Dog Problem

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Problem

There are 49 dogs signed up to compete in the dog show. There are 36 more small dogs than large dogs signed up to compete. How many small dogs are signed up to compete?

Solution

Let l and s represent the number of large and small dogs signed up to compete in the dog show, respectively. The problem states that there are 36 more small dogs than large dogs signed up to compete; hence,

$$s = l + 36. \tag{1}$$

If we assume that there are only small and large dogs signed up to compete in the dog show, we must have

$$49 = s + l. (2)$$

Using (1) this is equivalent to

$$49 = l + 36 + l$$
$$= 2l + 36.$$

Solving for l, the number of large dogs, we get

$$l = 6.5.$$

Assuming half a large dog doesn't exist, the premise represented by (2) must not hold since it was not explictly stated in the problem. Thus, at least one other type of dog is signed up to compete in the dog show. Let o represent the number of dogs that are neither large or small that are signed up to compete in the dog show. Instead of (2), we now have

$$49 = s + l + o. (3)$$

As before we use (1) to rewrite this as

$$49 = 2l + 36 + o.$$

This is equivalent to

$$13 = 2l + o$$
.

Since there are 2 unknowns and only one equation, there are multiple solutions. Again, assuming $l, o \in \mathbb{N}_0$ (i.e. l and o are non-negative integers) we have that $l \in \{0, 1, 2, 3, 4, 5, 6\}$. Thus, using (1), we have that

$$s \in \{36, 37, 38, 39, 40, 41, 42\}.$$

In words, the number of small dogs could be any number between 36 and 42.