

# Numerical Linear Algebra: Homework 1

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1. Prove the following statements:

(a)  $\|A\mathbf{z}\| \leq \|A\|\|\mathbf{z}\|$  for  $A \in \mathbb{R}^{n \times n}$  and  $\mathbf{z} \in \mathbb{R}^n$ .

(b)  $\|AB\| \leq \|A\|\|B\|$  for  $A, B \in \mathbb{R}^{n \times n}$ .

2. Solve the linear system  $A\mathbf{x} = \mathbf{b}$ , where

$$A = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & \ddots & \ddots & \ddots \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}_{100 \times 100} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{100 \times 1},$$

by Jacobi iterative method. Set the initial guess  $\mathbf{x}^{(0)} = (1, 0, 0, \dots, 0)^T$ , the tolerance  $TOL = 10^{-10}$  with the stopping strategy  $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty < TOL$ , and the maximum number of iterations  $N_0 = 10^5$ . Output the number of iterations  $N$  and the error  $\|\mathbf{x}^{(N)} - \mathbf{x}\|_\infty$  in your code (written in a Matlab file). [Hint: You may need to use the Matlab commands `tril`, `triu`, `diag`, and `norm`.]