UESA – Math (GE – 1)
DSC

2024

(Old)

Full Marks : 100

Time : 3 Hours

Candidates are required to give their answer in their own words as far as practicable. Their figures in the margin indicate full marks.

Answer from **both** the Sections as directed.

Section – A

(Compulsory)

1. Answer the following questions : $1 \times 10 = 10$

(a) State Maclaurin's Theorem.

(b) If y = Sin ax, then yn = ...

(c) The length of the sub tangent to the curve y = f (x) is ....

(d) Define curvature.

(e) Define Asymptotes.

P.T.O.

(f) Write down the polar equation of conic.

(g) Define Rotation of Axes.

(h) Write the equation of the normal to the Parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

(i) Write down the equation of the hyperbola in standard form.

(j) Write down the director circle of the ellipse.

2. Find the co-ordinates of a point in a plane when the origin is shifted to a new point (h, k), the new axes remaining parallel to the original axes. 5

3. Find yn when y = Sin (ax + b). 5

Section – B

Answer any **four** questions : $20 \times 4 = 80$

4. (a) State and Prove the Taylor's theorem on the expansion of (x + h). 10

(b) Apply Maclaurin's series to

$$\sin x = x - \frac{x^3}{\underline{|3}} + \frac{x^5}{\underline{|5}} . - \text{to } \infty.$$ 10

5. (a) Find the radius of curvature for the Cartesian curve y = f (x). 10

(b) Find the real asymptotes of the curve $x^3 + y^3 = 3axy$. 10

6. (a) State and prove the Euler's Theorem on partial differentiation of homogeneous function of two independent variables. 10

(b) Prove That : 10

(i) $$\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$$

(ii) $$\frac{1}{p^2}=u^2+\left(\frac{du}{d\theta}\right)^2$$

7. (a) State and prove Leibnitz's Theorem. 10

(b) If $y = {}_{e}a\sin^{-1} x$, prove that : 10

(i) $(1-x^2)y_2 - xy_1 - a^2y = 0.$

(ii) $(1-x^2)\,y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0.$

8. (a) Find the co-ordinates of a point in a plane when the axes are rotated through an angle α, the origin remaining fixed. 10

(b) Find the condition that the line $y = mx + c$ may touch the hyperbola. 10

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

9. (a) Show that the equation of the chord of a parabola $y^2 = 4ax$, whose middle poist is (x_1, y_1) is $(y - y_1)\, y_1 = 2a\,(x - x_1)$ 10

(b) Show that the equation $\frac{\ell}{r} = 1 + e\cos\theta$ and $\frac{\ell}{r} = -1 + e\cos\theta$, represent the same conic.

10

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