Fall 2018 – Practice Questions

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Course Introduction and Intro to Market Demand (Lecture 1)

1. *2016 Problem Set 1, Q1*
   United Airlines. United Airlines operates a non-stop route between New York City and Paris. There are two types of travelers: business and tourists. The annual demand for round-trip flights for these two groups are given by:

$$P_B(q) = 3{,}000 - \frac{1}{5}q$$

$$P_T(q) = 1{,}000 - \frac{1}{5}q$$

where B and T denote business and tourist, respectively. Quantities are the total number of round-trip tickets sold per month and prices are on a per-ticket basis.

a) What is the aggregate demand for this route? Express aggregate demand in terms of quantity as a function of price.

$$Q_{agg}(p) = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if } p \leq \_\_\_\_\_\_\_\_\_\_, \\ \_\_\_\_\_\_\_\_\_\_ & \text{if } p \geq \_\_\_\_\_\_\_\_\_\_, \end{cases}$$

Answer: First note that with two types of consumers that have different maximum willingness-to-pay, denoted by $\bar{p}$, the aggregate demand function is going to have a kink. We first solve for the maximum willingness-to-pay of each type by setting the quantity equal to zero and solving for price. For business travelers,

$$\overline{p_B} = 3{,}000$$

and for tourists:

$$\overline{p_T} = 1{,}000$$

The aggregate demand, therefore, has the following form (*):

$$Q(p) = \begin{cases} Q_B(p) + Q_T(p) & if\ 0 \leq p \leq 1{,}000 \\ Q_B\,(p) & if\ 1{,}000 < p \leq 3{,}000 \\ 0 & if\ p > 3{,}000 \end{cases}$$

where

$$Q_B(p) + Q_T(p) = (15{,}000 - 5p) + (5{,}000 - 5p)$$

$$= 20{,}000 - 10p$$

Replacing the variables in the demand curve, we get (*):

$$Q(p) = \begin{cases} 20,000 - 10p & if\ 0 \leq p \leq 1,000 \\ 15,000 - 5p & if\ 1,000 < p \leq 3,000 \\ 0 & if\ p > 3,000 \end{cases}$$

b) How many tickets would United sell if it priced this route at $500 per round-trip ticket?

Number of Tickets: ____________________

Answer: As the price is below both types maximum willingness-to-pay, we know United will sell these tickets to both groups. Therefore, we simply evaluate the aggregate demand function at this price

$$Q(500) = 20,000 - 10(500) = 15,000 \text{ tickets per month.}$$

In light of ongoing structural changes occurring in Europe due to Britain's exit from the European Union, United Airlines is evaluating the impact on business travel between New York and Paris. United has estimated that the relocation of some firms from London to Paris, will result in an increase in the demand for business travel between New York and Paris, with the resulting monthly demand for business travelers being:

c) How much is this growth in demand going to increase (in percentage terms) the total flights per-month demanded under United's original pricing of $500 per flight?

% increase = ____________________

Answer: Need to just resolve with the new business demand in the aggregate demand function and then express in percentage terms. Original quantity was 15,000, new quantity is 25,000, so increase is 66.6%.

d) If they keep the price fixed, would their customer mix (business versus tourist) shift towards more business travelers or towards more tourists?

towards more business travelers **X**

towards more tourists ☐

e) Discuss, in a few sentences, how United would likely adapt to this change and how such changes in United's pricing strategy would impact the welfare of tourists that use this route.

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Answer: United would adapt by increasing prices. This is going to lower the welfare of tourists since their demand function is unaffected, but they will be paying higher prices.
(Optional) They may also want to increase supply or alter their product to be more business-friendly.

2. *2013 Final Exam, Short question e)*
   Amazon.com has been testing a same-day product delivery program where they would charge an annual membership fee to "Amazon Ultra", and also a usage fee for each product delivered on the same day as ordered.

   The company estimates that they have two equally sized customer segments that value same-day delivery differently: "occasional" shoppers and "convenience-lovers". Annual demands for same-day deliveries when a same-day delivery costs p are given by

$$Q_{occ}(p) = 10 - p,$$
$$Q_{con}(p) = 20 - 2 \cdot p.$$

   Amazon's pricing department recommends a membership fee of \$39 per year and a usage fee of $p = \$3$ per delivery.

   a) If they offer "Amazon Ultra" at the suggested membership fee and usage fee, what will be the outcome?

No one will sign up	☐
Convenience-lovers will sign up	**X**
Occasional shoppers will sign up	☐
Convenience-lovers and occasional shoppers will sign up	☐

Answer: Convenience-lovers will sign up.

At the $3 price per shipment, occasional shoppers do not have sufficient CS to pay the membership fee:

$$\text{CS}_{\text{Occasional}} = (10 - 3)\left(\frac{1}{2}\right)(10 - 3) = 24.5 < 39$$

Occasional shopper will therefore not join. For convenience lovers,

$$\text{CS}_{\text{Convenience}} = (10 - 3)\left(\frac{1}{2}\right)(20 - 3 * 2) = 49 > 39$$

Since the surplus of the convenience shopper at this usage fee is more than the membership fee, the convenience shopper will join.

b) If Amazon's cost per delivery worked out to $6, will they make money off the convenience-lovers over the course of a year?

No, they will lose $42	☐
No, they will lose $3	**X**
They will break even on them	☐
Yes, they will make $3	☐
Yes, they will make $42	☐

Answer: No, they will lose $3:

$$\pi = 39 + 3 * 14 - 6 * 14 = -3 \text{ per shopper}$$

Equilibrium and Elasticity (Lecture 2)

3. *2014 Problem Set 1, Q3+Q4 (partial)*
   Chinatown buses. In the past ten years, low cost bus carriers have exploded as a transportation option for students, particularly in the Northeast. These buses pick up from less populated street locations, rather than central stations, and thus avoid expensive port fees. The weekly demand (in thousands of tickets) for low-cost bus travel between adjacent cities is

$$Q_D(p) = 30 - 0.5p,$$

   and supply (in thousands of tickets) is:

$$Q_S(p) = 2p - 20,$$

   where price is given in dollars.

   a) Calculate the equilibrium price and quantity, as well as the producer and consumer surplus.

      Quantity ________________

      Price ________________

      Consumer surplus ________________

      Producer surplus ________________

Answer: To determine the equilibrium price and quantity, we simply set supply equal to demand and solve for price, then plug that price into either the supply or demand functions to solve for the equilibrium quantity of tickets:

$$Q_D(p) = Q_S(p)$$

$$30 - 0.5p = 2p - 20$$

$$50 = 2.5p \rightarrow p^* = 20$$

$$q^* = 2\times 20 - 20 = 20$$

The inverse demand curve is given by $P_D(q) = 60 - 2q$ so the consumer surplus is given by

$$CS = \frac{1}{2}(60 - 20)\times 20 = 20\times 20 = 400{,}000$$

the inverse supply curve is given by $P_S(q) = 10 + \frac{1}{2}q$. Recall the producer surplus is the area *above* the supply curve and *below* the equilibrium price. This is given by

$$PS = \frac{1}{2}\left(p^* - \underline{p}\right) \times a^*$$

Where $\underline{p}$ is the y-intercept of the inverse supply curve. So we have

$$PS = \frac{1}{2}(20 - 10) \times 20 = 100{,}000$$

Construction on the Holland Tunnel increases delays on the bus route, leading to a temporary drop in demand. The demand during construction is reduced to

$$Q_D(p) = 25 - 0.5p$$

and the market arrives at a new equilibrium.

b) By what percent does consumer surplus drop due to the construction?

Percentage drop ________________

Answer: Given the new demand function, the new equilibrium price and quantity are

$$25 - 0.5p = 2p - 20$$
$$45 = 2.5p \rightarrow p^* = 18$$
$$q^* = 25 - 0.5 \times 18 = 25 - 9 = 16$$

the new inverse demand curve is $p = 50 - 2p$ so the new consumer surplus is

$$CS = \frac{1}{2}(50 - 18) \times 16 = 256$$

so the percent change in consumer surplus is

$$\%\Delta CS = \frac{256 - 400}{400} \times 100 = -36\%$$

Thankfully, the construction ends and demand for buses returns to normal. The bus carriers have enjoyed great success, at the expense of pricier alternatives such as Amtrak.

Rail carriers are lobbying the government to restrict low-cost bus carriers, and now the government is proposing a $2.50 tax on each intercity ticket for a bus not departing from a bus station (the tax would be paid by the bus companies for each ticket sold). The government would use the tax proceeds as a subsidy for the rail companies.

c) What are the elasticities of demand and supply at the equilibrium you calculated in 3a?

Elasticity of demand __________________

Elasticity of supply __________________

Answer: The elasticity of demand from part 3a is given by

$$\varepsilon_D = \frac{p^*}{q^*} \cdot \frac{dQ_D(p)}{dp} = \frac{20}{20} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$$

$$\varepsilon_S = \frac{p^*}{q^*} \cdot \frac{dQ_S(p)}{dp} = \frac{20}{20} \cdot (2) = 2$$

d) Which side of this market would bear most of the tax burden – consumers, or bus companies? Why?

X Consumers ☐ Bus Companies

Answer: Notice that the price elasticity of supply is

$$\varepsilon_S = \frac{p^*}{q^*} \cdot \frac{dQ_S(p)}{dp} = \frac{20}{20} \cdot (2) = 2$$

Since $|\varepsilon_D| < 1 < \varepsilon_S$ consumer demand is inelastic and consumer demand is much less sensitive to price changes than producer supply at the equilibrium price and quantity. That is, the increase in price due to the tax doesn't affect consumer demand for tickets very much so the producers can pass more of the tax on to consumers, making them bear the most of the tax burden.
<u>Note: merely comparing CS and PS from the latter part of the question and seeing which decreases more is not a complete solution here and only gets partial credit.</u>

4. *2016 Final, Q3 (partial)*

Pandora allows members to stream music over the internet. To support the "free" version of their service, Pandora sells advertising that then appears periodically in-between songs. Users, unsurprisingly, do not like listening to ads.

Pandora currently displays, on average, one advertisement per ten songs and streams about 30 billion songs per month to these customers. In the process of evaluating the attractiveness of alternatives to the current advertising approach, their data science team ran an experiment and found that if they raised the frequency of advertisements to one advertisement for every eight songs, users would stream only 21 billion songs per month.

a) What is the "elasticity" of demand for streaming music through Pandora with respect to the number of advertisements? Note: compute the elasticity using the percentage changes relative to the baseline with one advertisement per ten songs.

Elasticity = ___________________________

Answer: The elasticity of demand for music streamed (q) through Pandora with respect to advertisements (n) is given by:

$$\epsilon_D = \frac{\%\Delta q}{\%\Delta n}$$

where:

$$\%\Delta q = \frac{(21-30)}{30} = -0.3$$

$$\%\Delta n = 0.25$$

$$\epsilon_D = -1.2$$

Note that percentage increase in the number of adds is 25% and not 20% as it may seem. To see this, consider any number of songs streamed, denoted by S. The percentage change in the number of adds is:

$$\%\Delta n = \frac{\left(\frac{S}{8} - \frac{S}{10}\right)}{\frac{S}{10}}$$

$$= \frac{\left(\frac{10S - 8S}{80}\right)}{\frac{S}{10}}$$

$$= \frac{S(10-8)}{80}\left(\frac{10}{S}\right)$$

$$= \frac{20}{80} = 0.25.$$

Pandora, of course, makes money from the display of advertisements. They currently receive \$0.025 per advertisement they show and have to pay, on average, \$0.0014 per song streamed to the artist that created the content.

b) Assuming content costs are the only meaningful cost, what percent of Pandora's ad revenue per 1,000 songs is profit?

% of revenue that is Profit = ___________________________

Answer: For each 1,000 songs streamed, Pandora plays a total 100 ads. For each ad it receives $0.025, implying a total revenue of 100*0.025 = $2.5 per 1,000 songs. The total payments to content provides to stream 1,000 songs is 1,000*0.0014 = $1.4. The profit from 1,000 streams is, therefore, $2.5 – $1.4 = $1.1. This implies that the markup is $1.1/$2.5 = .44 and so 44% of its ad revenue is profit.

3) Government Interventions: Taxes and other Regulations

5. *2014 Problem Set 1, Q3*

   Cigar tax. The weekly supply, Q_s, and demand, Q_d, of cigars in Philadelphia are given by

$$Q_s(p) = p$$

and

$$Q_d(p) = 102 - 2p$$

where prices are in dollars and quantities are in hundreds of thousands of cigars.

a) How many cigars are consumed in equilibrium and at what price?

   Quantity ________________

   Price ________________

Answer: In equilibrium, we have $Q_s(p) = Q_d(p)$, and substituting:

$$p^* = 102 - 2p^*$$
$$p^* = 34.$$

Plug p^* into the supply to get $q^* = 34$.

b) What are the consumer surplus and producer surplus?

   Consumer surplus ________________

   Producer surplus ________________

Answer: Recall that

$$CS = \frac{1}{2} \times q^* (\text{demand's price intercept} - p^*)$$
$$PS = \frac{1}{2} \times q^* (p^* - \text{supply's price intercept})$$

By plugging in $Q_D = 0$ into the demand, we see that the demand's price-intercept is $\frac{102}{2} = 51$. Since the supply does not have any constant term, the supply has intercept 0:

$$CS = \frac{1}{2} \times 34 \times (51 - 34) = 289.$$
$$PS = \frac{1}{2} \times 34 \times 34 = 578.$$

In March, Pennsylvania Governor Tom Wolf proposed a 40% tax on cigars for the 2015 state budget. Although the proposal was soundly defeated, a cigar tax is still in consideration. Assume that a compromise is reached and a 20% tax will be levied on retail cigar sales.

c) After the introduction of the tax, how many cigars are consumed in equilibrium and what price do consumers pay? What price do producers receive?

Quantity ________________

Price paid by consumers ________________

Price paid by producers ________________

Answer: Due to the tax intervention, there is now a discrepancy between the price paid by the consumers and the price received by the producers at the equilibrium. These two prices are related by the following equation:

$$P_s \times (1 + 0.2) = P_d.$$

However, even with the government intervention, the quantities sold by the producers still equals the quantities demanded by consumers in equilibrium:

$$Q_s(p) = Q_d(p)$$
$$p_{exc} = 102 - 2\times(p_{exc}\times 1.2)$$
$$p_{exc} = p_s = 30.$$

The price received by producers is 30, while the price paid by consumers is

$$p_{inc} = p_d = 30\times 1.2 = 36.$$

d) Using the changes in equilibrium prices and quantities compared to (a), compute the price elasticity of demand and supply.

Elasticity of demand __________________

Elasticity of supply __________________

Answer: Using the changes in equilibrium prices and quantities, the demand elasticity is:

$$\varepsilon_D = \frac{q^{tax} - q^{no\ tax}}{q^{no\ tax}} \times \frac{p^{no\ tax}}{p_{inc} - p^{no\ tax}}$$
$$= \frac{30 - 34}{34} \times \frac{34}{36 - 34} = -2.$$

The supply elasticity is

$$\varepsilon_S = \frac{q^{tax} - q^{no\ tax}}{q^{no\ tax}} \times \frac{p^{no\ tax}}{p_{exc} - p^{no\ tax}}$$

$$= \frac{30 - 34}{34} \times \frac{34}{30 - 34} = 1.$$

e) What is the economic incidence of the tax hike on cigar consumers?

Fraction ___________________

Answer: The fraction that is paid by consumers is

$$\frac{\varepsilon_S}{\varepsilon_S + |\varepsilon_D|} = \frac{1}{1+2} = \frac{1}{3}.$$

Another way to compute the incidence is to use the formula

$$\frac{p_{inc} - p^*}{p_{inc} - p_{exc}} = \frac{36 - 34}{36 - 30} = \frac{1}{3}.$$

f) With the tax, what are the consumer surplus, the producer surplus, and the tax revenue collected by the government?

Consumer surplus _________________

Producer surplus _________________

Tax revenue _________________

Answer:

$$\begin{aligned} CS &= \frac{1}{2} \times q^{tax} \times (51 - p_{inc}) \\ &= \frac{1}{2} \times 30 \times (51 - 36) = 225. \\ PS &= \frac{1}{2} \times q^{tax} \times (p_{exc} - 0) \\ &= \frac{1}{2} \times 30 \times 30 = 450. \\ \text{Tax Revenue} &= (p_{inc} - p_{exc}) \times q^{tax} \\ &= 6 \times 30 = 180. \end{aligned}$$

g) Calculate the deadweight loss (DWL) induced by the tax. Briefly explain why the DWL can be interpreted as "inefficiency".

DWL ____________________

Reasoning:

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Answer: The DWL is equal to the difference between the total surplus before and after the tax intervention. The total surplus before the tax was

$$\begin{aligned} TS^{no\ tax} &= CS^{no\ tax} + PS^{no\ tax} \\ &= 289 + 578 + 867. \end{aligned}$$

The total surplus after the tax is

$$\begin{aligned} TS^{tax} &= CS^{tax} + PS^{tax} + \text{Tax Revenue} \\ &= 225 + 450 + 180 = 855. \end{aligned}$$

Therefore, $DWL = TS^{no\ tax} - TS^{tax} = 867 - 855 = 12.$
An alternative way to compute the DWL triangle is to use the formula

$$\begin{aligned} DWL &= \frac{1}{2}\times(p_{inc} - p_{exc})\times(q^{no\ tax} - q^{tax}) \\ &= \frac{1}{2}\times(36 - 30)\times(34 - 30) = 12. \end{aligned}$$

As no negative externalities exist in this market, the DWL represents the loss in total surplus due to distortionary taxation.

6. *2014 Exam, Q4*

High-fructose corn syrup. High-fructose corn syrup (HFCS) is the industrial sweetener of choice in the US. As the technology for producing HFCS is well known, the market is perfectly competitive. Let the supply for the American HFCS market be given by

$$P_S(q) = 225.5 + 0.0001q,$$

and the demand by

$$P_D(q) = 1{,}130 - 0.02q.$$

where quantity is in thousands of pounds, and price is in dollars.

a) What is the market equilibrium price and quantity of HFCS?

Price ____________________

Quantity ____________________

Answer: Solving the inverse supply and demand curves for the regular supply and demand curves gives

$$Q_S(p) = 10{,}000p - 2{,}255{,}000$$

$$Q_D(p) = 56{,}500 - 50p$$

Then the equilibrium price is found setting $Q_S = Q_D$:

$$10{,}000p^* - 2{,}255{,}000 = 56{,}500 - 50p^*$$

$$10{,}050p^* = 2{,}311{,}500$$

$$p^* = \$230$$

Then plugging this equilibrium price into either the supply of demand function gives

$$q^* = 10{,}000p^* - 2{,}255{,}000 = 45{,}000$$

$$q^* = 56{,}500 - 50p^* = 45{,}000$$

HFCS has been linked to obesity and rising health care costs. This negative externality has led the government to consider taxing it.

b) Suppose a per-unit tax of \$19.50 per thousand pounds were imposed on the production – what would the new market equilibrium be, and what fraction of the tax revenue would be "paid" by consumers? Which side of the market is more inelastic? You may round the equilibrium quantity value to the nearest hundred.

Price ____________________

Quantity ____________________

Share of tax revenue coming from consumer surplus ____________________

Check one of these two boxes:

Demand is more inelastic ☐ Supply is more inelastic ☐

Answer: Given the per unit tax on producers, we can think of the price the consumer pays as the price the producer charges plus the tax

$$P_D = P_S + 19.5$$

or equivalently, the price the producer receives will be the price the producer pays *less* the tax

$$P_S = P_D - 19.5$$

Either will give the correct solutions. We can re-solve for the equilibrium where $Q_S = Q_D$ given these expressions for the tax's effect on prices:

$$\begin{aligned}
10{,}000p_{exc} - 2{,}255{,}000 &= 56{,}500 - 50(p_{exc} + 19.5) \\
10{,}000p_{exc} &= 2{,}311{,}500 - 50(p_{exc} + 19.5) \\
10{,}050p_{exc} &= 2{,}310{,}525 \\
p_{exc} &= p_s = 229.90 \\
p_{inc} &= p_d = p_{exc} + 19.50 = 249.40.
\end{aligned}$$

Solving for the equilibrium quantity gives

$$Q_D(p_{inc}) = 56{,}500 - 50 \times p_{inc} = 56{,}500 - 50 \times 249.40 = 44{,}030$$

or using the supply curve

$$Q_S(p_{exc}) = 10{,}000 \times p_{exc} - 2{,}255{,}000 = 44{,}000.$$

it should be the case that $q^{tax} = Q_D = Q_S$ (where q^{tax} is the equilibrium quantity under the tax) but they differ here due to rounding of the prices. The simplest way to compute the share of tax revenue coming from the consumer surplus is

$$\frac{(p_{inc} - p^*)}{(p_{inc} - p_{exc})} = \frac{249.40 - 230}{249.40 - 229.90} = \frac{19.40}{19.50} \approx 99.5\%$$

Alternatively, students could compute this as follows:

$$TR = (p_{inc} - p_{exc}) \times q^{tax} = (249.40 - 229.90) \times 44{,}030 = 19.50 \times 44{,}030 = 858{,}585$$

The portion of this tax revenue coming from consumer surplus is

$$TR_D = (p_{inc} - p^*)\times q^{tax} = (249.40 - 230)\times 44{,}030 = 19.40\times 44{,}030 = 854{,}182$$

So the share of tax revenue coming from consumer surplus is

$$\frac{TR_D}{TR}\times 100 = \frac{854{,}182}{858{,}585}\times 100 \approx 99.5\%$$

Lastly, since the majority of the share of tax revenue is coming from consumer surplus, we know that demand is more inelastic. To see this explicitly, the elasticities of supply and demand at the equilibrium price and quantity under the tax are

$$\varepsilon_D = \frac{dQ_D}{dp}\times\frac{p_{inc}}{q^{tax}} = -50\times\frac{249.40}{44030} \approx -0.28$$

$$\varepsilon_S = \frac{dQ_S}{dp}\times\frac{p_{exc}}{q^{tax}} = 10{,}000\times\frac{229.90}{44030} \approx 52.21$$

since $\varepsilon_D < -1$ and $\varepsilon_S > 1$ demand is relatively more inelastic than supply.

c) The government received a report from the FDA stating that a 3% reduction in the consumption of HFCS (relative to the equilibrium in (a) above) would yield great benefits to society. What is the per-unit tax (per thousand pounds of HFCS) that would achieve that exact reduction in consumption?

Tax per thousand pounds which reduced consumption by 3%: $____________________

Answer: We use the same relationship above $Q_S = Q_D$ but now let the tax be a variable T in the equation

$$P_D = P_S + T$$

so we have

$$10{,}000p_{exc} = 2{,}311{,}500 - 50(p_{exc} + T) = 2{,}311{,}500 - 50p_{exc} - 50T$$

$$10{,}050p_{exc} = 2{,}311{,}500 - 50T$$

$$P_S^*(T) = 230 - \frac{50}{10{,}050}T$$

Relative to the equilibrium in part (a) we know we want the new quantity, call this $\tilde{q}$, to be

$$\tilde{q} = 0.97\times q^* = 0.97\times 45{,}000 = 43{,}650$$

since we know the expression for $P_S^*(T)$ must hold in equilibrium, we need to solve for the T that solves

$$\tilde{q} = 10{,}000P_S^*(T) - 2{,}255{,}000 = 10{,}000\times\left(230 - \frac{50}{10{,}050}T\right) - 2{,}255{,}000$$

$$\Rightarrow 43{,}650 = 2{,}300{,}000 - \frac{500{,}00}{10{,}050}T - 2{,}255{,}000 = 45{,}000 - 49.75T$$

$$\Rightarrow 49.75T = 45{,}000 - 43{,}650 = 1{,}350$$

$$\Rightarrow T = 27.14$$

4) Externalities

7. *2017 Problem Set 1, Q3 (partial)*
   Wine. Imagine the demand for wine in Pennsylvania (in millions of bottles per year) is given by:

$$P_D(q) = 28 - \frac{1}{4}q$$

And supply is given by:

$$P_S(q) = \frac{1}{3}q$$

a) Find the equilibrium price and quantity with no taxes.

   Price ____________________

   Quantity ____________________

Answer: The equilibrium is found by equating supply and demand:

$$q_D = q_S$$

We can simply plug in the demand and supply and then solve for the equilibrium price:

$$112 - 4p^* = 3p^*$$
$$112 = 7p^*$$
$$p^* = \$16\ per\ bottle$$

Now we substitute this price into the demand curve to get the equilibrium quantity:

$$q^*(16) = 112 - 4(16) = 48$$

As the quantity units are in millions, 48 million bottles of wine are expected to be sold in the state each year.

In order to fund rebuilding of the city of Johnstown after a large flood in 1936, liquor in Pennsylvania became subject to an 18% tax by the state. Although the city was rebuilt by the mid 1940's the tax has continued to this day. For simplicity, assume this same tax is applied to all wine and no other taxes are included on wine.

b) With this tax, what is the equilibrium price to consumers, price earned by producers, and quantity transacted? Be careful to note that this is a percentage tax, not a per unit tax.

   Price for Producers ____________________

   Price for Consumers ____________________

   Quantity ____________________

Answer: Begin by shifting the supply curve back to reflect the additional tax. The original curve was:

$$P_S(q) = \frac{1}{3}q$$

Multiplying we get a new supply curve of:

$$P_{inc}(q) = 1.18P_S(q) = \frac{1.18}{3}q$$

Setting supply equal to demand again we get a price to consumers of:

$$112 - 4p_{inc} = \frac{3}{1.18}p_{inc}$$
$$132.16 - 4.72p_{inc} = 3p_{inc}$$
$$132.16 = 7.72p_{inc}$$
$$17.12 = p_{inc}$$

This means the price to producers is the price to consumers minus the tax, or:

$$p_{exc}(1.18) = 17.12$$
$$14.51 = p_{exc}$$

The quantity sold can be found by inserting the price to consumers into the demand curve:

$$17.12 = 28 - \frac{1}{4}q^{tax}$$
$$43.52 = q^{tax}$$

Or, equivalently, by inserting the price to producers into the supply curve:

$$14.51 = \frac{1}{3}q^{tax}$$
$$43.53 = q^{tax}$$

Note: We will accept answers with slight rounding errors as well.

c) How much more do consumers pay due to the tax? What percentage increase is this in the price they pay?

Change in Price ______________________

Percentage increase ______________________

Answer: Before the tax, the price is $16 per bottle. After the tax, the price to consumers is $17.12. So, the consumers pay an extra $1.12 per bottle. Hence, the price to consumers increased by 7% (1.12/16).

d) The PA Department of Revenue states on its website: "The tax is borne by the consumer, but manufacturers, distributors and importers remit the tax to the commonwealth." Is this statement:

True. The consumer bears/pays the full cost of the tax. The retailer simply passes the payment on to the state. ☐

False. While the tax is included in the price the consumer pays, the incidence is shared by both the consumer and the producer. **X**

e) Imagine that the externality from alcohol is proportional to the amount of alcohol consumed, regardless of its quality. Would the optimal Pigouvian tax (expressed as a percentage of price) be higher for a 750 ml bottle of 90 proof Old Crow (retail price $9.99) or a 750 ml bottle of 90 proof Old Forrester (retail price $39.99)? Give 1-2 sentences of reasoning.

Higher for Old Crow as a percentage of price **X**

Higher for Old Forrester as a percentage of price ☐

Reasoning:

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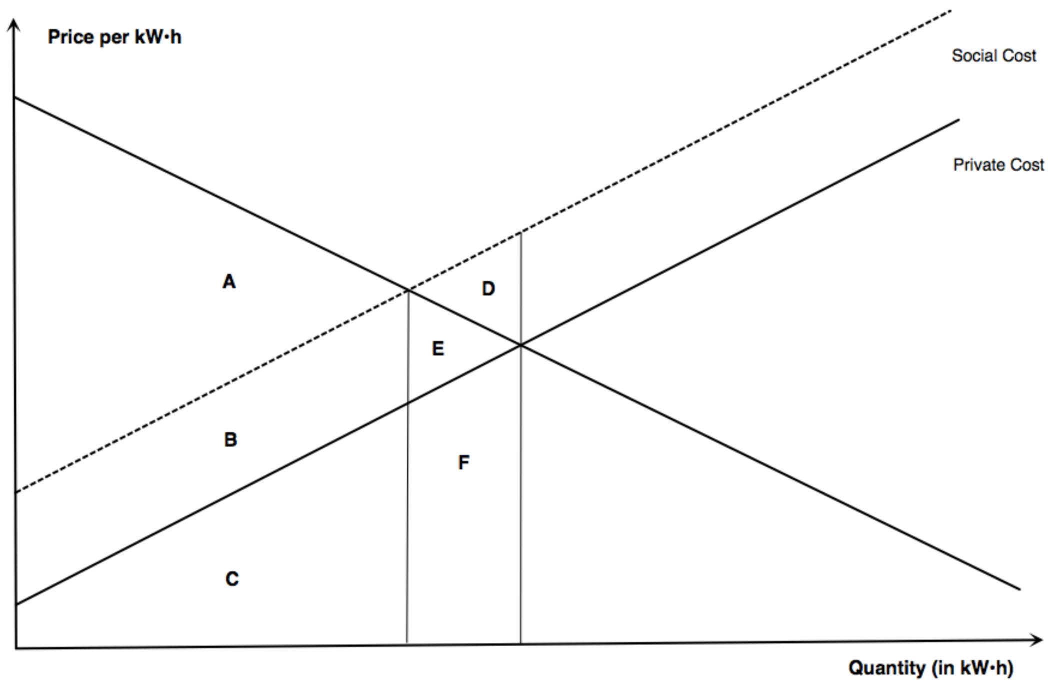
Answer: The cost to society is the same for both, but the price of Old Crow is lower. Hence, a higher percentage tax would be needed to perfectly “counteract” to externality.

8. *2015 Final, Short Answer Question*
Electricity. A recent government study finds that the production of electricity causes a negative externality of $0.10 per kilowatt hour (increased levels of acid rain reduce public health and destroy local crops). This is depicted in the graph below.



a) In general, if a good imposes a negative externality on society, ________ will be produced in the absence of intervention. If a good imposes a positive externality on society, _______ will be produced in the absence of intervention.

Too much; too much	☐
Too much; too little	**X**
Too little; too much	☐
Too little; too little	☐

b) Write the social welfare (or surplus) generated by this market with no intervention in terms of the labeled regions on the graph above (For example, A+B):

Social Welfare ____________________________

Answer: *Social* welfare is defined as

$$\begin{aligned} SW &= (\text{Total Surplus}) - (\text{Cost to Society}) \\ &= (A + B + E) - (B + E + D) = A - D. \end{aligned}$$

Notice areas B and E is part of the consumer and producer surpluses, but should be thought of as transfers from society to private agents (because they are below the social cost curve).

c) What would be the optimal tax per kilowatt hour for the government to levy?

Tax ____________________________

Answer: $0.10 per kWh; the optimal/Pigouvian tax is a per-unit tax equal to the negative externality.

d) Write the social welfare (or surplus) generated by this market if the optimal tax were levied in terms of the labeled regions.

Social Welfare ___________________________

Answer: *Social* welfare is again

$$\begin{aligned} SW &= (\text{Total Surplus}) - (\text{Cost to Society}) \\ &= (A) - 0 = A. \end{aligned}$$

5) Production and Supply Curves

9. *2017 Final, Q6*
   Mobile Apps. LeewayHertz is a software developer hired by major brands. Over the past decade they have built hundreds of mobile apps and have figured out the relationship between the number of engineers they have on staff and the quantity of apps those engineers can build each year. In particular, their production function is:

$$q = f(S,J) = S^{\frac{1}{4}} \cdot J^{\frac{3}{4}}$$

   where S is the number of senior engineer full-time equivalents (i.e., S need not be a whole number), J is the number of junior engineer full-time equivalents (i.e., J need not be a whole number), and q is the number of apps produced each year, where any fractional part represents an app that is only partially completed (i.e., q need not be a whole number).

   a) Compute the marginal productivity of senior engineers (S) and junior engineers (J) in terms of S and J.

   $MP_S\ (S,J) =$ ________________________________

   $MP_J\ (S,J) =$ ________________________________

Answer: The marginal productivity is the derivative of the production function with respect to that input, so:

$$MP_S\ (S,J) = \frac{\partial q}{\partial S} = J^{\frac{3}{4}} \Big/ 4S^{\frac{3}{4}} = \frac{1}{4}\left(\frac{J}{S}\right)^{\frac{3}{4}}$$

$$MP_J\ (S,J) = \frac{\partial q}{\partial J} = 3S^{\frac{1}{4}} \Big/ 4J^{\frac{1}{4}} = \frac{3}{4}\left(\frac{S}{J}\right)^{\frac{1}{4}}$$

   b) Compute the marginal productivity of investment for senior engineers (S) and junior engineers (J) in terms of S, J, p, and w.

   $MPI_S\ (S,J) =$ ________________________________

   $MPI_J\ (S,J) =$ ________________________________

Answer: The marginal productivity of investment is the marginal productivity divided by the price. The price for senior engineers is \$p and for junior engineers is \$w.

$$MPI_S = \frac{MP_S}{p} = \frac{1}{4p}\left(\frac{J}{S}\right)^{\frac{3}{4}}$$

$$MPI_J = \frac{MP_J}{w} = \frac{3}{4w}\left(\frac{S}{J}\right)^{\frac{1}{4}}$$

c) How many junior engineers should LeewayHertz hire for each senior engineer they hire in order to minimize the cost of producing q apps. Your answer should be in terms of p and w.

For every senior engineer, LeewayHertz should hire ________________ junior engineers.

Answer: Equating the two MPIs give us the ratio: $J = \frac{3ps}{w}$, so for every senior engineer (that is, when S=1), the firm should hire $\frac{3p}{w}$ junior engineers.

In 2017, the cost for Junior engineers was $w = \$150{,}000$ and for senior engineers was $p = \$800{,}000$.

d) Assuming LeewayHertz was engaged in cost-minimizing production and hired 3 senior engineers in 2017, how many junior engineers J did they hire, what were their total engineering costs, C, and how many apps, q, did they produce?

$J =$ ________________________________

C (in dollars) $=$ ________________________________

$q =$ ________________________________

Answer: Plugging w=$150k and p=$800k into the ratio from part c give us J = 48.

With these salaries, total cost is: (48 junior engineers*$800k)+(16 senior engineers*$800k)=$9.6 million. Finally, plugging in J=48 and S=3 into the production function gives: q = 24.

e) Say LeewayHertz wanted to produce twice as many apps as were produced in Part (d). What would be the cost minimizing number of senior engineers (S) and junior engineers (J) to have?

$S =$ ________________________________

$J =$ ________________________________

Answer: They are already at the cost minimizing ratio for those input costs, so to double production, they just need to double each input.

In 2018, due to competition from other app developers, the cost for junior engineers will double to $300,000 and the cost of senior engineers will go up 50% to $1,200,000.

f) Assume LeewayHertz ultimately chose to build the number of apps from Part (d) in 2017. If they want to build the same number of apps in 2018 at minimum cost, how must their staffing adjust relative to part (d)? (Note: No work needs to be shown.)

Hire fewer junior engineers and hire fewer senior engineers	☐
Hire fewer junior engineers and hire more senior engineers	**X**
Hire more junior engineers and hire fewer senior engineers	☐
Hire more junior engineers and hire more senior engineers	☐

Answer: Hire fewer junior and hire more senior engineers because after the price change, the MPI for senior engineers will be greater than the MPI for junior engineers (since the price of senior engineers only went up 50% while the price of junior engineers doubled).

10. *2013 Final, Q5*

Aluminum Smelting. Aluminum is produced by adding large amounts of electricity to aluminum oxide. Combining e thousand kilowatt-hours of electricity and o metric tons of oxide creates

$$q = f(e, o) = 20\times(4\times e\times o)^{1/4}$$

metric tons of aluminum metal.

a) The price of oxide is currently \$240 per metric ton, and the price of electricity is \$60 per thousand kilowatt-hours. In a cost-minimizing production plan, how many thousand kilowatt-hours of electricity should be used for every metric ton of oxide?

Thousand kilowatt-hours ______________________________

Answer: Four thousand kilowatt-hours per every metric ton of oxide.

$$\frac{MP_e}{MP_o} = -\frac{p_e}{p_o} = \frac{\frac{df(e,o)}{de}}{\frac{df(e,o)}{do}} = \frac{20\times(4\times e\times o)^{-\frac{3}{4}}4e}{20\times(4\times e\times o)^{-\frac{3}{4}}4o} \to \frac{o}{e} = \frac{60}{240} \to$$

$$e = 4o$$

b) Suppose the manager of a smelting plant wants to produce 1,000 tons of aluminum. How many thousand kilowatt-hours of electricity and how many tons of oxide should be used?

Thousand kilowatt-hours ______________________________

Tons of oxide: ______________________________

Answer:

$$1{,}000 = 20(4\times 4o\times o)^{1/4} = 40\sqrt{o}$$

$$o = 625; e = 4\times 625 = 2{,}500$$

A smelting plant producing q metric tons of aluminum has a daily variable cost of

$$VC(q) = 0.3\times q^2$$

Throughout, let p denote the price per metric ton of aluminum.

c) What is the plant's short-run supply curve, $Q^S_{plant}(p)$?

Supply Curve: ______________________________

Assuming that there are 60 such plants (all with identical cost structure) in the world, what is the global supply of aluminum, $Q^S_{global}(p)$?

Global supply of aluminum: ______________________________

Answer:

$$VC(q) = 0.3q^2$$

$$MC(q) = 0.6q = P \rightarrow q(P) = 1\frac{2}{3}P \rightarrow$$

$$Q^S_{global}(p) = 60\left(1\frac{2}{3}P\right) = 100P$$

Daily demand for aluminum in the global market is given by:

$$Q^D_{global}(p) = 1{,}000{,}000 - 500p$$

d) What is the equilibrium price and daily quantity of aluminum sold?

Equilibrium Price ______________________________

Quantity ______________________________

Answer:

$$Q^D_{global}(p) = Q^S_{global}(p)$$

$$1{,}000{,}000 - 500P = 100P$$

$$p = \frac{1{,}000{,}000}{600} \approx 1{,}666.6\overline{7}, Q = \frac{1{,}000{,}000}{6} \approx 166{,}666.6\overline{7}$$

e) Find a Chinese plant's new supply curve $Q^S_{Chinese}(p)$ once the subsidy takes effect. The supply of the remaining 30 firms is unaffected. Find the world supply curve under the subsidy program.

Single Chinese plant's new supply curve:

$q^S_C(p)$ = ____

New global supply curve:

$Q^S_{Global}(p)$ = ____

What would the equilibrium price and daily quantity of aluminum sold be now?

Equilibrium Price ____

Quantity ____

Answer:

Chinese firms:

$$VC(q) = 0.3q_C^2 - 200q_C$$

$$MC(q) = 0.6q_C - 200 = P$$

$$q_C = \frac{(P + 200)}{0.6}\ , Q^C = 30 * \frac{(P + 200)}{0.6} = 50P + 10{,}000$$

Non-Chinese firms:

$$Q^{-C} = 30 * \left(1\frac{2}{3}P\right) = 50P$$

World Supply:

$$Q^S_{global}(p) = Q^C + Q^{-C} = 100P + 10{,}000$$

$$1{,}000{,}000 - 500P = 100P + 10{,}000$$

$$P = \$1{,}650, Q = 175{,}000$$

f) Aluminum buyers and Chinese producers share the benefits of the subsidy: buyers pay a lower price and Chinese producers receive a higher price (inclusive of the subsidy). Examine the fraction of the $200 per ton subsidy of Chinese producers that aluminum buyers see in lower prices.

Fraction buyers see: ________________________________

What does this suggest about who benefits most from the subsidy (no further calculations required)?

Consumers ☐

Chinese producers **X**

Other producers ☐

Answer*:*

Change in price paid by consumers = $16.67
Share of the change out of total subsidy of $200 = 8.3%

This suggests consumers only benefit by a little, most of the benefits goes to Chinese producers.

6) Producing in Perfectly Competitive Industries

11. *2015 Problem Set 2, Q1*

    Golf carts. Golf carts were initially an innovative product produced by few manufacturers (Sears Roebuck among them), but as they gained popularity, many firms making other small vehicles (everything from motorcycles to riding mowers) realized they could join in the market. Each individual firm is itself insignificant to global supply, and their golf cart production can be considered nearly identical. Golf cart production involves a few principal costs: rent for factory space, equipment, materials, and worker time. This is the annual total cost function for an individual firm in dollars:

$$TC(q) \;=\; 14400 + 144q^2$$

where q is the quantity of golf carts in millions. You are going to analyze the market for golf carts and the behavior of individual firms.

a) First, solve for a firm's marginal cost and average cost functions.

   $MC(q)$: ______________________________

   $AC(q)$: ______________________________

Answer:

$$MC(q) = \frac{\partial TC}{\partial q} - 288q$$
$$ATC(q) = \frac{TC}{q} = \frac{14400}{q} + 144q$$

If units of q are not ignored (so q is divided by 1,000,000 in the functions), the cost functions should be deflated by a factor of 10^{-6}.

b) In the short run, the current prevailing wholesale market price is \$3456 per golf cart. How many golf carts will each firm produce, and how much will they earn in profits?

   Output: ______________________________

   Profits: ______________________________

Answer: A firm, independent of market competition, chooses its quantity q^* such that $MR(q^*) = MC(q^*)$. A competitive firm is a price taker, so $MR(q^*) = p$. Therefore, a competitive firm chooses its quantity such that:

$$p = MC(q^*)$$

$$3456 = 288q^*$$
$$q^* = 12.$$

If units of q are not ignored,

$$p = MC(q^*)$$
$$3456 = 288q^* \times 10^{-6}$$
$$q^* = 12 \times 10^6.$$

Where $q^* = 12 \times 10^6$ are in millions (so the actual output is 12×10^{12}).

Each firm's profit is (ignoring units)

$$\begin{aligned}\pi &= pq^* - TC(q^*) \\ &= 3456(12) - (14400 + 144(12)^2) \\ &= 6336,\end{aligned}$$

or with units,

$$\begin{aligned}\pi &= pq^* - TC(q^*) \\ &= 3456(12 \times 10^{12}) - (14400 + 144(12 \times 10^6)^2).\end{aligned}$$

c) There are no barriers to entry for firms in this golf cart market, and so we would expect entry if firms are profitable. After entry, how much will each firm produce? What will be the long-run wholesale price for a golf cart? What will each firm earn in profits?

Output: ______________________________

Long-run price: ______________________________

Profits: ______________________________

Answer: When firms in a perfectly competitive market are making positive profits, entry will happen until the firms in the market make zero profit. Therefore, in the long-run, competitive firms' profits will be zero. The long-run price which makes firms earn zero profit is computed from the following formula:

$$p^{LR} = \min_q ATC(q)$$

Once we compute $\min_q ATC(q)$, we know what the long-run price should be. To find the minimizing quantity, take the derivative of $ATC(q)$ with respect to q and set it equal to 0:

$$\frac{\partial ATC}{\partial q} = -\frac{14400}{q^2} + 144 = 0$$
$$\Rightarrow q = 10$$

With ATC minimized at $q = 10$, we have $p^{LR} = ATC(100) = 2880$.

As stated above, a competitive firm chooses its quantity such that $p = MC(q)$ holds:

$$p^{LR} = 288q^{LR}$$
$$q^{LR} = \frac{2880}{288} = 10.$$

Taking units into account, $p^{LR} = 2.88 \times 10^{-3}$, $q^{LR} = 10$ in millions.

One of these small manufacturers, Zip Golf Ltd., discovers an innovation that allows them to make golf carts cheaper. Zip Golf Ltd.'s new cost function is:

$$TC(q) = 14400 + 100q^2$$

d) The wholesale price remains at the level found in (c). How much will Zip Golf Ltd. produce, and what is its profit?

Output: ______________________________

Profits: ______________________________

Answer: The firm will choose quantity such that $p = MC(q)$ holds:

$$2880 = 200q$$
$$q^* = 14.4.$$

Accounting for units, the 10^{-6} factors in $p, MC(q)$ cancel out, so $q^* = 14.4$.
Its profits, ignoring units of q, is

$$\begin{aligned}\pi &= pq^* - TC(q^*) \\ &= 2880(14.4) - (14400 + 100(14.4)^2) \\ &= 6336,\end{aligned}$$

or not ignoring units:

$$\begin{aligned}\pi &= pq^* - TC(q^*) \\ &= 2880\times10^{-6}(14.4\times10^6) - (14400 + 100(14.4)^2) \\ &= 6336.\end{aligned}$$

The following year, all other firms adopt this technology, and the resulting long-run equilibrium price for golf carts falls to $2400. Unfortunately, Slow Carts Co. has been unable to convince their workers to adopt the innovation, and so still has the old cost function. To make matters worse, they were locked into a lease on their factory space and equipment, and so have to keep paying their fixed costs until the end of this year no matter what.

e) How many golf carts will Slow Carts decide to produce this year, if any? How many would you expect them to produce next year (after their long-term lease expires) if they still don't adopt the new cost-cutting manufacturing trick?

Output this year: ______________________________

Output next year will be:

Greater than this year ☐

Same as this year ☐

Less than this year, but greater than zero ☐

Zero (the firm will exit). ☐

Answer: The firm will produce

$$2400 = 288q$$
$$q^* = \frac{2400}{288}.$$

The firm earns profits

$$\pi = 2400(\frac{2400}{288}) - \left(14400 + 144\left(\frac{2400}{288}\right)^2\right)$$
$$= -4400.$$

If units of q are not ignored,

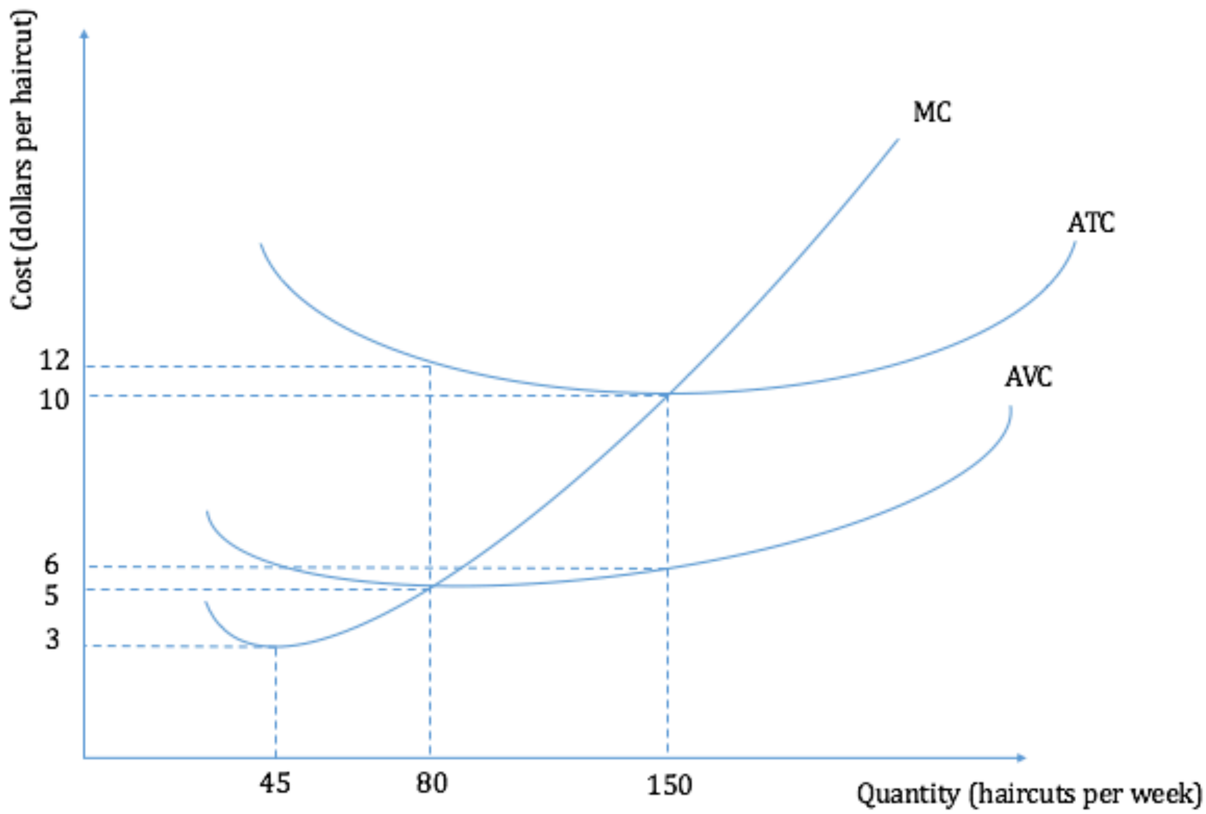
$$\pi = 2400\left(\frac{2400}{288}\times 10^{12}\right) - \left(14400 + 144\left(\frac{2400}{288}\times 10^{6}\right)^2\right) > 0.$$

In the first expression, profits are negative but this is better than producing nothing, in which case the firm will lose the entire (sunk) fixed cost, 14400. The firm will exit the market next year, since by doing that the firm makes zero profits, which is better than maintaining a loss of 4400.

In the second expression, the firm will produce the same output next year if it doesn't adopt the new technology, and it'll produce more if it adopts the new technology.

12. *2016 Final, Short question a)*

Hair Cuts. Amazing Clips provides budget hair cuts in Center City. Their costs (in dollars per haircut) can be summarized by the graphs below. Assume that the market for budget hair cuts in Center City is perfectly competitive (therefore anyone can enter).



a) What is the lowest price above which Amazing Clips cleaners makes a profit in the long run?

Price ____________________________

Answer: They make a profit as long as the price exceeds $10 per haircut, which is the bottom of the average total cost curve.

b) Between which two prices per haircut would Amazing Clips continue to produce in the short run, but shut down in the long run (before the next fixed costs are due)?

Prices: ______________and _________________

Answer: They would produce in the short-run, but shut down in the long run, for prices between $5 and $10 per haircut.

c) Below which price per haircut would Amazing Clips shut down even in the short run?

Price ____________________________

Answer: They would shut down in the short run if the price fell below $5 per haircut.

d) Assume that the price is currently exactly high enough to stay open in the long run, but not higher. What quantity does Amazing Clips produce?

Quantity ______________________________

Answer: Note that this price is \$10 per haircut. So we are interested in the total number of haircuts per week they would supply at \$10 per cut, which is 150 per week.

7) Producing with Market Power: Monopoly

13. *2014 Final, Q5*

    Copay Coupons. Pharmaceutical companies are clever about maximizing the profits from their drugs, even after patents have expired. Consider Lipitor, a blockbuster cholesterol drug whose patent expired at the end of 2011. Pfizer, the company behind Lipitor then faced a market where they had generic competitors (same product, sold under a different name).

    There are two types of consumers in the market for this drug: well-insured, and poorly-insured. The well-insured are not very price-sensitive, and prefer the branded Lipitor over the generic. The poorly-insured face a larger price difference due to their insurance contracts, and so are more likely to switch to a generic or other rival depending on the price. There are currently 4 poorly-insured individuals in the market for every 1 well-insured individual.

    The monthly demand curve per million well-insured consumers is

$$P_W(Q) = 210 - 0.5Q$$

    While for a million poorly-insured consumer it is

$$P_P(Q) = 110 - 2Q$$

    Price is in dollars for a monthly supply, and quantity is in thousands of monthly supplies of the drug. Pfizer's marginal cost is a constant \$20 per monthly supply for all types.

    a) How would Pfizer like to price Lipitor to each group if they could select a price for each type of consumer?

       Price for well-insured*:* \$__________________________

       Price for poorly-insured: \$__________________________

Answer: If Pfizer can set different prices for each type, they will choose prices where marginal revenues for each type are equal to marginal cost:

$$R_W = p_W \times Q_W = 210Q_W - 0.5Q_W^2 \rightarrow MR_W = 210 - Q_W$$

$$R_P = p_P \times Q_P = 110Q_P - 2Q_P^2 \rightarrow MR_P = 110 - 4Q_P$$

So that we have

$$MR_W = 20 \rightarrow 210 - Q_W = 20 \rightarrow q_W^* = 190$$

$$MR_P = 20 \rightarrow 110 - 4Q_P = 20 \rightarrow q_P^* = 22.5$$

so that

$$P_W^*(q) = 210 - 0.5q_W^* = 210 - 95 = 115$$

$$P_P^*(q) = 110 - 2q_P^* = 110 - 45 = 65$$

b) For contractual reasons, Pfizer cannot actually charge different prices to these different types of consumer (an insurer negotiates a price for all of its covered consumers). What is aggregate demand for a set of 5 million consumers? What is the optimal price if Pfizer must set a single price to the entire market?

$$Q_{agg}(p) = \begin{cases} \_\_\_\_\_\_\_\_\_\_ & \text{if } p \geq \_\_\_\_\_\_\_\_\_\_, \\ \_\_\_\_\_\_\_\_\_\_ & \text{if } p < \_\_\_\_\_\_\_\_\_\_, \end{cases}$$

Price for all: $ ____________________

Answer: First, we solve for the regular demand curves from the inverse demand curves provided above

$$q_W = 420 - 2p$$

$$q_P = 55 - \frac{1}{2}p$$

where I've used little q_W and q_P to denote *individual* demand curves. We are told that there are 4 poorly-insured individuals for every 1 well insured individual in the market. So we can assume there are 4 of the former and just 1 of the latter. Recall that we aggregate demand curves by horizontally summing (adding quantities not prices). First, since every individual within each market segment is identical, we can just multiply by the number of people to get the aggregate market segment demand. Let $Q_P = 4 \times q_P$ and $Q_W = 1 \times q_W$ so we have

$$Q_W = 420 - 2P$$

$$Q_P = 220 - 2P$$

finally, we need to sum these aggregate market segments demands to get the aggregate market demand curve

$$Q(P) = \begin{cases} Q_W = 420 - 2P & if\ 110 < P \leq 210 \\ Q_W + Q_P = 640 - 4P & if\ 0 < P \leq 110 \end{cases}$$

and the inverse demand curve is

$$P = \begin{cases} 210 - \frac{1}{2}Q_W & if\ 110 < Q_W \leq 200 \\ 160 - \frac{1}{4}Q & if\ 200 < Q \leq 640 \end{cases}$$

where $Q = Q_W + Q_P$. To get the equilibrium price, assume that the demand and supply curve cross on the portion of the inverse demand curve where $420 < Q \leq 640$. In this case, the revenue and marginal revenue curves are

$$R = 160 \times Q - \frac{1}{4}Q^2 \rightarrow MR = 160 - \frac{1}{2}Q$$

setting marginal revenue equal to marginal cost gives

$$160 - \frac{1}{2}Q = 20$$

$$\frac{1}{2}Q = 140 \rightarrow Q^* = 280$$

since $200 < Q^* \leq 640$, our assumption was correct and we can solve for the equilibrium price

$$P^* = 160 - \frac{1}{4} \times 280 = 90$$

c) To get around the contractual obligation to charge the same price to the insurer for all consumers, Pfizer came up with the idea of "copay coupons".

Since consumers only pay a fraction of the actual prices (the "copay"), with insurers paying the rest, a copay coupon makes Lipitor cheaper to the decision-maker (the patient) without changing the price charged to the insurer. Pfizer devised a system where consumers of the poorly-insured type could apply for a rebate on the copay, which effectively made many "poorly-insured" types act like "well-insured" types. They are so effective at targeting poorly-insured customers with coupons that they manage to reach almost all of them.

As a result, the effective market becomes only 1 poorly insured per 6 well-insured, instead of 4 poorly insured per well-insured, as before. This takes into account the cost of the coupons, so marginal cost is still $20. What is the optimal price to set now? Who will buy Lipitor?

Price: $__________________________

Check the box for each group that will purchase Lipitor:

Well-insured **X**

Poorly-insured with copay coupons **X**

Poorly-insured without copay coupons ☐

Answer: Since the coupons effectively change the market composition to be 1 poorly insured per 6 well-insured individual, let $Q_P = 1 \times q_P$ and $Q_W = 6 \times q_W$ so we have

$$Q_W = 2{,}520 - 12P$$

$$Q_P = 55 - \frac{1}{2}P$$

again, sum these aggregate market segments demands to get the aggregate market demand curve

$$Q(P) = \begin{cases} Q_W = 2{,}520 - 12P & if\ 110 < P \leq 210 \\ Q_W + Q_P = 2{,}575 - \frac{25}{2}P & if\ 0 < P \leq 110 \end{cases}$$

and the inverse demand curve is

$$P = \begin{cases} 210 - \frac{1}{12}Q_W & if\ 0 < Q_W \leq 1200 \\ 206 - \frac{2}{25}Q & if\ 1200 < Q \end{cases}$$

where $Q = Q_W + Q_P$. To get the equilibrium price, assume that the demand and supply curve cross on the portion of the inverse demand curve where $0 < Q_W \leq 1{,}200$. In this case, the revenue and marginal revenue curves are

$$R = 210 \times Q_W - \frac{1}{12}{Q_W}^2 \rightarrow MR = 210 - \frac{1}{6}Q_W$$

setting marginal revenue equal to marginal cost gives

$$210 - \frac{1}{6}Q_W = 20$$

$$\frac{1}{6}Q_W = 190 \rightarrow Q_W^* = 1{,}140$$

since $0 < Q_W^* \leq 1{,}200$ our assumption was correct and we can solve for the equilibrium price

$$P^* = 210 - \frac{1}{12} \times 1{,}140 = 115$$

This is the price that well-insured individuals will purchase Lipitor at from part (a) but it is also the price that poorly-insured individuals with copay coupons will pay since they are effectively behave like like well-insured individuals and are included in the aggregation of the market segment demand curve $P = 210 - \frac{1}{12}Q_W$.

14. *2015 Problem Set 2, Q2*

Microsoft Office. Overwhelmingly, the office software suite used by Wharton students is Microsoft Office for Students. The company's marketing group currently estimates aggregate annual demand for Microsoft Office suite by Wharton students is

$$P(q) \ = \ 100 - \frac{1}{2} \cdot q$$

where the quantity is number of software packages and the price is in dollars. Its cost function is

$$TC(q) = 10q$$

a) Solve for Microsoft's revenue function, marginal revenue function, optimal price to charge in this market, and monthly profits.

$R(q) =$ ______________________________

$MR(q) =$ ______________________________

$P =$ ______________________________

Profits: ______________________________

Answer: A monopoly firm chooses its quantity such that $MR(q) = MC(q)$. Revenue in terms of q is:

$$R(q) = P(q) \times q = (100 - 0.5q)q$$

Taking derivative with respect to q

$$MR(q) = 100 - q$$

so marginal cost is simply $MC(q) = 100 - q$. We have:

$$100 - q = 10$$
$$q^* = 90.$$

Plugging q^* back into the demand function, we get:

$$p^* = 100 - 0.5 \times 90 = 55.$$

The annual profit is $\pi = (p^* - 10) * q^* = 4050$.

The monthly profit is then $4050/12.

b) Leslie, a Wharton MBA doing an internship at Microsoft, warns the company that students are increasingly converting to tablets that come with or do not allow Microsoft Office downloads. She devises an experiment where a random sample of incoming Wharton MBA students are given discount coupons for the Office software suite, and compared the uptake between that group and a control group. The results are below.

Group	Price	Uptake (purchase rate)
Control	As found in Part (a)	27%
Discount	$5 off price found in Part (a)	30%

Based on this, and what Leslie remembers about the Inverse Elasticity Pricing Rule from MGEC 611, what can you say about the current price being offered to Wharton students?

It is the correct price to be charging Wharton students. **X**

It is not the correct price, and the discounted price is better. ☐

It is not the correct price, and the discounted price is worse. ☐

Reasoning:

____

____

____

Answer: If a monopoly firm's price is profit-maximizing, then the following formula should hold:

$$\frac{p^* - MC}{p^*} = \frac{1}{\varepsilon_d}$$

The LHS is computed by the following:

$$\frac{p^* - MC}{p^*} = \frac{55 - 10}{55} = \frac{9}{11}$$

ε_d is computed by:

$$\varepsilon_d = \frac{0.30 - 0.27}{0.27} \Big/ \frac{50 - 55}{55} = -\frac{11}{9}$$

So the RHS of the inverse elasticity equation is: $-\frac{1}{\varepsilon_d} = \frac{9}{11}$.

Since the inverse elasticity pricing rule holds with an equality, we can conclude that the price is optimal.

8) Pricing in Vertical Markets

15. *2015 Exam, Q4*

New Release. Paramount Studios has struck a deal with Netflix to allow the streaming service to show its new movie line-up through a "New Release" channel. "New Release" would be available as a premium subscription service, where Paramount would license the content to Netflix, and Netflix would resell it to customers at a price of p dollars per subscriber per month. Beyond payment to Paramount for the content, Netflix does not incur any additional distribution costs. Additionally, Paramount's marginal cost for providing the movies to Netflix is zero, as in this case the movies are already made and streaming-ready.

The companies agree that the monthly market demand for the "New Release" channel is:

$$q^D(p) = 15 - \frac{1}{3}p$$

where q is in millions of subscribers.

Under one possible payment structure, Paramount would license its content to Netflix for a monthly charge, called a "carriage charge," of c dollars per subscriber per month.

a) In terms of the carriage charge c, what is Netflix's profit maximizing number of subscribers and price for "New Release"?

Subscribers: ______________________________________

Price: ____

Answer: Netflix's profit function is

$$\pi_N = (p-c)Q(p) = (p-c)\left(15 - \frac{1}{3}p\right).$$

Take the derivative with respect to p and set to 0, to get the monopolist demand

$$P^*(c) = \frac{45-c}{2}$$
$$Q^*(c) = 15 - \frac{1}{3}P^*(c) = \frac{45-c}{6}.$$

Plug $P^*(c), Q^*(c)$ back into the profit function to get $\pi_N(c) = \frac{(45-c)^2}{12}$.

b) What is Paramount's optimal per-subscriber carriage charge? How much profit will Paramount make from "New Release" per month?

Carriage Charge: ______________________________________

Profit: ____

Answer: Paramount's profit function is: $\pi_P = c \times Q(c) = \frac{c(45-6)}{6}$.

From the FOC, $c^* = 22.5$. Then, $q^* = q^*(22.5) = 3.75$, $\pi_P^* = 22.5 \times 3.75 = 84.375$.

c) Given the carriage charge set by Paramount in *(b)*, what price will Netflix charge customers for "New Release"? How much profit does Netflix earn per month?

Price: ______________________________________

Profit: ______________________________________

Answer: Setting $c = 22.5$, $P^*(22.5) = \frac{45+22.5}{2} = 33.75$. Profits will then be

$$\pi_N^* = \pi_N(22.5) = \frac{(45-22.5)^2}{12} = 42.1875.$$

Instead, Paramount decides that it would only be willing to sell content to "New Release" if Netflix pays a flat monthly fee, F (independent of the number of subscribers) *and* a per-subscriber carriage charge, c.

d) How should Paramount set c to maximize joint profit, and what will be the total profit between both companies?

Optimal carriage charge c: ______________________________________

Profit: ____

Answer: The joint profit is maximized by setting the carriage charge to Paramount's MC, which is 0. Plug $c = 0$ into Netflix's profit function:

$$\pi_N(0) = \frac{45^2}{12} = 168.75.$$

So Netflix earns 168.75 − F , and Paramount earns F . The total profit is 168.75.

e) What is the range of possible values of F, the fee paid by Netflix to Paramount, where both Netflix and Paramount would be willing to sign on to this new pricing scheme, and launch "New Release"?

Range: ______________> *F* > ________________

Answer: Netflix agrees if 168.75 − F ≥ 42.1875, so F ≤ 126.5625.

Paramount agrees if F ≥ 84.375, its profit in part (b).

16. *2016 Problem Set 2, Q4*
The Chaddsford Winery can currently produce up to 35,000 cases annually; due to its location in suburban Philadelphia, it cannot easily increase its capacity beyond 35,000 cases. It sells its wine to retailers in only two markets: Pennsylvania and New Jersey, and the marginal cost of producing a case of wine is \$200; the winery does not incur any additional marginal cost to distribute the wine to retailers. The demand in Pennsylvania is given by $Q_{PA} = 400(400 - p_{PA})$ and the demand in New Jersey is given by $Q_{NJ} = 200(300 - p_{NJ})$.

a) Find the number of cases that Chaddsford should sell in each state and the price it will charge. Calculate the winery's profits. Assume that cases sold in one market cannot be resold in the other market.

Cases in PA: ____

Cases in NJ: ____

Price in PA: \$____

Price in NJ: \$____

Profits: \$____

Answer: First consider whether Chaddsford is able to meet demand in each of the states to maximize profit. If so, it would set marginal revenue per case in each state equal to its marginal cost of \$200.

For PA, this would mean setting $MR_{PA} = MC$:

$$400 - \frac{2}{400}Q_{PA} = 200 = MC$$
$$Q_{PA} = 40{,}000$$

Similarly, for NJ

$$
\begin{aligned}
&300 - \frac{2}{200} Q_{NJ} = 200 = MC \\
&Q_{NJ} = 10{,}000
\end{aligned}
$$

The total number of cases demanded between the states at each state's profit maximizing prices is greater than the 35,000 capacity, so the cases should be allocated across states such that $MR_{NJ} = MR_{PA}$.

Set MR's equal and solve for allocations and prices:

$$
\begin{aligned}
&(1)\ \ 400 - \frac{2}{400} Q_{PA} = 300 - \frac{2}{200} Q_{NJ} \\
&(2)\ \ Q_{PA} + Q_{NJ} = 35{,}000
\end{aligned}
$$

Plugging in (2) into (1),

$$
\begin{aligned}
&400 - \frac{2}{400} Q_{PA} = \ 300 - \frac{2}{200}(35{,}000 - \ \ Q_{PA}) \\
&100 - \frac{1}{400} Q_{PA} = \frac{-35{,}000}{100} + \ \frac{1}{100}\ Q_{PA} \\
&\frac{3}{200} Q_{PA} = 450 \\
&Q_{PA} = 30{,}000\ ;\ Q_{NJ} = 5{,}000
\end{aligned}
$$

Plugging the allocations into the demand equations, $P_{PA} = \$325$ per case; $P_{NJ} = \$275$ per case. Profits therefore are

$$
\begin{aligned}
&= (30{,}000 \times\ 325) + (5{,}000 \times\ 275) - (35{,}000 \times 200) \\
&= 9{,}750{,}000 + 1{,}375{,}000 - (35{,}000 \times 200) = \$4{,}125{,}000.
\end{aligned}
$$

Regulations governing the sale of alcoholic beverages in both states mean that Chaddsford Winery -- instead of selling directly to retailers – actually has to sell its wine first to a wholesale wine distributor (in the case of Pennsylvania, that is the Pennsylvania Liquor Control Board; in the case of New Jersey, a private company) and the distributor then sells the wine to retailers.

b) If Chaddsford charges the distributors a price per case to be resold by them to retailers in each of the two states, what should that price be? How many cases will Chaddsford sell to the distributor in each state? What will be Chaddsford's profit? (Assume the distributors' marginal cost is zero and that again, cases sold to a distributor in one market cannot be resold by that distributor in the other market.)

Cases in PA: ____

Cases in NJ: ____

Price in PA: $____

Price in NJ: $____

Profits: $____

Answer: Let $P_{PA}^{w} =$ Chaddsford' s wholesale price in PA and P_{NJ}^{w} = Chaddsford's wholesale price in NJ that it charges per case to the distributor.

The distributor sets its marginal revenue equal to its marginal cost, which is just the wholesale price per case it pays to the winery. So:

$$400 - \tfrac{2}{400} Q_{PA} = P_{PA}^{w} \text{ and } 300 - \tfrac{2}{200} Q_{NJ} = P_{NJ}^{w}$$

This is Chaddsfords' new demand that relates number of cases sold to the wholesale price it charges. To set the optimal wholesale price, Chaddsford now sets its marginal revenue equal to its marginal cost of $200 per case. Chaddsford's total revenue in PA and NJ is:

$$\text{Revenue}_{PA} = \left(400 - \frac{2}{400} Q_{PA}\right) Q_{PA}$$

$$\text{Revenue}_{NJ} = \left(300 - \frac{2}{200} Q_{NJ}\right) Q_{NJ}$$

Optimal pricing implies:

$$MR_{PA}^{w} = 400 - \tfrac{4}{400} Q_{PA} = 200 = MC, \text{ so } Q_{PA} = 20{,}000$$
$$MR_{NJ}^{w} = 300 - \tfrac{4}{200} Q_{NJ} = 200 = MC, \text{ so } Q_{NJ} = 5{,}000$$

Note that this allocation falls within Chaddsford's capacity (20,000 + 5,000 < 35,000)

To find prices, plug these allocations into the demand equations for Chaddsford and for the distributor:

$$P_{PA}^{w} = 400 - \frac{2}{400}(20{,}000) = \$300$$
$$P_{NJ}^{w} = 300 - \frac{2}{200}(5{,}000) = \$250$$
$$P_{PA}^{R} = 400 - \frac{1}{400}(20{,}000) = \$350$$

$$P_{NJ}^{R} = 300 - \frac{1}{200}(5{,}000) = \$275$$

Then profits are:

$$\text{Distributor Profits} = 50 \times 20{,}000 + 25 \times 5{,}000 = \$1{,}125{,}000$$
$$\text{Chaddsford Profits} = 100 \times 20{,}000 + 50 \times 5{,}000 = \$2{,}250{,}000$$

c) In the US, the three-tier distribution system that prohibits any form of vertical integration between producers, distributors, or retailers in the sale of alcoholic beverages continues to exist in many states. Based on the above, who benefits and who loses due to the producers' inability to circumvent distributors?

Producers:	☐ benefit	☐ lose	☐ are indifferent
Distributors:	☐ benefit	☐ lose	☐ are indifferent
Consumers:	☐ benefit	☐ lose	☐ are indifferent

Answer: Producers and Consumers lose (higher prices, lower quantity).

Whether distributors benefit or are harmed depends on what the law exactly does. If the law is preventing the distributors from merging with producers, it harms them. If it requires them to exist instead of a single vertically integrated firm, it benefits them. Either distributors benefiting or distributors losing is acceptable and receives full credit.

d) Instead of selling individual cases to each distributor, Chaddsford can make them a take-it-or-leave-it offer of X cases for a fixed fee F for all X cases. What will be the number of cases Chaddsford will offer to each distributor and how much will they charge each of them for their package? Calculate Chaddsford's profit. As before, resale is not possible and the distributor's marginal cost continues to be zero.

Cases in PA: ____________________

Cases in NJ: ____________________

Fee charged in PA (for all units): $____________________

Fee charged in NJ (for all units): $____________________

Profits: $____________________

Answer: We will accept two answers for Part (d), since the amount that Chaddsford can demand in each take-it-or-leave-it offer depends on the outside option available to each distributor, and there are two reasonable assumptions to make.

If the assumption is that absent the purchase arrangement, the distributors will not exist (as in Part (a)), then Chaddsford can demand all of the profit generated in each market. If the alternative for the distributors is to continue to resell under the arrangement in Part (b), then the take-it-or-leave-it offer will have to leave the distributors with at least as much profit as they received in Part (b).

Under either assumption, Chaddsford will choose the number of cases to offer to the distributors in each state to maximize total possible profits, which — as calculated in Part (a) — means $X = 30{,}000$ cases for the PA distributor and $X = 5{,}000$ cases for the NJ distributor.

If the outside option of each distributor is that they do not exist, Chaddsford can demand each distributor pay a fee that extracts all of the profit generated in their market, which implies a fee of $F = \$9{,}750{,}000$ from the PA distributor (we will also accept an answer of \$325 per case) and a fee of $F = \$1{,}375{,}000$ from the NJ distributor (we will also accept \$275 per case).

With these fees, total profits for Chaddsford are:

$$= 9{,}750{,}000 + 1{,}375{,}000 - (35{,}000 \times 200) = \$4{,}125{,}000$$

which is the same profit as in Part (a) when there is no distributor in the supply chain.

If the outside option of each distributor is to remain in the supply chain as in Part (b), then the offer must provide them with at least as much profit as they would get under Part (b). In Part (b), the PA distributor buys 20,000 cases at a price of \$300 each and sells them for a price of \$350 each, and so makes $20{,}000 \times (350 - 300) = \ 1{,}000{,}000$ in profit.

The NJ distributor buys 5,000 cases at a price of \$250 and sells at a price of \$275 and so makes $5{,}000 \times (275 - 250) = \ 125{,}000$ in profit. Subtracting these numbers from the fees indicated above, the most Chaddsford can demand for a fee is $F = \$8{,}750{,}000$ from the PA distributor for the $X = 30{,}000$ cases (we will also accept an answer of \$291.67 per case) and a fee of $F = \$1{,}250{,}000$ from the NJ distributor for the $X = 5{,}000$ cases (we will also accept \$250 per case).

With these fees, total profits for Chaddsford are:

$$= 8{,}750{,}000 + 1{,}250{,}000 - (35{,}000 \times 200) = \$3{,}000{,}000$$

which is larger than they earned in Part (b) because they have solved the double marginalization problem and are reaping all the additional profits resulting from solving it.

Since in both cases above, the distributor is indifferent between paying the fee in the take-it-or-leave-it offer or not paying it, we will also accept an answer in which Chaddsford offers \$1 less in fees to make the distributors strictly better off paying the fee in the take-it-or-leave-it offer than walking away.

9) Simultaneous-move Games

17. *2016 Problem Set 2, Q4*
    Wharton Socializing. Your learning team, and another, are both headed out for dinner in Philadelphia tonight. Both learning teams are separately considering having dinner at Casta Diva or at Audrey Claire.

    Going to either restaurant will net either team a value of 100. However, if they end up at the same restaurant, both teams would get a bonus of 20 value from having more Wharton friends around.

    a) Fill out the payoff table below.

Other Team

Your Team	Casta Diva	Audrey Claire
Casta Diva		
Audrey Claire		

Answer:

	Casta Diva	Audrey Claire
Casta Diva	120, 120	100, 100
Audrey Claire	100, 100	120, 120

b) While it would be nice to coordinate, cell phone reception in Huntsman can be spotty, and so you have no way of communicating with the other team. Are there any strictly dominant strategies?

Yes ☐ No **X**

c) What are the one-shot Nash equilibria of this game, if any?

One-Shot Nash Equilibria: ______________________________

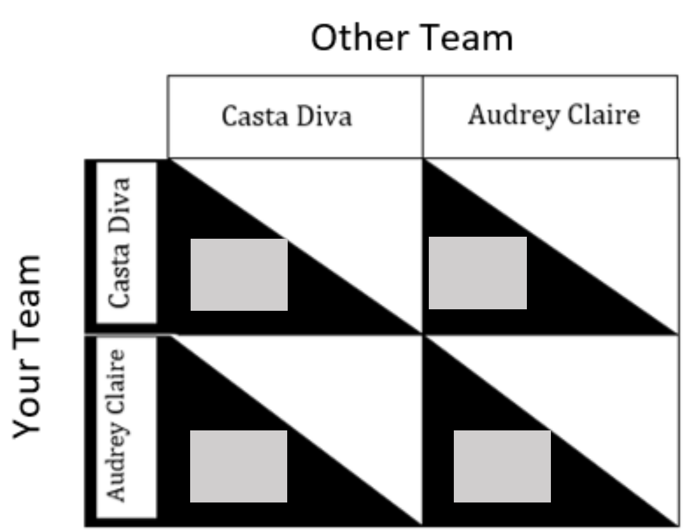
Answer: There are two Nash equilibria: (Casta Diva, Casta Diva) and (Audrey Claire, Audrey Claire).

d) Would your answer to (c) change if Casta Diva actually had a higher base value than Audrey Claire (because cash-only is a pain at Audrey Claire)?

Yes ☐ No ☐

Answer: It would not change for a SMALL increase in base value (up to 20). However, it would be the case that one of the Nash equilibria had a higher overall surplus than the other equilibrium.
(Multiple answers accepted here due to the question not making clear that only a small increase should be considered)

a) It turns out that tastes differ, and while your team gets a base value of Casta Diva of 110, the other team gets a base value of Audrey Claire of 110 (before the bonus of 20 if you all end up at the same place. Fill in the updated game board below.



Answer:

	Casta Diva	Audrey Claire
Casta Diva	120 130	110 110
Audrey Claire	110 100	130 120

b) Suppose you're now crossing the Walnut Street bridge, and so phones are working again. If you call up the other learning team, will you easily be able to agree on a location?

Yes ☐ No **X**

Answer: NO—a negotiation would ensue because each team prefers a different equilibrium.

10) Sequential and (Finitely) Repeated Games

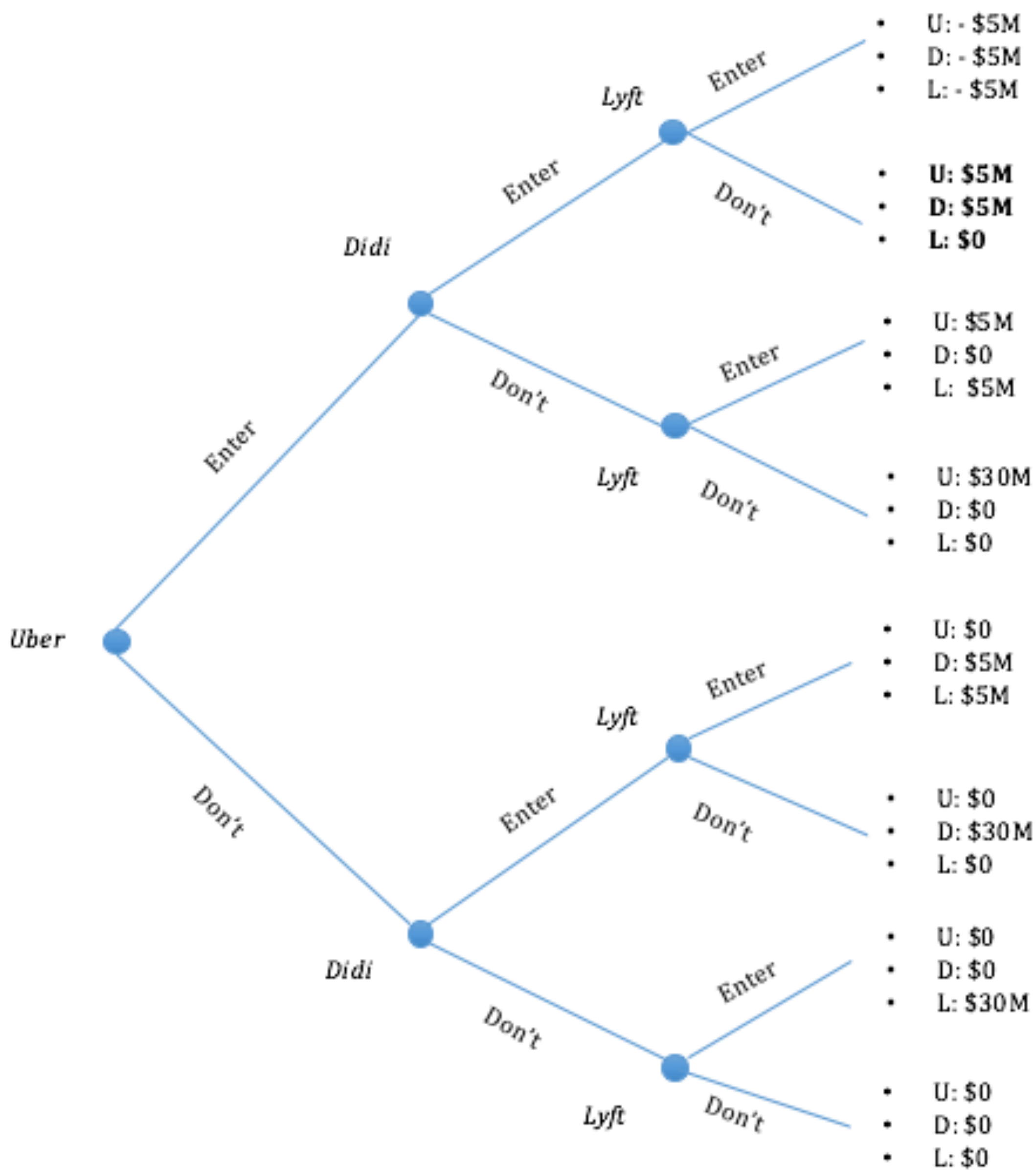
18. *2016 Problem Set 3, Q1*
    Entry Decisions. Uber, Didi and Lyft are three large ride-sharing services that are considering entering the ride-sharing market in the Philippines. They will decide one at a time, in the order Uber, then Didi, then Lyft whether to enter or not. If only one firm enters, expected revenues to that firm are \$50M; if two firms enter, expected revenues per firm are \$25M, and if all three enter, \$15M. All three firms face entry costs of \$20M.

    a) Draw a game tree and determine the equilibrium entry decisions.

       Equilibrium decisions:

Uber:	**X** Enter	☐ Not enter
Didi:	**X** Enter	☐ Not enter
Lyft:	☐ Enter	**X** Not enter

Answer: Uber and Didi will enter, but Lyft will not enter.



b) In an effort to compete with Uber's global dominance, Lyft and Didi enter into an agreement, whereby they will share entry costs by sharing a driver and customer network in this market only. Thus, if one enters a market, because the entry costs will already be paid, the other will too, at an entry cost of only $10million each. Redraw the game tree under this new agreement and determine the new equilibrium decisions.

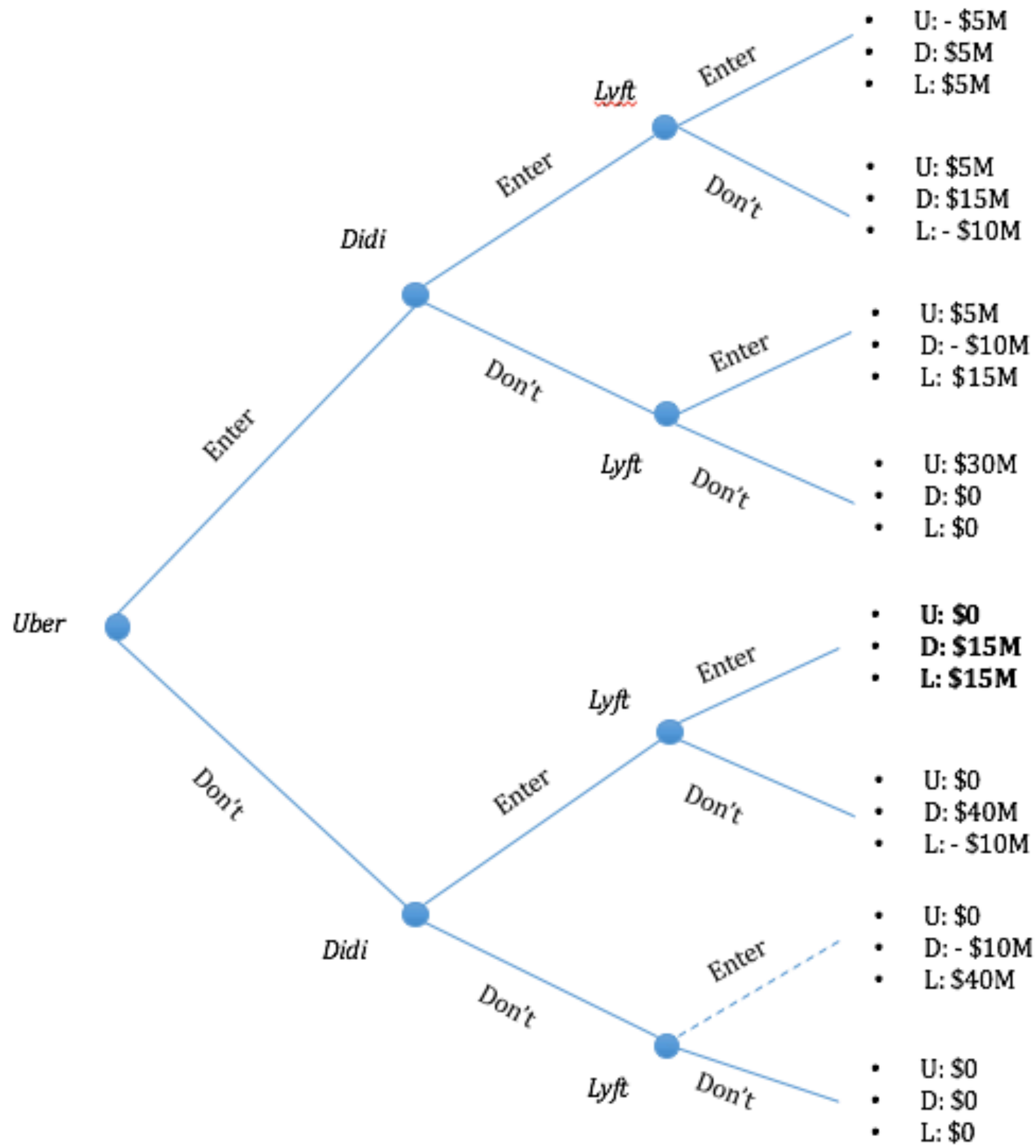
Equilibrium decisions:

Uber: ☐ Enter **X** Not enter

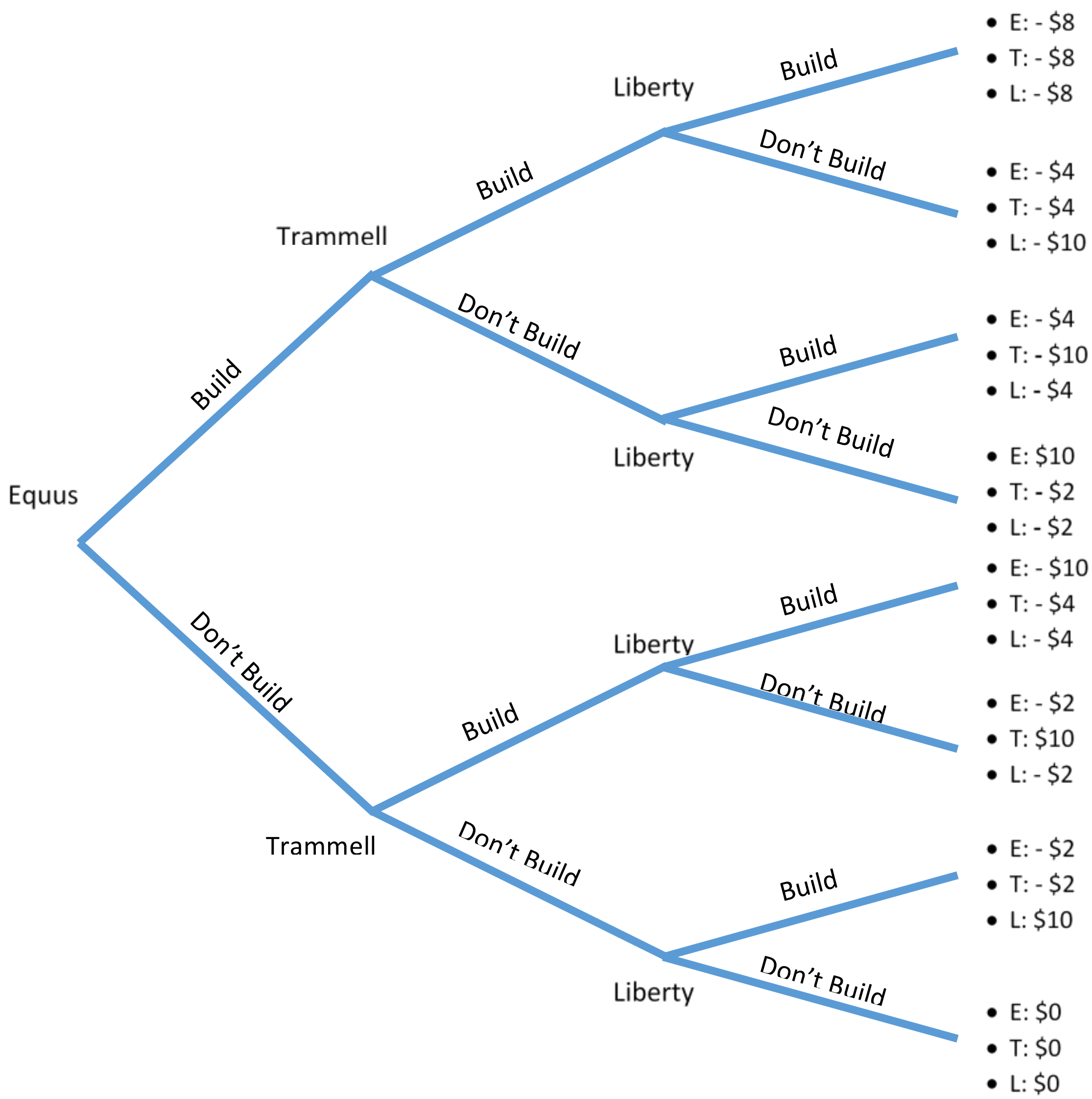
Didi: **X** Enter ☐ Not enter

Lyft: **X** Enter ☐ Not enter

Answer: Uber will not enter, but Didi and Lyft will enter.



19. *2017 final, Q2 (short)*
Real Estate. The three largest Philly real estate development firms are each deciding whether to build a new office building in Center City. Equus Capital Partners (E), Trammell Crow (T), and Liberty Property Trust (L) will decide one at a time, in that order. Payoffs are in millions of dollars, depend on the firms' strategies, and are given by the following tree. Note that it is possible for firms to lose money, even if they don't build (because the new buildings affect rents for existing units that these firms own).



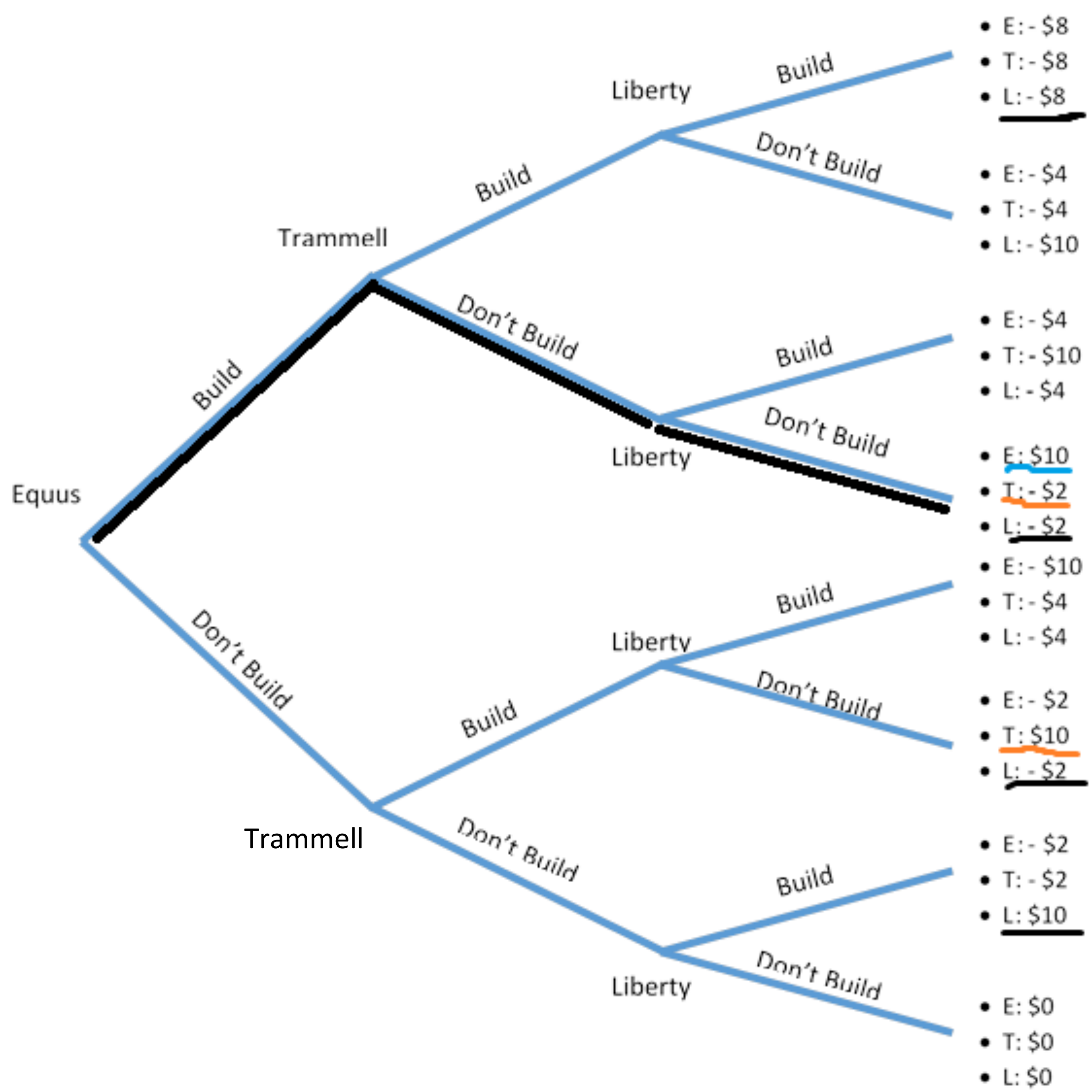
a) Please indicate the action that will be taken by each firm (showing your work on the tree above):

Equus: **X** Build ☐ Don't

Trammell: ☐ Build **X** Don't

Liberty: ☐ Build **X** Don't

Answer:



The Philly government is considering creating (and publicly announcing) a tax credit of $5M for Trammell if Equus decides not to build and Trammell decides to build. Note that this subsidy only changes payoffs; it does not change the order in which the firms move.

b) What would the equilibrium be if this tax credit were available to Trammell? (Note: No work needs to be shown.)

Equus:	**X** Build	☐ Don't
Trammell:	☐ Build	**X** Don't
Liberty:	☐ Build	**X** Don't

Answer: This tax credit has no change because Equus will build, and because if Equus does not build Trammell already would build anyway.

11) Infinitely Repeated Games

20. *2017 Practice Problems*
    Grocery Stores. In a small town, two grocers play the following game every quarter, where the payoffs are in thousands of dollars. For parts (a) to (c) of this problem, you can ignore the e and f in the payoff grid below (i.e. until told otherwise, assume that $e = f = 0$).

Smaller grocer

Larger grocer	High prices	Low prices
High prices	$100-f$, $10+f$	$80-e$, $20-e$
Low prices	$200-f$, 8	90, 9

a) Is there a dominant strategy for the Larger Grocer in the one-shot game (i.e. played just for one quarter)? For the smaller grocer? (Note: Show your work in the game board above.)

Dominant strategy for the Larger grocer (CIRCLE ONE): Yes No

Dominant strategy for the Smaller grocer (CIRCLE ONE): Yes No

Answer: Yes and Yes. (see (b) for explanation)

b) What are the strategies played at the Nash equilibrium of the one-shot game (i.e. played just for one quarter)?

The Larger grocer will set (CIRCLE ONE): High Prices Low prices

The Smaller grocer will set (CIRCLE ONE): High prices Low prices

Answer: Low and Low

To see this, let's start with the Larger Grocer. If the Smaller Grocer plays High, the Larger Grocer will play Low (because payoffs of 200k are greater than 100k). If the Smaller Grocer plays Low, the Larger Grocer still play Low (90k>80k). Thus the Larger Grocer has a dominant strategy: always play Low.

Now, we can check if the Smaller Grocer has a dominant strategy. Assume first that the Larger Grocer plays High Prices. Then the Smaller Grocer will play Low (20k>10k). If the Larger Grocer instead plays Low, the Smaller Grocer faces payoffs of 8 vs. 9, so again the Smaller Grocer will play Low.

Thus, the Smaller Grocer also has a dominant strategy to play Low. Since both players have a dominant strategy, the outcome of carrying out these strategies will be the Nash Equilibrium, so (Low, Low) are the strategies played at the NE.

c) Assume that both grocers have the same quarterly discount rate. What is the highest such rate for which grim trigger strategies could sustain both firms setting high prices as a Nash equilibrium of the infinitely repeated game? (Note: assume ties are broken in favor of setting high prices.)

Highest Discount Rate: ____________________

Answer:_ 10%. Recall that to sustain cooperation, we need the the discounted payoffs of the cooperative state to be greater than the value of a one period deviation plus the future discounted payoffs of the "grim" state. For the Larger Grocer, we need

$$\begin{aligned}&100 + \frac{100}{r} \geq 200 + \frac{90}{r},\\ &\frac{10}{r} \geq 100,\\ &10\% \geq r.\end{aligned}$$

The same rate works for the smaller grocer, since its payoffs are just 10% of (aka proportional to) the larger grocer's.

To confirm this, for the Smaller Grocer to sustain cooperation we need:

$$\begin{aligned}&10 + \frac{10}{r} \geq 20 + \frac{9}{r},\\ &\frac{1}{r} \geq 10,\\ &10\% \geq r.\end{aligned}$$

The discount rate (r) must be less than or equal to 10%. So the highest discount rate needed to sustain cooperation of High Prices is 10%.

In the past, both grocers have had discount rates of 5% per quarter. Unfortunately, the smaller grocer has run into cash flow issues, which has driven its discount rate up to 15% per quarter. The larger grocer's discount rate stays at 5% per quarter. To deal with this development, the Larger grocer is considering two options.

d) The Larger grocer could (credibly) threaten to bring a frivolous lawsuit against the Smaller grocer (say for some sort of zoning violation) in the case that the Smaller grocer sets Low prices when the Larger grocer sets High prices.

   Assume that the Larger grocer can choose the "intensity" of its legal action (which we denote by e), but that a lawsuit of intensity e costs both grocers the same amount, namely e, in every quarter. This lawsuit would go on indefinitely, so that the game-board would be permanently changed.

   What is the lowest e that the Larger grocer could threaten that would restore collusion as an equilibrium of the infinitely-repeated game, implemented by grim trigger strategies? How much will the Larger grocer ultimately pay in legal fees in this collusive equilibrium? (Note: Assume $f = 0$.)

   Smallest Intensity of legal action e: **$**___________________

   Legal costs paid by the Larger grocer in equilibrium: **$**___________________

Answer: \$3,333 and \$0. The cost e does not affect the Larger grocer's incentives. To see this, notice that if both grocers have to pay e and lose the same amount, the Larger Grocer's payoff under (High, Low) is now 80-e.
The Larger Grocer still has Low as a dominant strategy in a one-shot game, but still would prefer to cooperate and play High in an infinitely repeated game. As such, we can just look back to Part (c) to see that the Larger grocer is willing to collude when its discount rate is 5% per quarter.

For the Smaller grocer to keep High prices, however, now we need

$$
\begin{aligned}
10 + \frac{10}{15\%} &\geq 20 - e + \frac{9}{15\%}, \\
\frac{1}{15\%} &\geq 10 - e, \\
1 &\geq 1.5 - 15\% \cdot e, \\
15\% \cdot e &\geq 0.5, \\
e &\geq 3.333.
\end{aligned}
$$

So, the *smallest* legal threat that renews collusion on High Prices as an equilibrium is $e = 3.333$ (i.e. threatening a \$3,333 lawsuit). Of course, in a collusive equilibrium, this threat of a lawsuit is never realized, so the legal costs to the Larger grocer will be \$0.

e) Alternatively, the Larger grocer could commit to pay the Smaller grocer a reward f every time the Smaller grocer chooses to set High prices. As in Part (d), this reward would permanently change the game-board. If the reward is \$4,000, i.e. $f = 4$, what is the Nash equilibrium of the one-shot version of the altered game? (Note: Assume $e = 0$.)

The Larger grocer will set (CIRCLE ONE): High prices Low prices

The Smaller grocer will set (CIRCLE ONE): High prices Low prices

Answer: The single-shot game now admits an equilibrium where Larger sets Low prices and Smaller sets High prices. To see this, notice the Larger grocer still has a dominant strategy to play Low, but now, given Large will play Low, the Smaller Grocer would prefer to play high (as 12k>9k, once we incorporate the \$4,000 reward). This is different than the unaltered game board's single-shot NE of (Low, Low). Notice *both* players are better off under this new altered game.

f) Assume that the Larger grocer goes with the plan from Part (e). If it becomes common knowledge that both grocers will simultaneously retire in exactly 10 quarters (i.e. the game is only finitely repeated, not infinitely repeated), what equilibrium do we expect in each quarter?

The Larger grocer will set (CIRCLE ONE): High prices Low prices

The Smaller grocer will set (CIRCLE ONE): High prices Low prices

Answer: (Larger sets Low prices , Smaller sets High prices)
is an equilibrium of the single-shot game, so it can still be sustained in finite repetition. Working through the backward induction of the finitely repeated game, in the 10^{th} and final quarter, both players will play the single-shot NE, which is (Low, High). Then in the 9^{th} quarter, both players, knowing what will be the outcome of the 10^{th} quarter, will try to maximize their immediate payoffs and again they'll play (Low, High). This line of reasoning can be repeated for all quarters, so we will always expect the Larger Grocer to play Low and the Smaller Grocer to play High.

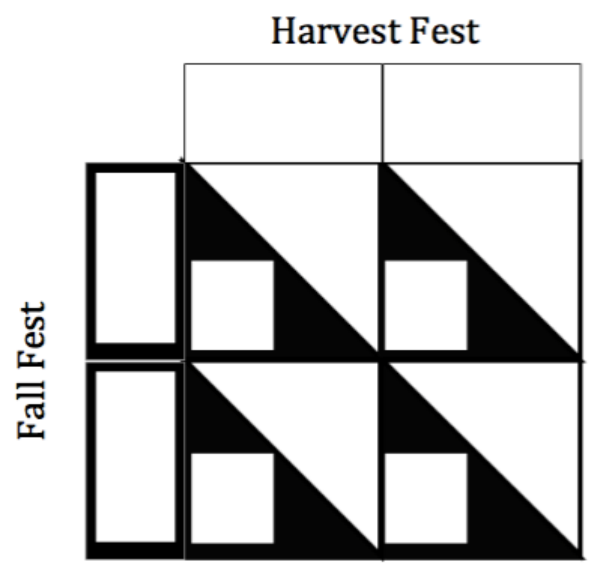
		Smaller grocer	
		High prices	Low prices
Larger grocer	High prices	96, 14	80, 20
	Low prices	196, 12	90, 9

21. *2016 Final, Q2*
Every summer in Pawnee, Indiana the city's legendary Harvest festival attracts thousands of attendees. Given this success, a new festival, Fall Fest, is being developed that would compete with the Harvest Festival.

Based on past experience, there are 7,000 people that would attend a festival in Pawnee each summer, and each person is only willing to attend a single festival per summer. To make giving change easy, festivals only charge one of two possible prices: $20 or $15 per attendee. Each fest can handle 6,000 people at capacity, and wants to maximize its own revenue.

As each festival must announce its price in the Sunday newspaper, they must simultaneously decide what entrance price to charge. If they charge the same price, then each festival will have a total 3,500 people that attend. If one festival charges a lower price, then 6,000 people will attend that festival with the lower price and the remaining 1,000 will go to the festival with the higher price.

a) Draw the game board including payoffs for the simultaneous move game being described above.



Answer: The payoffs are as follows:

		Harvest Festival	
		$20	$15
Fall Fest	$20	70,000; 70,000	90,000; 20,000
	$15	20,000; 90,000	52,500; 52,500

b) If the new festival operated for one weekend, what prices will each festival charge and how much revenue does each make?

Harvest Fest: Price____________________ Revenue____________________

Fall Fest: Price____________________ Revenue____________________

Answer: As each festival has a dominant strategy to charge the low price, the unique Nash equilibrium is for each festival to chart \$15 per ticket. Each festival will make \$52,500.

c) Let's say the festivals plan to operate for three weeks in October. Knowing they will be competing for exactly three weekends (and assuming demand is the same for each of the three weekends as in the one-week case), what is the Nash equilibrium for the *first* weekend?

Harvest Fest: Price____________________

Fall Fest: Price____________________

Answer: With the two festivals competing a finite number of weekends, we know that they will still choose their dominant strategy from the one-shot game and charge the lows prices. To see this, note that in the last period each festival will play operate as in (b). Looking forward from an earlier period, they know they would never be able to co-operate with only three periods. The unique Nash equilibrium is once again for each festival to chart \$15 per ticket. Each festival will make \$52,500.

Now, suppose due to the popularity, the festivals decide to operate every week year-round. Continue to assume demand is the same every week (people in Pawnee really love festivals!). Suppose the weekly discount rate is δ, so one dollar next week is worth $\frac{1}{1+\delta}$ today, and that the two festivals announce their prices simultaneously in the paper each week.

d) What is the highest discount rate δ such that the two festivals can sustain an equilibrium of charging \$20 indefinitely, using a grim trigger strategy (that is: each festival chooses the high price, but switches to the low price forever if the other festival charged the low price in the previous period)?

$\delta \leq$ ____________________

Answer: The grim trigger will sustain setting the high price as long the following condition holds:

$$70{,}000 + \frac{70{,}000}{\delta} \geq 90{,}000 + \frac{52{,}500}{\delta}$$

We can multiply both sides by δ:

$$70{,}000\delta + 70{,}000 \geq 90{,}000\delta + 52{,}500$$

and solving for δ, we get $\delta \leq 0.875$.