

Booth School of Business  
University of Chicago

Professor Quentin Vandeweyer

BUSN 20400

Problem Set #4  
CAPM & APT

You may work on this problem set in groups of up to five students. Hand in one solution per group. You may discuss the problems only with members of your group. Answers should be typed. This problem set is due on Canvas at **11:59pm on May 22, 2022**.

**For the 2nd and 4th problems you will need to obtain Excel spreadsheets from Canvas.**

**Problem 1.** You are given the following information: the variance of the rate of return on stock 1, stock 2, and the market portfolio are,  $\sigma_1^2 = 0.16$ ,  $\sigma_2^2 = 0.09$ , and  $\sigma_M^2 = 0.04$ . The covariances between these assets are  $\sigma_{1,2} = 0.02$ ,  $\sigma_{1,M} = 0.064$ , and  $\sigma_{2,M} = 0.032$ . Consider forming a portfolio that has 75% invested in asset 1 and 25% invested in asset 2. Call this portfolio  $P$ .

- (a) What is the variance of portfolio  $P$ ?
- (b) What are the betas of 1, 2, and  $P$  relative to the market (i.e. what are  $\beta_{1,M}$ ,  $\beta_{2,M}$ , and  $\beta_{P,M}$ )?
- (c) What are the  $R^2$  values in regressions of the return on 1, 2, and  $P$  on the market portfolio?

**Problem 2. (Computer Exercise)** Obtain the data file **ps4\_problem2.xls** from Canvas. This file contains historical monthly rates of return on several stock indices, corporate and government bonds, and individual stocks. The rate of return (**expressed as a percent**) is defined as:

$$100 \times \frac{P_t + D_t - P_{t-1}}{P_{t-1}}$$

where  $P_t$  is the stock or bond price at time  $t$  and  $D_t$  is any cash paid by the security (e.g., a dividend for stocks or an interest payment for bonds) over the period. For example, if you paid \$100 for a stock at time  $t - 1$  and it paid no dividend but the stock appreciated to \$110 at time  $t$ , the rate of return on the stock would be 10%. The following calculations can be performed in Excel.

- (a) Set up four different portfolios, finding monthly rates of return on each. **The first portfolio is evenly invested in GM and IBM. The second portfolio is evenly invested in three stocks (GM, IBM, and Anheuser Busch) while the third portfolio is evenly invested in four stocks (GM, Toyota, IBM, and Anheuser Busch). The last portfolio is equally invested in all 13 common stocks.** The rate of return on any portfolio is just the weighted-average of the rates of return on the underlying stocks.

- Calculate the average monthly return and the return variance on these four portfolios (you do not need to use any complicated formulas here, just apply Excel functions AVERAGE and VAR to the portfolio returns).
  - For each of these four portfolios, *estimate* the beta and  $R^2$  of the portfolio versus the “World Market” portfolio. (If you are working in Excel, the functions SLOPE and LINEST will both estimate the beta. LINEST provides other regression statistics as well.)
- (b) How does the variance of the portfolio change as we add assets to the portfolio? How does the beta change as we add assets to the portfolio?
- (c) What do the results in part (a) tell us about the behavior of the components of risk and return as we add assets to the portfolio?

**Problem 3.** True or False? Explain your reasoning.

- (a) APT assumes that investors only care about mean and variance of their end-of-term portfolio wealth.
- (b) CAPM assumes that returns are generated by a one-factor model, and that factor is the market return.
- (c) You analyze GM monthly stock returns during the last 20 years. Suppose you do a regression on the three Fama–French factors, and you find that, whichever 3-year period you choose, the alpha of the regression is negative and significant. This would indicate that APT is not true.
- (d) Suppose you do some APT regressions on a set of given factors, and each time you do a regression, you get an  $R^2$  of at least 80%. This indicates that APT is true with those factors.

**Problem 4. (Computer Exercise)** Use the Excel spreadsheet “FF\_IBM\_IMMU.xls” to compare the APT- and CAPM-predicted risk premia on IBM and IMMU (Immunomedics Inc). It contains monthly returns data for the two stocks, and for the three Fama–French portfolios (MKT, SMB, and HML) between 01/1994 and 12/1998.

- (a) Find the  $\alpha$ , the  $\beta$ , and the  $R^2$  of the monthly CAPM regression for the two stocks. (Hint: Use the Excel command LINEST. When using the formula, keep in mind that the result is an array, so you have to press **Ctrl-Shift-Enter**. Also, make sure you pay attention to the order of the coefficients, which LINEST for some obscure reason places in **reverse order**! ) Write down the standard errors and the t-statistics.
- (b) Which regression provides a better fit? (I.e. which  $R^2$  is larger?) How do you explain that?
- (c) Are the coefficients significantly different from zero? (*Hint*: look at the standard errors given by LINEST.) Do these results make you believe that CAPM is true when applied to IBM and IMMU?

- (d) Find now the intercept  $a$ , and the factor loadings  $b_{MKT}$ ,  $b_{SMB}$ , and  $b_{HML}$  for the APT monthly regression. Which of these are significantly different from zero? Interpret the signs of  $b_{SMB}$  and  $b_{HML}$  for the two companies: Does IBM behave like a small or large stock? Value or growth stock? Same question for IMMU.
- (e) Show that for both stocks the market factor loading loses significance in the Fama–French APT, when compared to the CAPM. Can you tell from this whether APT or CAPM do a better job of predicting expected returns?