CASACT Exam 7 Notes

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1 Benktander Method

1.1 General Relationship Between Reserve & Ultimate Loss Estimates

Suppose that C_k is the actual claims amount paid after k years of development. Given a **reserve estimate** $\widehat{R}$ and **ultimate loss estimate** $\widehat{U}$, we have the following general relationship:

$$\widehat{U} = C_k + \widehat{R} \tag{1}$$

This relationship always holds. Note that C_k is the cumulative paid amount.

1.2 Bornhuetter/Ferguson (BF) Method

The **Bornhuetter/Ferguson (BF) Method** estimates reserves based on an **a priori** expectation of ultimate losses. Mathematically:

$$R_{BF} = q_k * U_0 \tag{2}$$

where:

- R_{BF} is the **reserve estimate**
- $q_k = 1 - \frac{1}{CDF}$ is the proportion of the ultimate claims amount which is expected to remain unpaid after k years of development
- U_0 is the a priori expectation of ultimate losses

Since R_{BF} uses U_0, it assumes that the current claims amount C_k is **NOT predictive of future claims**.

Using the general relationship described above, we obtain the **BF ultimate loss**:

$$U_{BF} = C_k + R_{BF} \tag{3}$$

1.3 Chain Ladder Method

The **Chain Ladder Method** estimates ultimate losses and reserves based on claims to date. In other words, it assumes that the current claims amount C_k is **fully predictive of future claims**. Mathematically:

$$U_{CL} = \frac{C_k}{p_k} \tag{4}$$

$$R_{CL} = q_k * U_{CL} \tag{5}$$

1.4 Benktander Method

Since the CL and BF methods represent extreme positions, where the CL method fully believes C_k and the BF method does not rely on C_k at all, Gunnar Benktander replaced U_0 with a credibility mixture:

$$U_c = c * U_{CL} + (1 - c) * U_0 \tag{6}$$

where c is the credibility weight.

As the claims C_k develop, credibility should increase, Benktander proposed the following:

- Set $c = p_k$
- Set $R_{GB} = R_{BF} * \frac{U_{pk}}{U_0}$

$$\begin{aligned} R_{GB} &= R_{BF} && * \frac{U_{pk}}{U_0} \\ &= (q_k * \cancel{U_o}) && * \frac{U_{pk}}{\cancel{U_o}} \\ &= q_k * U_{pk} \end{aligned} \tag{7}$$

1.4.1 BF Method as a Credibility-Weighted Average

Using our credibility mixture with $c = p_k$, we can show the following:

$$\begin{aligned} U_{pk} &= p_k * U_{CL} + (1 - p_k) * U_0 \\ &= p_k * U_{CL} + q_k * U_0 \\ &= C_k + R_{BF} \\ &= U_{BF} \end{aligned} \tag{8}$$

then

$$R_{GB} = q_k * U_{BF} \tag{9}$$

Hence, the **BF method** is a credibility-weighted average of the CL method and the a priori expectation.

1.4.2 Benktander Method as a Credibility-Weighted Average

The **Benktander method** is a credibility-weighted average of the CL and BF methods:

$$\begin{aligned} U_{GB} &= C_k && + R_{GB} \\ &= C_k && + q_k * U_{BF} \\ &= (1 - q_k) * U_{CL} && + q_k * U_{BF} \end{aligned} \tag{10}$$

The **Benktander reserve** is also a credibility-weighted average of the CL and BF methods:

$$R_{GB} = (1 - q_k) * R_{CL} + q_k^2 * R_0 \tag{11}$$

We can also express the Benktander method as a credibility-weighted average of the CL method and the a priori expectation:

$$
\begin{aligned}
U_{GB} &= C_k + R_{GB} \\
&= U_{CL} - R_{CL} + q_k * U_{pk} \\
&= U_{CL} - q_k * U_{CL} + q_k * ([1 - q_k] * U_{CL} + q_k * U_0) \\
&= U_{CL} - \cancel{q_k * U_{CL}} + \cancel{q_k * U_{CL}} - q_k^2 * U_{CL} + q_k^2 * U_0 \\
&= U_{CL} - q_k^2 * U_{CL} + q_k^2 * U_0 \\
&= (1 - q_k^2) * U_{CL} + q_k^2 * U_0 \\
&= U_{1-q_k^2}
\end{aligned} \tag{12}
$$