Final Design Report

Section A03 Spring 2016

Authors: Scott Kresie, Jeronimo Mora, Dominique True

Team Name: The Embedded

Project Title: Design of Embedding Sensors in Additively Manufactured (AM) Parts for Structural Health Monitoring

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Abstract

Lawrence Livermore National Laboratory (LLNL) has tasked us with investigating the effects of embedding a strain gauge into a 3D printed part. Strain gauges are conventionally used on the surface of structures to measure strain. Embedding these sensors requires special internal geometry for their seamless placement. Our goal is to determine an embedding process as well as experimental evidence to show that an embedded strain gauge can accurately sense strain within a structure. Due to the nature of the embedding process, embedded sensors may not provide an accurate depiction of strain within a 3D printed part. By comparing our experimental results to a theoretical model we can determine the validity of the data received from the embedded strain gauges. We hope to explain any discrepancies we discover between our experimental results and theoretical model to the best of our ability as well as provide ideas for future research to answer questions that are still present after the completion of this project.

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Introduction

Strain gauges are conventionally used on the surfaces of structures to measure strain. However, Lawrence Livermore National Laboratory (LLNL) has research interests involving embedded sensors. For our senior design project, LLNL tasked us with investigating the effects embedding a strain gauge into a 3D printed part has on the accuracy of the strain measurement. In this research project both the behavior of 3D printed material when put under bending load as well as the validity of embedded strain gauges will be investigated.

Our project can be split into three main areas:

1. Method for embedding the strain gauge
2. Learning how 3D printed cantilever beams behave under a point load
3. Learning the effectiveness of measuring strain from embedded strain gauges.

Strain gauges are conventionally used on the surface of structures to measure strain. Embedding these sensors requires special internal geometry for their seamless placement. The embedding process we will use takes advantage of the layer by layer additive manufacturing process of 3D printing by directly printing material over the adhered sensor to embed it.

We have two experiments that we hope will tell us about the behavior of a 3D printed cantilever beam under a point load as well as the effectiveness of embedded strain gauges. The first is a deflection experiment in which for a given load, we measure the amount the beam deflects in different positions along the length of the beam. The second is an experiment where we measure strain using our embedded strain gauges which are connected to a wheatstone bridge circuit.

Our goal is to find experimental evidence to show the accuracy and repeatability of an embedded strain gauge and present reasonable evidence for any error that is present. Accuracy is determined by comparing experimental measurements to a theoretical value. Theoretical values are determined with hand calculations using theory from our coursework or by using a finite element analysis (FEA) simulation. Due to several factors including the nature of the 3D printing process and the behavior of 3D printed materials which are put under load, embedded sensors may not provide accurate strain measurements within a 3D printed part.

We are providing the groundwork for future research LLNL plans to do in the area of embedded sensors.

Research Needs From LLNL

From our research we hope to answer as many of these research needs (questions) for LLNL as we can to the best of our ability and provide ideas for future research to investigate any unanswered questions:

1. How was strain calculated for the theoretical model, what assumptions were made?
2. If you increase fidelity of model (closer representation of strain gauge and adhesive), is there a notable change?
3. How would you minimize difference between theoretical and test setup?
4. Address possible causes for mismatch between predicted value and test measurement?
5. Did you do a mesh convergence of model?
6. Are your results repeatable?
7. What are the material properties of the beam, epoxy?
8. At what load, does the beam bending behavior stop being linear?

Deflection Experiment

Objectives

Prove that the second moment of area of the cross section also known as area moment of inertia, is constant along the long axis of a 3D printed PLA cantilever beam by loading the beam and measuring deflection along the beam.

Description

Deflections caused by bending in beams can be described by the Euler-Bernoulli bending theory. Some key assumptions of this theory are:

1) The beam is straight and slender
2) The material is isotropic and homogeneous
3) The material is linearly elastic (it will not deform plastically)
4) Only small deflections are considered

Isotropic materials have uniform material properties in all directions. Because the cantilever beam is 3D printed, the beam may not be isotropic since it was additively manufactured layer by layer and not molded or extruded all at once. The goal of the experiment is to show that Euler-Bernoulli bending theory and its assumptions can be used to describe bending in the tested 3D printed beams. A straight cantilever beam will be loaded at one end. The beam will be homogeneous and will be loaded so it remains in the linearly elastic range. Deflection data along the long axis will be recorded and used to show the amount of variation in the second moment of area along the long axis. If there is minimal variation the Euler-Bernoulli bending theory for isotropic materials can be used to describe the behavior of the beam.

Background

From the Euler-Bernoulli bending theory:

The equation describing deflection (w) in an isotropic beam under bending is:

$$\frac{d^2y(x)}{dx^2} = \frac{M(x)}{EI(x)} \qquad (1)$$

Solving for the second moment of area:

$$I(x) = \frac{M(x)}{E\,\frac{d^2y(x)}{dx^2}} \qquad (2.1)$$

The load on the beam will be approximated as a point load and the distance L is from the fixed edge to the location of the applied load. This leads to Eq. 2.2.

$$I(x) = \frac{F\,(L-x)}{E\,\frac{d^2y(x)}{dx^2}} \qquad (2.2)$$

$\frac{d^2y(x)}{dx^2}$ can be found numerically from the deflection data using finite difference methods and taking two consecutive first derivatives (Eq. 3.1, 3.2, 3.3). The central difference method will provide the most accurate results and thus will be used.

Backward Difference Method $$\frac{dy(x)}{dx} \approx \frac{y_{i-1} - y_i}{\Delta x} \qquad (3.1)$$

Forward Difference Method $$\frac{dy(x)}{dx} \approx \frac{y_{i+1} - y}{\Delta x} \qquad (3.2)$$

Central Difference Method $$\frac{dy(x)}{dx} \approx \frac{y - y}{2\Delta x} \qquad (3.3)$$

Using Eq. 3.3 in Eq. 2.2 with a known Young's modulus of the material and measured F and x, I(x) can be plotted and any variation in I(x) can be observed. This will be known as **"Method 1"**

Another method which uses finite differences would be to directly calculate $\frac{d^2y(x)}{dx^2}$ (Eq. 4.1-3). This will be known as **"Method 2"**

Backward Difference Method $$\frac{d^2y(x)}{dx^2} \approx \frac{y_{i-2} - 2y_{i-1} + y_i}{\Delta x^2} \qquad (4.1)$$

Forward Difference Method $$\frac{d^2y(x)}{dx^2} \approx \frac{y_{i+2} - 2y_{i+1} + y_i}{\Delta x^2} \qquad (4.2)$$

Central Difference Method $$\frac{d^2y(x)}{dx^2} \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} \qquad (4.3)$$

Both finite difference methods can be effective as Δx approaches 0. However, due to limitations in our measurement resolution, inaccuracies may arise when using finite difference methods to determine if I(x) is constant.

An alternative to using finite difference is to use Excel to generate a 3rd order polynomial line of best fit with an intercept set to 0 and to take the double derivative of that function along with using Eq. 2.2 to find I(x). The 3rd order polynomial is appropriate as two derivatives should still leave $\frac{F\,(L-x)}{EI}$ which is a function of x. This will be known as the **"LOBF (Line of best fit) Method".**

For further analysis of beam behavior, the theoretical deflection at each point along the beam for each load can be plotted using Eq. 5.1. These values can be compared to their measured counterparts.

Theoretical Deflection in a Cantilever Beam $$y(x) = \frac{F\,x^2}{6EI}(3L - x) \qquad (5.1)$$

Data Analysis

1. Compute an average initial deflection from the two "no load" trials taken at steps 2 and 3 of the *Taking Data* section. This average will be known the *initial deflection.*
2. Take the difference in values between the *initial deflection* and the recorded deflection when the beam was under load. These values will be known as the *corrected deflection*.
3. Plot the *corrected deflection* as a function of position along the beam. For each of the load scenarios and mark those that were taken when weight was ascending or descending.
4. Use "Method 1", "Method 2", and the "LOBF (line of best fit) Method" along with Eq. 2.2 to find an experimentally derived I(x).
5. Compare the experimentally derived values for the second moment of area to the textbook definition for a rectangular beam , $\frac{1}{12}bh^3$, and discuss discrepancies.
6. Use Eq. 5.1 and plot theoretical deflection against the measured deflection and discuss discrepancies.

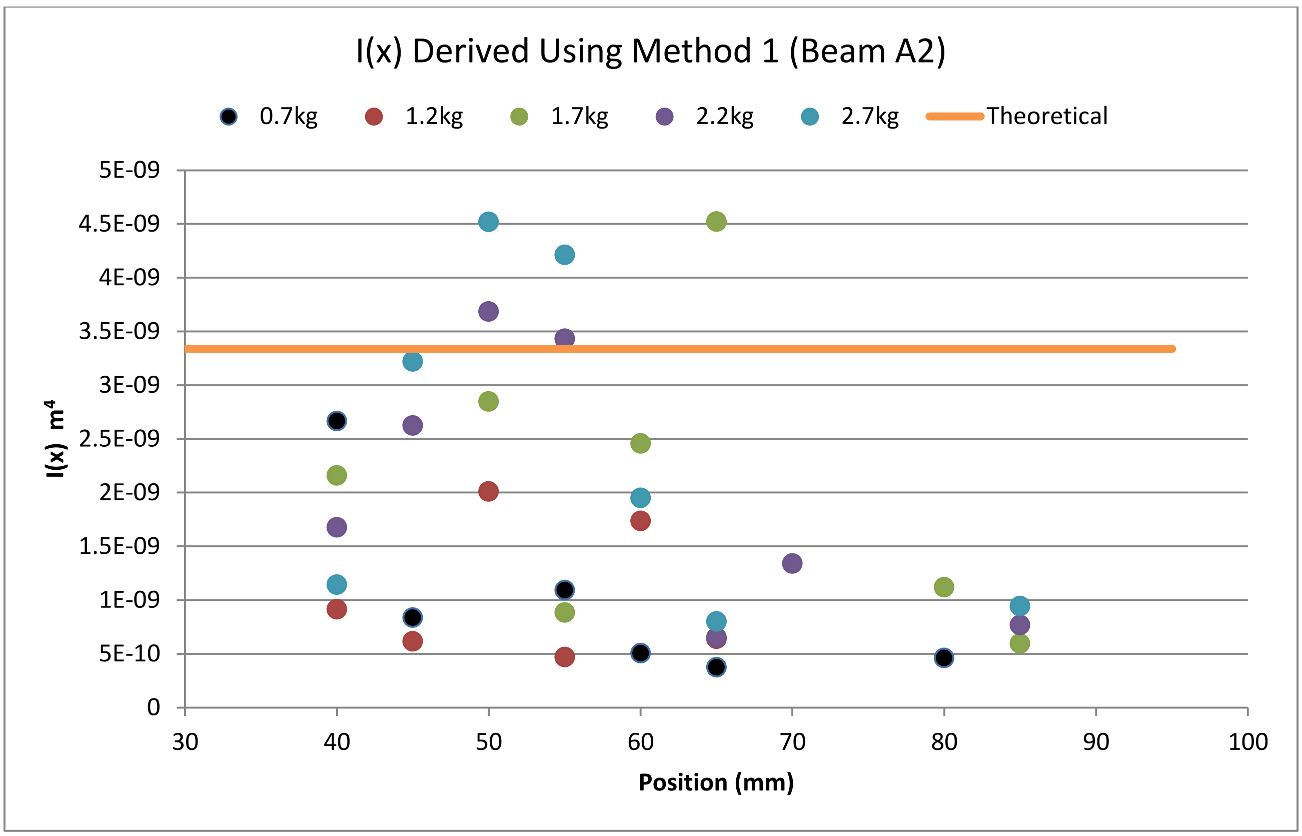


Figure 1 I(x) plotted at different points along the beam using Method 1. Some points could not be plotted for some loading scenarios because they were negative. This is likely due to limitations in the finite difference methoed.

Figures 1-3 show the experimentally derived functions I(x) using Method 1, Method 2, and the LOBF method. There were issues with acquiring data points and accuracy for all three methods that should be addressed. Table 1 summarizes the pros and cons of each method.

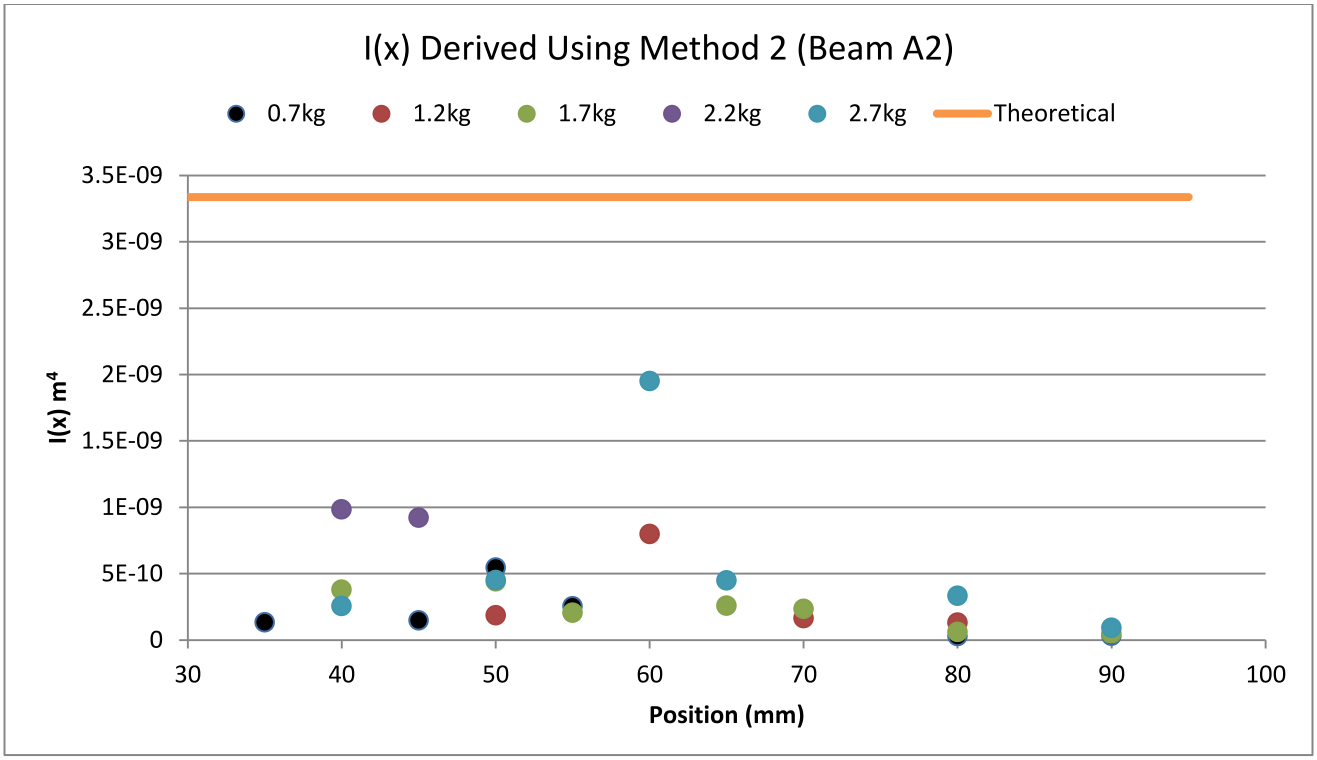


Figure 2 I(x) plotted at different points along the beam using Method 2. Like Method 1, some points are nonsense and are negative and thus not plotted. This is likely due to limitations in the finite difference method.

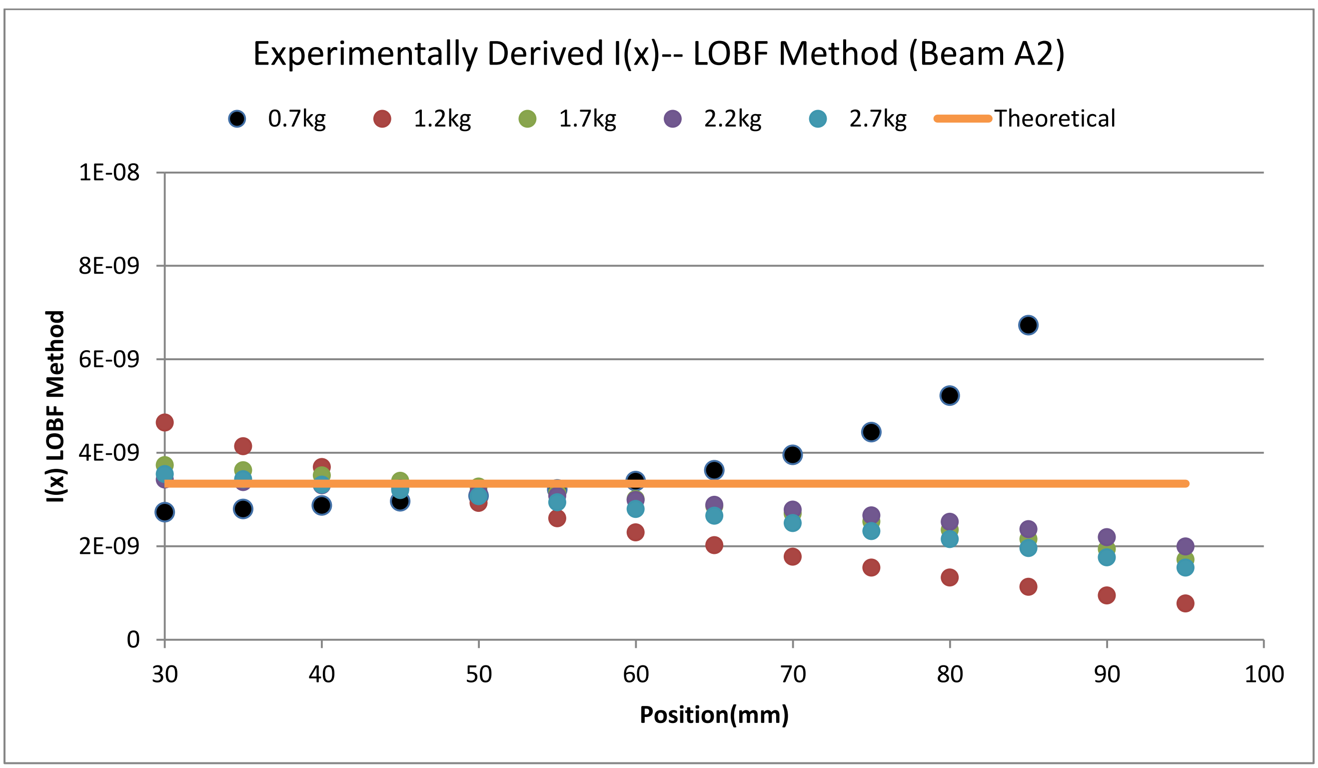


Figure 3 I(x) plotted using the LOBF Method. Slight downward trend for all loading scenarios except 0.7kg. The most accurate method when comparing to the expected theoretical value for second moment of area.

Method 1 does not provide any evidence that I(x) is constant, however the calculated I(x) is much closer to theoretical than it is for the values calculated using Method 2. In both cases, I(x) is consistently smaller than the theoretical value. The LOBF Method seems to be the most accurate in approximating I(x). I(x), again, generally seems to be less than the theoretical prediction and also has a slight downward trend as position increases along the beam. The data from the 0.7kg run result in an opposite trend. It is important to note that resolution error of our measurement devices has a greater effect at lower load scenarios since smaller deflections are produced.

The data from all three methods suggests that I(x) may actually be smaller than the theoretical values calculated using $\frac{1}{12}bh^3$. This has some merit since even a 3D print set with 100% infill (the closest approximation to a solid beam with no air gaps) may still have small gaps between layers and lines of extruded filament as seen in Figure 4. This effect could be magnified with prints that have more layers. We also assumed E, Young's modulus, was 3368 x 10^6 MPa, a value taken from a research paper from Michigan Technological University. The filament used for our cantilever beams may have a slightly different value for E. Furthermore, the printing settings may affect the rate at which different parts of the cantilever beam cool which can have an effect on E along the length of the beam for example. The material properties of 3D printed structures could be an area of future research to be investigated further.

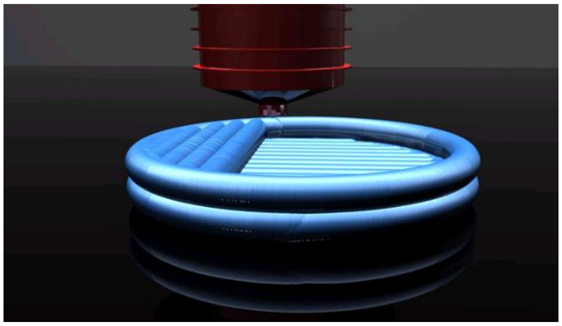


Figure 4 Animated image[4] of extruded filament. Notice the inherent gaps that occur when lines of filament are laid out.

Table 1 – I(x) Methods, Pros and Cons

Method	**Pros**	**Cons**
Method 1	Seems to be more accurate than Method 2 in calculating I(x).	Some calculated values of I(x) come out negative and don't make sense to be plotted. We also lose the first and last two data points due to using the central difference method twice.
Method 2	Second derivative can be calculated directly. Thus, only the first and last data points are lost due to using the central difference method.	Much less accurate calculation of I(x). Many more data points for I(x) come out negative due to limitations in the finite difference method and cannot be plotted.
LOBF Method	Seems to be the most accurate method with calculated I(x) closest to the theoretical calculation.	Relies on a line of best fit generated by Excel.

Theoretical Deflection and Measured Deflection

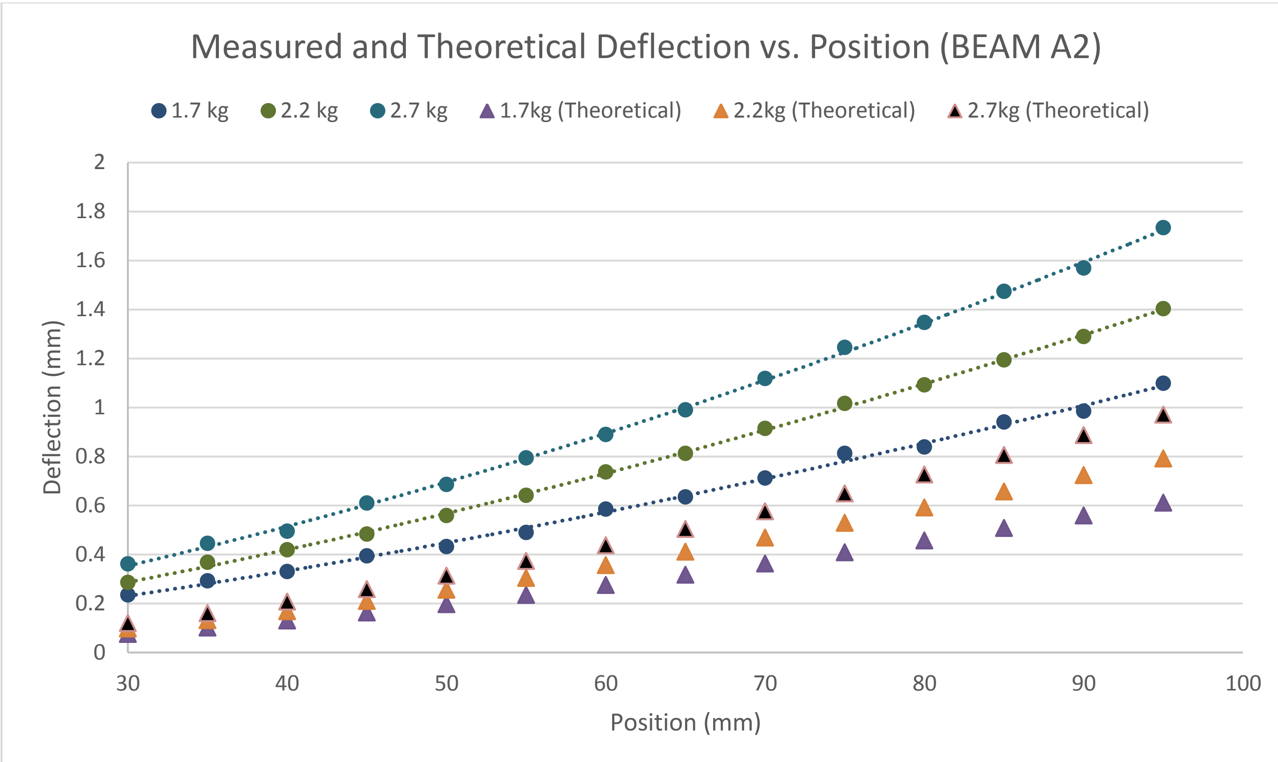


Figure 5 Theoretical deflection plotted with the measured deflection for Beam A2 (V2 type beam in appendix).

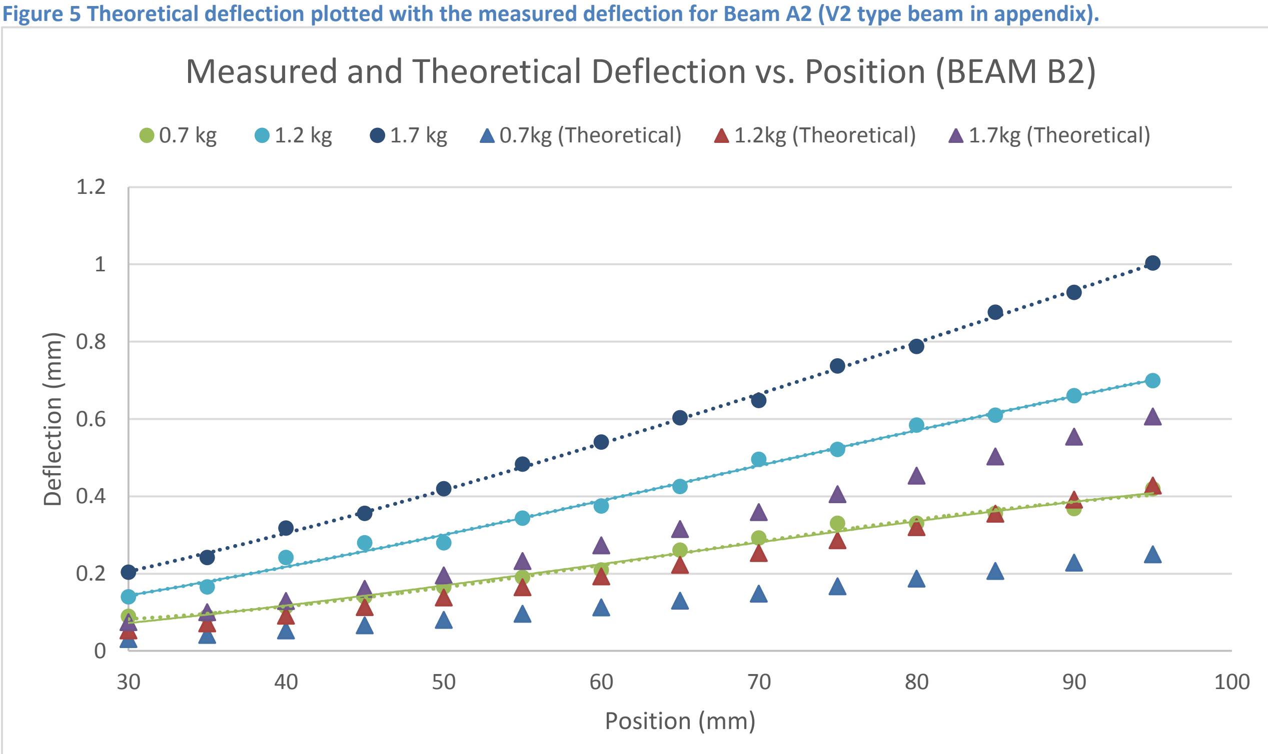


Figure 6 Theoretical deflection plotted with the measured deflection for Beam B2.

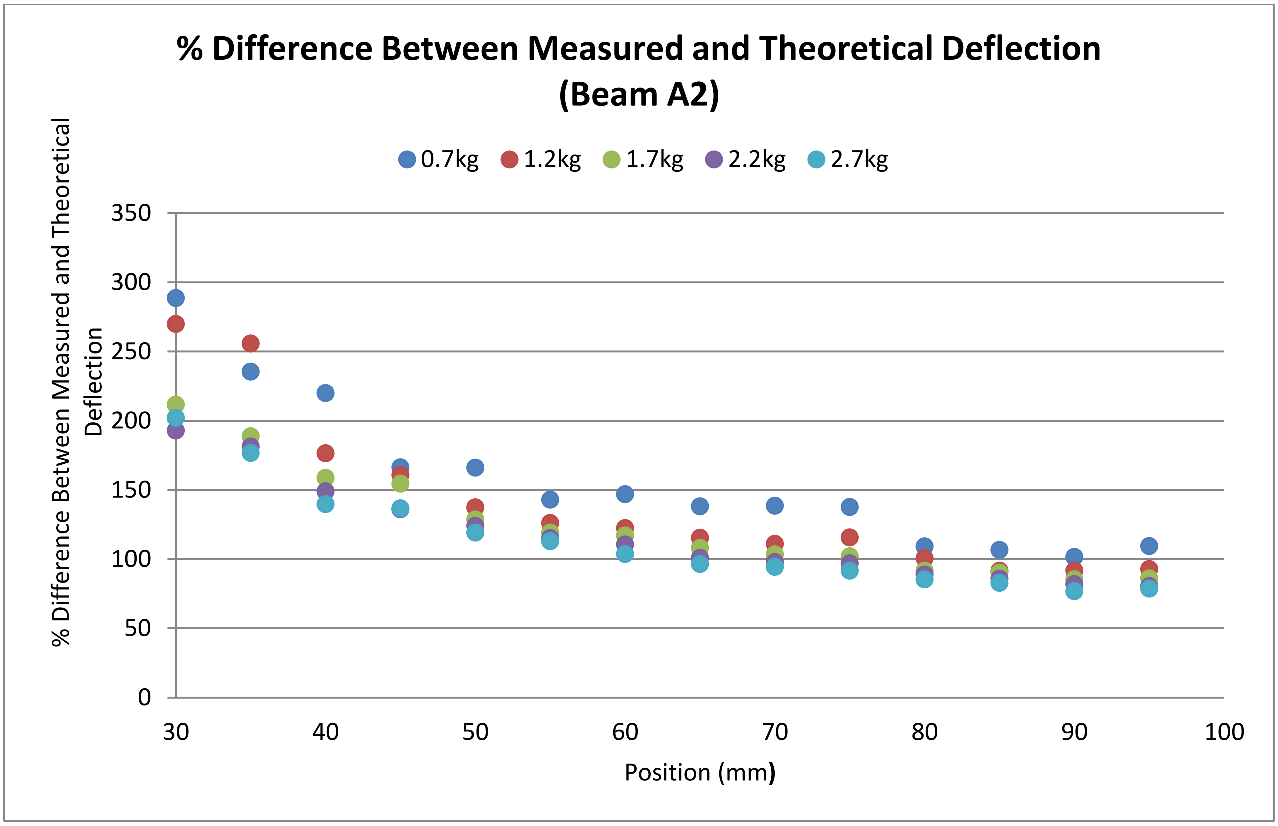


Figure 7 Percent difference between the measured deflection points and the theoretical deflection.

The measured deflections are consistently greater than all of the theoretical deflections at each specified load. Figure 5 and 6 compare deflections from two different beams that were tested and the results are fairly similar. Figure 7 seems to reveal a relationship between the position along the beam and how large the discrepancy in deflection is (Beam B2 had a similar trend). Nearest to where the load is applied is the minimal discrepancy and nearest to the fixed end of the beam is the maximum discrepancy. There also appears to be some variation in the percent difference between measured and theoretical deflection of the beam for different load scenarios nearest the fixed end. This data supports the idea described by Figures 1-3 that I(x) for 3D printed beams could be less than the theoretical value of $\frac{1}{12}bh^3$. Greater deflection corresponds with smaller values for I(x) according to equation 5.1.

Deflection Experiment Conclusion

Overall, the analyzed data from the deflection experiment reveal some major things. The 3D printed beams may have an I(x) that is smaller than the theoretical $\frac{1}{12}bh^3$. The magnitude of this difference is difficult to determine from the data available so further investigation is needed. This difference is supported by the finite difference, LOBF methods as well as the measured vs. theoretical deflection comparisons. Furthermore, the beam may have properties for I and E that may not be constant with x. . This is supported by the decreasing discrepancy between measured and theoretical deflections along the length of the beam seen in Figure 7. The reason for the variation of material properties is not certain, but the heating and cooling inherent in the 3D printing process may be to blame.

Strain Gauge Experiment

Strain Experiment Assumptions

- The strain gauge is accurately measuring the strain of the beam material, PLA, and not of the adhesive used to hold it in place.
- Young's modulus, E, and the second moment of area, I, are constant along all dimensions for theoretical calculations.
- The neutral axis of the beam is halfway between the top and bottom surfaces.
- Young's modulus for PLA is 3368 MPa. [3]
- Poisson's ratio for PLA is 0.36
- Assumptions from the Euler-Bernoulli Bending Theory, from Deflection Experiment.
- Theoretical strain uses an I that assumes a solid beam, thus I $=\frac{1}{12}bh^3$
- The beam is perfectly cantilevered where it is clamped to the table and the "beginning" of the beam is the edge of the table.
- The presence of gauge wires inside the beam has negligible effect on the data collection.
- A point load can be used for the theoretical models from SolidWorks and textbook equations.
- Lacking a perfectly balanced bridge does not affect our results if the bridge is "re-zeroed" between each loading scenario.

We do not account for the fact the epoxy filled channel has slightly different material properties than the rest of the beam. If we assumed the channel was empty the effect on I would be very small due to the small geometry of the channel. The effect of this assumption on strain is small, making our theoretical strain values approximately 0.1-0.15% smaller than they should be. The theoretical values of strain assume isotropic material properties, which may not necessarily be the case in a directional and temperature varying manufacturing process like 3D printing.

Strain Experiment Discussion

Table 2: Embedded Strain Gauge Tested Beams

Beam	Printing Direction Type	Distance of Strain Gauge from Neutral Axis (mm)
E1	90	3.125
E2	90	2.985
E3	45	2.56
E4	45	2.81
E5	45	2.195
E6	45	4.02

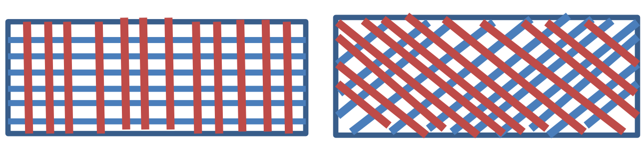


Figure 8 Printing lay up directions Type 90 (left) and Type 45 (right). The printing process alternates red and blue layers until print is completed.

After analyzing the strain data from the bending experiments and plotting strain vs. load, we see that the data all sat on linear trends, which is what we would expect. Theoretical strains at the location of the embedded sensor for each beam were calculated using equation 6. Almost all of the beams with embedded sensors tested had strains that deviated from the theoretical curve's data by up to 15%. Printing direction type also did not seem to have an effect on this deviation as Type 45 beams had deviations above and below the theoretical of similar magnitude to beams with Type 90 print directions. (See Table 2 and Figure 9).

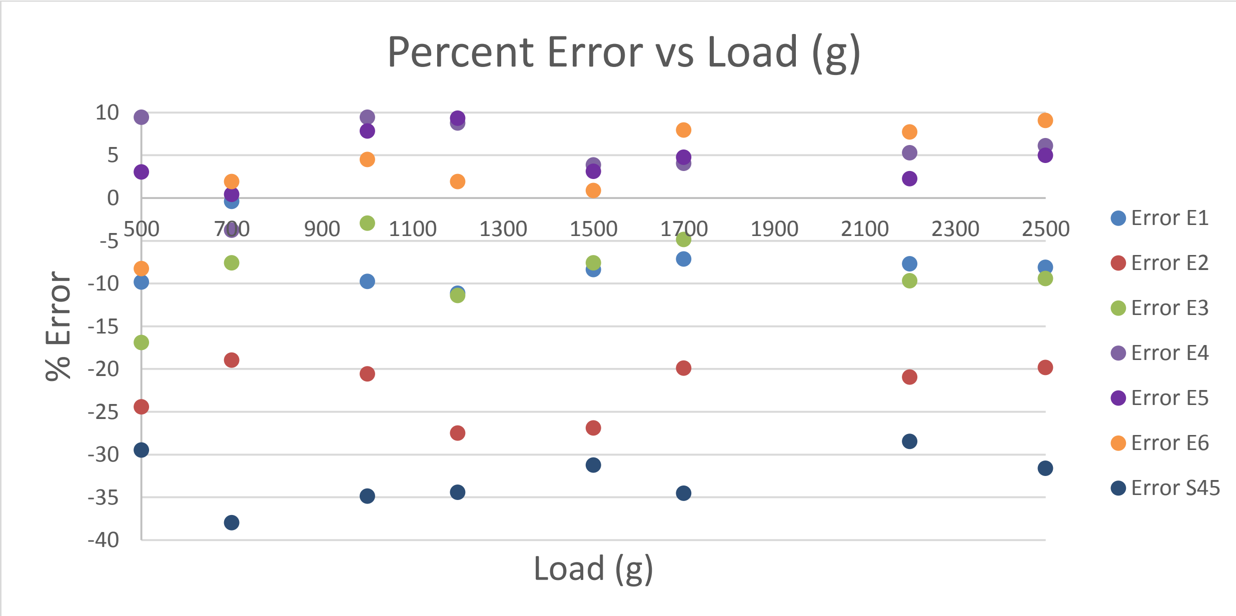


Figure 9 Positive percent refers to a measured strain that was greater than theoretical and vice versa for negative percent error.

Looking at the results from multiple beams, there was deviation in the measured strain that was both above and below theoretical. Beams E1, E2, and E3 had measured strains below the theoretical and beams E4, E5 and E6 had measured strains above the theoretical.

Our methods are repeatable, but not perfectly so. One of our embedded sensor beams was tested three times in one session (Fig. 10). Between these trials, the strain values were essentially identical. This beam was tested on a prior date, however, and the percent difference between that lone trial and these three was an average of 14%.

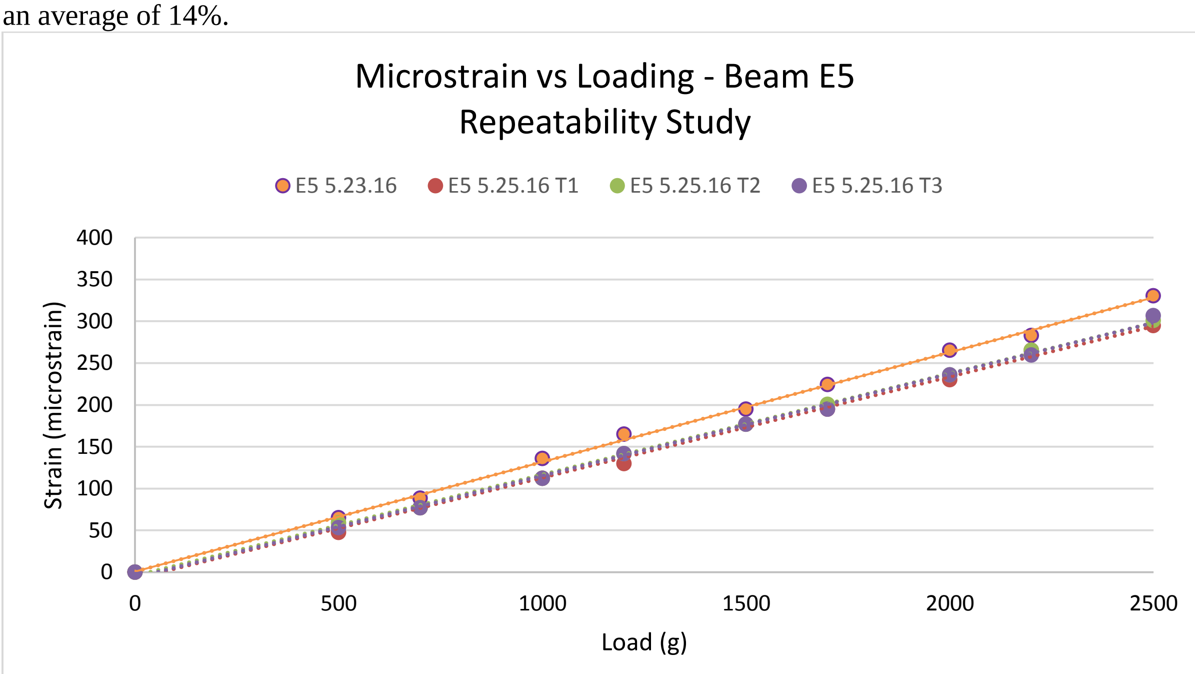


Figure 10 Strain data taken 4 times. One trial was done on May 23rd and three more trials were done on May 25th.

$$\epsilon = \frac{mg(L-x)y}{EI}$$

Equation 6: Theoretical strain equation for a cantilever beam under a point load. m is mass, g is the acceleration due to gravity, L is the length of the beam, x is the distance from the cantilevered edge to the location of interest, y is the distance from the neutral axis, E is the Young's modulus and I is the second moment of area.

Changes in setup conditions and an unsteady circuit may be responsible for this. Assessing repeatability between *different* beams was not possible because of the tolerances of the 3D printer. Each beam had similar but slightly different (0.5-1mm) dimensions which makes it hard to directly compare the strain values of experiments done with different beams. This can be mitigated by using a well-tuned higher end 3D printer. The tighter fabrication tolerances will lead to beams being closer to each other in dimensions and better repeatability in the embedding process.

We tested two beams with surface mounted sensors (one with sensors on both the top and bottom surface). Unlike the embedded sensors, which on average had errors of approximately 10%, the surface mounted beams had errors that deviated between 20% and 30% from the theoretical calculation.

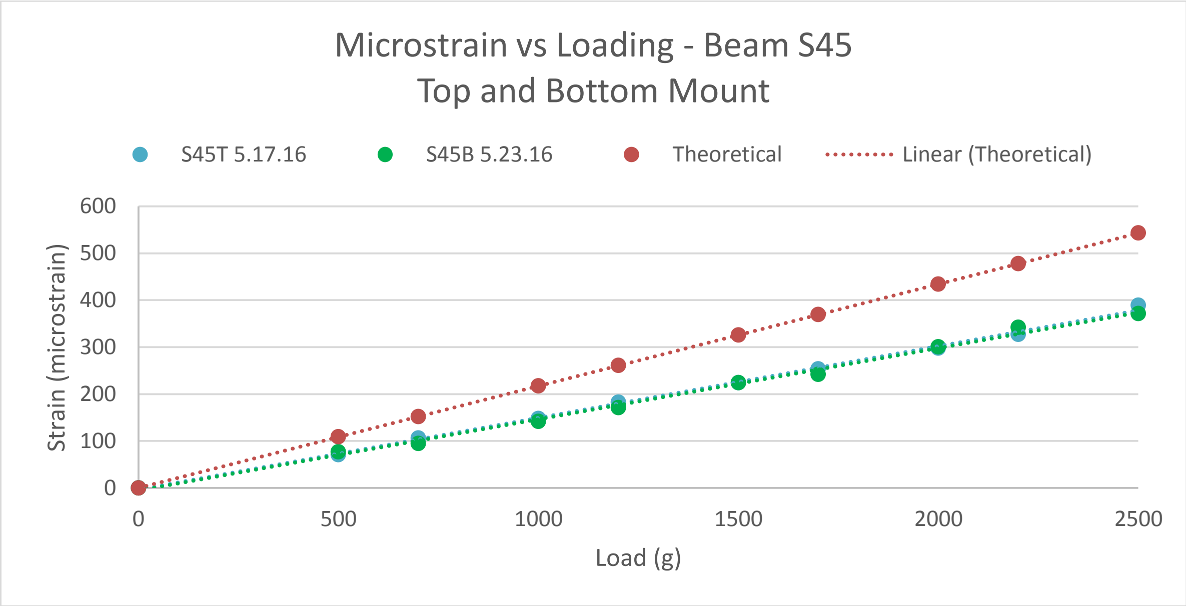


Figure 11 Strain data from surface mounted strain gauges. They were mounted on Type 45 printing orientation, however surface strain gauges mounted on Type 90 print orientation had very similar results.

It is unclear why this might be the case. One theory is that our strain gauges are under-reporting the true strain values and transverse shear strain is making it appear that embedded gauges are reporting more accurate values. For Beam E3, a 1kg load at the location of the strain gauge would have theoretical transverse shear stress of 0.0288MPa and a normal stress of 0.427 MPa. Transverse shear stress would be 6.74% of the normal stress. Theoretical transverse strain would be 11.78 $\mu\varepsilon$ and normal strain would be 126.8 $\mu\varepsilon$. Transverse shear strain would be 9.29% the value of normal strain. It is not certain that transverse stresses and strains can directly translate to strains the gauge is measuring, but their theoretical values are significant enough to warrant further investigation.

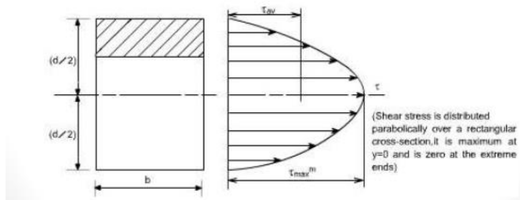


Figure 12 Transverse shear stress[6] within a beam of rectangular cross section. Shear stress is 0 at the surface.

The surface mounted sensors were well below their theoretical comparisons and the embedded sensors measured strain slightly above or below the theoretical. On the surface of the beam, there is no shear stress, which is why the sensors' output was lower. Inside of a beam, there is shear stress which causes additional strains and raises the experimental values to make it appear as though the experimental values are closer to the theoretical values.

One way to mitigate the effect of shear stress in our experiments would be to use strain gauge orientations and configurations which are not sensitive to axial strains. This would ensure that only bending stresses are recorded by our measurements.

We ran additional experiments to try and show the effect of shear stress and strain on our measurements by embedding strain gauges further and closer to the neutral axis than we had before. E5's strain gauge and E6's strain gauge was 2.195mm and 4.02mm from the neutral axis, respectively. This is compared to E1-E4's strain gauges that were placed nearly in the same location, 2.87mm from the neutral axis on average. If shear strain were a contributing factor we should see that the measured strain should deviate further below from theoretical, the further you are from the neutral axis (where there is less shear strain). However the data from these two beams show that the measured strains were above the theoretical. This implies that either shear strain is not much of a contributing factor, or that there is another factor we are not accounting for that affects surface mounted and embedded gauges differently.

Another way to make the theoretical strain values match up with the experimental values would be to find a way of accurately quantifying the second moment of area and Young's modulus of our 3D printed materials. Also, in our theoretical SolidWorks model we did not account for the fact that 3D printed materials are essentially many thin layers of plastic fused together. The multi-layer geometry of our beams which is different from the solid beam our theoretical analysis assumes may also be an area of further research to help explain our results.

Another explanation for the results from our surface strain gauge results could depend on the fact that the embedded strain gauges are covered in epoxy during the embedding process. The epoxy have an effect that allows for the strain gauge to be adhered and flex with the beam better inside the beam rather than on the surface, leading to more accurate data. A way to test this would be to embed the strain gauge without filling the channel with epoxy and then take measurements like before.

Our theoretical calculations assume a cantilever beam, however the method in which we have clamped the beams to the table may not perfectly resemble a cantilever beam. In future research it would be nice to have a SolidWorks model that more closely resembles the experimental setup with a rigid table and applied clamp forces as part of the model rather than a beam with one fixed end.

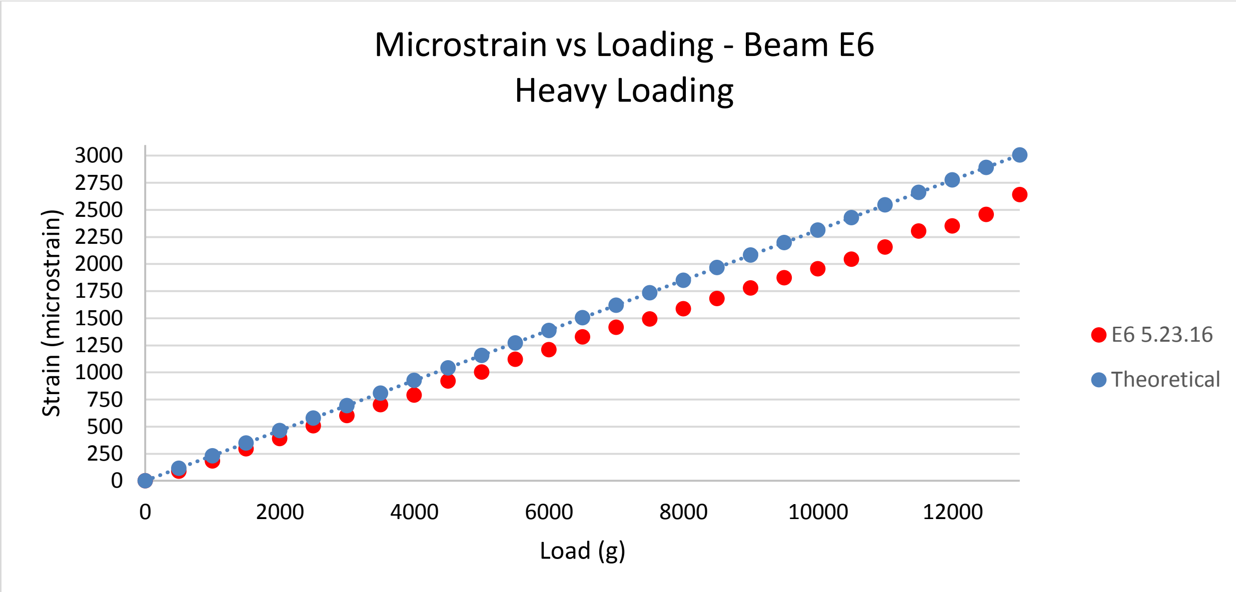


Figure 13 Beam E6 was loaded with 13kg in attempt to yield the beam. Strain data is plotted here along with the theoretical strain. It follows a fairly linear slope up until about 7000g.

Lastly, we attempted to put one of our beams with an embedded strain gauge under heavy load to try and yield the material in order to determine yield stress (Figure 13). At a load of about 7000g, the measured strain seems to deviate from the original linear slope which corresponds to a normal stress at the location of the strain gauge of 5.45 MPa. This is well below the yield strengths for PLA found online which are about 60Mpa. Since the beam may not have been close to yielding or failing our results from this test may be misleading. Future research should be done to load the beam until definite yielding and failure occurs.

Strain Gauge Embedding Process Description

Strain gauges were embedded by strategically pausing the print at an appropriate layer. The model for the cantilever beam had an internal geometry of a channel that extended from where the gauge would be placed to the end of the beam. All dimensions of this internal geometry were measured with calipers to be for theoretical strain calculations. The gauge was adhered to the channel using the adhesive suggested by the manufacturer (Loctite 496). The channel was then filled with JB Weld Plastic Weld two-part epoxy. This epoxy was allowed to set before printing of the next layer was resumed. JB Weld Plastic Weld was chosen due to its good adhesive compatibility with plastics like PLA.

Deliverables

The main deliverable that our sponsor requested was this report which describes in detail our experimental methodologies, and our data and error analyses. Furthermore, this report provides ideas for more future work in the area of embedded sensor and hopefully lays the groundwork for this research.

Conclusion

From all of our analysis here answers to the questions proposed in the "Research Needs From LLNL" section.

1. How was strain calculated for the theoretical model, what assumptions were made?
   Standard textbook formula was used($\frac{Mc}{EI}$) along with a SolidWorks Simulation. Both assumed a solid beam with constant E and I along all dimensions. . For the SolidWorks model, a model of a beam in question was created using measurements taken with calipers. Material was removed from the beam to model the shortened cantilevered section only. That end was fixed and a load was applied at the hole on the opposite end of the beam. After using the finest mesh available, a section was taken at the plane that contains the strain gauge. The probe tool was used to probe strain at the strain gauge location.
2. If you increase fidelity of model (closer representation of strain gauge and adhesive), is there a notable change?
   A 1mm difference in the recorded position of the strain gauge results in a 4 microstrain difference in theoretical output. This is within our expected error. The strain gauge and epoxy should have a minimal effect on the behavior of the beam. The difference in the second moment of area a solid beam and a beam with an unfilled channel is between 0.1% and 0.15% due to the small geometry of the channel. We assume this to have a negligible effect on the beam.

3. How would you minimize difference between theoretical and test setup?
   We would experimentally find more accurate values for E of the PLA and adhesive and epoxy to use in computer simulations and theoretical calculations. Achieving a balanced circuit consistently without having to "re-zero" after each loading scenario would likely reduce error and give us more confidence in the data we acquire. Using a power supply that can provide a consistent voltage would be optimal. For the SolidWorks simulation, we could model the experiment more closely by modeling a rigid table where a clamping force holds the beam in place. Being able to accurately model the interaction between the strain gauge and the adhesive and epoxy would also improve the validity and accuracy of our data and can reduce the discrepancies we observed.

   We may also want to run more experiments to achieve a more accurate picture of how the second moment of area is different for 3D printed beams as opposed to solid beams. The deflection experiment provided evidence to suggest that I varies with x and is also slightly less than the geometrical definition of $\frac{1}{12}bh^3$.

4. Address possible causes for mismatch between predicted value and test measurement?
   The presence of transverse shear stress and strain within the beam could possibly explain the large error when measuring strain from a surface gauge. If we assume we are under reporting strain for all measurements the addition of transverse shear strain could increase the values of strain we measure. We did not account for shear strain in our theoretical analysis, but found that shear strain values are about 9% the value of normal strain values which can be significant. It is not certain by how much the strain gauge would be influenced by shear strain, but this would be an area for future research to investigate.

   We make many assumptions which are covered in the *Strain Experiment* section. Some of which are related to material properties and properties of the beam which may be inaccurate. The second moment of area should be investigated further by testing a solid beam of PLA with our Deflection Experiment methods to rule out poor experimental design as a cause for the results for I(x) we received from that experiment.

5. Did you do a mesh convergence of model?

The finest mesh was used in the SolidWorks Simulation. However this may not have been fine enough of a mesh to produce reliable results.

6. Are your results repeatable?

We tested Beam E1 once on two different days and on the second day, the strain values were lower. We tested Beam E5 one on one day, and three times on a different day, and on the day with three trials, the data is lower than on the first day. The data for the three trials performed on the same day are essentially identical with very little variation between them. (See Figure 10). The fact that when tested on subsequent days, beams report lower values of strain may just be due to slight changes in the experimental setup and likely does not represent something systematic. The amount of overhang can vary slightly as we are lining up a mark on the beam with the edge of the table so human error is present. We do not believe we are plastically deforming the beams.

7. What are the material properties of the beam, epoxy?

Tensile tests should be performed on tensile test coupons made with bot the PLA that is used to 3D print the beams as well as the JB Weld Plasticweld epoxy to obtain a more accurate theoretical model. On that note, a beam of solid PLA should also be tested in the deflection experiment as well as with a surface strain gauge to ensure that our experimental methods are sound and that the 3D printing manufacturing process is to blame for the discrepancies in our data.

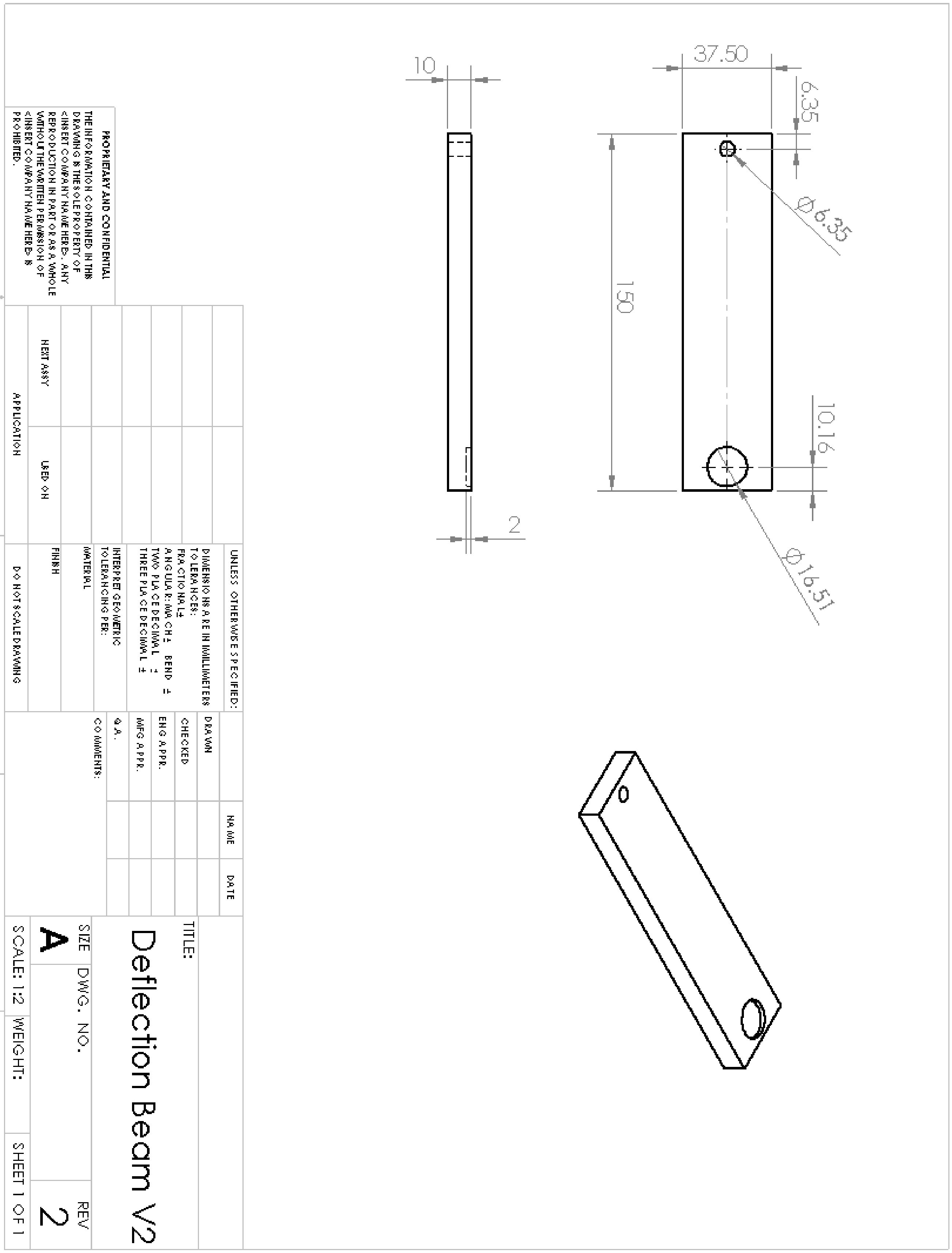
8. At what load, does the beam bending behavior stop being linear?

One beam was loaded with 13 kg and appears to cease to be linear at approximately 7 kg. Calculating the yield stress at this point comes out to be 5.45 MPa. This is about 10 times lower than it is reported to be. It may be that we are still not loading it enough to see the true shape of the curve. It is not impossible to envision it being linear but it randomly appearing curved. The change in slope at 7kg is not dramatic. As stated before, further tests with larger loads are necessary to assure yielding and potential failure in the beam.

References

1) Budynas, Richard G., J. Keith. Nisbett, and Joseph Edward. Shigley. *Shigley's Mechanical Engineering Design*. New York: McGraw-Hill, 2011. Print.
2) https://en.wikipedia.org/wiki/Finite_difference
3) Tymrak, B.m., M. Kreiger, and J.m. Pearce. "Mechanical Properties of Components Fabricated with Open-source 3-D Printers under Realistic Environmental Conditions." *Materials & Design* 58 (2014): 242-46. Web.
4) CreateItReal. “3D Printing Process”. Online video clip. YouTube. 2 April 2013. https://www.youtube.com/watch?v=U-8-lu4mq8o
5) Aerospace, Mechanical & Mechatronic Engineering, *University of Sydney* http://web.aeromech.usyd.edu.au/AMME2301/Documents/
6) “Transverse Shear Stress” by http://nptel.ac.in/ CC BY-SA

Appendix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SHEET 1 OF 1
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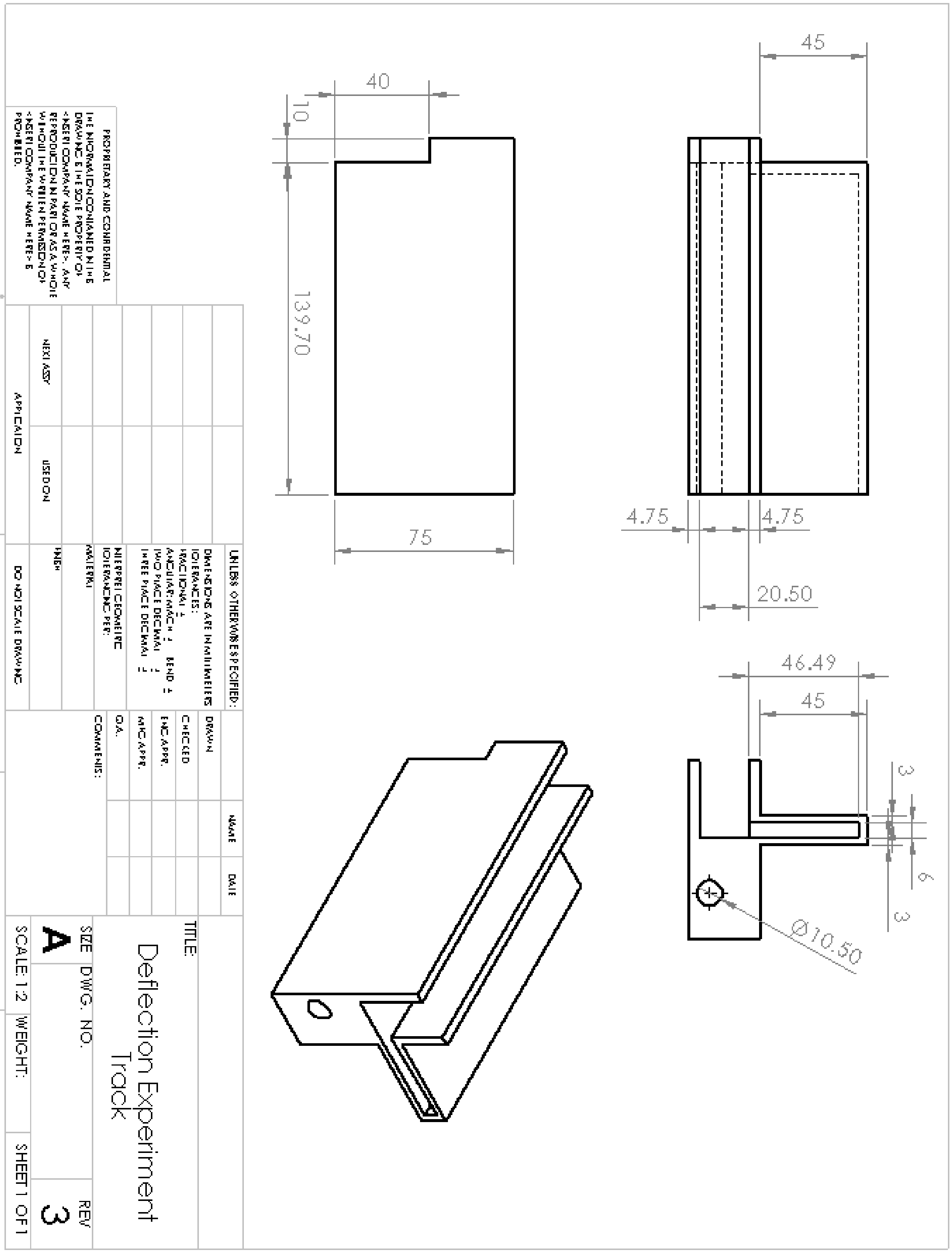
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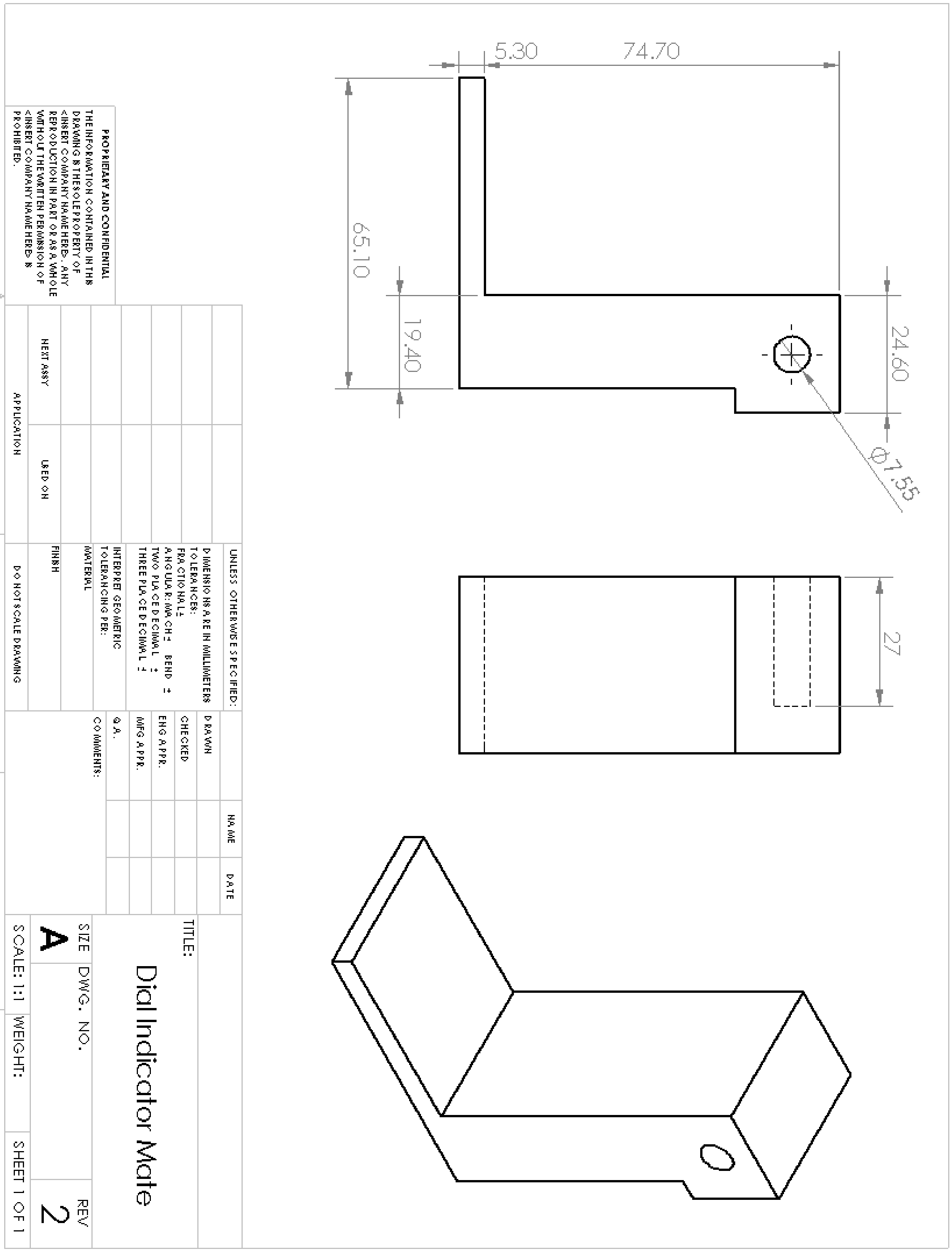
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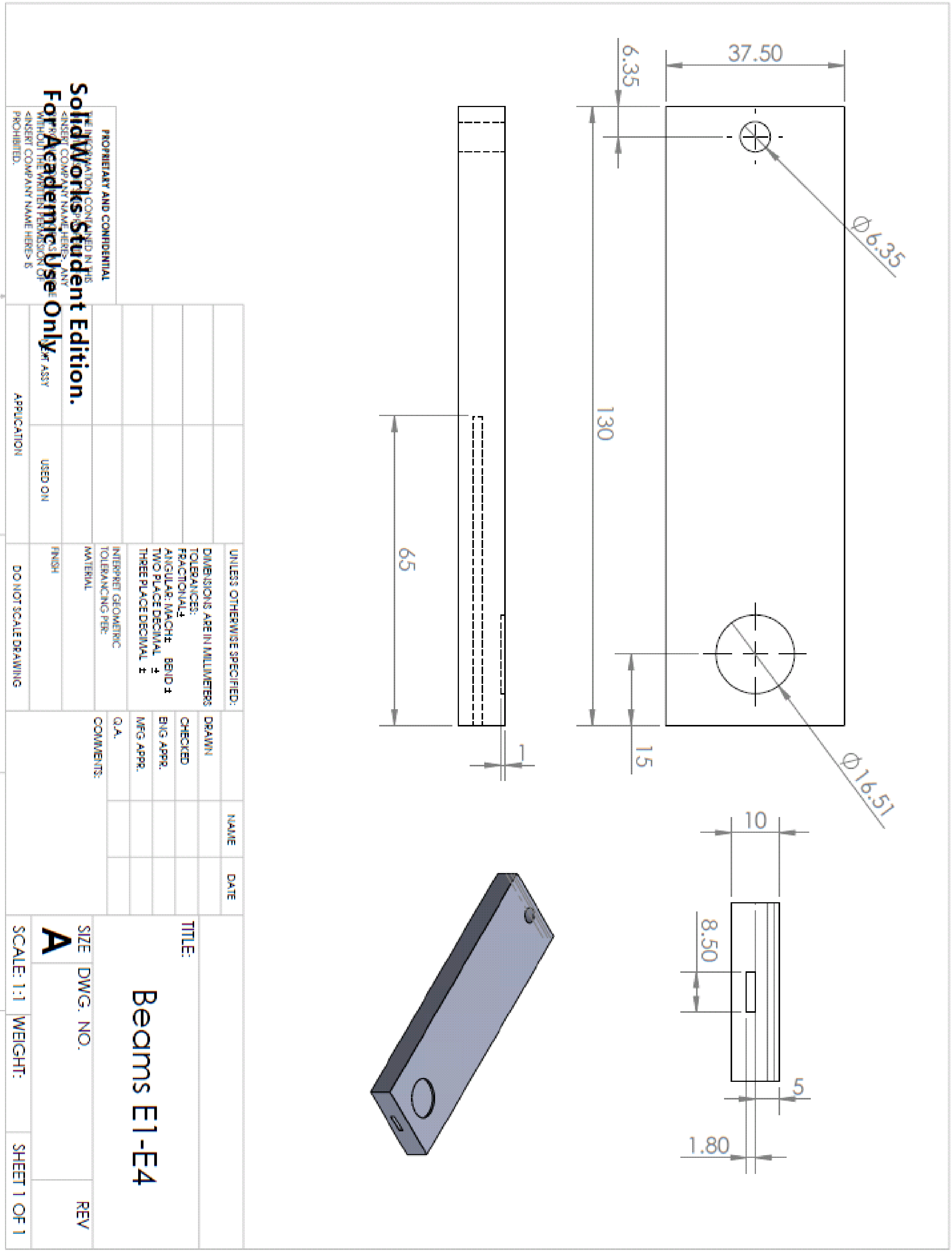
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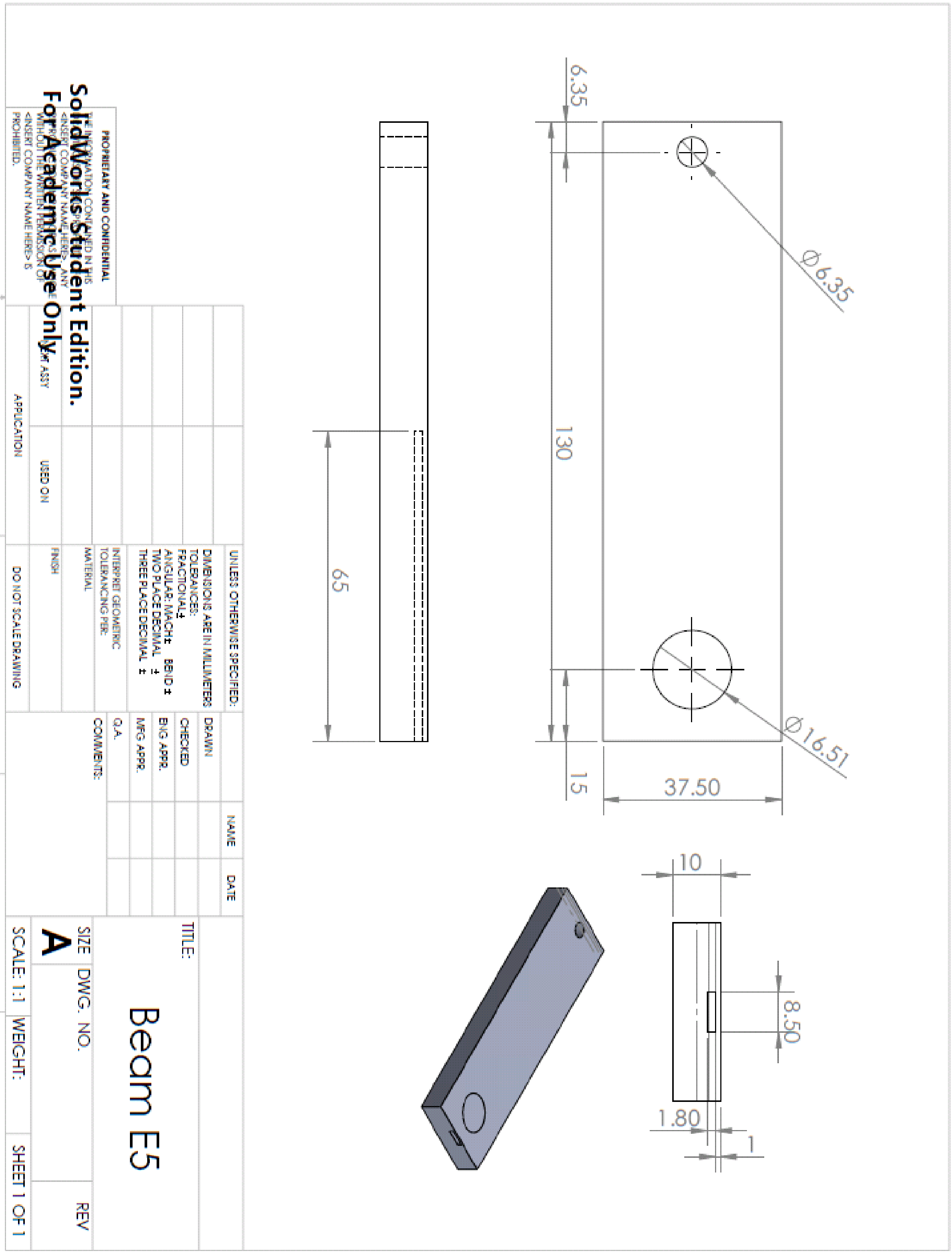
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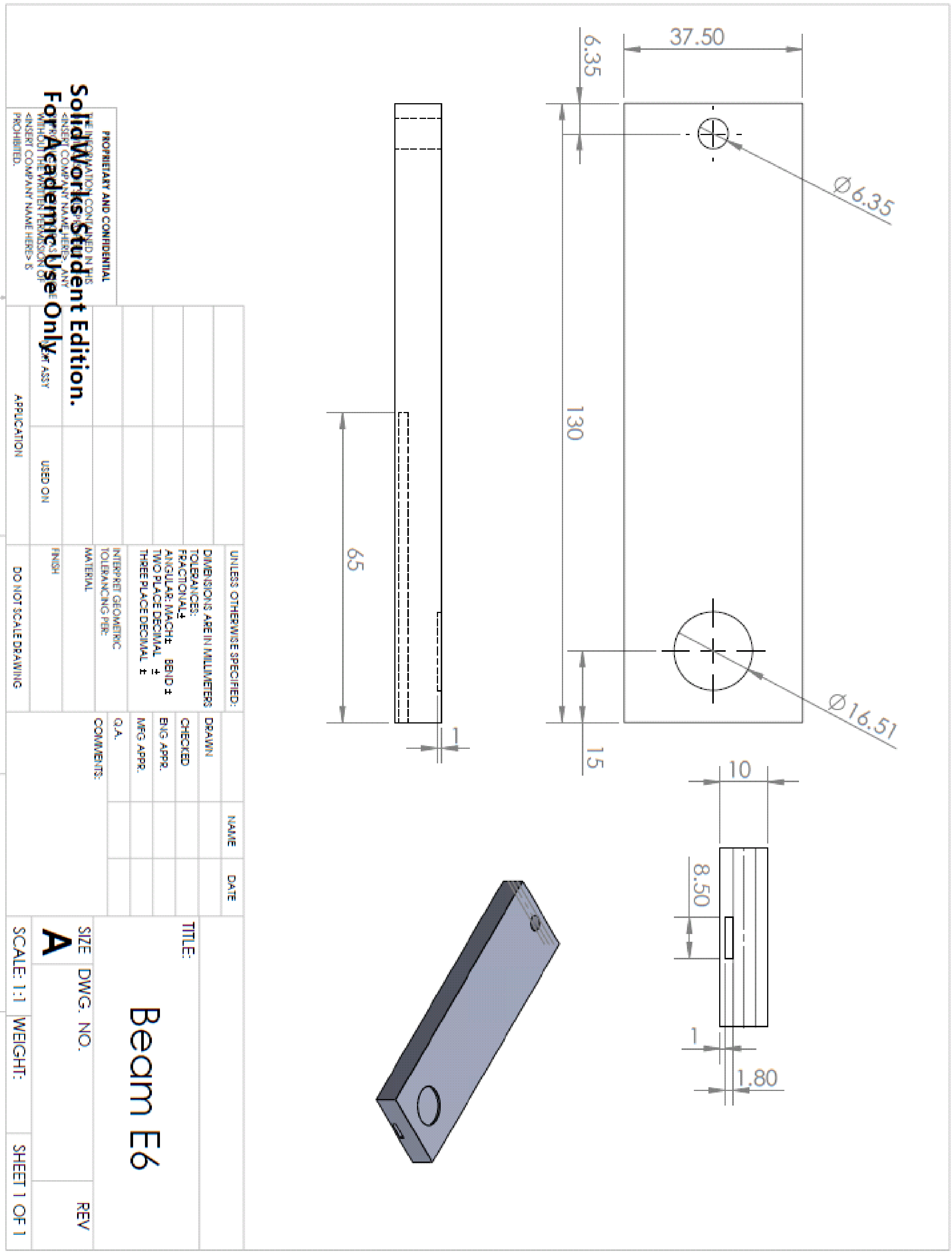

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Bill of Materials

Item	Description	Qty	Price	Subtotal	Part Number	Vendor
Eyebolt w/nut 3/16"X2.5"	Hardware	2	$0.99	$2	58292	Ace Hardware
Washers	Hardware	5	$0.05	$1	5780	Ace Hardware
Pony C-Clamp	Clamp	2	$4.99	$10	20519	Ace Hardware
J-B Weld Plastic Weld 50132	Adhesive	4	$6.59	$30	043425501325	Ace Hardware
United Scientific WHSBE9 Black Enamel Hooked Weight Set, Set of 9 Weights, Black Enamel Finish	Weights	2	$31.62	$65	B00ES3ID4K	Amazon
PTFE Thread Seal Tape 0.75in.	Tape	1	$5.99	$6	754362402926	Ace Hardware
Breadboard	Breadboard	1	$7.99	$8	B017U551T2	Amazon
140 Wires	140 Wires	1	$5.86	$6	B014JOV4TI	Amazon
Strain Gauge	Strain Gauge x 10	2	$195	$390	KFH-1.5-120-D16-11L1M2S	OMEGA quote# WC231839
Loctite 496	Adhesive	2	$33	$66	SG496	OMEGA quote# WC231839
Box of Nitrile Gloves	Protective Gloves	1	$14.99	$15	697383941077	Ace Hardware
Scrap Material, Aluminum Plate	5"x12"x1/8"	1	$0 (From EFL)	$0	N/A	N/A
String	2 feet	1	$0 (From EFL)	$0	N/A	N/A
3D Printer Filament	PLA Natural	N/A	$0 (From ESSC)	$0	MH0000005002	Matterhackers
Multimeter	N/A	1	$0 (from ESSC)	$0	N/A	N/A
Misc. Electroinic Components	Potentiometer, capacitor,resistors INA126 OpAmp					EME 107A Lab Room
Total:	**$599**					

Strain Gauge Embedding Process Procedure

1. Import the 3D model (.stl file) into RepetierHost Software.
2. In the Slicer tab, select settings for 0.2mm layer height and 100% infill and slice the model
3. Go into the G-Code Editor tab, click on the Visualization tab, and find the layer that will print over the embedded sensor/channel using the sliders. The pause statement must be sent to the printer before the start of this layer and at the end of the previous layer's g-code.

   IMPORTANT NOTE: The G-Code Editor starts at LAYER 0, however the layer visualization starts at layer 1. Therefore, subtract 1 from the layer number that the visualization showed the pause statement needed to be inserted.
   (ex. The visualization shows that the pause statement must be inserted at the end of layer 19, before layer 20. Therefore, in the G-Code Editor the pause statement should be inserted at the end of layer 18, before layer 19.)

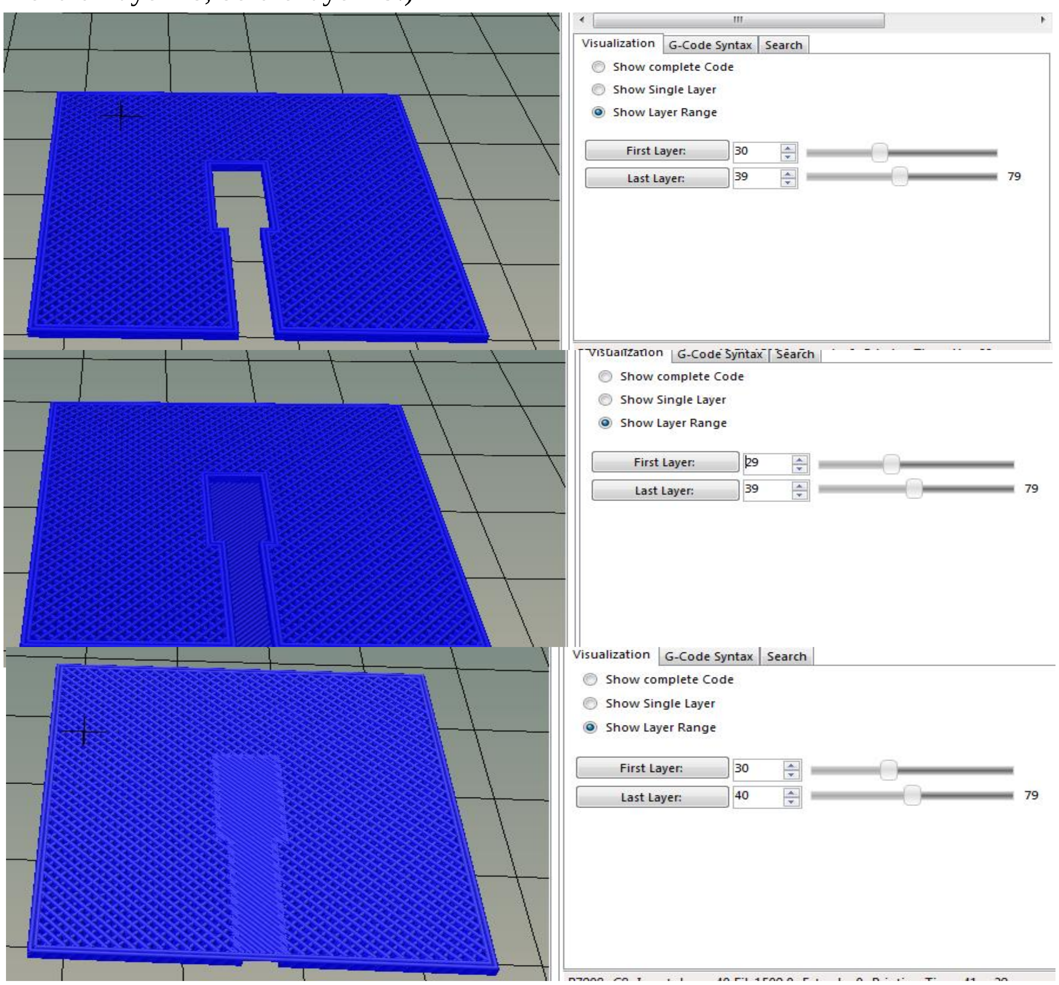

Figure 14 In this model, layer 30 is the first layer that is printed that has the special geometry for embedding. Layer 40 is the layer that will cover up this geometry. G-Code must be edited between layers 38 and 39 due the G-Code Editor starting the layer count at Layer 0.

4. Use the Search tab to look for "LAYER:40". And add the following lines of code seen in Fig. 15.

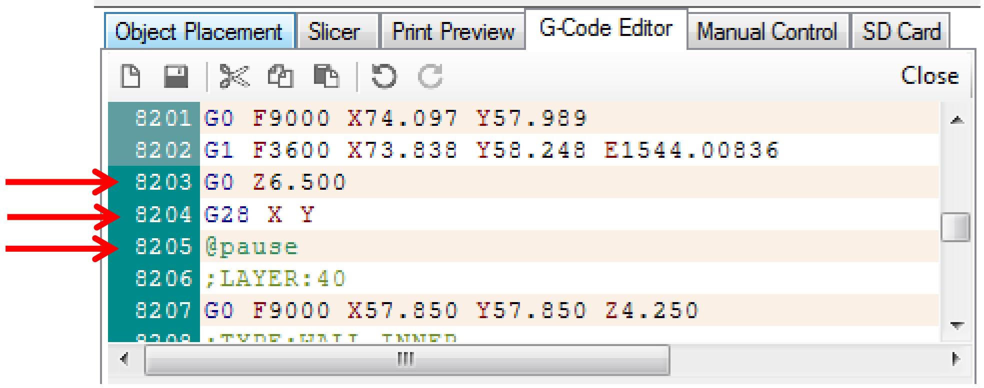

Figure 15 Line 8203 raises the extruder head away from the print. Line 8204 homes the X and Y axes. Line 8205 pauses the print. The sensor can now be embedded at this point.

5. Start the print as usual and wait for the pause command to be executed.
6. To see step 5 in action go here: https://www.youtube.com/watch?v=SMvVc_SIafw
7. With the print paused, you can use the manual controls to move the extruder head to a desired location to allow for more room during the embedding of the strain gauge.
8. Put on nitrile gloves and wrap teflon tape (thread seal tape) around the index of your dominant hand. The teflon will prevent adhesion of the gloves to the Loctite 496. (Figure 16)

Figure 16 Wrap PTFE (Teflon thread seal) tape over an index finger. PTFE tape does not stick to the Loctite 496 adhesive.

9. Prep the channel for adhering the strain gauge by roughening the surface where the strain gauge will be adhered with sandpaper. (Figure 17)

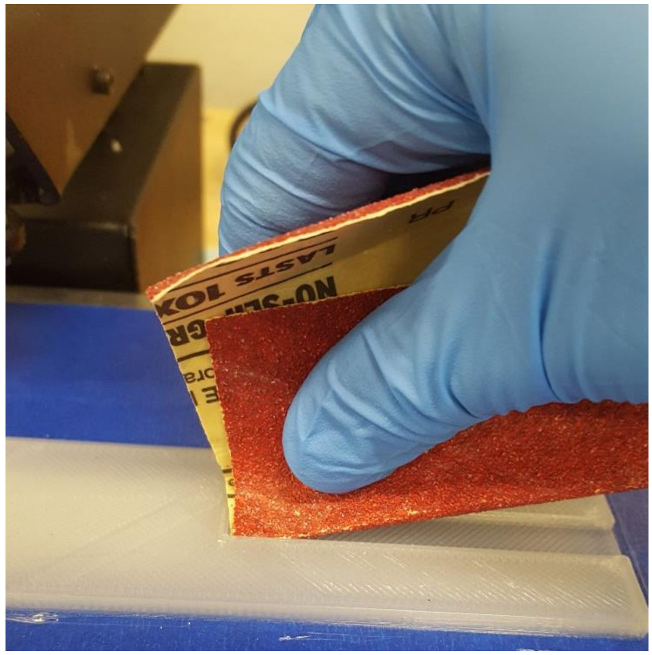

Figure 17 Use sandpaper to roughen the location where the strain gauge will be adhered to ensure a better bond.

10. Use the Loctite 496 to adhere the strain gauge to the beam. Apply firm pressure to the strain gauge with the teflon taped index finger for ~ 1 minute.
11. Tape down the strain gauge wires to the print bed to minimize their interference with the next printed layers. (Figure 18)

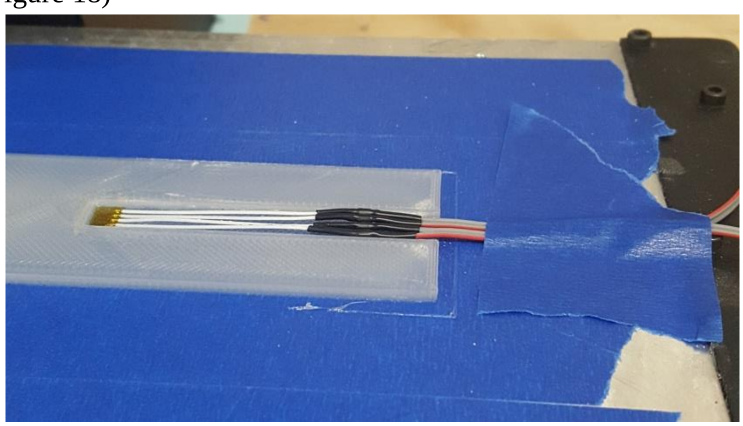

Figure 18 Wires are taped down to prevent interference of printing process and to keep them down within the channel.

12. Squeeze out 2 quarter size blobs of the JB-Weld Plasticweld 2-part epoxy adhesive. Mix the two parts of the epoxy together using a popsicle stick and start a timer for 5 minutes. Mix thoroughly for 30 seconds.

13. Using the pospsicle stick, slowly pour the epoxy into the channel over the strain gauge and wires. Use the popsicle stick to drag the epoxy along the channel so that it fills the channel completely without overflowing. (Figure 19)

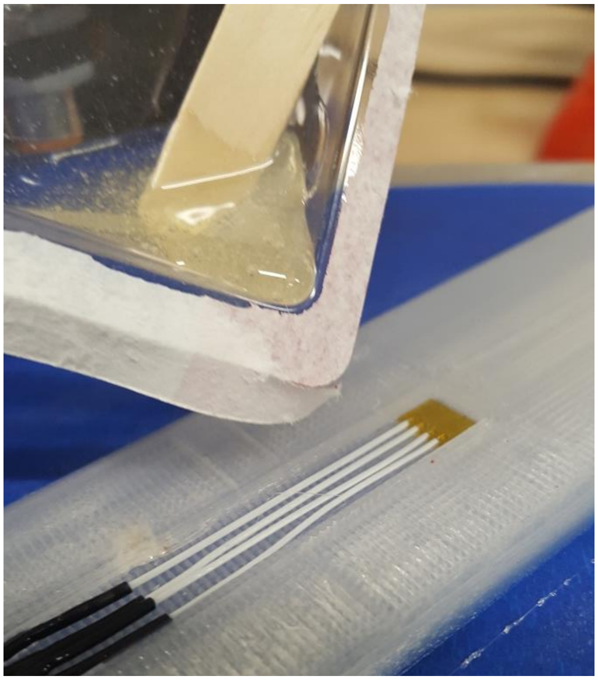

Figure 19 Pour epoxy into the channel using a pospsicle stick.

14. Allow the epoxy to set as until the 5 minute timer goes off. To ensure filament flows out of the print head smoothly on the next layer, use the manual controls in Repetier host to extrude about 50mm of filament on the printbed away from the beam.
15. Un-pause the print and allow normal printing to resume.

Deflection Experimental Procedure

Prior to the Experiment

1. Mark the beam as shown Figure 1. Make tick marks down the center of the beam every 5mm with a ruler starting 20mm in from the edge that will be clamped to the table.
2. Measure and record the dimensions of the beam with calipers. (Fig. 20)

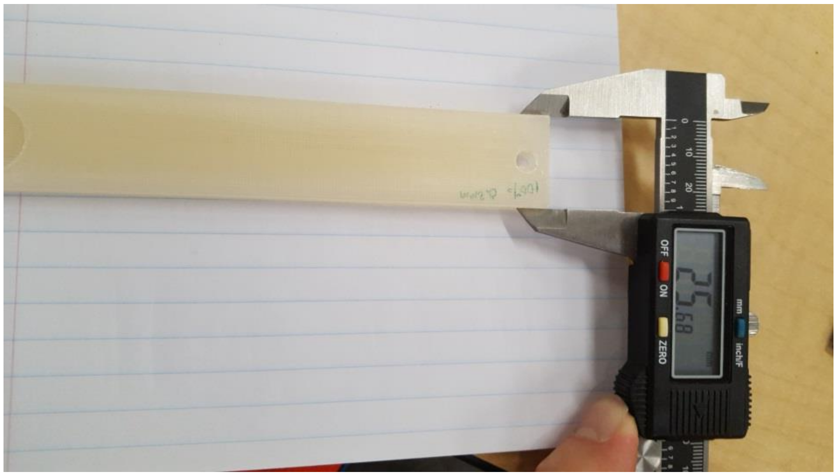

Figure 20 Measuring dimensions of the 3D printed beam.

Setting up the Experiment

1. Lay the flat aluminum plate flush against the edge of the table using the level from the combination square.
2. Align the 20mm line of the 3D printed beam with the front edge of the aluminum plate using a combination square so that it is perpendicular to the edge. The beam should also be about 6 inches from the side edge of the table. This is done so that the dial indicator will properly line up with the beam after the track is setup.
3. Clamp the beam to the table. Double check it is properly aligned and perpendicular to the edge of the plate using the level from the combination square. (Fig.21)

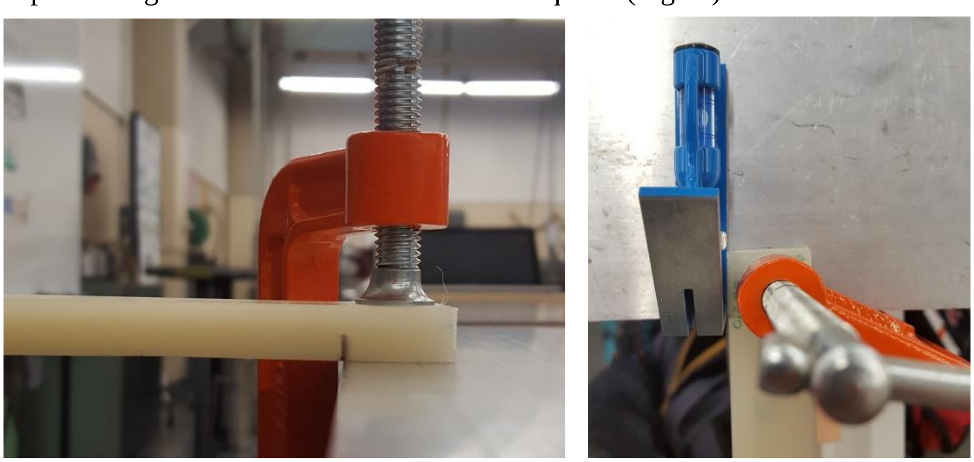

Figure 21 Clamping the beam to the table.

4. Slide the dial indicator onto the dial indicator mate piece.
5. Slide the track onto the magnetic base and attach the magnetic base onto the leg of the table.
6. Slide the dial indicator and mate piece into the track.
7. Adjust magnetic base as necessary to make the track level and the so that the dial indicator lines up with the center of the beam and such that the indicator is depressed about a quarter inch. (Fig.22)

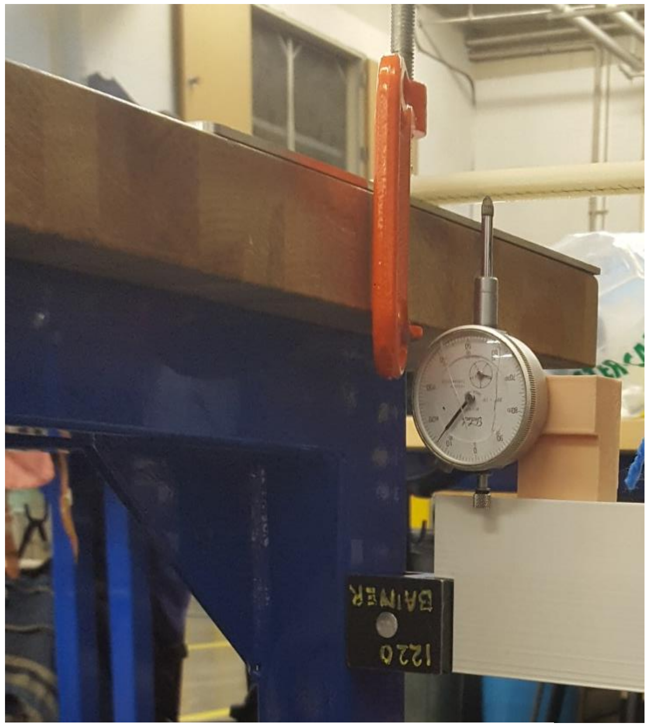

Figure 22 Setup as described in steps 4-7.

Taking Data

1. With no load, zero the dial indicator at the point closest to the clamped end of the beam. (Note: some of the marked points cannot be measured due to interference between the clamp and/or the attached weights.)
2. Take deflection data at each marked point along the beam. Record positive deflection in the direction towards the floor and negative deflection in the direction towards the ceiling. (Note: Even with no load, the beam may be very slightly warped due to the 3D printing process leading to non-zero measurements in this step.)
3. Repeat step 2 and take another data set with no load.
4. Attach string to the eyebolt such that the hook/weights can be hung from the string. Attach the eyebolt to the end of the beam using nuts and washers.
5. Take deflection at the marked points after the beam is loaded.
6. Increase the load in 0.5 kg increments up to 2.7kg and take deflection data.
7. Repeat steps 4 and 5 until max load is reached.
8. Gradually decrease the load by the same increments as in steps 4 and 5 until there is no load. Take deflection data as before.
9. Repeat steps 1-8 for each beam being tested.

Strain Experiment Procedure

After the beam is finished being printed, and enough time has elapsed for the adhesive to cure, bending experiments are performed on the beam in order to acquire data from the sensor. The beams were marked 25mm of the beam would be directly above the table and the rest of the beam would overhang without support of the table. The beams were clamped in the same way as done for the deflection experiments.

The aim of these experiments is to compare theoretical strain values at the location of the strain gauge with experimentally derived values from the sensor. A beam is aligned with the edge of a table such that the strain gauge and the location of the load application are at predetermined distances from the edge. A clamp is used to fix the beam's location and orientation relative to the table. A wheatstone bridge circuit is created in order to amplify the gauge signals such that we can easily measure them. A schematic of the circuit assembled on a breadboard is provided below in Figure 23. Not shown is a 0.1 μF capacitor in parallel with the wheatstone bridge [left] which is used for noise reduction. The strain gauges are wired such that the axial strain gauge [nodes 1-2] is in series with the transverse strain gauge [nodes 2 - 3]. Power is supplied by a National Instruments MyDAQ.

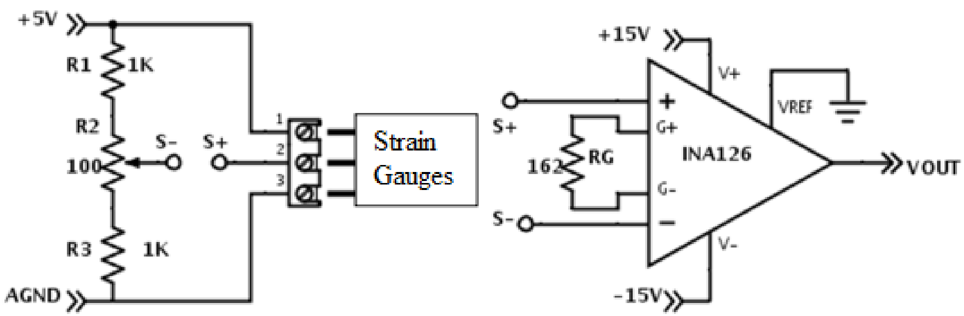

Figure 23 Circuit schematic. Resistors R1 and R3 are the bridge completion resistors, while the strain gauge block is placed in parallel to the resistors. R2 is a potentiometer used to balance the bridge. [1]

Once the circuit is completed, the wheatstone bridge is balanced. This is done by adjusting the potentiometer until the circuit output voltage is as close to zero as possible. In order to induce strain on the beam, weights are attached to a hook at the end of the beam. The output voltage is then recorded using a multimeter. In between every loading step, the output voltage without any load on the beam is measured. The respective offset of every data point is subtracted to account for the fact that a balanced bridge was regularly unattainable. Once we have a set of data which relates mass to output voltage, a conversion to strain is necessary. Equation A below shows this relation.

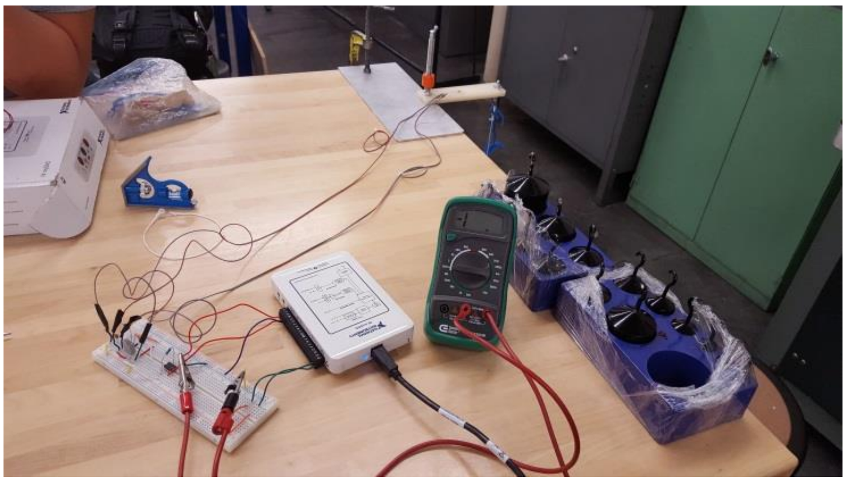
Figure 24 Strain Gauge Experimental Setup.

$$\varepsilon = \frac{4V_0}{A_v F E_i K_b}$$

Equation A: The relationship between strain and voltage where $\mathcal{E}$ is strain, V0 is the opamp output voltage, Av is the amplifier gain (499), F is the gauge factor (2), Ei is the bridge input voltage (5V), and Kb is the bridge constant (1.36).

The amplifier gain Av is given in the INA126 documentation as 5 + (80000/Rg) where Rg is 163. The bridge constant is shown below in Equation B. Many online sources claim for the Poisson's ratio of PLA to be 0.36 and that is what is assumed here. This property, along with the input voltage and gain, are assumed to be constant.

$$K_b = 1 + \nu$$

Equation B: Bridge constant equation for one longitudinal strain and one transverse strain. ***ν*** *is the Poisson's ratio.*

Derivation of Strain Relation

If one begins with $E_0 = E_i \frac{F}{4} K_b \varepsilon$ and $V_0 = A_v E_0$, it is easy to arrive at equation X.

Derivation of Bridge Constant Kb

In the second configuration, in the Wheatstone bridge circuit, gauge 1 and 2 is active. For gauge 1:

$$\frac{\delta R_1}{R_1} = \varepsilon_a (GF)_1$$

Where εa is the axial strain gauge 1 experiences. For gauge 2, assume GF1 = GF2, **ν** is Poisson's ratio

$$\frac{\delta R_2}{R_2} = -\upsilon_p \varepsilon_a (GF)_2 = -\upsilon_p \frac{\delta R_1}{R_1}$$

With only one gauge active:

$$\frac{\delta E_0}{E_i} = \frac{\delta R_1/R_1}{4 + 2(\delta R_1/R_1)} \approx \frac{\delta R_1/R_1}{4} \qquad (1)$$

With both gauges active:

$$\frac{\delta E_0}{E_i} = \frac{(\delta R_1/R_1)(1+\upsilon_p)}{4 + 2(\delta R_1/R_1)(1+\upsilon_p)}$$

Since δR/R<<1, the equation above can be changed to

$$\frac{\delta E_0}{E_i} = \frac{(\delta R_1/R_1)(1+\upsilon_p)}{4} \qquad (2)$$

The ratio of equation (1) and (2) is the bridge constant, and the bridge constant is $\kappa = 1 + \upsilon_p$.

References

[1] EME107B W16 Lab 3 Strain A Handout
[2] EME107B W16 Lab 4 Strain B Handout
[3] Figliola, R. S., & Beasley, D. E. (2015). *Theory and design for mechanical measurements*.
[4] INA126 datasheet, Texas Instruments: http://www.ti.com/lit/ds/symlink/ina126.pdf

Transverse Shear Stress/Strain Derivation:

$$\tau_{xy} = \frac{V(x)}{It(y)} \int_{y}^{y_{top}} yt(y)dy = \frac{V(x)Q(y)}{It(y)} = \frac{VQ}{It}$$

where:

$V(x)$ the shear force carried by the section, found from the shear force diagram

I the second moment of area

$t(y)$ the sectional width at the distance y from the *N.A.*

$Q(y) = \int_{y}^{y_{top}} yt(y)dy = \bar{y}'A'$ *A'* is the top (or bottom) portion of the member's cross-sectional area, defined from the section where $t(y)$ is measured, and $\bar{y}'$ is the distance to the *centroid of A'*, measured from the Neutral Axis.

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

Where τ_{xy} is shear stress and G is the shear modulus.