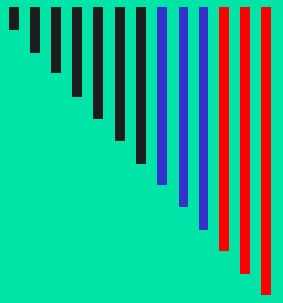


# Chapter 10

# *Multiway Trees*

## Objectives

*Upon completion you will be able to:*

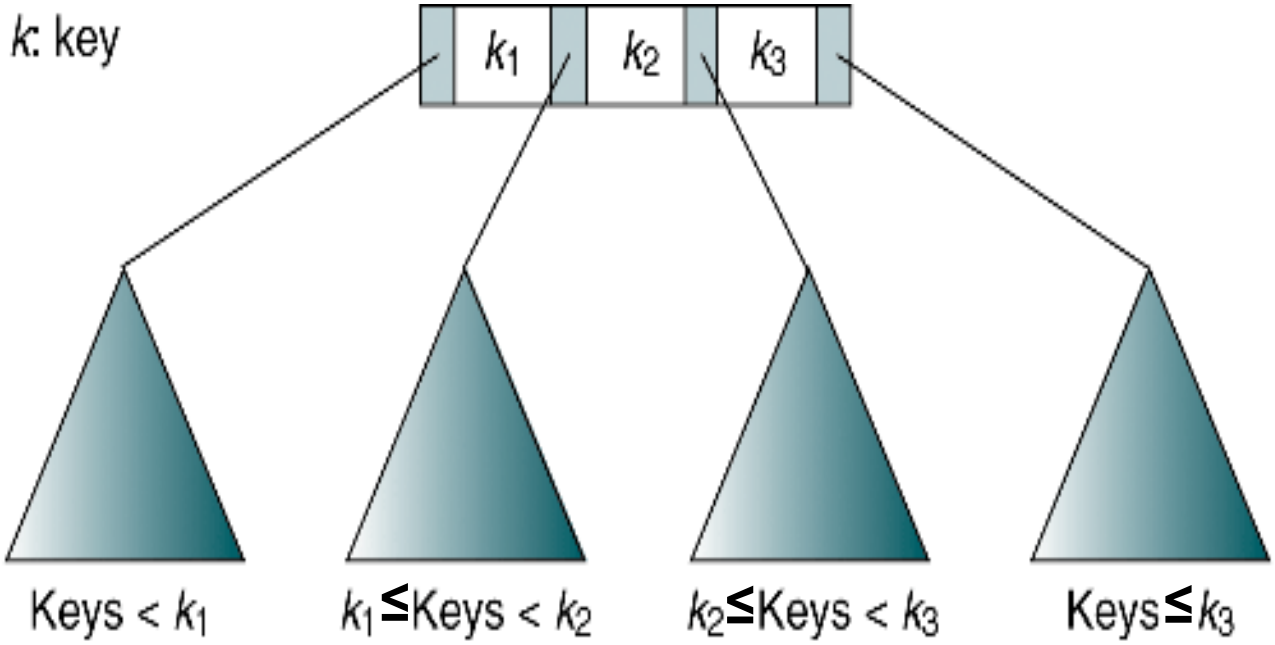
- Define and discuss multiway tree structures
- Understand the operation and use of the B-tree ADT
- Discuss the use and implementation of simplified B-trees
- Compare and contrast B-trees, B*trees, and T+trees
- Discuss the design and use a lexical search trees (Tries)

# 10-1 M-way Search Tree

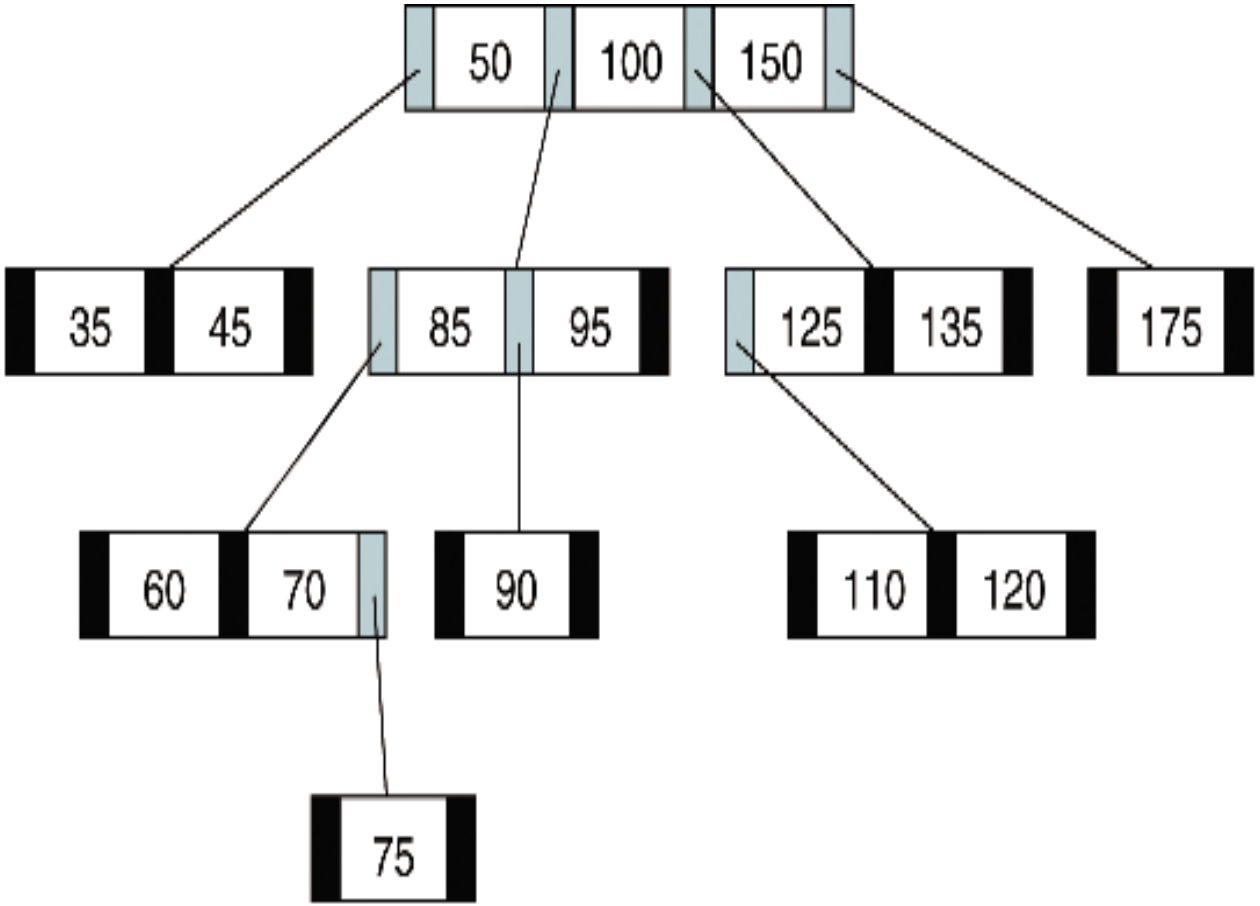
***This section explore trees whose outdegree is not restricted to 2 but that retain the general properties of binary search trees; Multiway trees have multiway entries in each node and thus may have multiple subtrees.***

- **M-way tree**
- **B-trees**
- **B*trees**
- **T+trees**

# M-way Tree

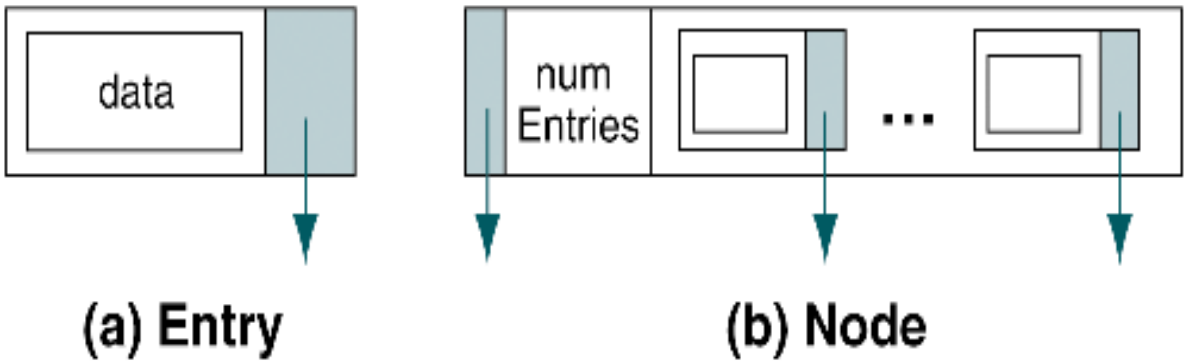


# 4-way Tree



# M-way Node Structure

```
entry
    data
    rightPtr
end entry
node
    firstPtr
    numEntries
    entries    array of entry
end node
```




(a) Entry (b) Node

# 10-2 B-trees

*This section discusses the B-tree structure which is a balanced version of m-way tree.*

- Insertion
- Deletion
- Traversal
- Search

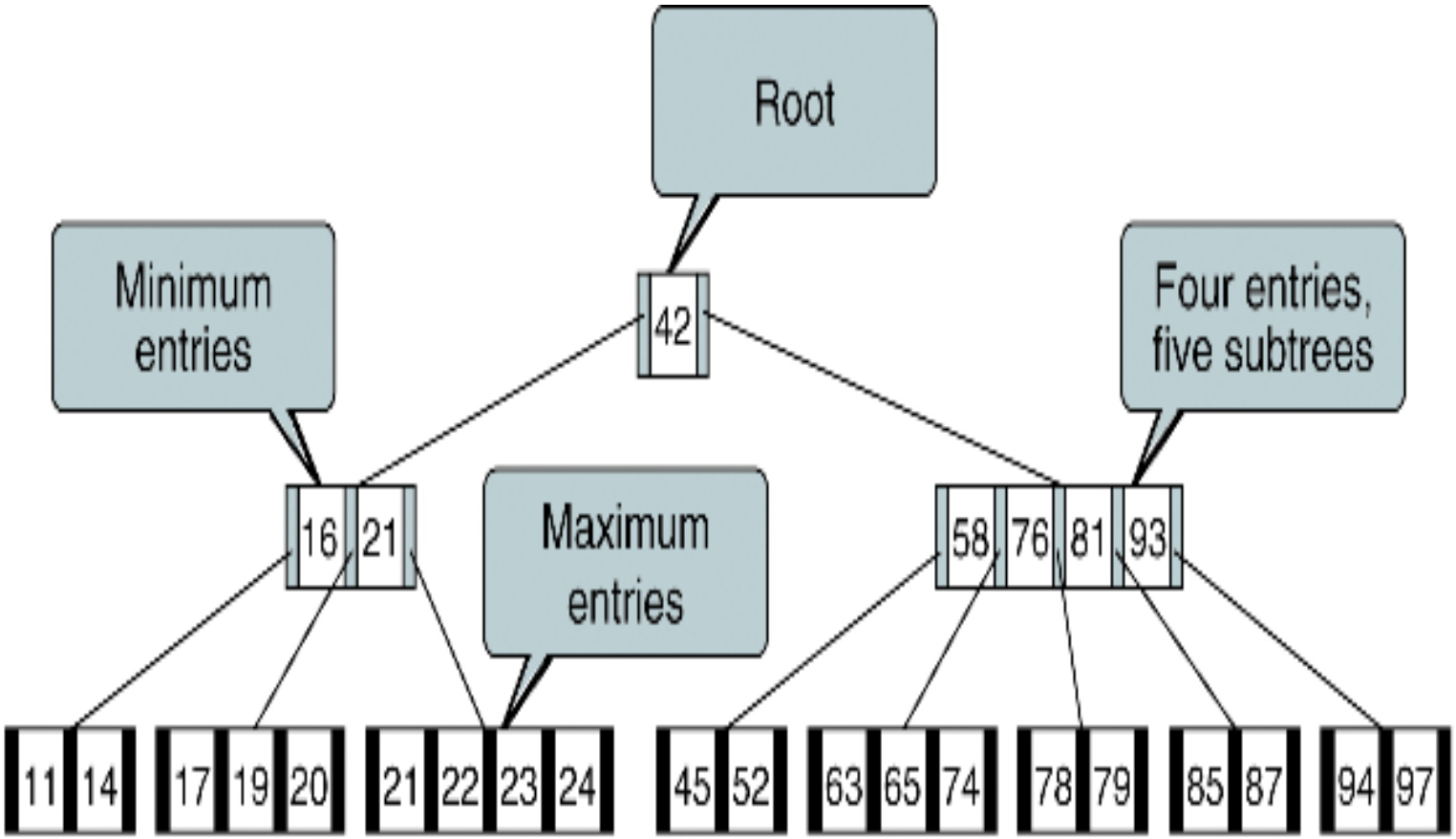
# Entries in B-trees of Various Orders

- Root: a leaf or it has 2…m subtrees
- Internal node: $\lceil m/2 \rceil$… m subtrees ($\lceil m/2 \rceil$-1 … m-1 entries)
- Leaf node: $\lceil m/2 \rceil$-1 … m-1 entries
- All leaf nodes are at the same level (perfectly balanced)

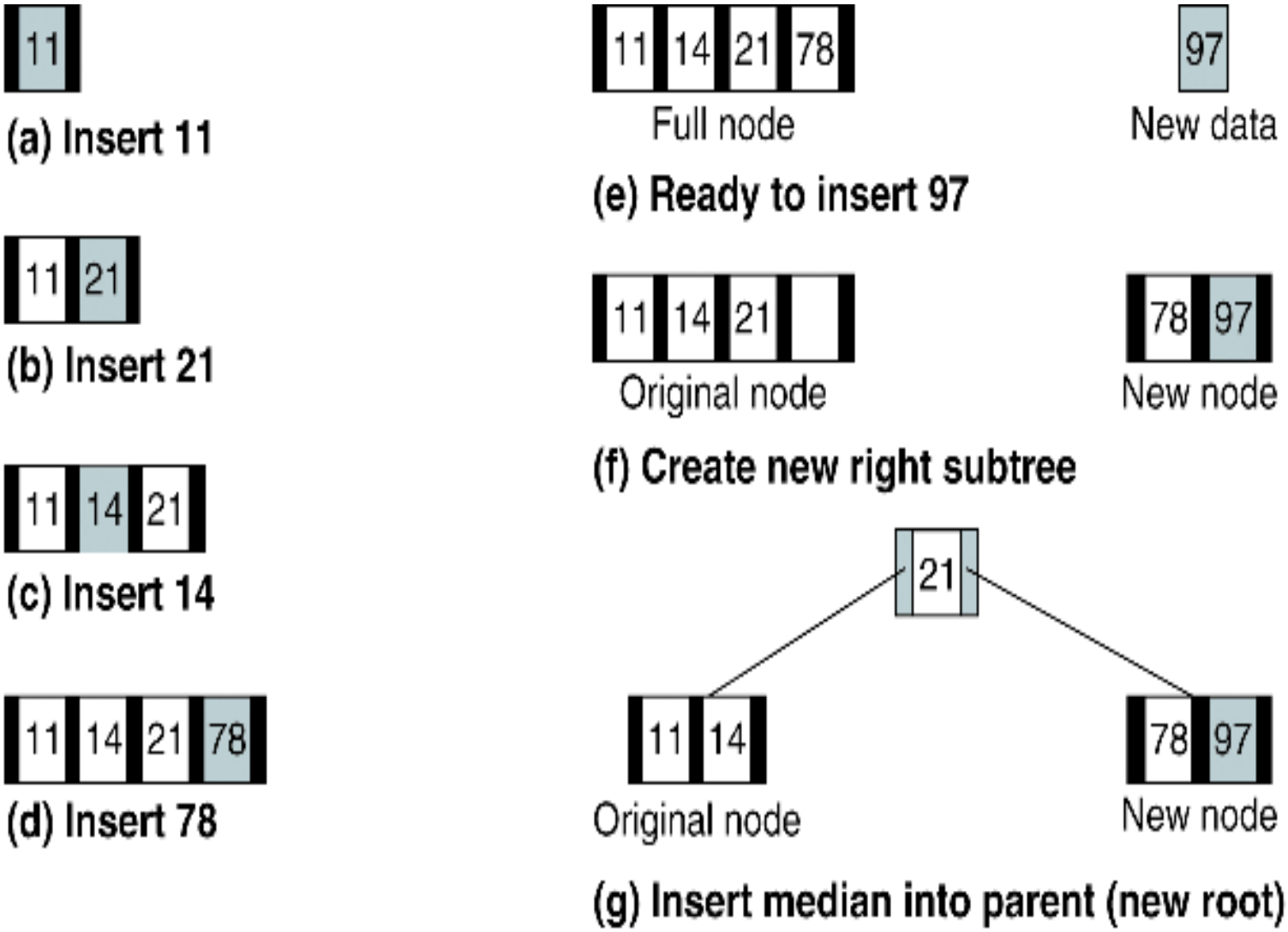
| Order | Number of subtrees | | Number of entries | |
|---|---|---|---|---|
| | Minimum | Maximum | Minimum | Maximum |
| 3 | 2 | 3 | 1 | 2 |
| 4 | 2 | 4 | 1 | 3 |
| 5 | 3 | 5 | 2 | 4 |
| 6 | 3 | 6 | 2 | 5 |
| … | … | … | … | … |
| $m$ | $\lceil m/2 \rceil$ | $m$ | $\lceil m/2 \rceil - 1$ | $m-1$ |

# A B-tree of Order 5

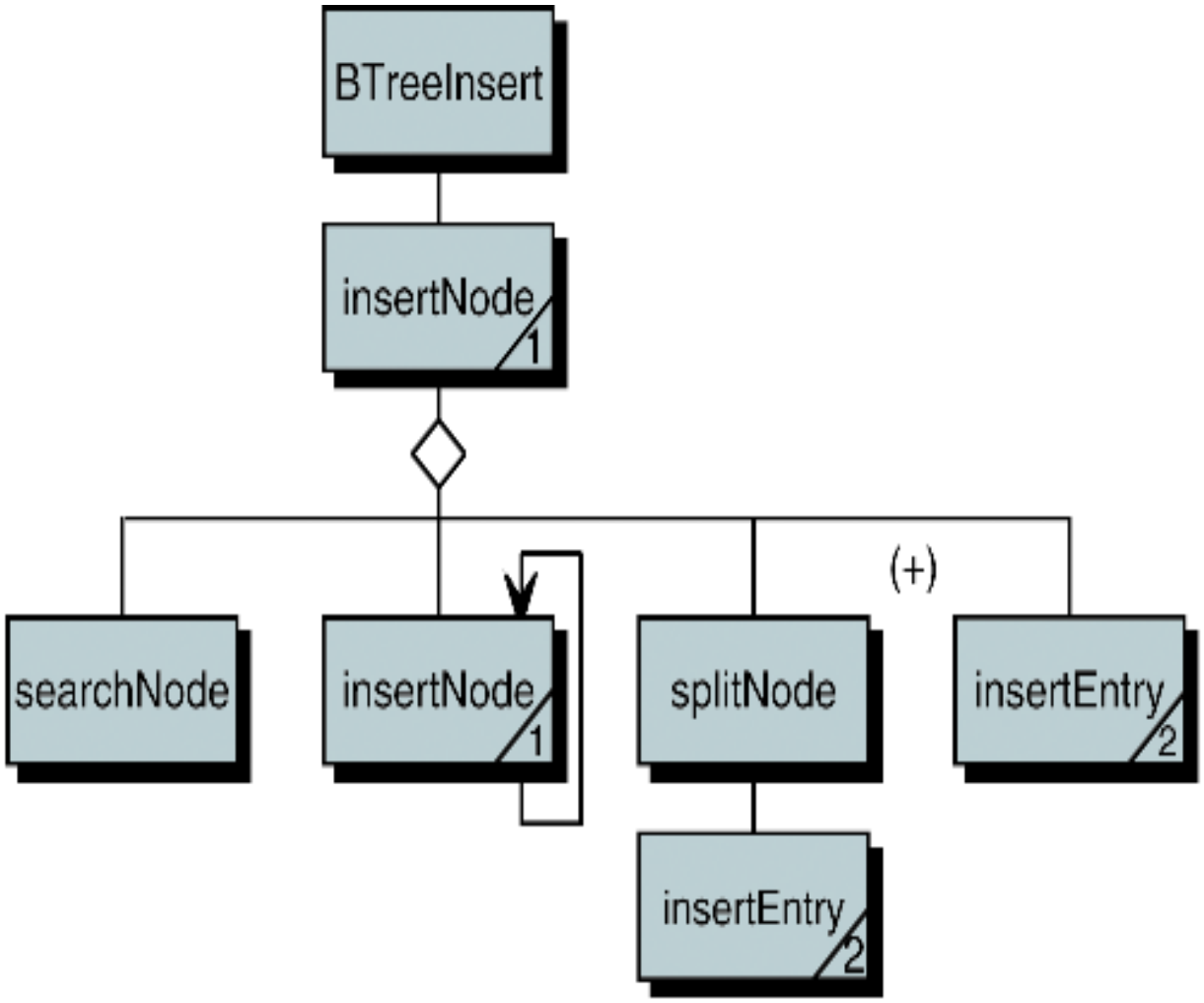
最多幾個 subtree



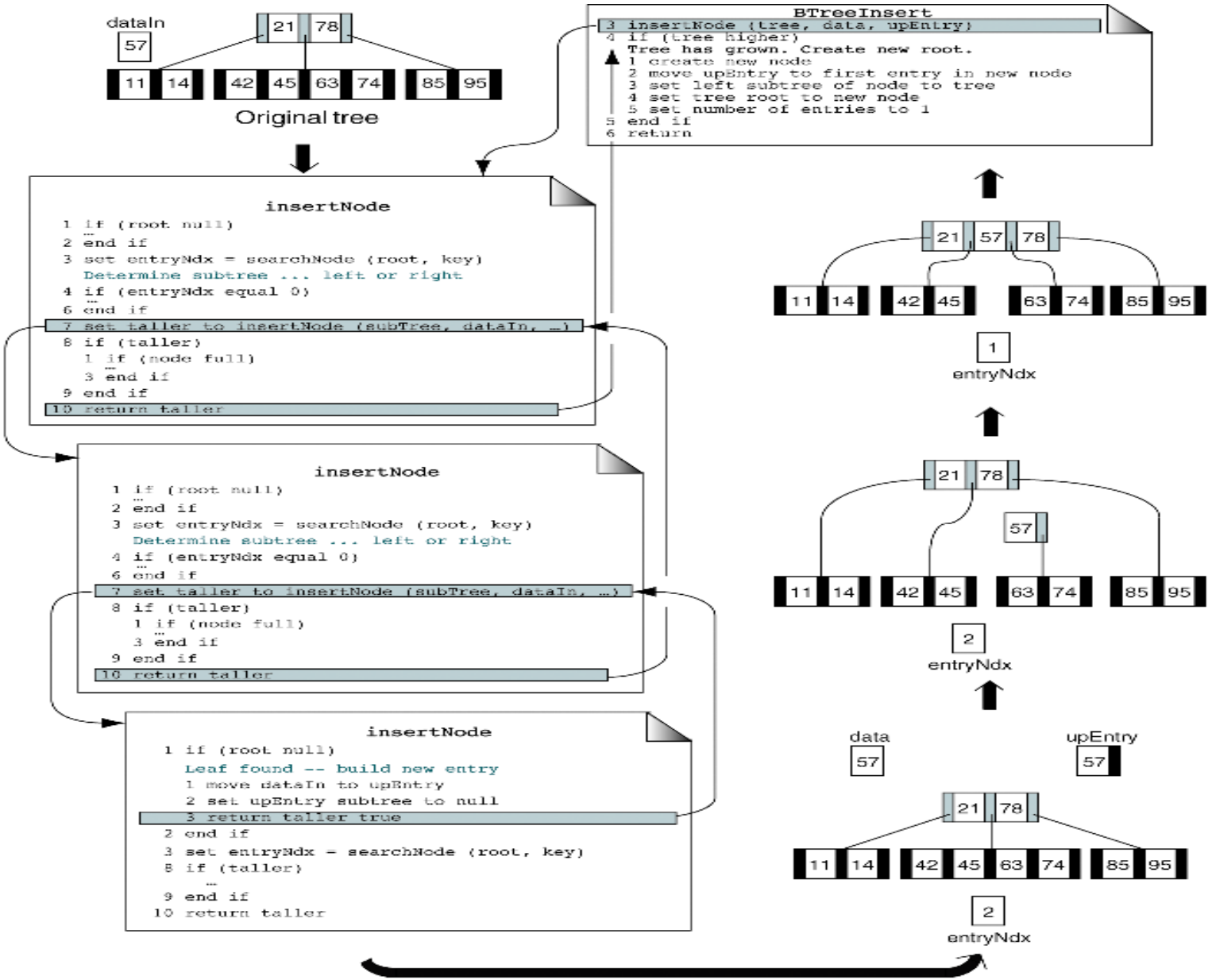
# B-tree Insert Overview



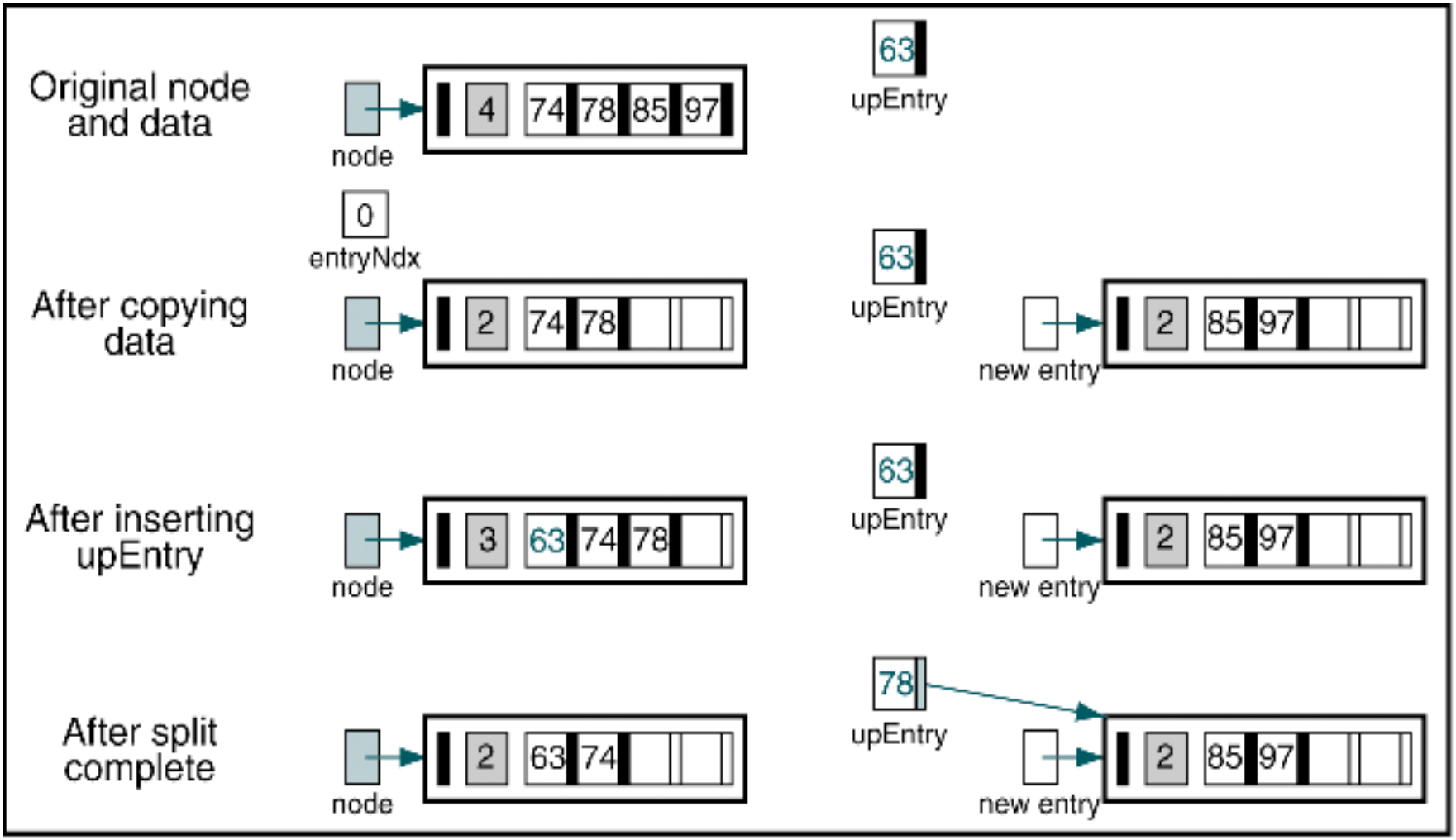
# B-tree Insert Design



# Build B-tree with Overflow

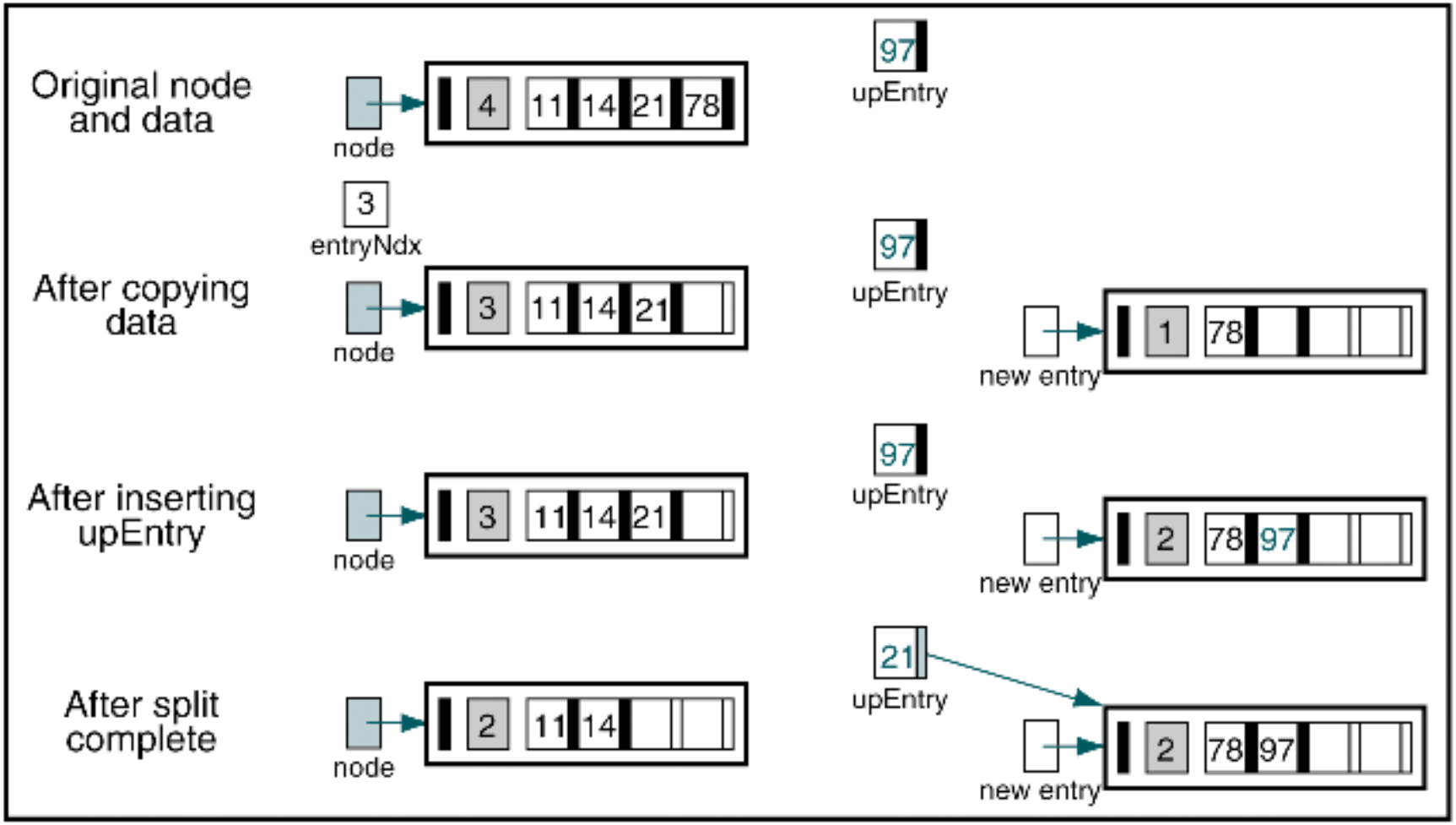


# Split Node B-tree Order of 5




(a) New entry ≤ median

# Split Node B-tree Order of 5 (cont.)

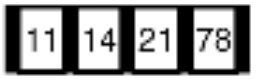



**(b) New entry > median**

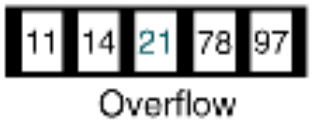
# Building a B-tree of Order 5

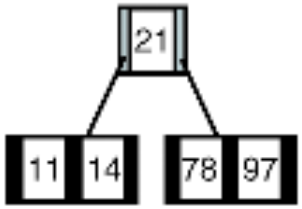
(a) Insert 78, 21, 14, 11

Trees after insert

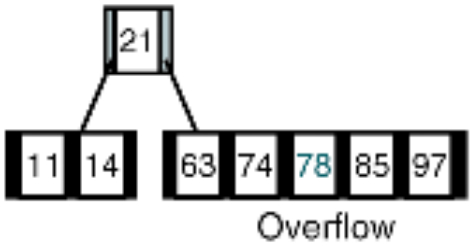


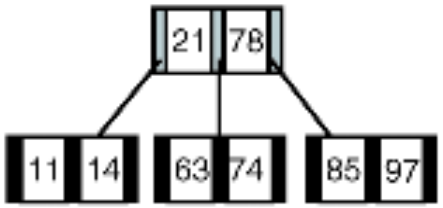

(b) Insert 97



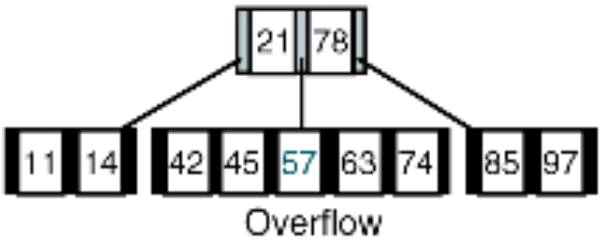




(c) Insert 85, 74, 63



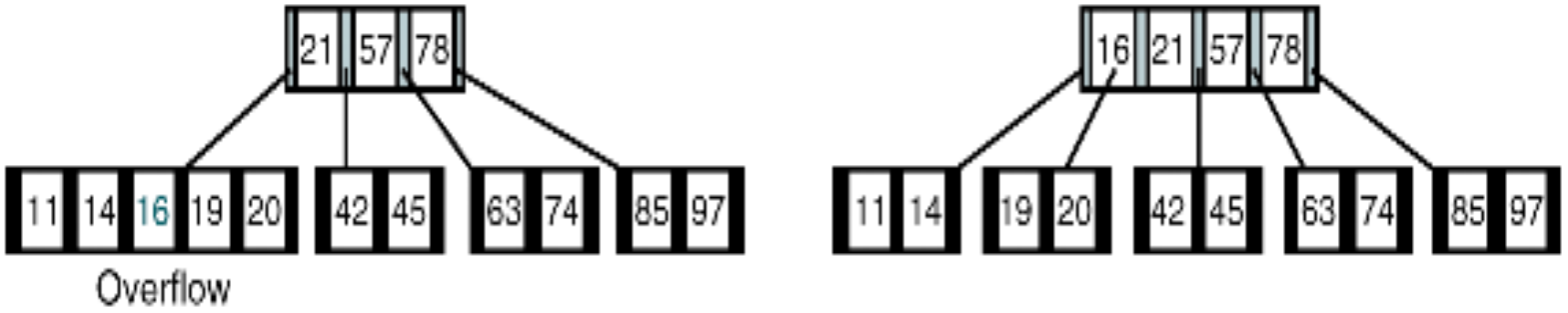




(d) Insert 45, 42, 57

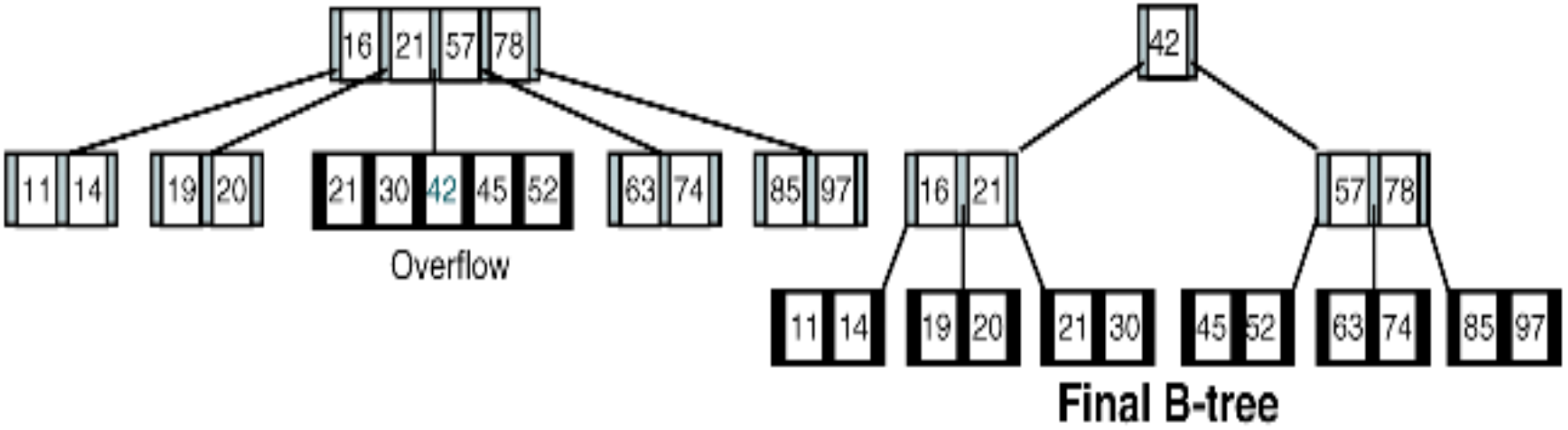



# Building a B-tree of Order 5 (cont.)

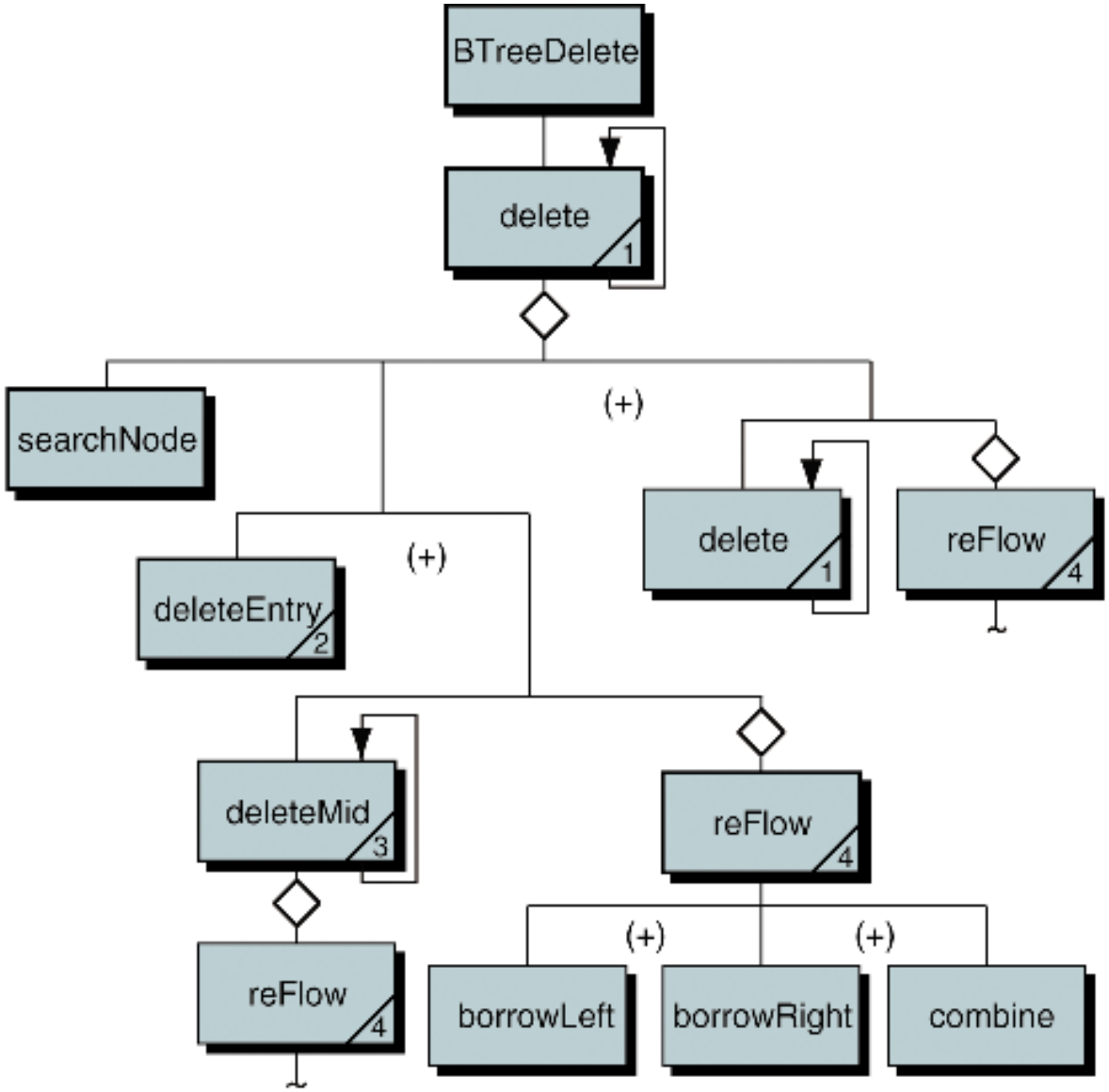
(e) Insert 20, 16, 19



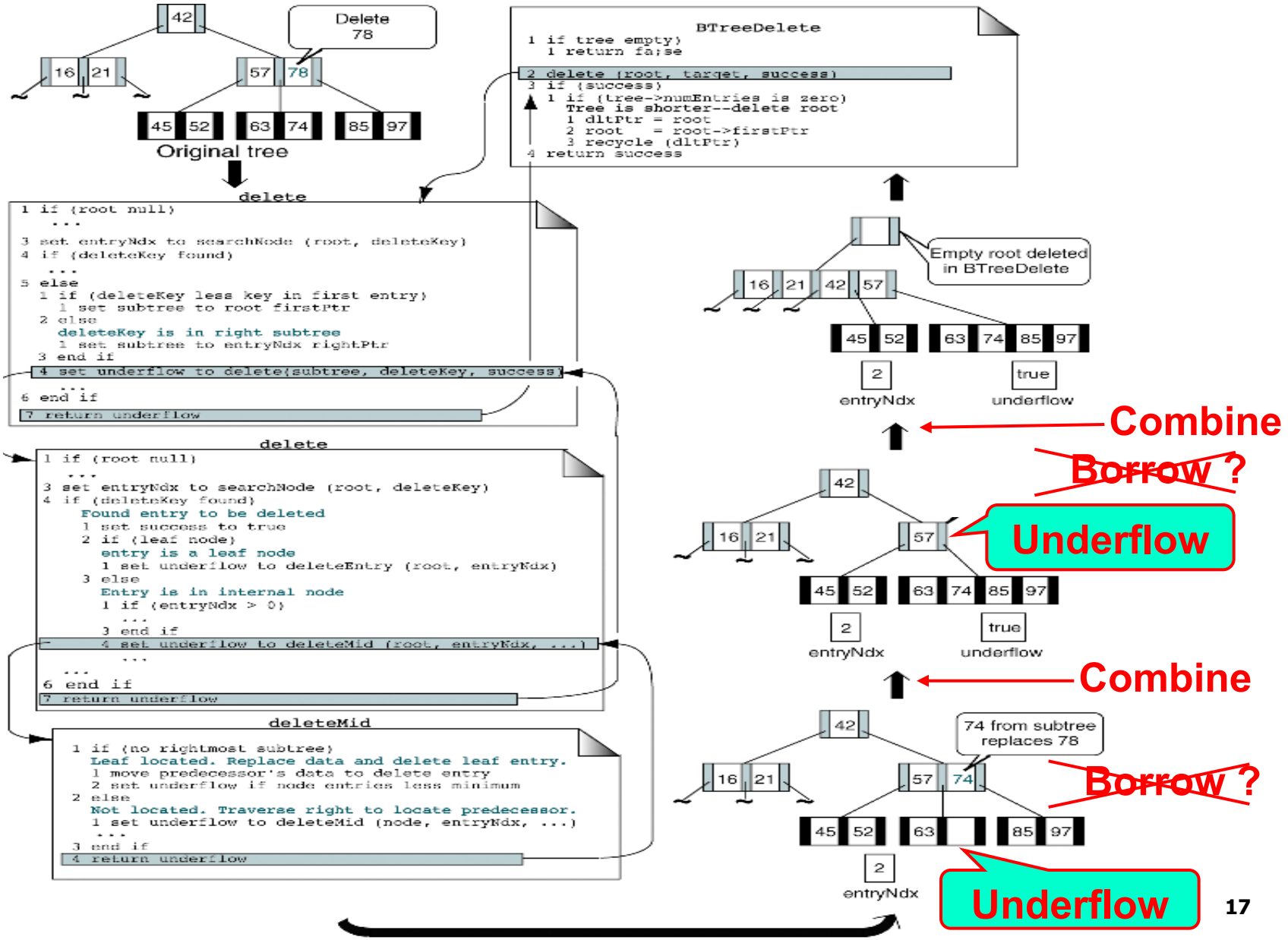

(f) Insert 52, 30, 21



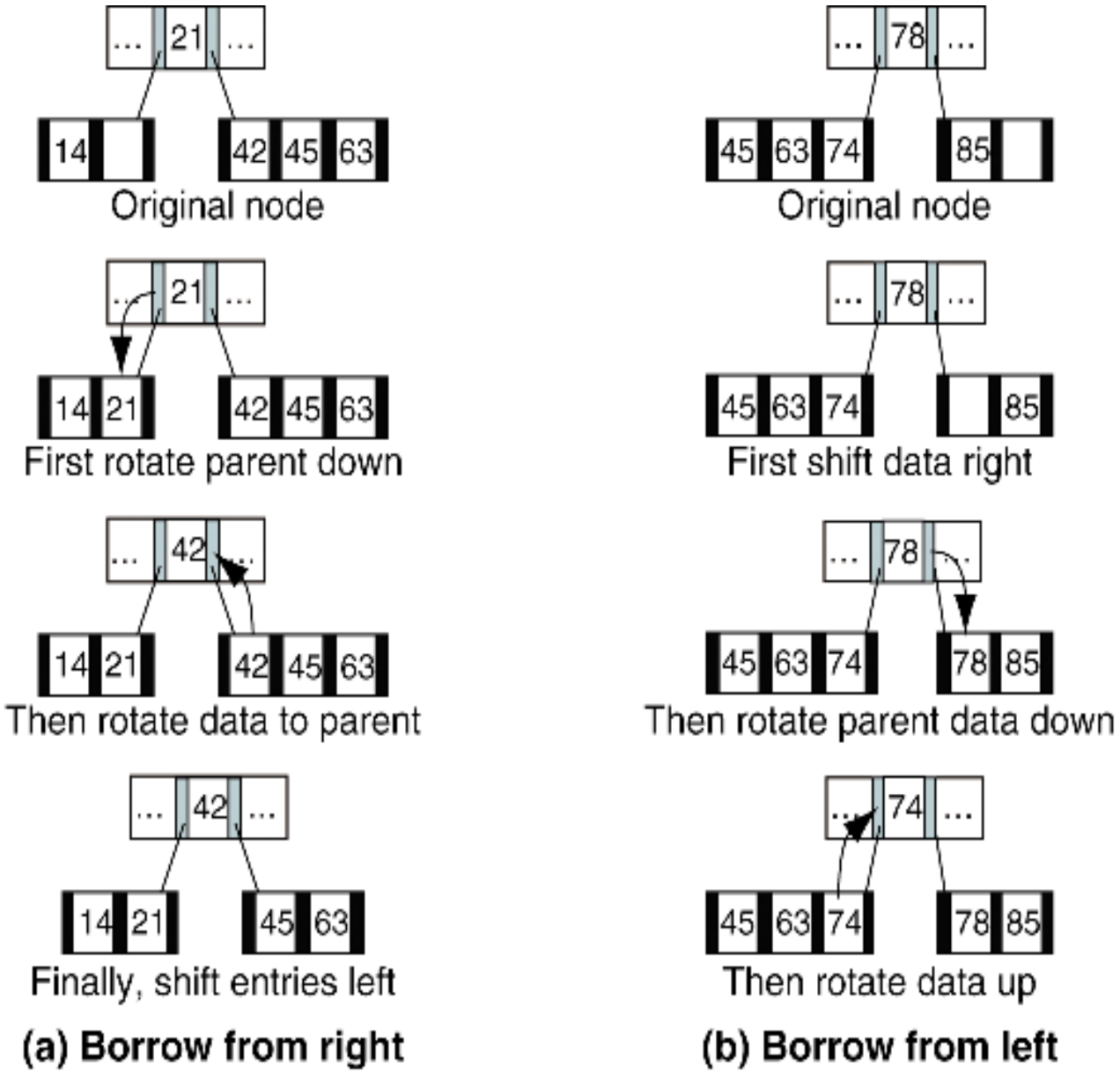
# B-tree Delete Design



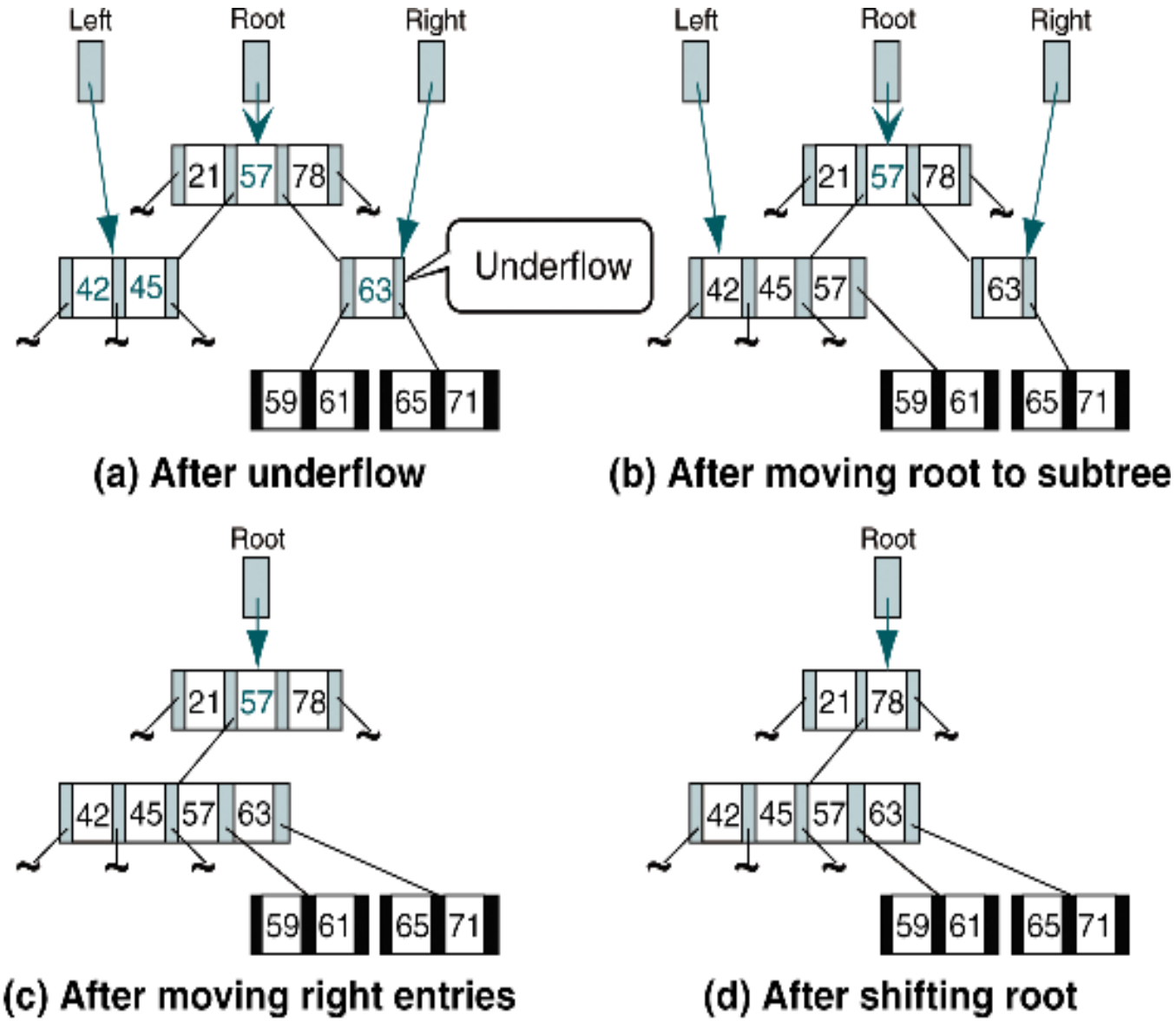
# B-tree Deletions



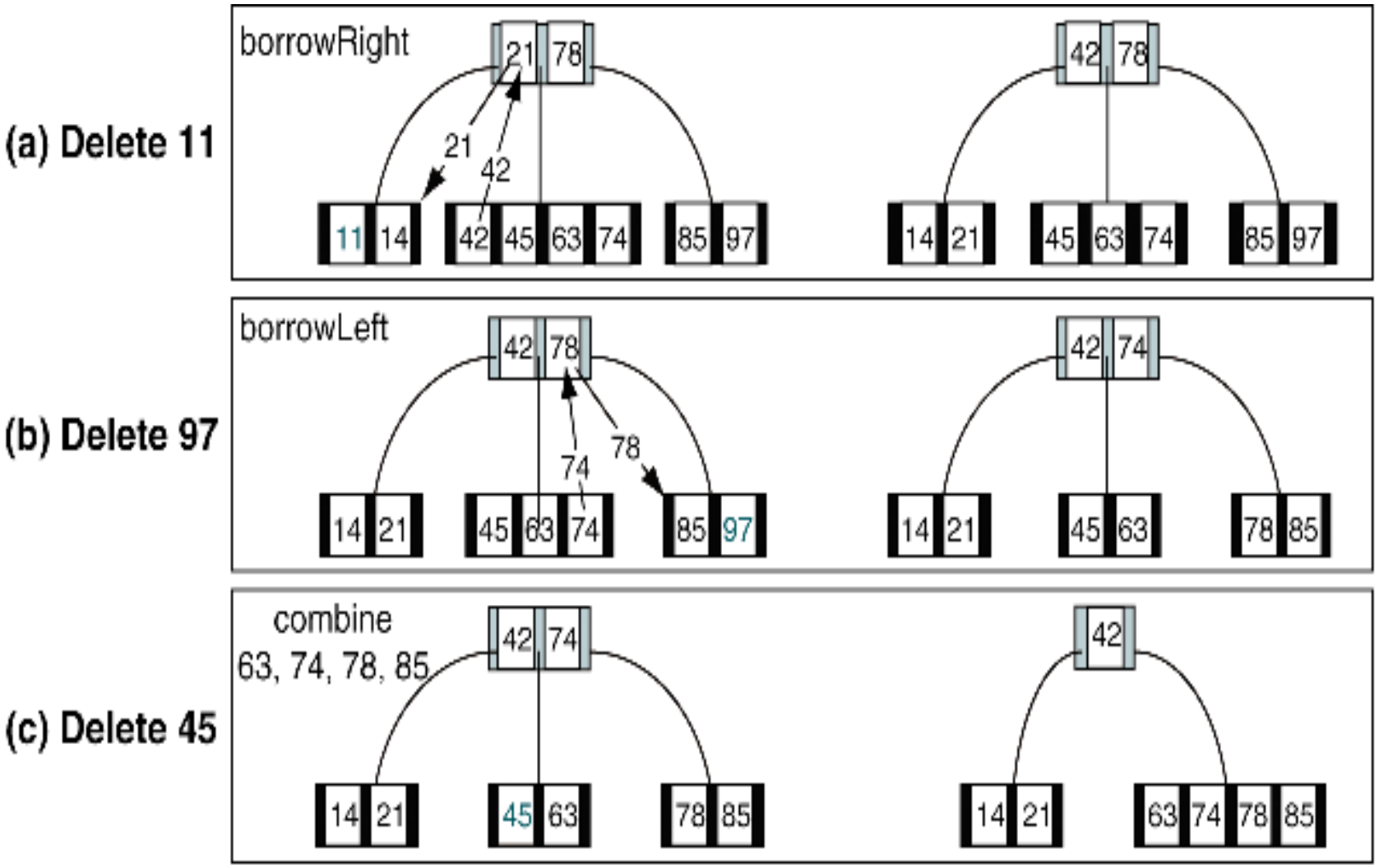
# Restoring Order by Borrowing



# B-tree Combine

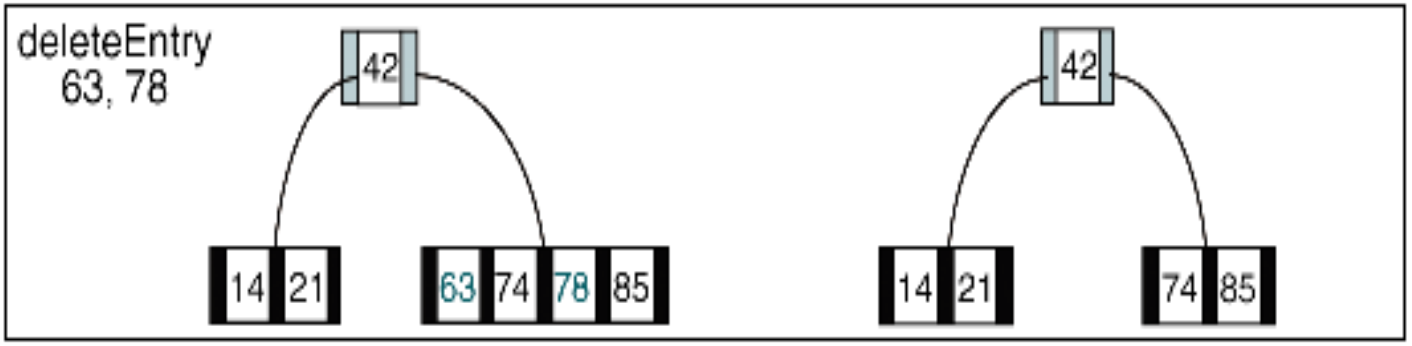


# B-tree Deletion Summary

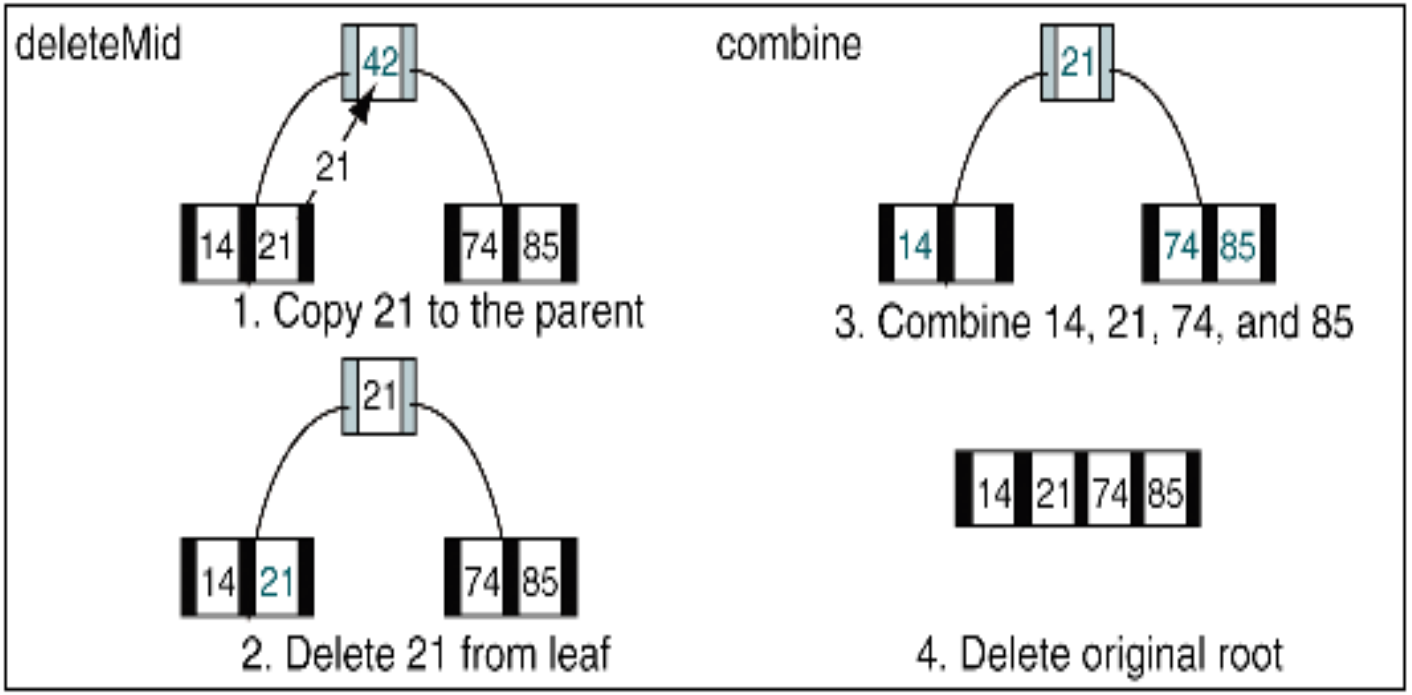


# B-tree Deletion Summary (cont.)

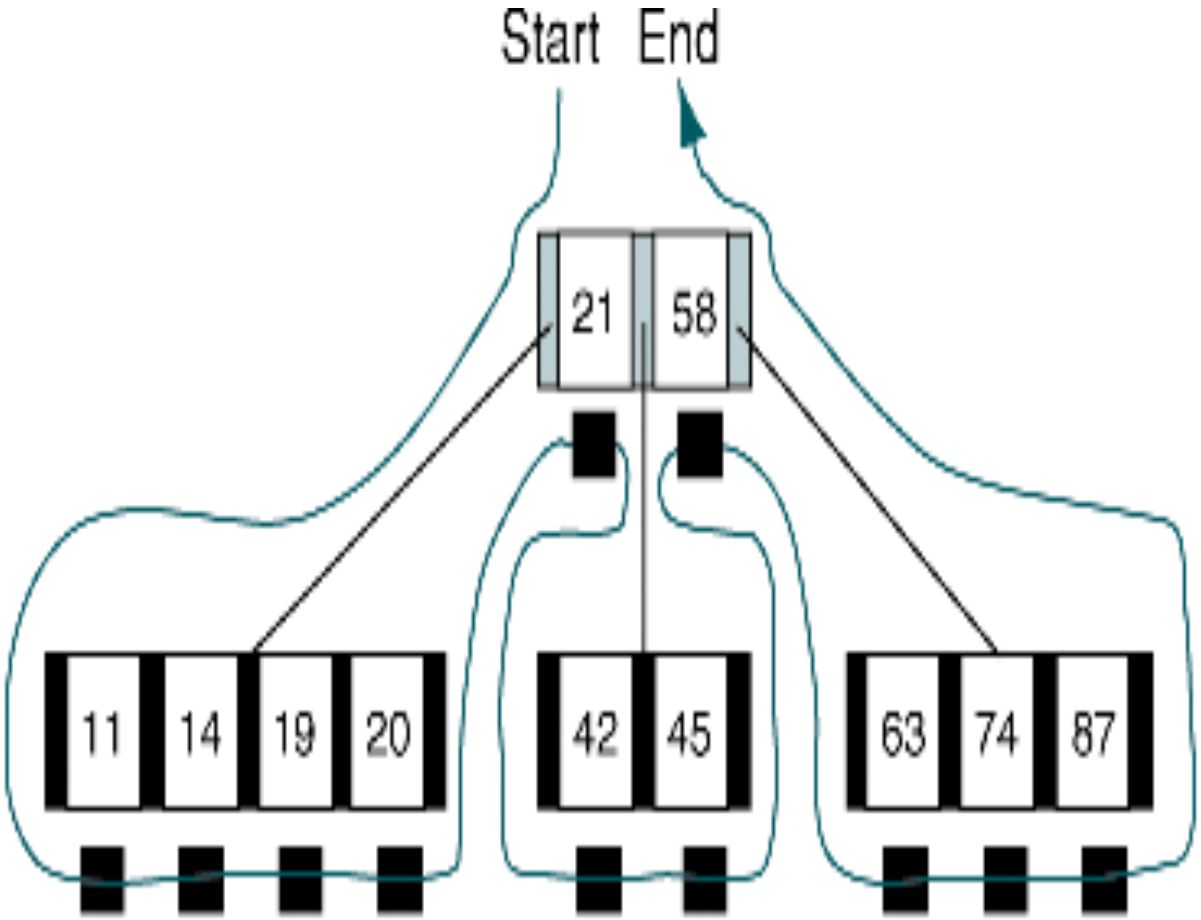
(d) Delete 63 and 78



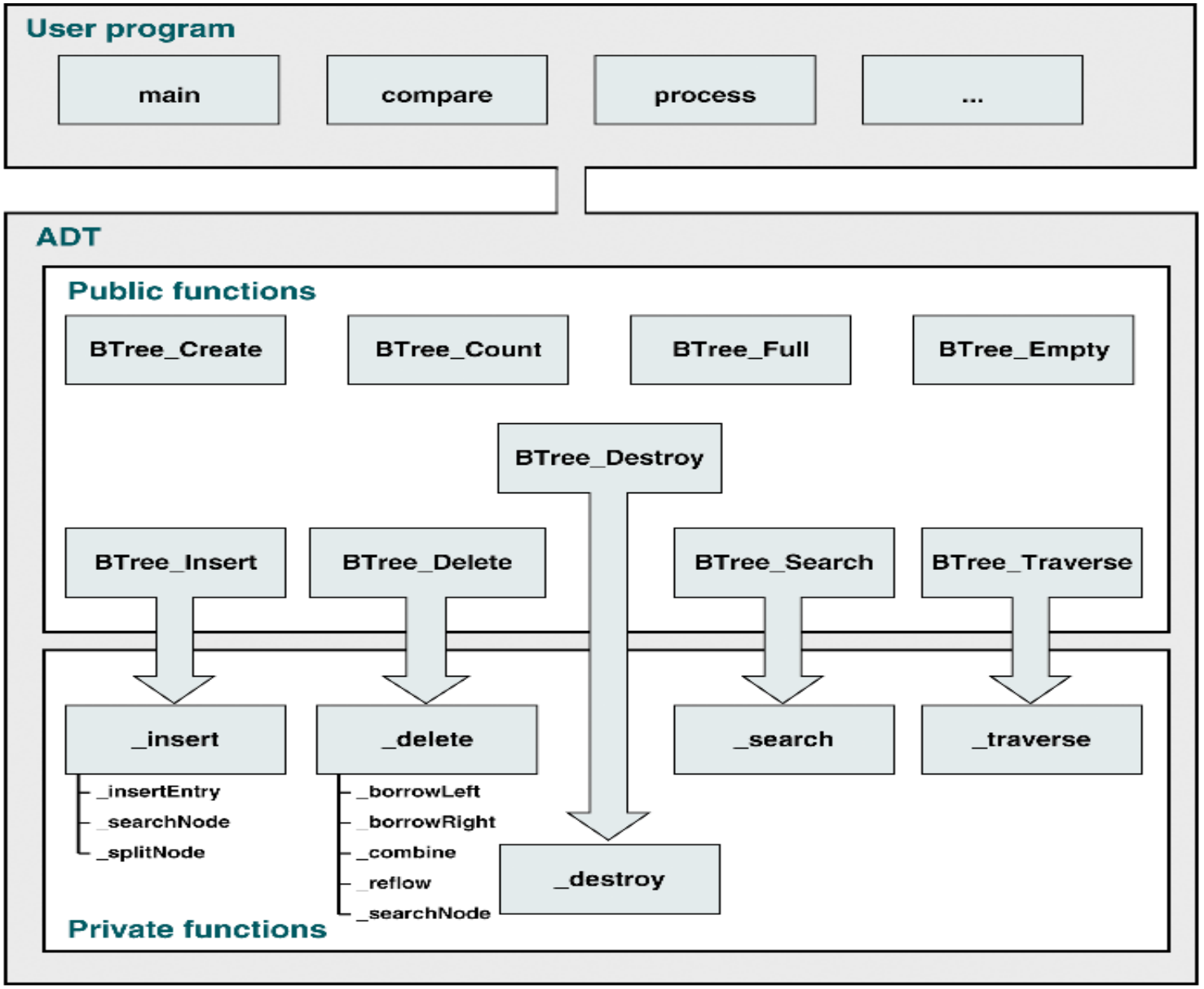

(e) Delete 42



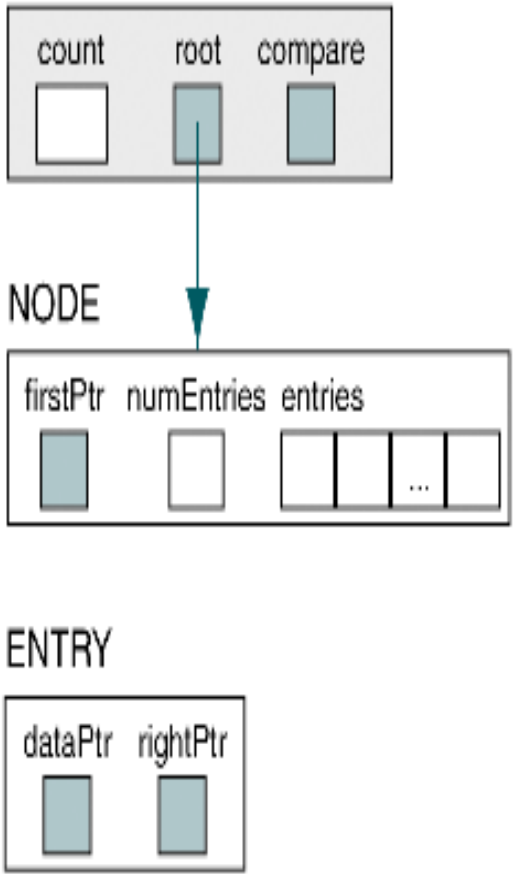
# Basic B-tree Traversal



# 10-3 B-tree ADT



# B-tree Data Structure




```
typedef struct
 {
  void*        dataPtr;
  struct node* rightPtr;
 } ENTRY;

typedef struct node
 {
  struct node* firstPtr;
  int          numEntries;
  ENTRY        entries[ORDER - 1];
 } NODE;

typedef struct
   {
    int   count;
    NODE* root;
    int  (*compare) (void* argu1, void* argu2);
   } BTREE;
```

# B-tree Declaration

```
1  /* ==================== B-Tree.h ====================
2     This header file contains the functions for the AVL
3     Tree abstract data type.
4        Written by:
5        Date:
6  */
7  #include <stdlib.h>
8  #include <stdbool.h>
9
10 // =============== CONSTANTS & MACROS ==============
11 const int ORDER = 5;
12 const int MIN_ENTRIES = (((ORDER + 1) / 2) - 1);
13
14 // ================== STRUCTURES ==================
15 struct node;
16
17 typedef struct
18    {
19     void*        dataPtr;
20     struct node* rightPtr;
21    } ENTRY;
22
23 typedef struct node
```

# B-tree Declaration (cont.)

```
24      {
25       struct node* firstPtr;
26       int          numEntries;
27       ENTRY        entries[ORDER - 1];
28      } NODE;
29
30  typedef struct
31      {
32       int   count;
33       NODE* root;
34       int  (*compare) (void* argu1, void* argu2);
35      } BTREE;
36
37  // =========== Prototype Declarations ===========
38
39  // User interfaces
40  BTREE* BTree_Create
41          (int  (*compare) (void* argu1, void* argu2));
42  void   BTree_Traverse
43          (BTREE* tree, void (*process) (void* dataPtr));
44  BTREE* BTree_Destroy (BTREE* tree);
45  void   BTree_Insert  (BTREE* tree, void* dataInPtr);
46  bool   BTree_Delete  (BTREE* tree, void* dltKey);
47  void*  BTree_Search  (BTREE* tree, void* dataPtr);
48  bool   BTree_Empty   (BTREE* tree);
49  bool   BTree_Full    (BTREE* tree);
50  int    BTree_Count   (BTREE* tree);
```

# B-tree Declaration (cont.)

```
51
52  // Internal BTree functions
53  static void* _search
54                      (BTREE* tree, void*  targetPtr,
55                       NODE*  root);
56  static int   _searchNode
57                      (BTREE* tree, NODE* nodePtr,
58                       void*  target);
59  static bool  _delete
60                      (BTREE* tree,       NODE* root,
61                       void*  dltKeyPtr, bool* success);
62  static bool _insert
63                      (BTREE* tree,       NODE*  root,
64                       void*  dataInPtr, ENTRY* upEntry);
65  static void _traverse
66                      (NODE* root,
67                       void  (*process)(void* dataPtr));
68  static void _splitNode
69                      (NODE*  root,     int    entryNdx,
70                       int compResult, ENTRY* upEntry);
71  static void _insertEntry
```

# B-tree Declaration (cont.)

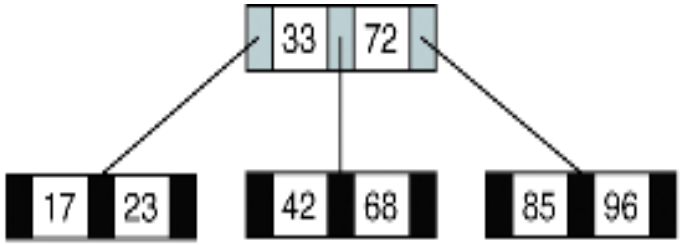
```
72                  (NODE*  root, int entryNdx,
73                   ENTRY  upEntry);
74  static bool _deleteEntry
75                  (NODE*  node, int entryNdx);
76  static bool _deleteMid
77                  (NODE*  root, int entryNdx,
78                   NODE*  leftPtr);
79  static bool _reFlow
80                  (NODE*  root, int entryNdx);
81  static void _borrowLeft
82                  (NODE*  root,     int   entryNdx,
83                   NODE*  leftTree, NODE* rightTree);
84  static void _borrowRight
85                  (NODE*  root,     int   entryNdx,
86                   NODE*  leftTree, NODE* rightTree);
87  static void _combine
88                  (NODE*  root,       int   entryNdx,
89                   NODE*  leftTree,   NODE* rightTree);
90  static void _destroy (NODE* root);
```

# 10-4 Simplified B-tree

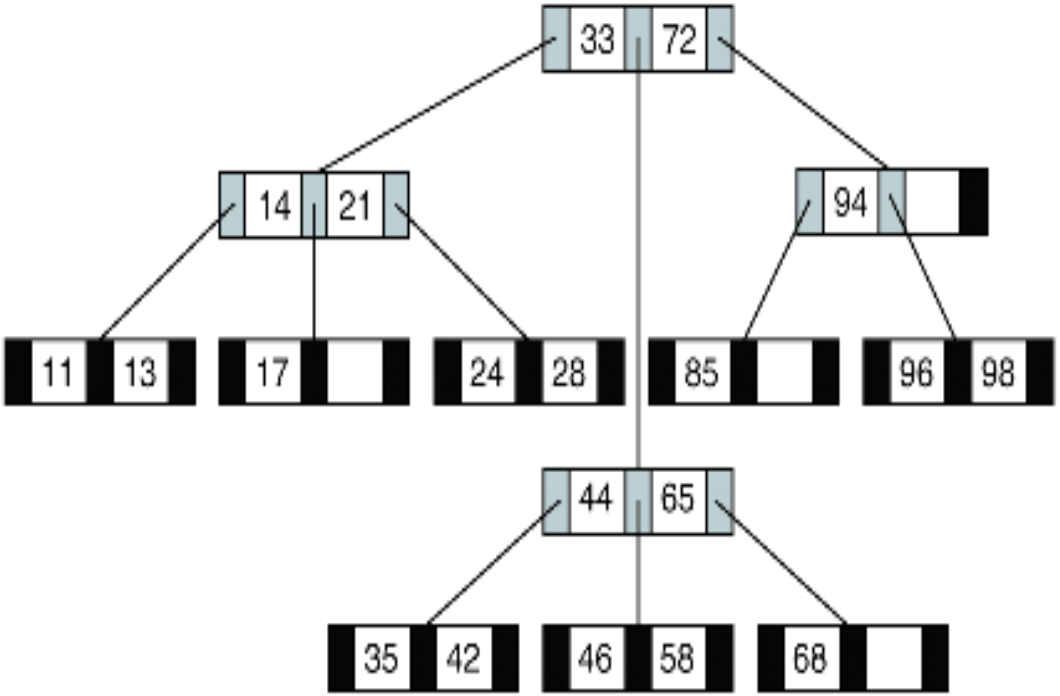
***This section discusses 2 specialized B-tree which have been assigned unique names by computer scientists***

- **2-3 tree**
- **2-3-4 tree**

# 2-3 Trees

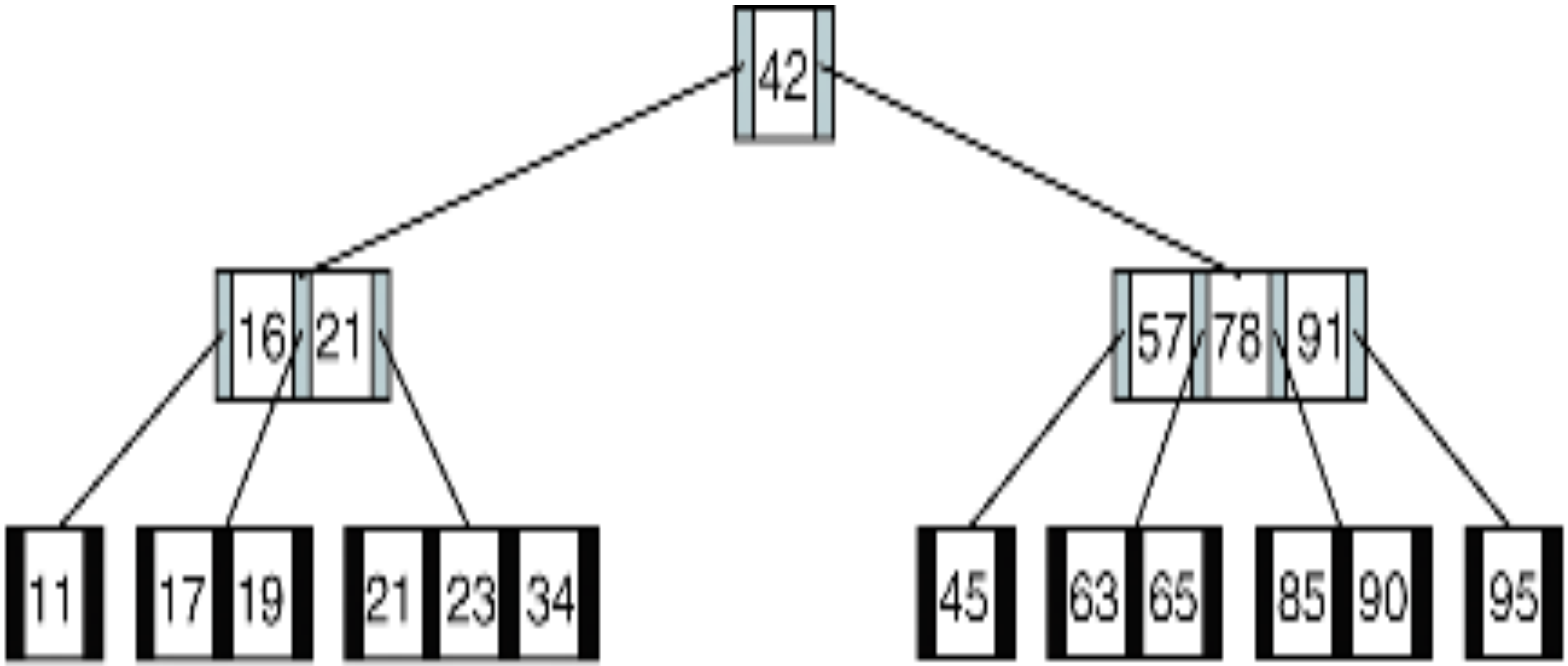



(a) Complete 2-3 tree




(b) 2-3 tree with empty entries

# 2-3-4 Tree



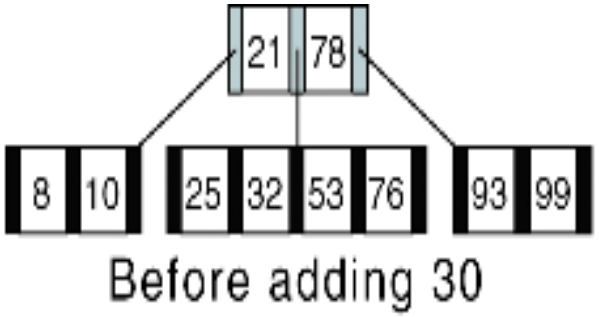
# 10-5 B-tree Variations

*This section discusses two popular variations on the B-tree*

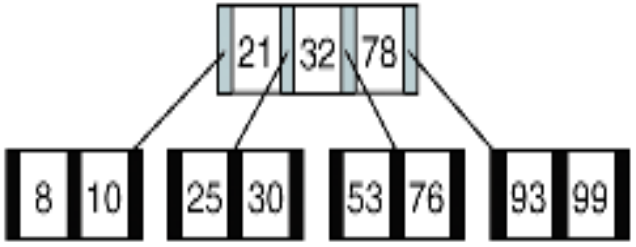
- B* tree
- B+ tree

# B* tree Insertion


Before adding 30

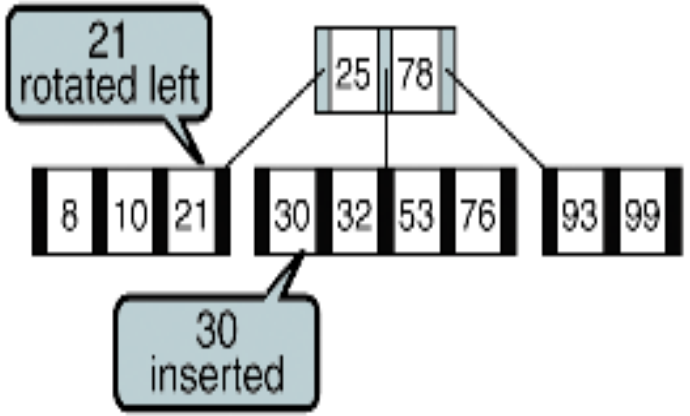



Before adding 30




Result after overflow

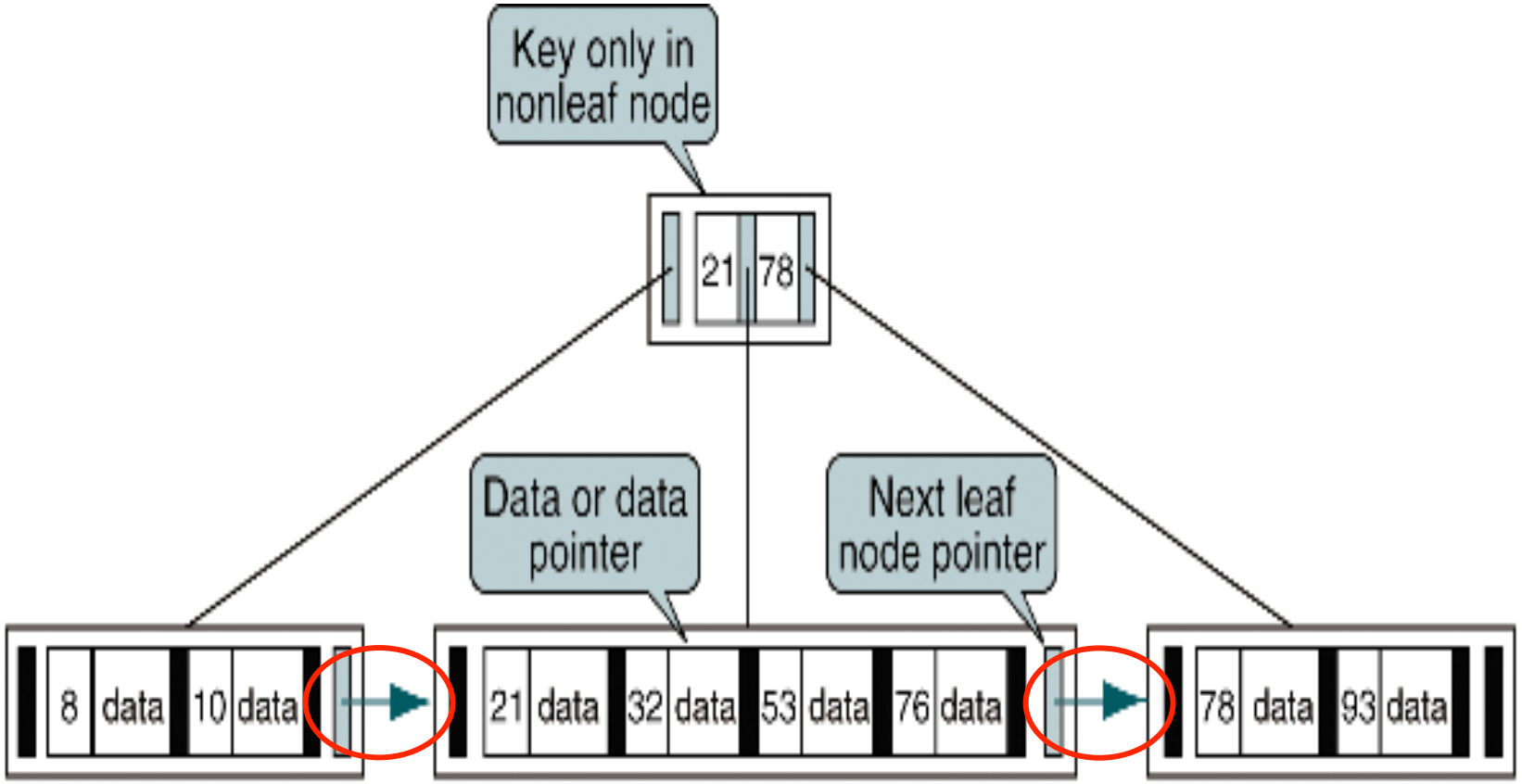
(a) Insertion into B-tree of order 5




Result after redistribution

(b) Insertion into B*tree of order 5

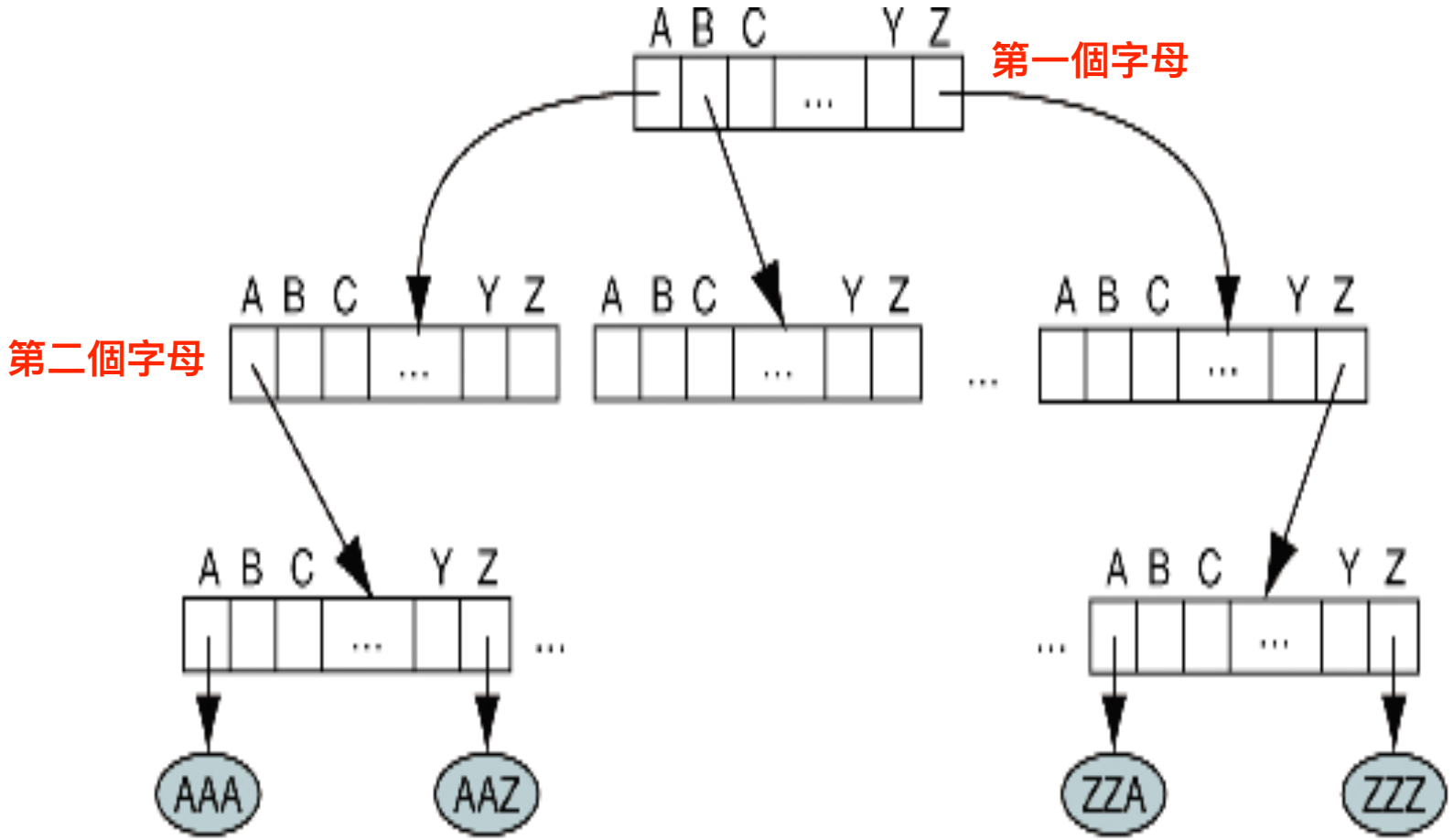
# B+ tree 結合了linear list跟tree的優點




leaf node才有data

# 10-6 Lexical Search Tree

字典可能會用這種結構

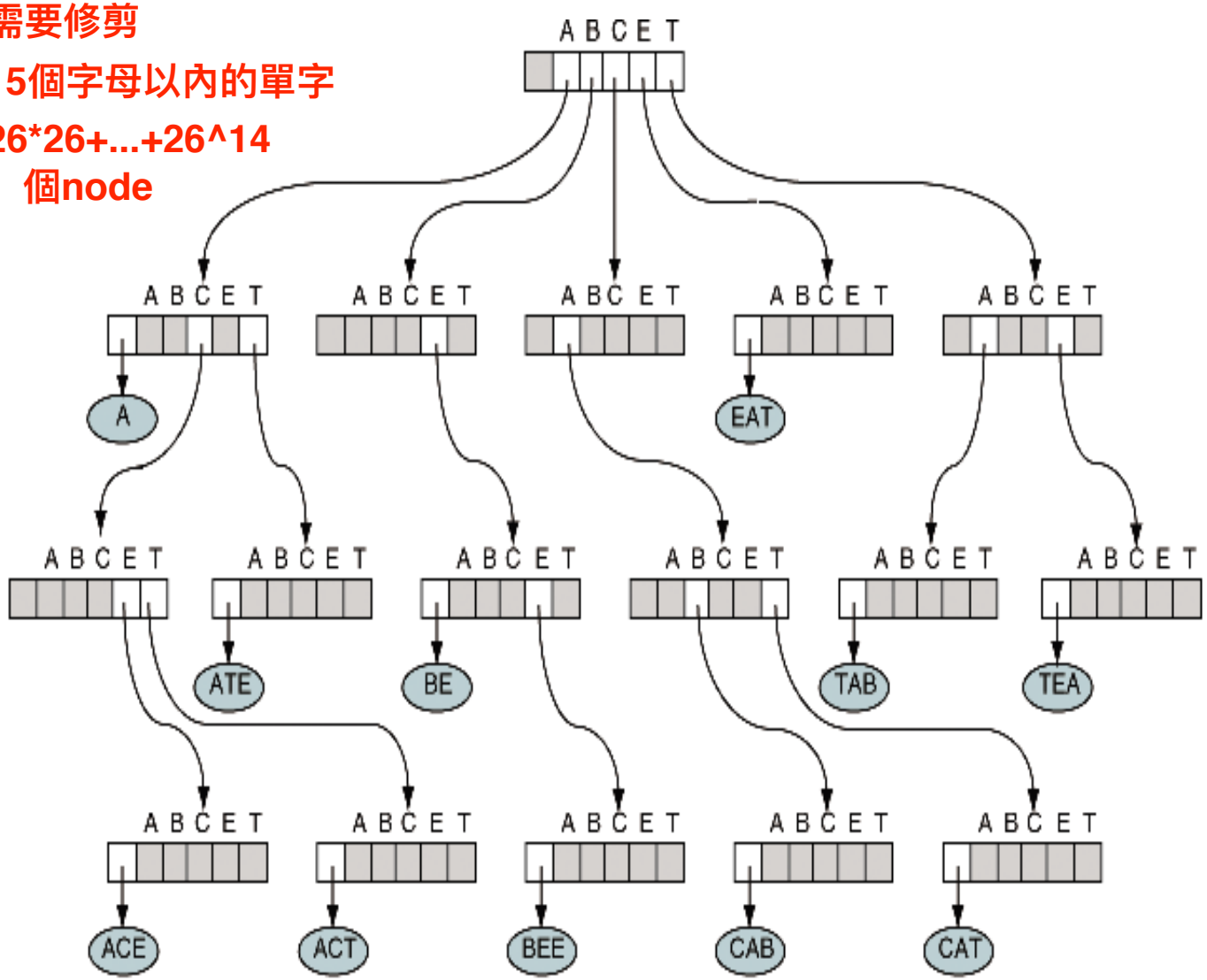


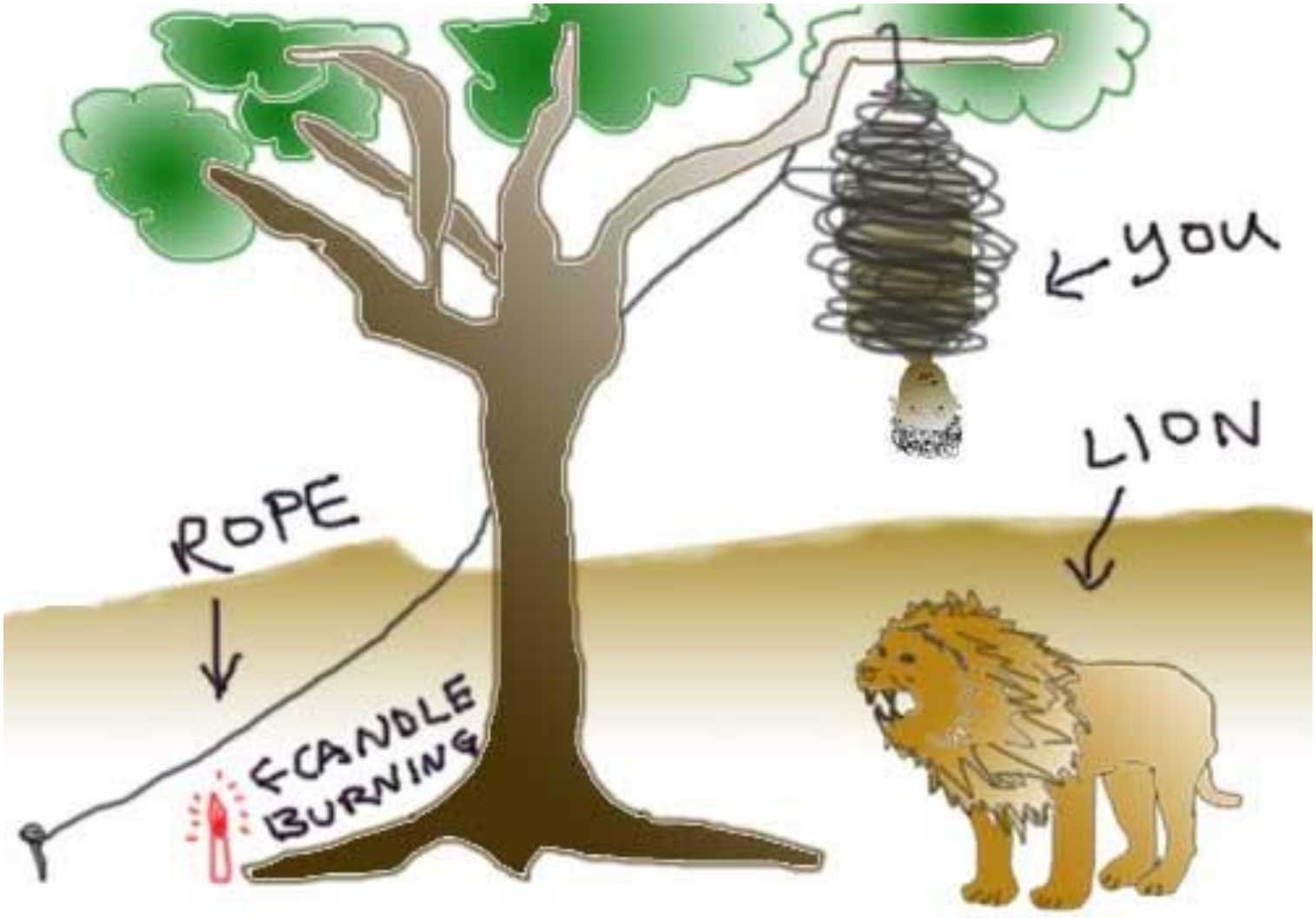
# Spell Checker Trie

實務上可能需要修剪

要儲存所有15個字母以內的單字

需要1+26+26*26+...+26^14（約10^19）個node




you
LION
ROPE
CANDLE
BURNING


HAPPY BIRTHDAY TO YOU
CANDLE NO FIRE!
HAPPY LION


NEED CASH
FOR ALCOHOL
RESEARCH

**With the high rate of attacks on women in secluded parking lots, especially during evening hours, the Minneapolis City Council has established a "Women Only" parking lot at the Mall of America. Even the parking lot attendants are exclusively female so that a comfortable and safe environment is created for patrons.**

**Below is the first picture available of this world-first women-only parking lot in Minnesota.**