Vector :-

* Scalar field :- If a scalar changes from point to point in a region is called scalar field.
  eg.: Temperature, electric potential.
* Vector field :- If a vector changes from point to point in a region is called vector field.
  eg. electric intensity, velocity of liquid flowing through a tube.

$$\vec{\nabla} = \text{del operator or nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Gradient :- When we operate a scalar with del we get a vector which is called the gradient of scalar.

Let ϕ be a scalar then,

$\therefore$ gradient of ϕ = Grad ϕ = $\nabla\phi$

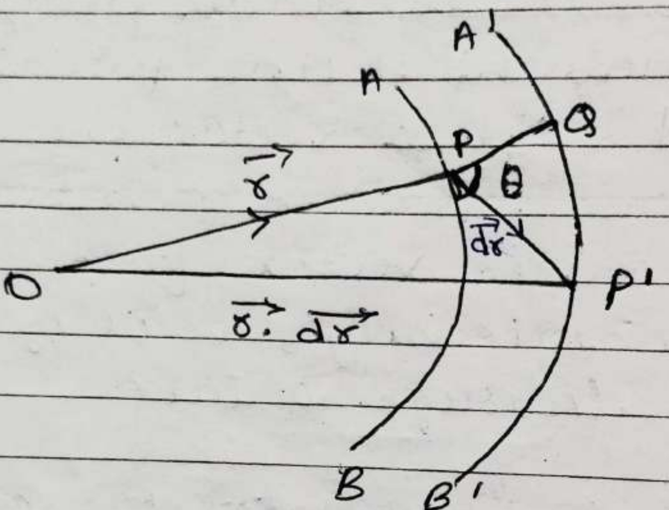
$$\therefore \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}$$

Physical Meaning (defination)
It is defined as a rate of space variation along the normal to the surface on

which it remains constant

$$\nabla \phi = \frac{d\phi}{dn}\hat{n}$$

Proof :-



$$\vec{OP'} = \vec{OP} + \vec{PP'}$$

Let us a draw a surface AB and a point P is taken whose position vector $\vec{r}$. Another surface A'B' parallel to AB. Let P' any point whose position vector is $\vec{r}.\vec{dr}$. Then PP' are equal to $d\vec{r}$. Let us draw a perpendicular PP' on AB which is also perpendicular to the surface A'B' such that

$$\angle QPP' = \theta$$

then $PQ = dn$

Also $\cos\theta = \frac{PQ}{PP'}$

or $PQ = PP' \cos\theta$

$= \vec{dr} \cos\theta$

$$\therefore dn = \vec{dr}\cos\theta$$
$$= \hat{n} \cdot \vec{dr}$$

Also $d\phi = \frac{d\phi}{dn} \cdot dn$

$$= \frac{d\phi}{dn}(\hat{n} \cdot \vec{dr}) \quad \text{———(I)}$$

Since ϕ is a function of x, y and z then

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz \quad \text{———(II)}$$

From eqn (I) and (II)

$$\frac{d\phi}{dn}(\hat{n} \cdot \vec{dr}) = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dx + \frac{\partial\phi}{\partial z}dz$$

$$= \left(\hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}\right) \cdot (\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$\frac{d\phi}{dn}(n \cdot \cancel{\vec{dr}}) = \left(i\frac{\partial}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \frac{\partial}{\partial z}\right)\phi \cdot \cancel{\vec{dr}}$$

$$\frac{d\phi}{dn}\hat{n} = \nabla\phi$$

$$\boxed{\text{grad } \phi = \nabla\phi}$$

* Gradient of Sum of two Scalars

Let u and v be two scalar point-function then,

the sum $= u + v$

grad $(u+v) = \nabla (u+v)$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(u+v)$$

$$= \hat{i}\frac{\partial(u+v)}{\partial x} + \hat{j}\frac{\partial(u+v)}{\partial y} + \hat{k}\frac{\partial(u+v)}{\partial z}$$

$$= \hat{i}\frac{\partial u}{\partial x} + \hat{i}\frac{\partial v}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{j}\frac{\partial v}{\partial y} + \hat{k}\frac{\partial u}{\partial z} + \hat{k}\frac{\partial v}{\partial z}$$

$$= \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z} + \hat{i}\frac{\partial v}{\partial x} + \hat{j}\frac{\partial v}{\partial y} + \hat{k}\frac{\partial v}{\partial z}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)u + \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)v$$

$$\therefore \nabla (u+v) = \nabla u + \nabla v$$

* Divergence of a vector or div of a vector

It we operate del dot on a vector we get a scalar which is called the divergence of a vector.

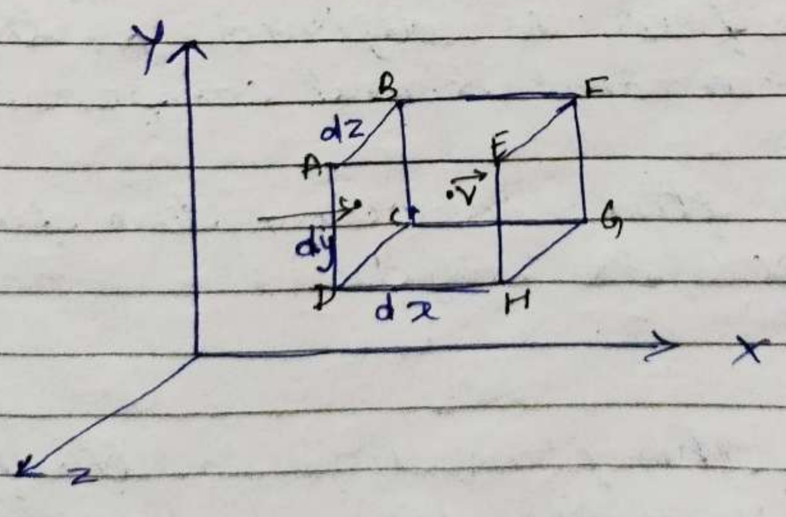
Let $\vec{A}$ be a vector, then

$$\text{div } \vec{A} = \nabla \cdot \vec{A}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Geometrical meaning :- The divergence of a vector function of any point in the vector field is the ammount of flux per unit volume diverging from that point.



Let us consider as small rectangular parallel point with sides dx, dy and dz along x-axis, y- axis, and z-axis respectally. Let $\vec{V}$ be the vector which

represent the vector of a moving fluid at the centre of the parallel piep.
Let V_x, V_y, V_z be the components of the vector B along x axis y-axis and z-axis respectly. Thus the value of x-component of $\vec{V}$ at the center of the face ABCD. Is given that,

$$V_x - \frac{\partial V_x}{\partial x}\frac{dx}{2}$$

And similarly at the centre of the face EFGH is

$$V_x + \frac{\partial V_x}{\partial x}\frac{dx}{2}$$

The value can be taken uniform all over the faces which are vanishingly small.

Since the flux is defined as the product of the area and normal component of the vector.
Therefore flux entering the face ABCD
$= \left(V_x - \frac{\partial V_x}{\partial x}\frac{dx}{2} \cdot dy\,dz\right)$

Similarly the flux leaving the face EFGH $= \left(V_x + \frac{\partial V_x}{\partial x}\frac{dx}{2}\right) \cdot dy\,dz$

The net flux leaving the parallel piep over that entering it along x-direction is given by.

$$\left(V_x + \frac{\partial V_x}{\partial x}\frac{dx}{2}\right)dy\,dz - \left(V_x - \frac{\partial V_x}{\partial x}\frac{dx}{2}\right)dy\,dz$$

$$\left[\cancel{V_x} + \frac{\partial V_x}{\partial x}\frac{dx}{2} - \cancel{V_x} + \frac{\partial V_x}{\partial x}\frac{dx}{2}\right]dy\,dz$$

$$\frac{\cancel{2}\,\partial V_x}{\partial x}\frac{dx}{\cancel{2}}\,dy\,dz$$

$$\frac{\partial V_x}{\partial x}\,dx\,dy\,dz$$

Similarly the flux leaving the parallelopip along y- direction and z-direction are respectivelly.

$$\frac{\partial V_y}{\partial y}\,dx\,dy\,dz \text{ and } \frac{\partial V_z}{\partial z}\,dx\,\partial y\,dz$$

$$\frac{\partial V_x}{\partial x}\,dx\,dy\,dz + \frac{\partial V_y}{\partial y}\,dx\,dy\,dz + \frac{\partial V_z}{\partial z}\,dx\,dy\,dz$$

$$\Rightarrow \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)dx\,dy\,dz$$

The volum of a given parallelopiped $\delta = dx\,dy\,dz$

The ammount of flux diverging per unit volume

$$= \frac{\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)dx\,dy\,dz}{dx\,dy\,dz}$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

By defination of div of $\vec{V}$

$$\text{div}\ \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

* Divergence of Sum of two vectors :-

Let $\vec{u}$ and $\vec{v}$ be two vector point functions given by.

$$\vec{u} = \hat{i} u_x + \hat{j} u_y + \hat{k} u_z$$
$$\vec{v} = \hat{i} v_x + \hat{j} v_y + \hat{k} v_z .$$

Then $\nabla \cdot (\vec{u} + \vec{v}) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot (\vec{u} + \vec{v})$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left[\hat{i}(u_x + v_x) + \hat{j}(u_y + v_y) + \hat{k}(u_z + v_z)\right]$$

$$= \frac{\partial (u_x + v_x)}{\partial x} + \frac{\partial (u_y + v_y)}{\partial y} + \frac{\partial (u_z + v_z)}{\partial z}$$

$$= \frac{\partial u_x}{\partial x} + \frac{\partial v_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial v_y}{\partial y} + \frac{\partial u_z}{\partial z} + \frac{\partial v_z}{\partial z}$$

$$= \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}\right) + \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)$$

$$= \nabla \cdot \vec{u} + \nabla \cdot \vec{v} .$$

Curl of a vector.

If we make del cross operation on a vector we get a vector which is called the curl of a vector.

Let the given vector be $\vec{v}$

Then curl $\vec{v} = \nabla \times \vec{v}$

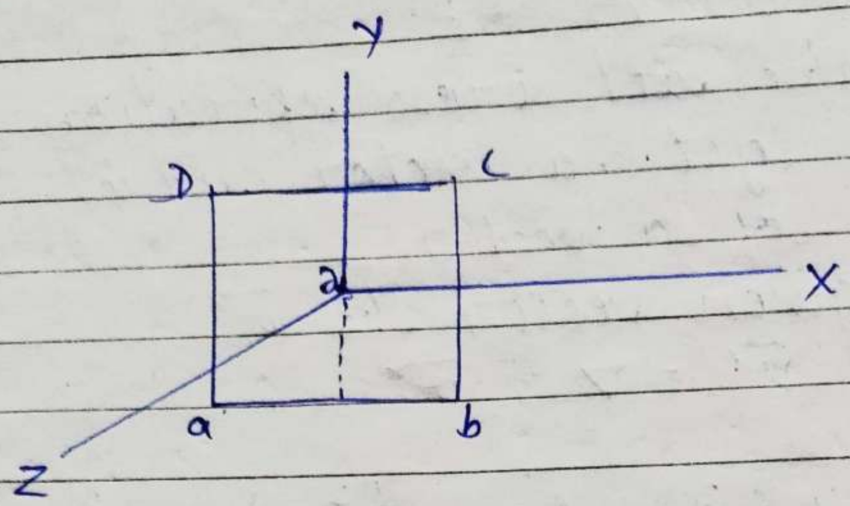
$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\hat{i}v_x + \hat{j}v_y + \hat{k}v_z\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{j}\left(\frac{\partial V_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{k}\left(\frac{\partial v_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

Physical meaning :- It is design as a vector along the normal to the elementry area centre of the point of magnitude equal to the limiting value of the ratio of the line integral of vector along the boundry of the elementry area to the area itself.

i.e curl $\vec{v} = \lim_{\Delta S \to 0} \frac{\oint \vec{v} \cdot \vec{dl}}{\Delta S} \hat{n}$



Let us consider a rectangular elementary area abcd of sides dx, dy its normal being along z-axis which is perpendicular to the elementry surface. The sides of the elementry. The side of the rectangle are venitialy

So the numarical value of component of vector of the $\vec{v}$ at middle point of any side may be take a avarage value along that sides.

So, the avarage value along the sides ab, bc, dc and ad are

$V_x - \frac{\partial V_x}{\partial y}\frac{dy}{z}$, $V_y + \frac{\partial V_y}{\partial x}\frac{dx}{z}$, $V_x + \frac{\partial V_x}{\partial y}\frac{dy}{z}$

and $V_y - \frac{\partial V_y}{\partial x}\frac{dx}{z}$ respectively.

The line intigral around counter abcd in the direction of arrows shown in the rectangle is given by.

$$= \left[\left(v_x - \frac{\partial V_x}{\partial y}\frac{dy}{z} \right) dx - \left(v_x + \frac{\partial V_x}{\partial y}\frac{dy}{z} \right) dx + \right.$$
$$\left. v_y + \frac{\partial V_y}{\partial x}\frac{dx}{z} \right) dy - \left(v_y - \frac{\partial V_y}{\partial x}\frac{dx}{z} \right) dy \bigg]$$

$$= \left[\cancel{v_x} - \frac{\partial V_x}{\partial y}\frac{\partial y}{z} - \cancel{V_x} - \frac{\partial V_x}{\partial y}\frac{dy}{z} \right] dx + \left[\cancel{v_y} + \frac{\partial V_y}{\partial x}\frac{dx}{z} - \cancel{v_y} + \frac{\partial V_y}{\partial x}\frac{dx}{z} \right] dy$$

$$- \frac{\partial V_x}{\partial y} dy\, dx + \frac{\partial V_y}{\partial x} dx\, dy$$

$$- \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) dx\, dy$$

But the area of element abcd is $dx\, dy$
So value of line intigral per unit area

$$= \frac{\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \cancel{dx\, dy}}{\cancel{dx\, dy}}$$

$$= \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}.$$

Its direction is along Z - axis

$$\therefore \text{curl}_z \vec{v} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right) \hat{k}$$

Similarly the other component is given by

$$\text{curl}_y \vec{v} = \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{j}$$

$$\text{Curl}_x \vec{v} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{i}$$

$$\therefore \text{curl}\,\vec{v} = \text{curl}_x\vec{v} + \text{curl}_y\vec{v} + \text{curl}_z\vec{v}$$

$$= \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)\hat{i} + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)\hat{j} + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{k}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

Date - 20/09

Gauss Law :- In a closed surface the total electric flux over in vacuum is equal to $1/\varepsilon_0$ times the total charge enclosed by it.

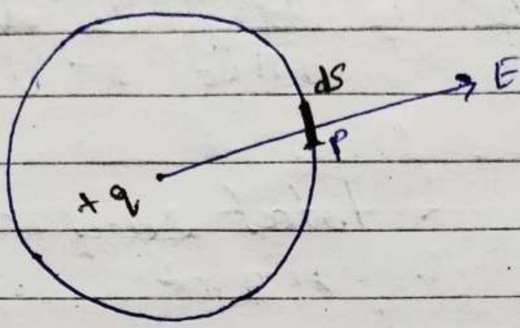
Let ϕ is the flux and q is the charge enclosed by the closed surface.

$$\phi = \frac{1}{\varepsilon_0} \times \text{total charge}$$

or $\boxed{\phi = \frac{1}{\varepsilon_0} \times q}$

electric flux (ϕ) = E ds

Proof :-



Let us take a charge +q at the centre of a spherical ~~all~~ shell of radius r. Let us consider a small element of surface area ds at point p on the shell. Let E be the electric intensity at p which is radial (along to the radius) then the electric flux over the element E ds.

Electric field at a point due to a point charge is given by.

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

∴ The electric flux over the element

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} ds.$$

$\therefore$ total electric flux over the spherical shell $= \int \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \, dS$

or $\phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \int dS$

$\because \int dS =$ total surface area $= 4\pi r^2$

or $\phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} 4\pi r^2$

$$\boxed{\phi = \frac{q}{\varepsilon_0}}$$

Case-I If the spherical shell contains charges $q_1, q_2, q_3, q_4 \ldots\ldots$

Then $\phi = \frac{1}{\varepsilon_0}$ (sum of charges)

$\phi = \frac{1}{\varepsilon_0} (q_1 + q_2 + q_3 + q_4 + \cdots)$

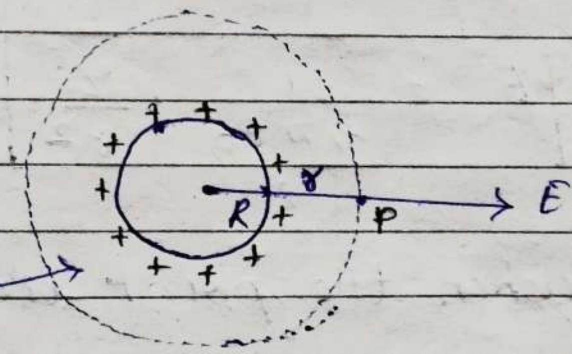
Case-II If the spherical shell contain charges $+q_1, -q_2, -q_3, +q_4, -q_5, +q_6 \ldots$

then $\phi = \frac{1}{\varepsilon_0} (q_1 + q_2 + q_6 + \cdots) + (-q_2 - q_3 - q_5 \ldots)$

Case-III If the charges lie outside the closed surface the electric flux over the closed surface is always zero.

* Electric field at a point due to a charged spherical shell.

Case-I : when the point lie outside the charged spherical shell.



Gaussian surface.

Let us consider a spherical shell of radius R being uniformally charge with charge +q. p is any point at a distance r from the centre outside the charged spherical shell at which the electric field is to be determine. Let us construct a Gaussian surface being sphericall of radius R which encloses the charged spherical shell. Since the Electric lines of ~~foco~~ force are normal to the surface of the charged spherical shell so they are also normal to the Gaussion surface and hence the direction of the electric field at p is also normal to the gaussion surface.

$\therefore$ Area of the Gaussion surface $= 4\pi r^2$

Total charge enclosed by the Gaussion Surface $= q$

Then according to gaussian theorem,

Total electric flux $= \frac{1}{\varepsilon_0} \times$ total charge

$$E\, 4\pi r^2 = \frac{1}{\varepsilon_0} \cdot q$$

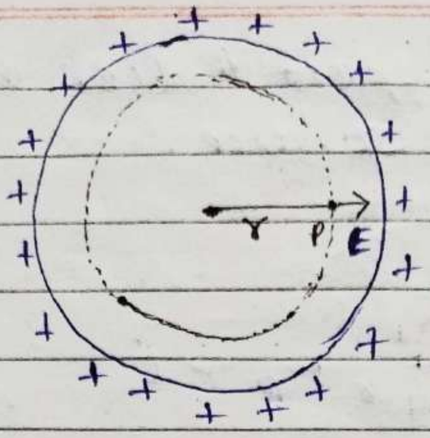
$$\text{Or} \quad \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

Case-II when the point lies on the surface

In this case $r = R$

$$\therefore \quad \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}}$$

Case-III when the point lies inside the spherical shell.



Let the point p lies inside the charged spherical shell. A gaussion surface being spherical of radius R which constructed Let ~~p~~E be the Electric field at p which is radial.

$\therefore$ Area of the gaussion surface $= 4\pi r^2$

$\therefore$ Electric flux over the Gaussion surface $= E\pi r^2$

Total charge ~~to~~ enclosed by Gaussion surface $= 0$

Then according to Gauss's law,

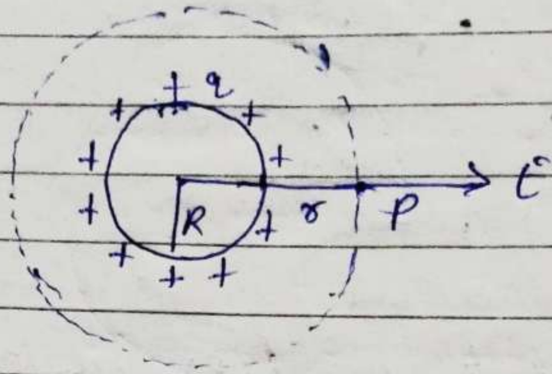
Electric flux $= \frac{1}{\varepsilon_0} \times$ total charge

$$E \cdot 4\pi r^2 = \varepsilon_0 \times 0$$

$$\boxed{E = 0}$$

* Electric field at a point due to a charged solid sphere:

Case-I when the point lies outside the charged sphere.



~~then $\rho = q$~~

Let us take a solid sphere of Radius R It is uniformally charged with charge +q. Let ρ (rho) be the surface density of charge.

then,

$$\rho = \frac{q}{4/3 \pi R^3}$$

Let the point p lie outside the charged sphere at a distance r from the centre A gaussion surface being spherical of radius r is constructed. Let E be the electric field at point p which is radial.

Then the area of the Gaussion Surface $= 4\pi r^2$

The electric flux of the Gaussion Surface $= E 4\pi r^2$

Total charged enclosed by " " $= q$

Then, from gauss's theorem

Electric flux $= \frac{1}{\epsilon_0} \times$ total charged

$$E 4\pi r^2 = \frac{1}{\varepsilon_0} q$$

or $$\boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}}$$

$$\rho = q/v \Rightarrow q = \rho v$$

Also, $$E = \frac{1}{4\pi\varepsilon_0} \frac{\rho \, 4/3 \, \pi R^3}{r^2}$$

or, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho 4\pi R^3}{3r^2}$$

or $$\boxed{E = \frac{1}{3\varepsilon_0} \cdot \frac{\rho R^3}{r^2}}$$

Case-II when the point lies on the Surface

In ~~its~~ this case, $r = R$

$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}}$$

Also, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho \, 4/3 \, \pi R^3}{R^2}$$

$$\boxed{E = \frac{\rho R}{3\varepsilon_0}}$$

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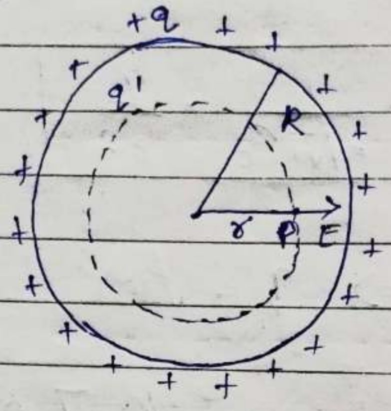
Case-III when the point lie inside the solid Sphere :-



$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

Let the point p lie inside the solid sphere at a distance r from the center. Let as construct a gaussian surface being spherical of radius r. Let this gaussian surface enclose the charges q' and E be the electric field at p which is radial. Let ρ be the charge density. ~~then~~

Then,

$\therefore$ the surface area of gaussian surface $= 4\pi r^2$

$\therefore$ Electric flux over the gaussian surface $= E\,4\pi r^2$

Total charge enclosed by Gaussian surface $= q'$

Then according to gauss's theorem,

Electric flux $= \frac{1}{\varepsilon_0} \times$ total charge

or $E . 4\pi r^2 = \frac{1}{\varepsilon_0} . q'$

$$E = \frac{1}{4\pi \varepsilon_0} . \frac{q'}{\varepsilon_0}$$

or, $$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q'}{r^2} \quad \text{———} (i)$$

$\because$ $q' = \rho \times$ Vol of inner solid sphere (Gaussian Surface)

$$q' = \rho \times \frac{4}{3}\pi r^3$$

$$q' = \frac{q}{\cancel{\frac{4}{3}\pi} R^3} \times \cancel{\frac{4}{3}\pi} r^3$$

$$q' = \frac{q r^3}{R^3}$$

on putting this value in eqn (i)

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q r^3}{r^2 R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q r}{R^3}$$

$$E = r \cdot \frac{q}{\frac{3}{3} \cdot 4\pi\varepsilon_0 R^3}$$

$$E = \frac{r}{3\varepsilon_0} \cdot \frac{q}{\frac{4}{3}\pi R^3}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0}} \qquad \therefore \rho = \frac{q}{4/3\, \pi R^3}$$

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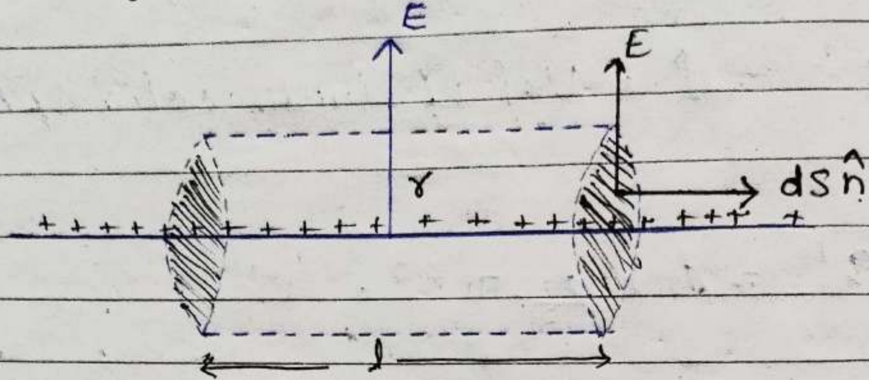
* Electric field at a point due to an infinite long charge of straight line:-



$\phi = Eds \cdot \cos$
$= E ds \cdot \cos$
$= 0$

Let us take an infinite long line of charge having linear density of charge. (charge per unit length) λ. p is any point at a distance r from the line of charges at which electric field is to be determind. Let us construct as gaussian surface being cilendrical having radius r electric line of charge passes through its axis. Since the electric lines of force emitting from the line of charges are perpendicular to it. They are also perpendicular to the curved surface of the gaussion surface. Let E be the electric field intensity are p which is along to Electric line of force and hence it is normal to the gaussion surface. Since the electric field intensity and the surface area at the cross-section are perpendicular to each other. So the Electric flux over the cross-section is always zero (0).

Then the curved surface area = $2\pi r l$

where l = length of the gaussian surface.

electric flux over the gaussian surface $= E\,2\pi r l$

$\therefore$ Total charge enclosed by gaussian surface $= \lambda l$

Then according to gauss's theorem,

Total electric flux $= \frac{1}{\varepsilon_0} \times$ total charge

$$E \cdot 2\pi r l = \frac{1}{\varepsilon_0} \times \lambda l$$

~~or $E \cdot 2\pi r l = \lambda l$~~

$$\text{or, } E = \frac{\lambda \cancel{l}}{2\pi r \cancel{l}\,\varepsilon_0}$$

$$\text{or, } \boxed{E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r}}$$

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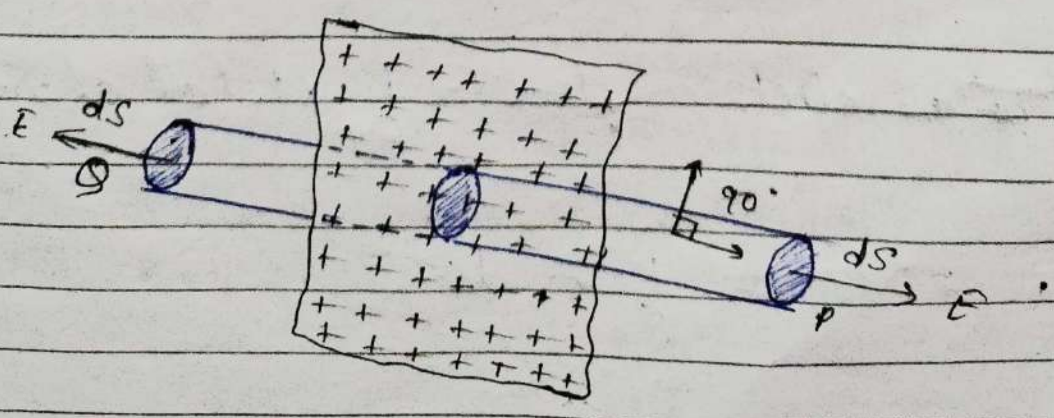
* Electric field at a point due to infinite thin sheet of charge :-



Let us consider a thin sheet of charge having charge per unit area σ. Let us take two point P and Q on either side of the sheet. Let us construct a gaussian surface being cylindrical of crossectional area ds such that it is perpendicular to the plane of the sheet. Then the electric field at P and Q are perpendicular to the crossections the gaussian surface. On the curved surface of the gaussian surface. The area and the electric field are perpendicular to each other so the electric flux over the curve surface equals zero.

$\therefore$ El. flux over the Gaussian surface

$$= E ds + E ds$$
$$= 2 E ds$$

Total charge enclosed by the Gaussian surface $= \sigma ds$

Then according to Gauss' theorem,

Total electric flux $= \frac{1}{\varepsilon_0} \times$ total charge

$$2 E ds = \frac{1}{\varepsilon_0} \sigma ds$$

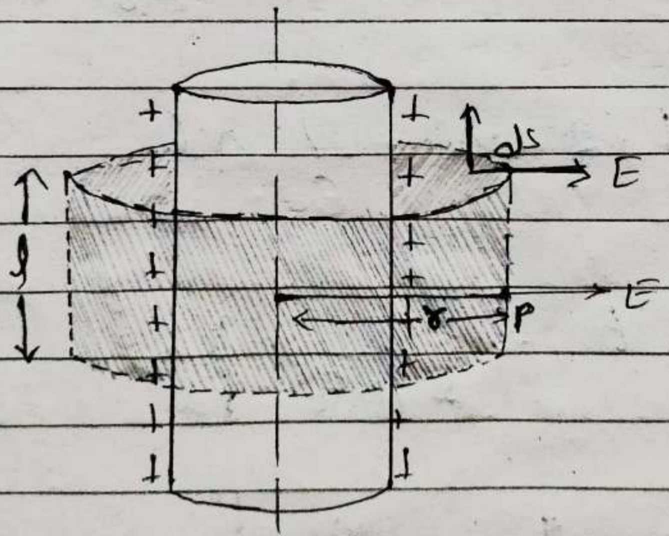
$$E = \frac{1}{\varepsilon_0} \frac{\sigma\, \cancel{dS}}{2\, \cancel{dS}}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

* Electric field at a point ~~dut~~ due to thick sheet of charge is given by

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

* Electric field at a point due to the charged cylinder :-



Let us take a celender being uniformally charged with charge per unit length λ. P is any point outside the cylinder at a distance r from the axis at which the electric feild is to be determind a gaussian surface being cylindrical of length l and radius r is constructed. Such that it co- ~~axil~~ axial with the cylinder. Let E be the electric field Intensity at p which is radial. The electric flux over the cross section of the gaussian surface becomes zero because the direction of electric field Intensity and the surface area are perpendicular to each other.

∴ Area of the curved surface = $2\pi r l$
∴ Electric flux over the Gaussian Surface = $E \cdot 2\pi r l$
Total charge enclosed by Gaussian Surface = λl

According to gauss's theorem
Total Electric flux = $\frac{1}{\varepsilon_0}$ x total charge

$$E \cdot 2\pi r l = \frac{1}{\varepsilon_0} \lambda l$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda l}{r l}$$

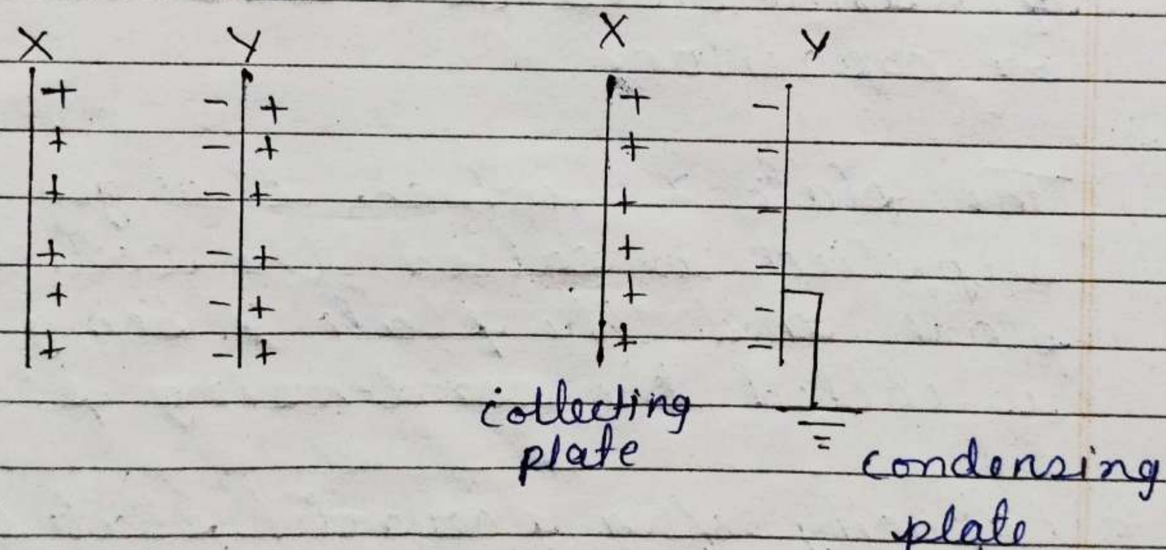
$$\boxed{E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{l}}$$

Capacity or Capacitance :- The number of charge required to raise the potential difference of a body by 1 volt is called the capacity.

Let q charge be supplied to a body which ranges the potential difference by V volt then,

$$\text{Capacity} = \frac{q}{V}$$

Condenser or capacitor :-



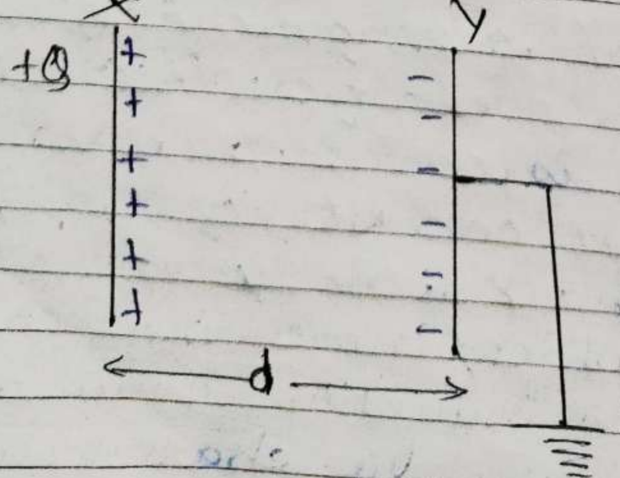
The artificial arrangment of two parallel plate to store charges their by increasing its capacity is called the condencer or capacitor. It consists of two parallel plate X and Y, the plate X is supplied but it is placed near the charged plate due to induction equal opposite charges and similar charges are produced on a inner surface or outer surface —

of the plate y. These charges produced negative potential and positive potential the plate x. Thus the net potential of x decreases. Therefore increasing the capacitor.

But when the plate Y is earthed, the free positive charges neutralized. So only negative potential is produced to plate x. thus its potential further increases. And Hence the capacity increases. This artificial arrangement of conductors is called the condenser or capacitor.

The plate x which charges are supplied is called the collecting plate and the plate Y being earthed is called the condensing plate.

* Capacity of a parallel plate condenser without dielectric.



It consist of two parallel plate X and Y seperated by a distance d. The plate X is ~~sp~~ supplied with +Q charge There as the plate Y is earthed. Let and σ be the surface area of the plate and the charged per unit area ~~rep~~ respectively.

The electric field intensity at any point between the plate is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

$\sigma = \frac{Q}{A}$

$\boxed{E = \frac{Q}{A\varepsilon_0}}$

$\therefore$ Potential difference between the plate = Ed.

(distance)

$$\text{or } V = \frac{\sigma}{\varepsilon_0} d$$

$$\therefore \text{Capacity} = \frac{Q}{V}$$

ε_0 = Permiability of Free space

$$\text{or } C = \frac{Q}{\frac{\sigma d}{\varepsilon_0}}$$

$$C = \frac{\varepsilon_0 Q}{\sigma d}$$

$$C = \frac{\varepsilon_0 \cancel{Q}}{\frac{\cancel{Q}}{A} d}$$

$\therefore \sigma = \frac{Q}{A}$

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$

Thus the capacity of a parallel plate condencer is inversally proportional to the distance between the plate.

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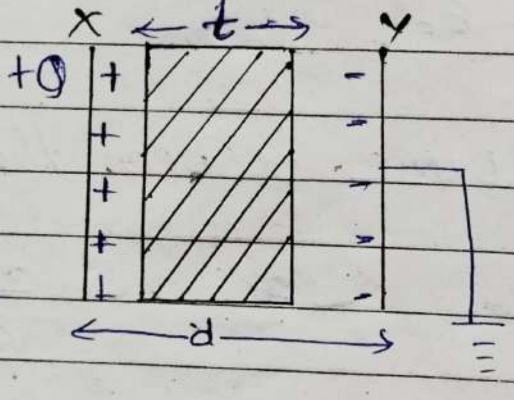
parallel plate condenser with a di-electric



Let us take two parallel plate X and Y seperated by a distance 'd'. plate X is supplied with +Q charge where as Y is earthed. Let a ~~distance~~ dielectric of permitively ϵ ($\epsilon_r \epsilon_0$) be placed between the plates. Then, the thickness of air is $d-t$.

Then, The electric intensity at any point in air is given by.

electric field

$E_1 = \frac{\sigma}{\epsilon_0}$ where σ change per unit area of the plates.

$\therefore$ pot. dif. (V) ~~acct~~ across the air medium.

$V_1 = \underset{\text{E field} \times \text{distance}}{E_1 (d-t)}$

$= \frac{\sigma}{\epsilon_0} (d-t)$

The electric intensity at any point in the dielectric is given by.

$E_2 = \frac{\sigma}{\epsilon}$

$= \frac{\sigma}{\epsilon_r \epsilon_0}$

$\therefore$ pot. dif. across the dielectric $= E_2 t$

$V_2 = \frac{\sigma}{\epsilon_r \epsilon_0} t$

Then, the total potential diff. between the plates X and Y is given by.

$V = V_1 + V_2$

$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{\epsilon_r \epsilon_0} t$

$= \frac{\sigma}{\epsilon_0}\left(d-t + \frac{t}{\epsilon_r}\right)$

$$= \frac{\sigma}{\epsilon_0}\left(d - t\left(1 - \frac{1}{\epsilon_r}\right)\right)$$

Let A be the surface of the plate x, then

$$\sigma = \frac{Q}{A}$$

then $\therefore V = \frac{Q}{A\epsilon_0}\left(d - t\left(1 - \frac{1}{\epsilon_r}\right)\right)$

Hence the cappacity is given by

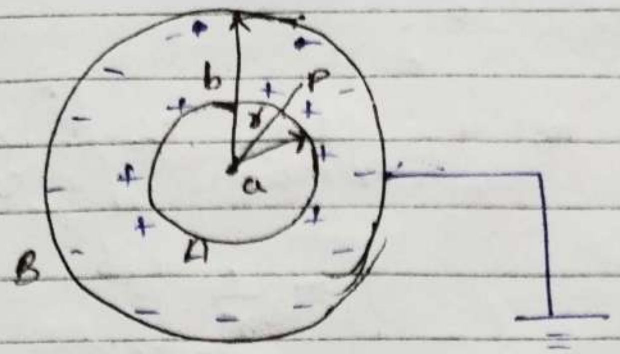
$$C = \frac{Q}{V}$$

$$= \frac{\cancel{Q}}{\cancel{Q}/A\epsilon_0\left[d - t\left(1 - \frac{1}{\epsilon_r}\right)\right]}$$

$$C = \frac{A\epsilon_0}{d - t\left(1 - \frac{1}{\epsilon_r}\right)}$$

Thus the capacity of the parallel plate condensor increases when a dielectric is placed between the two plates

Spherical Condenser



Let us take two constoric sphrical shell A and B of radii a and b ~~S~~ respectively. The Inner sphere is charged with +q charge where as the outer sphere is earthed. Let us take a point p at a distance r from the center. Such that

$a < r < b$

The ele. intencity at p is given by

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

∴ work done in shifting a unit +ve charge through a small distance dr is given by

$$dw = E\,dr$$

~~Then~~
$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,dr$$

Then the work done in shifting from A to B ~~be~~ is given by.

$$\int dw = \int_a^b \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr$$

$$\text{or} \quad w = \frac{1}{4\pi\varepsilon_0} \cdot q \int_a^b \frac{dr}{r^2}$$

$$\left[-\frac{1}{r}\right]_a^b$$

$$w = \frac{1}{4\pi\varepsilon_0} q \left[-\frac{1}{b} + \frac{1}{a}\right]$$

$$\text{or} \quad w = \frac{1}{4\pi\varepsilon_0} q \left[\frac{b-a}{ab}\right]$$

$\therefore$ Pot. diff between A & B = w

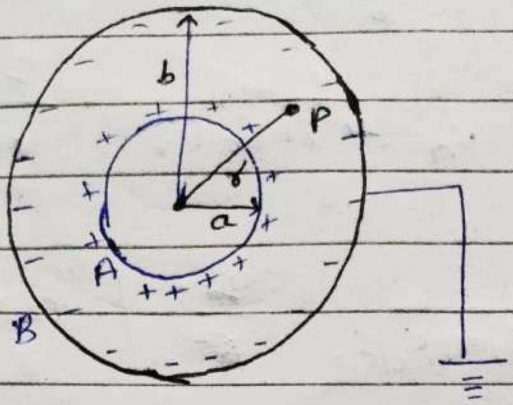
$$\text{or} \quad V = \frac{1}{4\pi\varepsilon_0} q \left[\frac{b-a}{ab}\right]$$

$\therefore$ capacity $= \frac{q}{V}$

$$\text{or} \quad C = \frac{q}{\frac{1}{4\pi\varepsilon_0} q \left[\frac{b-a}{ab}\right]}$$

$$C = 4\pi\varepsilon_0 \left[\frac{ab}{b-a}\right]$$

Spherical condenser ~~withe~~ filled with dielectric



Let us consider two ~~const~~ concentric spherical shell A and B having radii a and b respectivly. The inner sphere is charged with $+q$ and outer sphere is earthed. A dielectric of dielectric constant ε_r or K is filled between the two spherical shells. Let p be any point at a distance r from the centre where $a < r < b$.

The electric intensity at p is given by

$$E = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q}{r^2}$$

$\therefore$ work done in displacing a unit charge through a small distance dr is given by

$$dw = E\,dr$$

$$dw = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q}{r^2}\,dr$$

Total work done in displacing from A to B is given by

$$\int dw = \int_a^b \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{q}{r^2}\, dr$$

or $$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q}{} \int \frac{dr}{r^2}$$

or $$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot q \left[-\frac{1}{r}\right]_a^b$$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} q \left[-\frac{1}{b} + \frac{1}{a}\right]$$

$$w = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot q \left[\frac{-a+b}{ab}\right]$$

$$= \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot q \left[\frac{b-a}{ab}\right]$$

$\therefore$ pottential diff. $V = w$

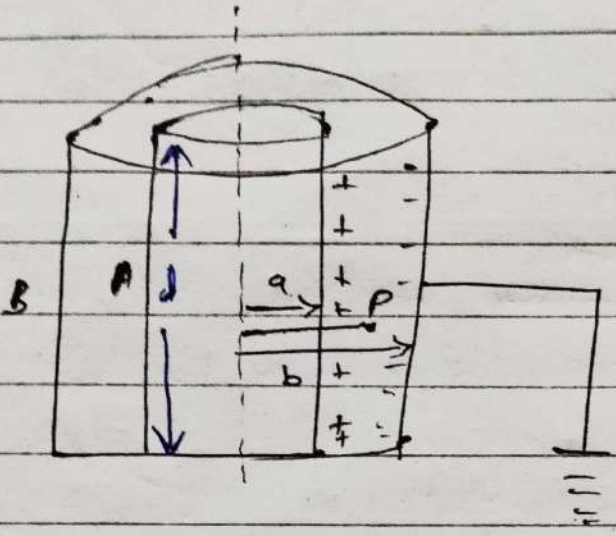
or $$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot q \left[\frac{b \cdot a}{ab}\right]$$

$\therefore$ capacity $\frac{q}{v}$.

$$= \frac{q}{\frac{1}{\pi\varepsilon_0\varepsilon_r} q \left[\frac{b-a}{a\ldots}\right]}$$

$$\boxed{C = 4\pi \varepsilon_0 \varepsilon_r \left(\frac{ab}{b-a}\right)}$$

Cylindrical Condenser :-



Let us take two co-axial cylender A and B having radii a and b. The inner cylender is charged with +q and the outer cylender is earthed
Let λ be the charge per unit Length
Let us take a point p at a distance r from the axis then the electric intensity at p is given by

$$E = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r}$$

where λ = charge per unit length.

∴ work done is shifting a unit +ve charge through a small distance dr is given by.

$$dw = E\, dr$$

$$dw = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r} dr$$

$\therefore$ Total work done is given by

$$\int dw = \int_a^b \frac{1}{2\pi\varepsilon_0} \cdot \frac{\lambda}{r} dr$$

$$w = \frac{1}{2\pi\varepsilon_0} \cdot \lambda \int_a^b \frac{dr}{r}$$

$$w = \frac{1}{2\pi\varepsilon_0} \cdot \lambda \left[\ln r\right]_a^b$$

$$w = \frac{1}{2\pi\varepsilon_0} \cdot \lambda \left[\ln b - \ln a\right]$$

$$w = \frac{1}{2\pi\varepsilon_0} \cdot \lambda \cdot \ln\left(\frac{b}{a}\right)$$

$\therefore$ pot. diff = w

$$= \frac{1}{2\pi\varepsilon_0} \lambda \ln\left(\frac{b}{a}\right)$$

$\therefore$ Capacity $= \frac{q}{v}$ $\quad \therefore q = \lambda l$

or $$C = \frac{\cancel{\lambda}\, l}{\frac{1}{2\pi\varepsilon_0} \cdot \cancel{\lambda} \ln\left(\frac{b}{a}\right)}$$

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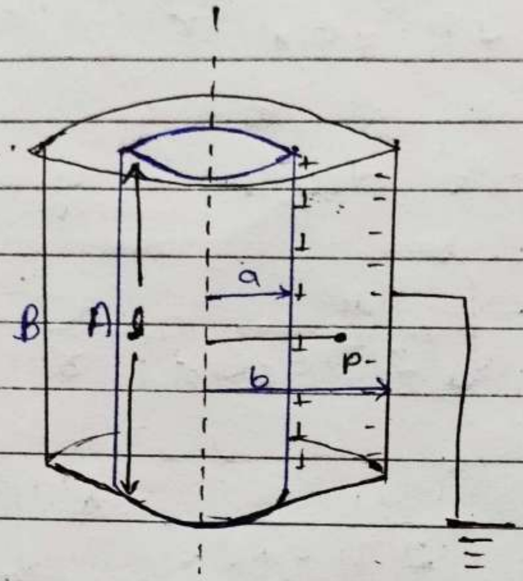
$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}}$$

Capacity of cylindrical condenser filled by dielectric :-



Let us take two co-axial cylenders A and B having radii a and b respectively. A dielectric is filled between the cyllenders whose dielectric constant is ϵ_r and K. The inner cylender is charge having charge per unit area λ. The outer cylender is earthed. Let p be the any point between the cylenders at a distance r from the axis such that $a < r < b$.

Then,
The electric intensity at a point P

$$= \frac{1}{2\pi\epsilon_0\epsilon_r}\frac{\lambda}{r}$$

$$E = \frac{1}{2\pi\epsilon_0\epsilon_r}\frac{\lambda}{r}$$

$\therefore$ Work done is displacing a unit +ve charge by a small distance dr

$$dw = E\,dr$$
$$dw = \frac{1}{2\pi\epsilon_0\epsilon_r}\frac{\lambda}{r}\,dr$$

$\therefore$ Total work done is given by

$$\int dw = \int_a^b \frac{1}{2\pi\epsilon_0\epsilon_r}\frac{\lambda}{r}\,dr$$

$$\int dw = \frac{1}{2\pi\epsilon_0\epsilon_r}\cdot\lambda\int_a^b \frac{dr}{r}$$

$$W = \frac{1}{2\pi\epsilon_0\epsilon_r}\cdot\lambda\left[\ln r\right]_a^b$$

$$W = \frac{1}{2\pi\epsilon_0\epsilon_r}\cdot\lambda\left[\ln b - \ln a\right]$$

$$W = \frac{1}{2\pi\epsilon_0\epsilon_r}\lambda\ln\left(\frac{b}{a}\right)$$

$\therefore$ Potential difference $= W$

$$= \frac{1}{2\pi\epsilon_0\epsilon_r}\lambda\ln\left(\frac{b}{a}\right)$$

$\therefore$ Capacity $= \frac{q}{V}$

$$= \frac{\lambda l}{\frac{\lambda}{2\pi\epsilon_0\epsilon_r} \ln\left[\frac{b}{a}\right]}$$

$$C = \frac{l \, 2\pi\epsilon_0\epsilon_r}{\ln\left(\frac{b}{a}\right)}$$

or. $$\boxed{C = \frac{2\pi l \epsilon_0 \epsilon_r}{\ln\left(\frac{b}{a}\right)}}$$

* Gauss' theorem in dielectric

According to gauss' theorem.

$$\Phi = \iint_S \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

Let us take parallel plate condenser having vaccum medium between the plates whose one plate is charged with $+q$ charged.

Then a gaussian surface may be constructed surrounding the charged plate. Then the electric flux over the Gaussian surface vaccum and medium b/w the plate of capacitor

$$\iint_S \vec{E_0} \cdot \vec{dS} = \frac{q}{\epsilon_0} \quad \text{——— (1)}$$

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Since ϵ_0 (external EMF) and $\vec{dS}$ are the ~~name~~ same direction so.

$$\vec{E_0} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

$$E_0 \, dS = \frac{q}{\epsilon_0}$$

$$\therefore \iint_S E_0 \, dS = \frac{q}{\epsilon_0}$$

$$\text{or, } E_0 \iint_S dS = \frac{q}{\epsilon_0}$$

$$E_0 A = \frac{q}{\epsilon_0} \qquad \text{where } \iint_S dS = A$$

$$E_0 = \frac{q}{A\epsilon_0} \qquad \text{(iii)}$$

Let us now introduce a dielectric material of dielectric constant K or ϵ_r then the induced charges appear on a surface of the dielectric. These induced charges produced their own electric field which opposes the external electric field E_0. Let E be the resultant electric field within the dielectric. If q_i is the induced surface charge then the net charge within the gaussian surface will be $q + (-q_i)$ total charge $= q - q_i$

$$\iint_S \vec{E}\,\vec{ds} = \frac{q - q_i}{\epsilon_0} \qquad \text{—— (iii)}$$

or, $$\iint_S E\,ds = \frac{q - q_i}{\epsilon_0}$$

$$E \iint_S ds = \frac{q - q_i}{\epsilon_0}$$

$$EA = \frac{q - q_i}{\epsilon_0}$$

$$E = \frac{q - q_i}{A\epsilon_0}$$

$$E = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0}$$

$$E = \frac{E_0}{K} \qquad \boxed{\because K = \frac{E_0}{E}}$$

on putting this value

$$\frac{E_0}{K} = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0}$$

$$\frac{q}{A\epsilon_0 K} = \frac{q}{A\epsilon_0} - \frac{q_i}{A\epsilon_0} \qquad E_0 = \frac{q}{A\epsilon_0} \text{ from eq. (II)}$$

$$\frac{q}{\cancel{A}\epsilon_0 K} = \frac{1}{\cancel{A}\cancel{\epsilon_0}}(q-q_i)$$

$$\frac{q}{K} = q - q_i$$

$$q_i = q - \frac{q}{K}$$

$$q_i = q\left(1 - \frac{1}{K}\right)$$

On putting the value of q_i in (III)

$$\iint_S \vec{E}\cdot\vec{dS} = \frac{q}{\epsilon_0} - \frac{q}{\epsilon_0}\left(1 - \frac{1}{K}\right)$$

$$= \cancel{\frac{q}{\epsilon_0}} - \cancel{\frac{q}{\epsilon_0}} + \frac{q}{K\epsilon_0}$$

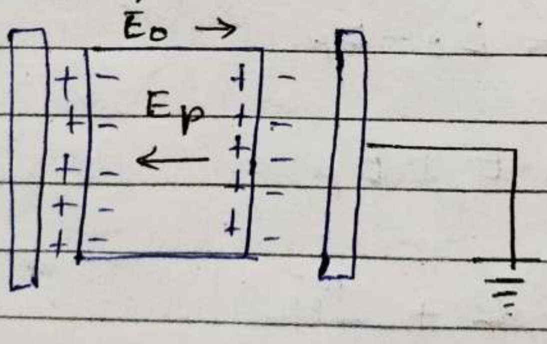
$$\boxed{\iint_S \vec{E}\cdot\vec{dS} = \frac{q}{K\epsilon_0}}$$

This is the gauss's theorem in dielectric

Polarization

When a dielectric slab which placed between two charged parallel plates then charged are induced on it when an external electric field is applied. The gravity of all positive charges (central gravity) shift in the direction of applied field and the gravity of all negative charges shifts in opposite direction to the applied field. Thus on the opposite faces of the dielectric charges are induced. The phenomenon of appearance of charges on the opposite faces of a dielectric slab induced by the external electric field is called. polarization.

Electric displacement vector :-



Let us consider in polarization of a dielectric slab placed between the plates of a charged parallel plate capaciter. Let σ_{free} be the surface charge density of free charges on capacitor σ_p is the surface charge density of bound of polarization charges. Then magnitude of electric field between the plates of charged capacitor without dielectric

$E = \frac{\sigma}{\epsilon_0}$ $\quad E_0 = \frac{\sigma_{free}}{\epsilon_0}$

magnitude of electric field between the plate of capasitor with dielectric due to induced charges.

$$E_p = \frac{\sigma_p}{\epsilon_0}$$

The net electric field within the dielectric is given by.

$$E = E_0 + (-E_p)$$
$$E = E_0 - E_p$$

or $E_0 = E + E_p$

$$E_0 = E + \frac{\sigma_p}{\epsilon_0}$$

$$\frac{\sigma_{free}}{\epsilon_0} = E + \frac{\sigma_p}{\epsilon_0}$$

$$\frac{\sigma_{free}}{\epsilon_0} = E + \frac{P}{\epsilon_0} \qquad \because P = \sigma_p = \frac{q_i}{A}$$

or, $\sigma_{free} = \epsilon_0 E + P$

or, $\sigma_{free} = D$

Where $D = \epsilon_0 E + P$ and it is called the electric displacement.

Date- 31/01/2024

Faraday's law's of electromagnetic Induction

According to faraday's :-

(I) Whenever their change in magnetic line of force (Magnetic flux) and emf. is induced and hence and induced current flows.

(II) The induced EMF last only for the time for which their is actual charge in magnetic flux.

(III) The ammount of induced EMF is directly proportional to the rate of change of magnetic flux.

Let ϕ be the magnetic flux and e the induced EMF (electro motive force).

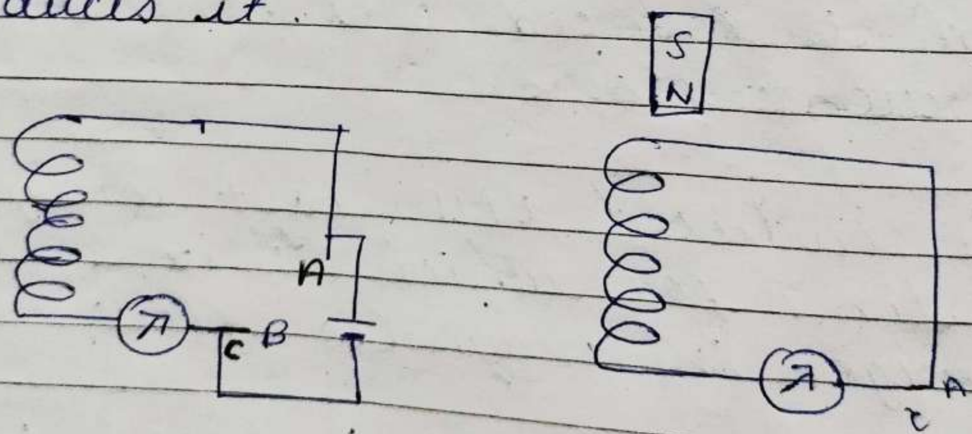
Then
$$e \propto -\frac{d\Phi}{dt}$$

$$e = -K\frac{d\Phi}{dt}$$

where K is a constant ~~on~~ or taking $K=1$

we have $\boxed{e = -\frac{d\Phi}{dt}}$

Lenz's law :-
The law states that the induced EMF produced in a coil flows in a such a direction. So, as to oppose the which produces it.



A coil is taken which is conected with a galvanometer and a cell as shown in the figure. Initially the point B and C are connected so the current flows through the coil the direction of current at the upper end is anti-clockwise direction.

At this state let the diflection in galvanometer be towords the right After that the cell is removed and the north pole of a bar magnet is brough towords the coil. The diflection ~~is~~ in the galvanometer is towords the right in this case which shows the flow of current in anticlock wise direction at the upperend of the coil. This produces a north pole at the upper end of the coil which oppose the comming magnet. When the magnet is takenlly from the coil the diflection in the galvanometer is towords left. It produces a south ~~pole to~~ pole to the upper end of the coil which attracts the magnet that is oppose the change in flux.

Self Induction :- The phenomenon of production of induced EMF and hence induced current in a coil due to the charge of its own flux is called ~~sef~~ self induction.

Let I be the current flowing through the coil and ϕ, the magnetic flux then The magnetic flux is directly proportion al to the current flowing through the coil.

$\phi \propto I$

on differentiating both side w.r.t t

$$\frac{d\phi}{dt} \propto \frac{dI}{dt}$$

from faraday's laws of electromagnetic induction

$$e = -\frac{d\phi}{dt}$$

$$-e \propto \frac{dI}{dt}$$

$$\text{or} \quad e = -L\frac{dI}{dt}$$

where L is a constant of proportionality and is called the cofficent of self induction.

If $\frac{dI}{dt} = 1$

then $e = -L$

Thus the cofficient of self induction is define as the amount of EMF induced in a coil due to unit rate of change current.

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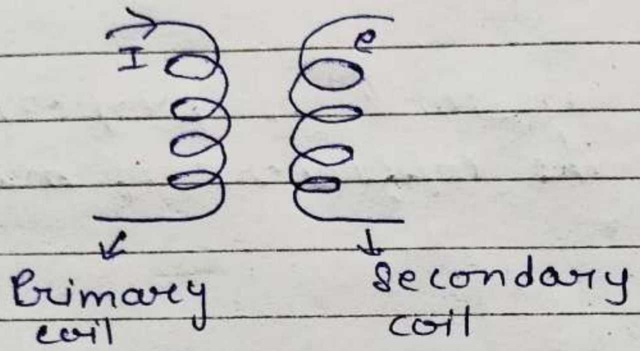
Mutual Induction :-

The phenomenon of production of induced EMF in a coil due to the change of current in another coil is called Mutual induction.



Let I be the current flowing through the primary coil and Φ is magnetic flux. Let e be the EMF induced in secondary then the flux linked to the secondary coil is directly proportional to the current flowing to the primary coil.

ie. $\Phi \propto I$

on differentiating both side w.r.t t

$$\frac{d\Phi}{dt} \propto \frac{dI}{dt}$$

But ~~form~~ from faraday's law of electromagnetic induction.

$$e = -\frac{d\Phi}{dt}$$

then

$$-e \propto \frac{dI}{dt}$$

or $$\boxed{e = -M\frac{dI}{dt}}$$

where M is a constant of proportonality and is called the cofficient of mutual induction.

If $\frac{dI}{dt} = 1$

then $e = -M$

Thus the cofficient of Mutual induction is defined as the amount of EMF induction in the secondary coil due to unit rate of change of current in the primary coil.

* Energy stored in a magnetic field (induction coil)

When a current is allowed to pass through a coil, EMF is induced in it ~~a~~ which opposes the increase in current. In order to attempt the pick value of current, work is to be done these work done is stored in the coil as the potential energy.

The magnitude of EMF induced in a coil due to the flow of current I is given by

$$e = L\frac{dI}{dt} \quad \text{——— (1)}$$

But the induced EMF is ~~done~~ defined as the work done per unit charge

i.e. $$e = \frac{dw}{dq} \quad \text{——— (2)}$$

from eqn (1) and (2)

$$\frac{dw}{dq} = L\frac{dI}{dt}$$

or $$dw = L\frac{dI}{dt}\,dq$$

$$dw = L\,dI\,\frac{dq}{dt} \qquad \therefore I = \frac{dq}{dt} \text{ (current)}$$

$$dw = L \cdot dI \cdot I$$

$$dw = LI\,dI$$

Hence the work done in increasing the current from 0 to I_0 is given by

$$\int dW = \int_0^{I_0} LI\, dI$$

$$W = L\int_0^{I_0} I\, dI$$

$$W = L\left[\frac{I^2}{2}\right]_0^{I_0}$$

$$W = L\frac{I_0^2}{2}$$

This work done is stored in the coil as the potential energy.

$\therefore$ Potential energy $(U) = W$

$$\boxed{U = \frac{1}{2} L I_0^2}$$