trajectories, instigating curtailed investment endeavours and the conceivable onset of economic deceleration.

Technology Shocks: Technology shocks signify unforeseen alterations in the production technology accessible to firms. These shocks can emanate from diverse origins, including technological advancements, shifts in knowledge, or abrupt enhancements in production process efficiency. Within the context of the RBC model, these shocks are posited as stochastic and inherently unpredictable. Upon the occurrence of a favourable technology shock, the productivity of firms undergoes augmentation, consequently fostering escalated output and economic growth. Conversely, adverse technology shocks curtail productivity, precipitating diminished output levels and economic contraction. The model delves into the intricate dynamics of how the economy responds and adapts to these undulating technology-driven fluctuations over time.

Equilibrium: Central to the RBC model is the pursuit of a comprehensive equilibrium within the economy, wherein all markets reach a state of balance. This equilibrium translates to a scenario in which the quantities demanded are equated with the quantities supplied across both goods and labour markets. The vitality of equilibrium conditions lies in their capacity to unravel the intricacies of the economy's reactions to an array of shocks and perturbations. The model performs the crucial task of elucidating how market recalibrations, encompassing alterations in prices and wages, function as the guiding forces that steer the economy back towards equilibrium in the aftermath of technological disturbances.

2.1 Calibrated Parameters

The model simulation is carried out through the calibration of all the structural parameters. A total of nine parameters are involved, and the assigned values, along with their respective sources, are detailed below.

These values have been meticulously calibrated to precisely match those presented in "MA Advanced Macroeconomics - The Real Business Cycle Model" by Karl Whelan [Whe16] for the corresponding variables. Moreover, I fix the equilibrium technology level at 1 (or 0 in logarithmic terms), a strategic choice intended to scrutinize the ramifications of technological innovations.

Table 1: Calibrated parameters

Parameters List			
Parameter	Description	Source	Value
α	Capital share in the production function	slides	1/3
β	preference for future consumption	slides	0.99
δ	Depreciation rate of capital	slides	0.015
ρ	Autoregressive parameter for exogenous technology process	slides	0.95
η	Elasticity of intertemporal substitution in consumption	slides	1.00
A	Initial value of the exogenous technology variable	personal calibration	1.00

2.2 Consumers

The representative consumer aims to maximize their anticipated utility, as expressed by:

$$\max_{c,n} \mathrm{U} = E_t[\sum_{i=0}^{\infty} \beta^i (U(C_{t+i}) - V(N_{t+i}))]$$

where utility is positively and concavely correlated with consumption while being adversely impacted by the aversion to labour. The primary conditions stemming from this optimization underscore two fundamental dilemmas confronted by households: the allocation of time between remunerative labour and gratifying leisure, and the allocation of resources between today's consumption and that of the future.

$$\frac{Y_t}{N_t} = \frac{a}{1-\alpha} C_t^{\eta}$$

$$C_t^{-\eta} = \beta E_t (C_{t+1}^{-\eta} R_{t+1})$$

The first equation equates the condition for optimal hours worked. The second condition is the usual Euler equation, also called the "Keynes-Ramsey Condition", describing the intertemporal trade-off between consumption and savings.

2.3 Final good producing firms

Firms produce the final good and sell it in a perfectly competitive market. The production process involves using capital and consumer labor as inputs. The relevant constraints are as follows:

$$Y_t = C_t + I_t \quad \text{(Goods market clearing)}$$

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \quad \text{(Production function)}$$

$$R_t = \frac{\alpha Y_t}{K_t} + 1 - \delta \quad \text{(Optimal behavior condition for firms)}$$

Commenting on the first equation, it asserts that the total demand for goods, represented as $C_t + I_t$, equals the total supply Y_t within a perfectly competitive market. Here, C_t stands for total consumption, and I_t denotes total investment at time t. The principle of goods market clearing ensures a state of equilibrium without excess goods or shortages, thereby contributing to economic stability.

Turning to the production function equation, firms are the driving force of the economy, responsible for generating the final goods and services consumed by households. The production process combines two pivotal inputs: capital (K_t) and consumer labour (N_t). Through the application of technology (A_t), firms transform these inputs into output (Y_t) in accordance with a production function. In this equation, Y_t signifies the total output at time t, A_t represents the level of technology, K_t symbolizes the capital stock, N_t denotes the labour input, and α is a parameter that captures the output elasticity of capital ($0 < \alpha < 1$), with ($1 - \alpha$) representing the output elasticity of labour.

Finally, we encounter the last equation, which establishes the optimal behaviour condition for firms. Essentially, firms strive to optimize their profits by selecting the optimal level of capital investment represented by (R_t), the interest rate. This interest rate (R_t) equates to the marginal return on capital $\frac{\alpha Y_t}{K_t}$, coupled with the depreciation rate of capital (δ) added to one. This condition underscores the firms' pursuit of profit maximization through judicious capital allocation, considering both the productivity of capital and the pace at which it depreciates.

2.4 Stochastic components

Finally, I posit that the prevailing technological level within the economy adheres to an AR(1) process in logarithmic form, taking the following structure:

$$\log(A_t) = (1-\rho)\log(A^*) + \rho \log(A_{t-1}) + \varepsilon_t$$

where $\log(A^*)$ denotes the steady-state level (with A^* held constant at 1).

3 Analysis of the Model and Solution

The aforementioned model comprises six equations, involving six unknowns: $\{c_t, r_t, y_t, k_t, n_t, i_t\}$.

3.1 Linearization and State Space Representation

The model undergoes log-linearization around the steady state, facilitating the interpretation of deviations from equilibrium as percentage divergences. This process, informed by DeJong and Dave 2011 (Chapter 2) [DD11], encompasses five sequential steps:

1. The natural logarithms of each equation constituting the system are taken on both sides.
2. Variables not logarithmically transformed in the preceding step are then subjected to logarithmic manipulation, leveraging the identity $x_t = e^{\ln(x_t)}$.
3. All terms are consolidated on the left-hand side of each equation.
4. The system is solved for its steady state by setting $x_{t+1} = x_t = \bar{x}$.
5. Utilizing gradient procedures, implemented via Python, derivatives around the steady state $\bar{x}$ are numerically computed with respect to x_{t+1} and x_t.

Rather than crafting the algorithm from scratch in Python, I adopt the package developed by Jenkins [Jen], a framework modeled upon the same literature. Consequently, the system takes on the form:

$$AE[x_{t+1}] = Bx_t$$

enabling its solution through the application of Klein's method.

3.2 Implementation of Klein's Method

To execute the solution procedure, the set of variables is partitioned into predetermined or state variables (comprising endogenous and exogenous) and non-predetermined or jumping ones. Jumping variables, by their essence, encompass those influenced by (potential) forecast errors. Within the aforementioned model, there exists one predetermined variable—specifically, the one exogenously determined: A_t. This implies $s_t = [A_t]$, and six jumping variables, $j_t = [c_t, r_t, y_t, k_t, n_t, i_t]$.

The solution is characterized by a pair of matrices governing the policy (dictating how state variables determine jumping ones) and transition functions (depicting how state variables of today shape those of tomorrow):

$$j_t = F_{s_t}$$

$$s_{t+1} = P_{s_t} + \epsilon_{t+1}$$

Here, ϵ denotes exogenous forcing shocks influencing the stochastic components of the technology model.

4 Technology Shock

Having delineated the model's attributes, the solution approach, and the methodology for parameter calibration, I now pivot to the simulation of the economy. The aim is to investigate, via impulse response functions, how a technology shock can influence our model. It's noteworthy that the technological shock demonstrates a pronounced level of persistence over successive periods.

The exploration of a technology shock's transmission involves scrutinizing the dynamic response of the model economy to a one-standard-deviation escalation in the technological level, departing from its equilibrium value. This increase, calibrated as a 1% rise from the steady-state value, is detailed through the charts presented in Appendix "C2".

Given that technology has been cast as labour-augmenting, the impulse engenders concurrent increments in both the volume of labour employed and wages. Furthermore, this wage increase spurs consumption. The positive shock's repercussions become apparent as it influences both labour hours and capital favourably. Consequently, this impetus drives an upward trajectory in our output.

5 Simulation of Technology Shock

Within the conclusion of my code, a stochastic simulation has been implemented to offer insights into the behaviour of variables within an RBC model in the wake of a technological shock. The simulation grants us a glimpse into how our actual business cycles manifest, revealing patterns that do not appear overly unrealistic. Notably, the model's parameters can be thoughtfully configured to approximate the scale of observed oscillations in output. Furthermore, the model is adept at capturing the discernible volatility gap between investment and consumption.

In spite of these accomplishments, it's worth acknowledging that RBC models have encountered some criticism. Part of this scrutiny arises from their inability to fully realize the aspirations of their early proponents. These models were initially touted for harbouring critical propagation mechanisms capable of transforming technology shocks into business cycles. The concept posited that technological enhancements could stimulate extra output by augmenting capital accumulation and encouraging increased labor participation. In essence, initial research suggested that even in a realm of independently and identically distributed (iid) technology levels, RBC models could still engender business cycles.

However, the final figure reveals a close alignment between output fluctuations in this model and the corresponding technology fluctuations. This observation underscores that the potency of these supplementary propagation mechanisms remains relatively modest.

6 Conclusion

This study has been centred on the formulation of an intuitive model that encompasses the role of technological shocks in shaping the business cycle. Although sparked by insights from the paper "Time to Build and Aggregate Fluctuations" by Prescott [KP82], it is important to underline that this work transcends mere replication. Notably, the original model's intricate details and complexity did not align with the present endeavour's scope. Consequently, I opted to build upon the foundational framework established by Karl Whelan's simpler RBC model [Whe16], while infusing it with pertinent facets culled from the original model.

While I believe the outcomes presented in this work offer thought-provoking insights, I am wary of presenting them as exhaustive or definitive. My intent has been to showcase how the theory introduced during the Numerical Methods course can be effectively applied to tackle intricately detailed models. Looking ahead, it would be compelling to delve into how banking operations and the decisions of monetary policy authorities might interplay with the outcomes engendered by a technological shock.

References

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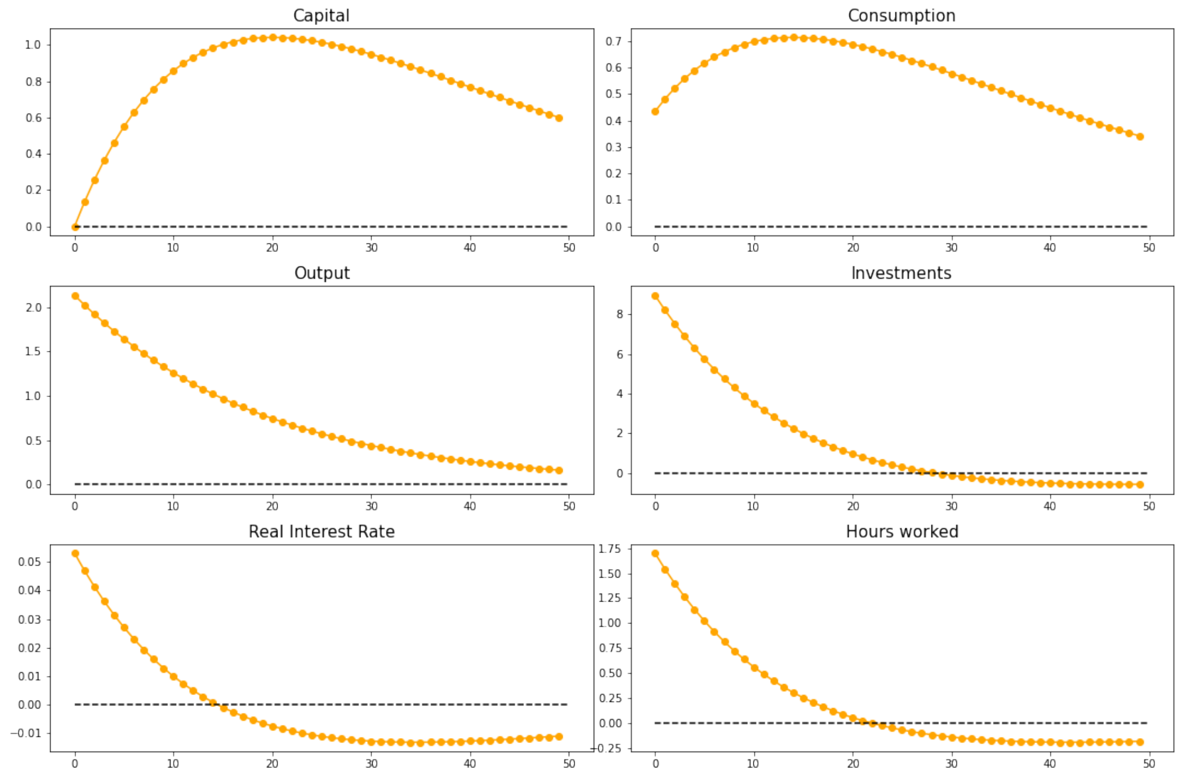
[Whe16] Karl Whelan. The real business cycle model, 2016. https://fep.up.pt/docentes/pcosme/S-E-1/kP-Econ.pdf/.

C. Impulse response functions

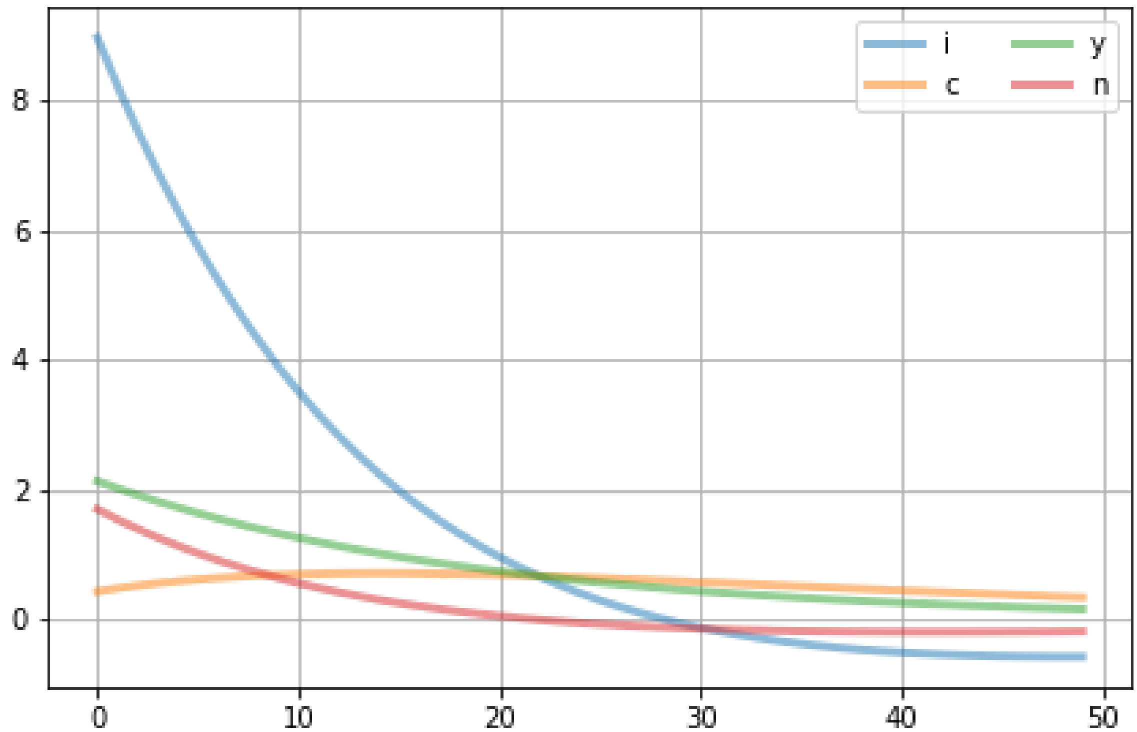
C.1 Technological shock - single impulse response functions

Figure 1: Impulse response function to the transmission of a technological shock



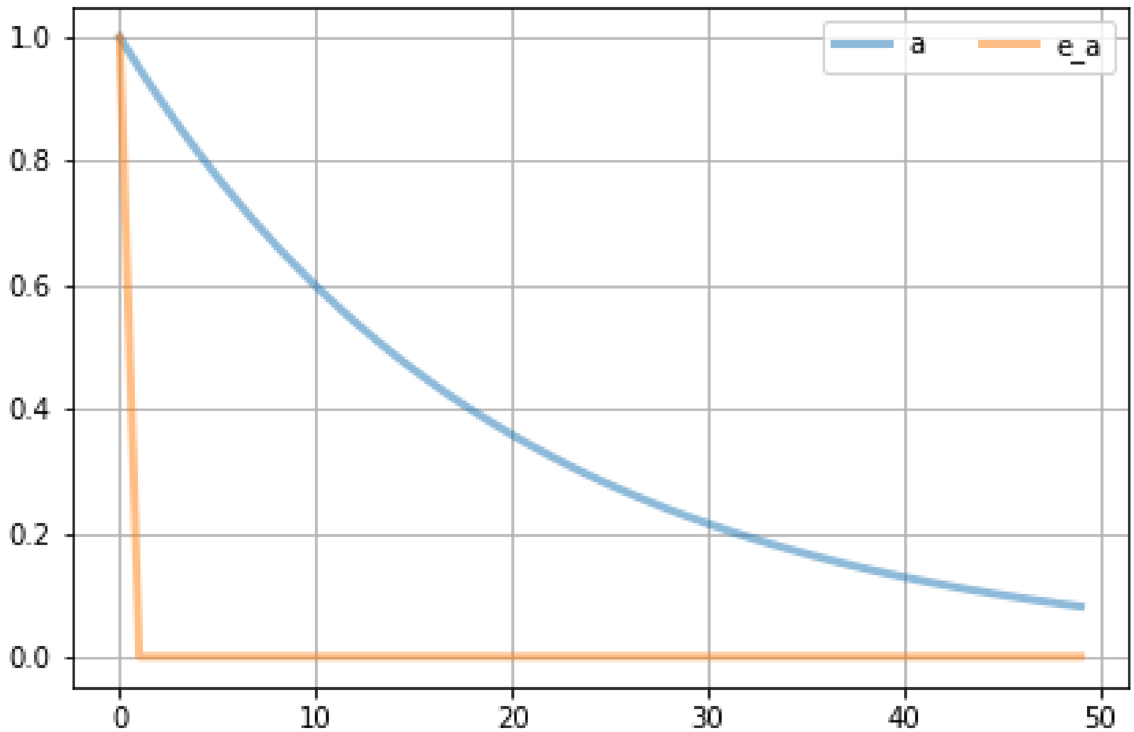
C.2 Technological shock - overall effect

Figure 2: Impulse response function to the transmission of a technological shock - overall, equal to the results of the slide



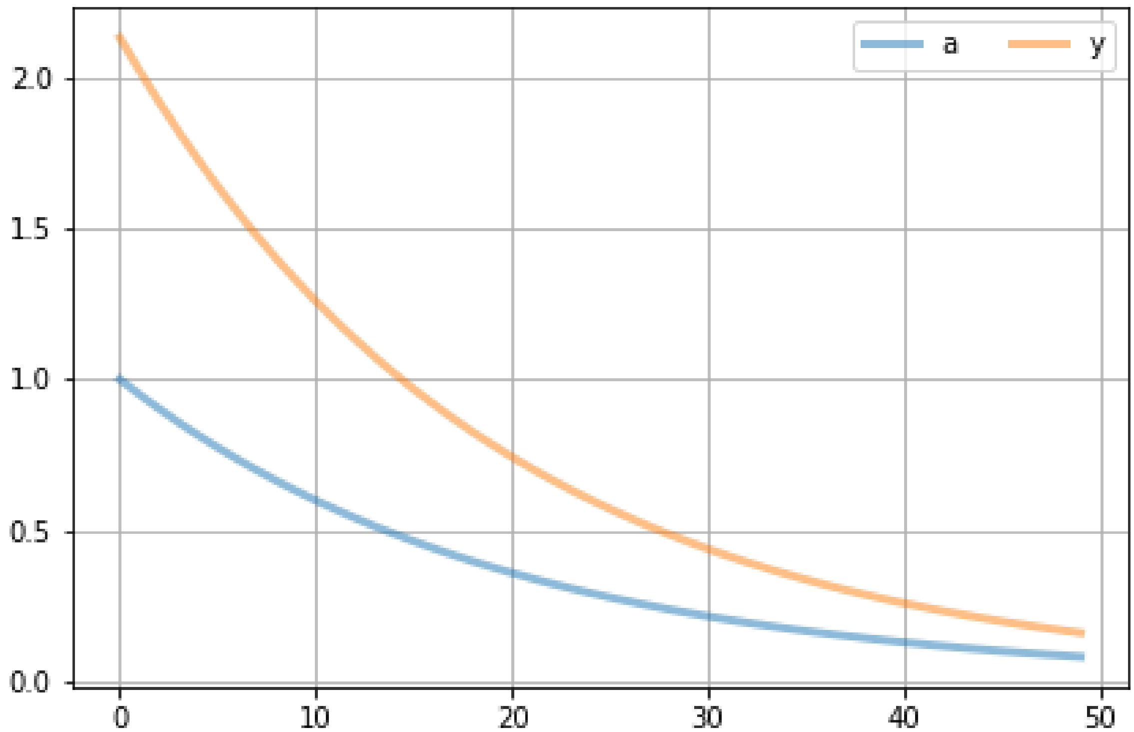
C.3 Isolate technological shock

Figure 3: Technology shock and its error



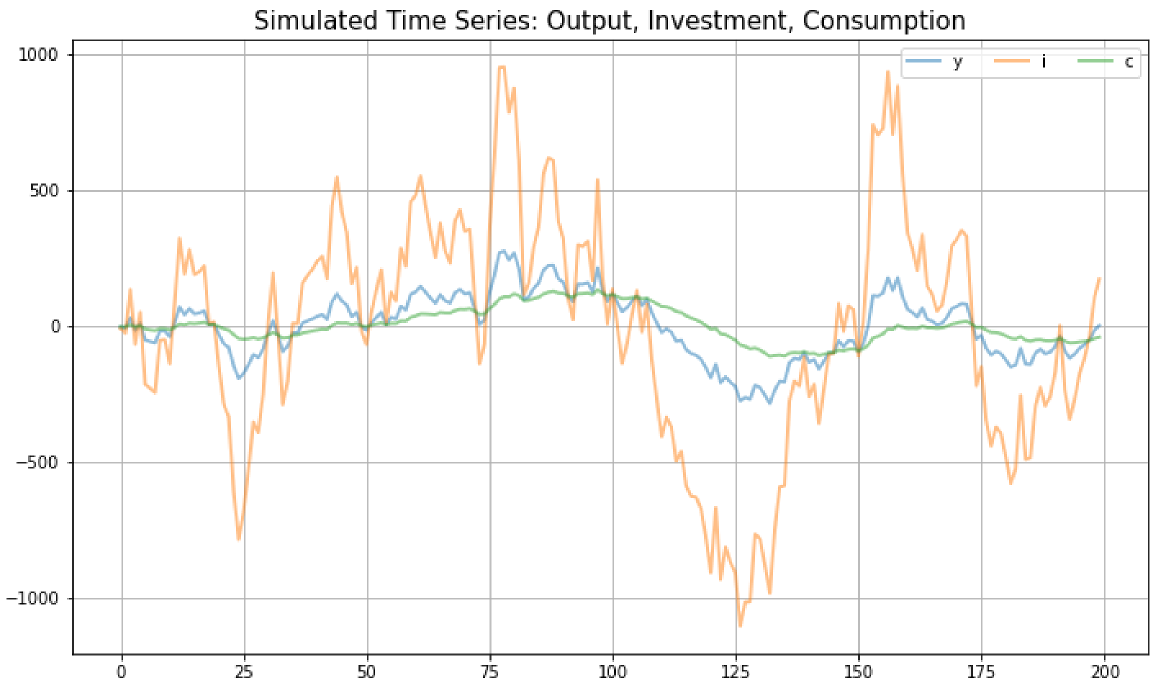
C.4 Technological shock - technology and output comparison

Figure 4: response of output to the technology shock pretty much matches the response of technology itself



C.5 Technological shock - simulation Output, Investments and Consumption

Figure 5: It generates actual business cycles and they don't look too unrealistic, the model can match the fact that investment is far more volatile than consumption



C.6 Technological shock - simulation technology and output

Figure 6: Output and technology fluctuations are quite closely: This shows that these additional propagation mechanisms are quite weak.