Q21. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$.

$\Rightarrow$ Given: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$

$\Rightarrow D^2y - 2Dy + 4y = e^x \cos x$

$\Rightarrow (D^2 - 2D)y + 4y = e^x \cos x$

$\frac{d}{dx} = D$

$\Rightarrow$ The auxiliary eqn is

$m^2 - 2m + 4 = 0$

$\therefore m = \frac{2 \pm \sqrt{4 - 4.1.4}}{} = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$

$\therefore C.F. = e^x (A \cos \sqrt{3}x + B \sin \sqrt{3}x)$.

Now, $P.I. = \frac{1}{D^2 - 2D + 4} e^x \cos x$

$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 4} \cos x$

$= e^x \frac{1}{D^2 + 2D + 1 - 2D - 2 + 4} \cos x = e^x \frac{1}{D^2 + 3} \cos x$

$= e^x . \frac{1}{D^2 + 2D + 1 - 2D - 2 + 4} \cos x = e^x \frac{1}{D^2 + 3} \cos x$

$= e^x .. \frac{1}{-1^2 + 3} \cos x = e^x \frac{1}{2} \cos x = \frac{1}{2} e^x \cos x$

Therefore the complete solution is

$y = e^x (A \cos \sqrt{3}x + B \sin \sqrt{3}x) + \frac{1}{2} e^x \cos x$

Proved

Q22 Solve $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$.

$\Rightarrow$ From given we have.

$$\frac{dx-dy}{(y+z)-(z+x)} = \frac{dy-dz}{(z+x)-(x+y)} = \frac{dz-dx}{(x+y)-(y+z)}$$

$$\Rightarrow \frac{dx-dy}{y-x} = \frac{dy-dz}{z-y} = \frac{dz-dx}{x-z}$$

$$\Rightarrow \frac{dx-dy}{x-y} = \frac{dy-dz}{y-z} = \frac{dz-dx}{z-x}$$

integrating the first two:

$$\log(x-y) = \log(y-z) + \log a$$

$$\Rightarrow \log(x-y) - \log(y-z) = \log a$$

$$\Rightarrow \log\frac{x-y}{y-z} = \log a$$

$$\Rightarrow \frac{x-y}{y-z} = a \longrightarrow (1)$$

Similarly from the last two equations, we have.

$$\frac{y-z}{z-x} = b \longrightarrow (2)$$

But (1) and (2) are not independent, because on adding 1 to both side of (1); we get (2).

P.T.O.

Again, $\frac{dx - dy}{y - x} = \frac{dx + dy + dz}{2(x + y + z)}$

$\Rightarrow -\frac{dx - dy}{x - y} = \frac{dx + dy + dz}{2(x + y + z)}$

integrating we get.

$-\log(x-y) = \frac{1}{2}\log(x+y+z) - \frac{1}{2}\log b$

$\Rightarrow -2\log(x-y) = \log(x+y+z) - \log b$

$\Rightarrow \log(x+y+z) + 2\log(x-y) = \log b$

$\Rightarrow \log(x+y+z) + \log(x-y^2) = \log b$

$\Rightarrow \log\{(x+y+z)(x-y)^2\} = \log b$

$\Rightarrow (x+y+z)(x-y)^2 = b \longrightarrow (III)$

Hence (1) and (3) together give the complete solution :

Proved

Q23. Solve the diffrential equation :

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$$

$$\Rightarrow \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}$$

C.F. $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right) + 2y = 0$

Auxillary equation is :

$$m^2 + 3m + 2 = 0 \Rightarrow (m+1)(m+2) = 0. \therefore m = -1, -2.$$

Therefore C.F.

$$= Ae^{-x} + Be^{-2x}$$

Now, we can write

$$(D^2 + 3D + 2)y = e^{2x}$$

So that P.I. is given by

$$y = \frac{1}{D^2+3D+2}e^{2x} = \frac{1}{2^2+3\cdot 2+2}e^{2x} = \frac{1}{12}e^{2x}.$$

Hence the general solution is

$$y = Ae^{-x} + Be^{-2x} + \frac{1}{12}e^{2x}$$

Proved

Page No.

Q24. Show that four roots of unity namely 1, -1, i, -1 form a group.

×	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-1	-i	+i	1	1

Clouser:

$G = \{1, -1, i, -i\}$

let a, b.

$a \star b \in G = ab \in G$

$a \cdot b \in G$ satisfied

② Associative:

let $a, b, c \in G$

$(a \star b) \star c = a \star (b \star c)$

It holds associative property

③ Existance of identity:

let $a \in G$

for $a \cdot e = e \cdot a = a$

$ae = a$

$e = \frac{a}{a} = 1$, $\boxed{e = 1}$ $\because 1 \in G \Rightarrow 1(a) = a \in G$

Satisfied

④ Inverse

let $a \in G$ then $^{-1}$ is the inverse of a

$a \cdot a^{-1} = e$

$aa^{-1} = e$

$a^{-1} = \frac{e}{a}$

$a^{-1} = \frac{1}{a}$

$a = 1 \Rightarrow a^{-1} = \frac{1}{1} = 1$ satisfied

⑤ Commutative

let $a, b = \in G$

$a \star b = b \star a$

let $a = 1, b = -1$

then $1 \star -1 = -1 \star 1$

$-1 = -1 \in G$ Proved

Solution: $G = \{1, \omega, \omega^2\}$

	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

(i) Clousure property:

Let $a, b \in G$
then $a * b = ab \in G$
Clousure property holds.

(ii) Associative property.

It holds the colusure property so it also hold the associative property.

(iii) Existance of identity.
let a
$\because$ $e\ 1 \in G$, let $a \in G$
$1(a) \Rightarrow a$
Satisfied.

(iv) Existance of inverse
let $a = 1$, $a^{-1} = \frac{1}{1}$

$a * a^{-1} = e$
$1\left(\frac{1}{1}\right) = e$
$1 = e$

$1(1) = 1$
$\omega \cdot (\omega^2) = \omega^3 = 1$
$\omega^2 \cdot (\omega) = \omega^3 = 1$
Satisfied

Hence $(G, *)$ is a subgroup.