

Chapter 11

Graphs

Objectives

Upon completion you will be able to:

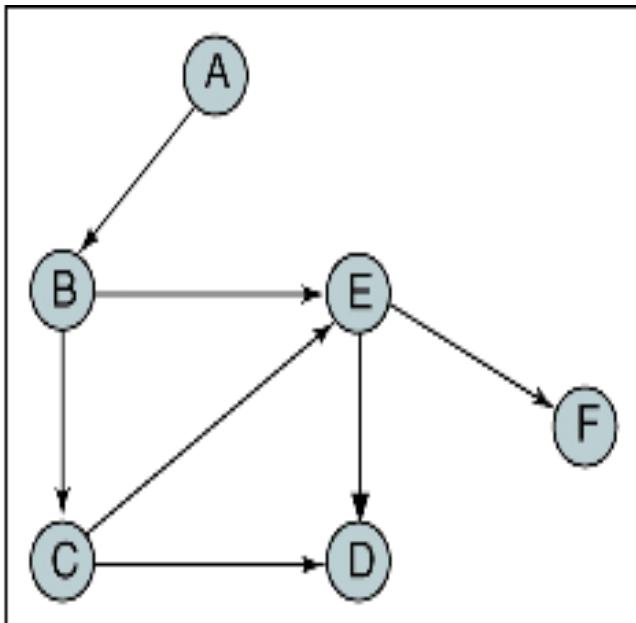
- Understand and use basic graph terminology and concepts
- Define and discuss basic graph and network structures
- Design and implement graph and network applications
- Design and implement applications using the graph ADT
- Define and discuss Dijkstra's shortest path algorithm

11-1 Basic Concepts

A graph is a collection of nodes, called vertices, and a collection of segments, called lines, connecting pairs of vertices. In Basic Concepts we develop the terminology and structure for graphs. In addition to the graph definitions, discussion points include:

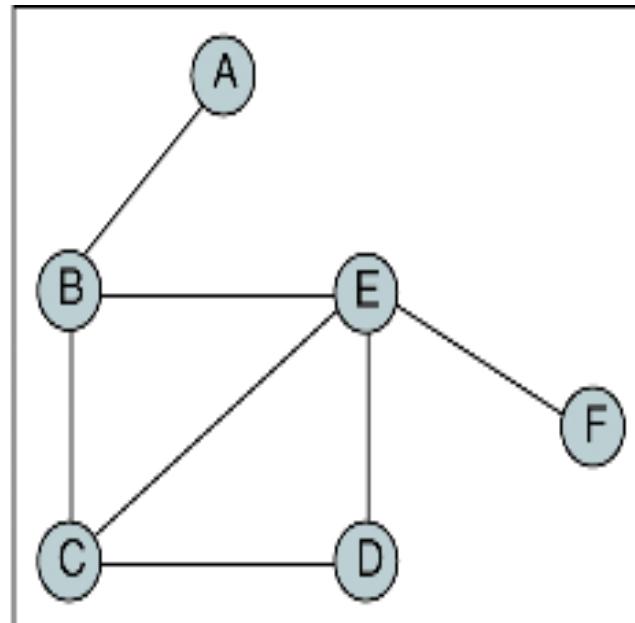
- **Directed and Undirected Graphs**
- **Cycles and Loops**
- **Connected and Disjoint Graphs**

Directed and Undirected Graphs



(a) Directed graph

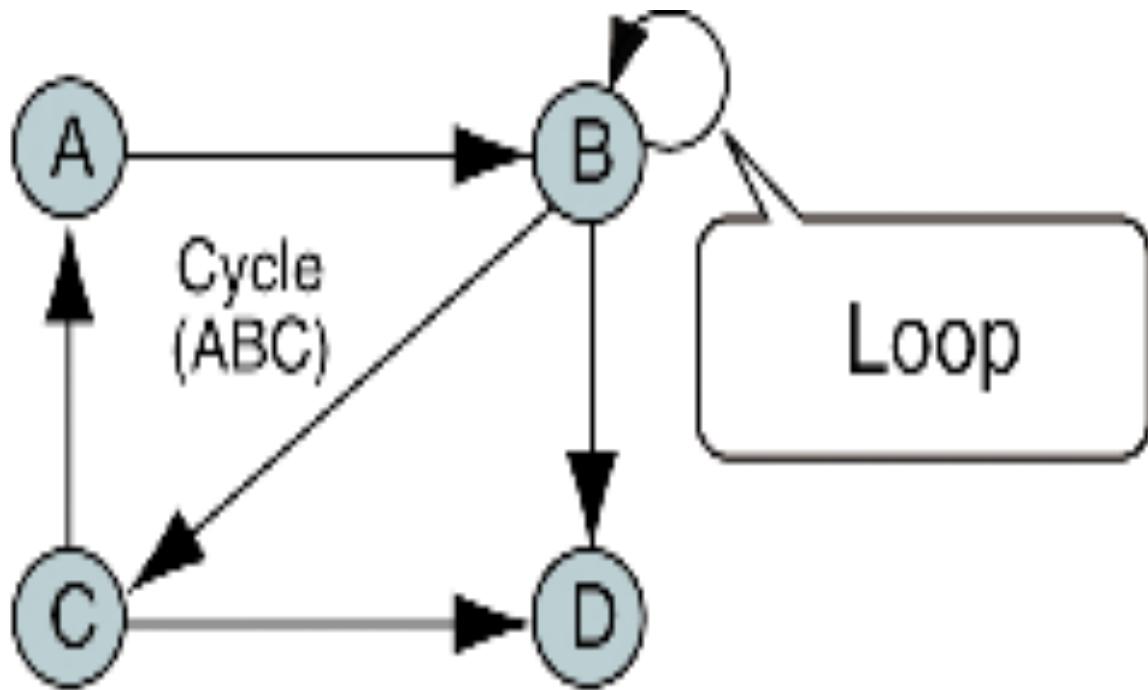
有向圖



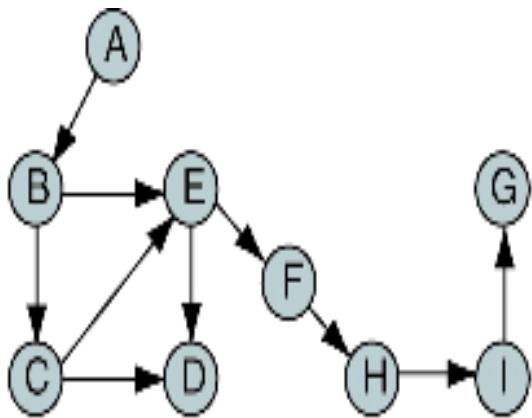
(b) Undirected graph

無向圖

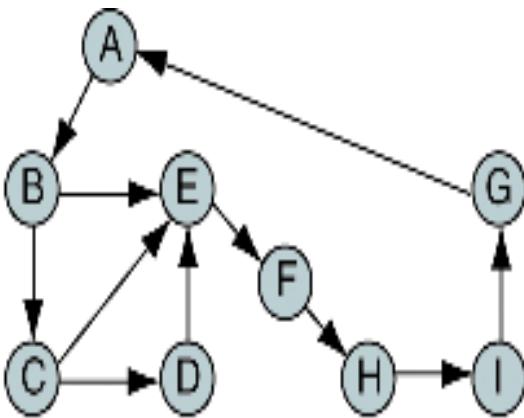
Cycles and Loops



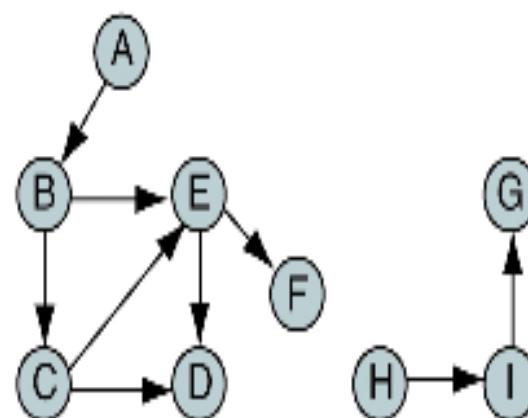
Connected and Disjoint Graphs



(a) Weakly connected



(b) Strongly connected



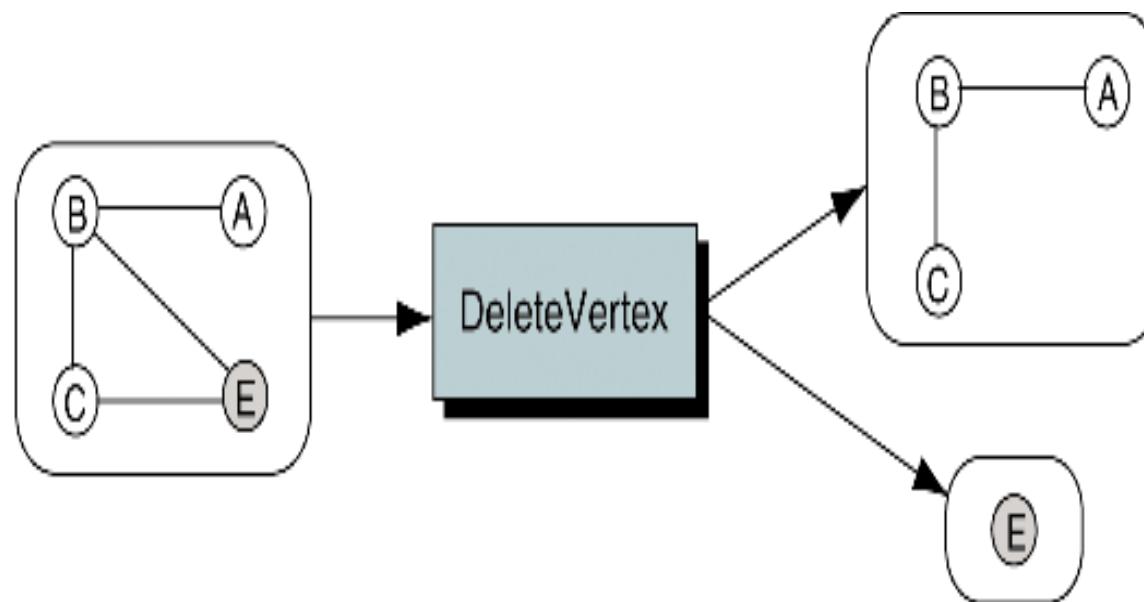
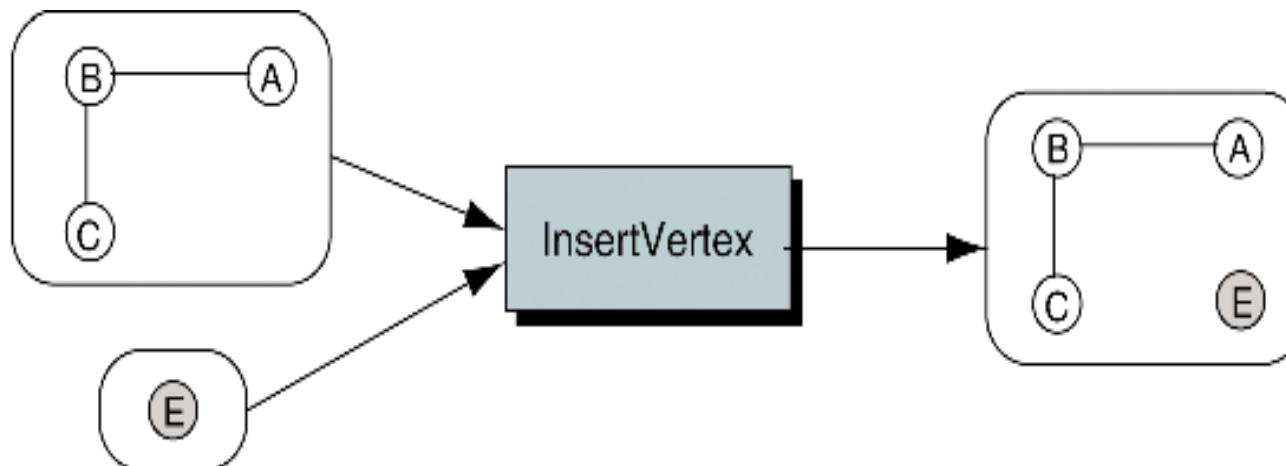
(c) Disjoint graph

11-2 Operations

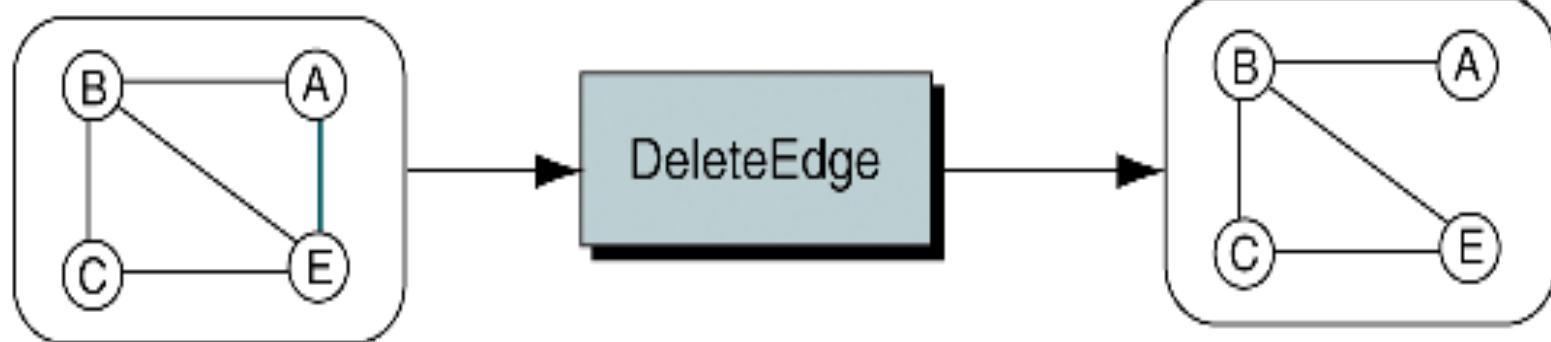
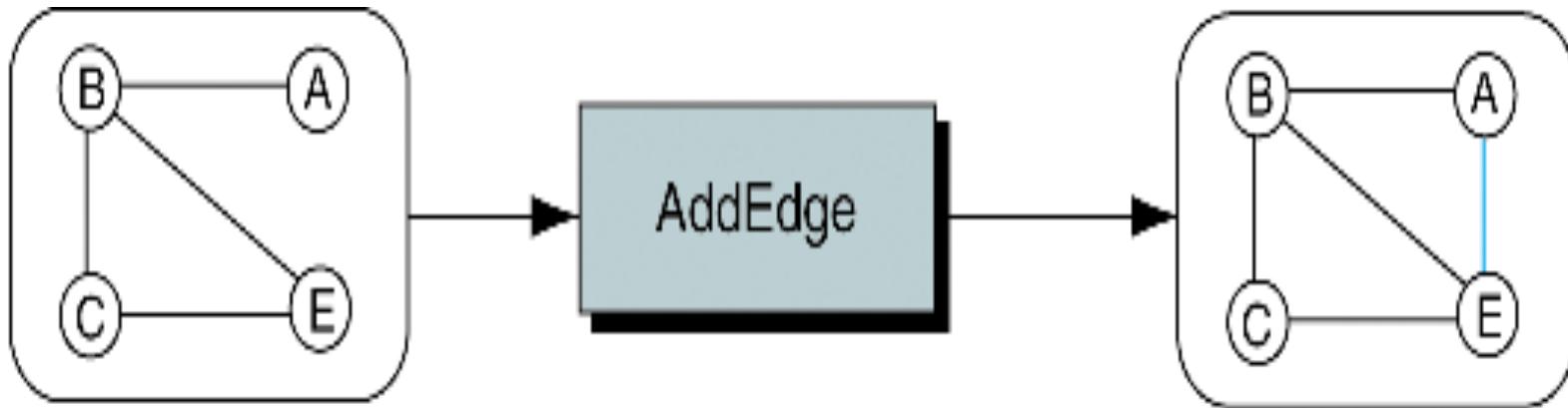
We define and discuss the six primitive graph operations required to maintain a graph.

- Insert Vertex 頂點
- Delete Vertex
- Add Edge
- Delete Edge
- Find Vertex
- Traverse Graph

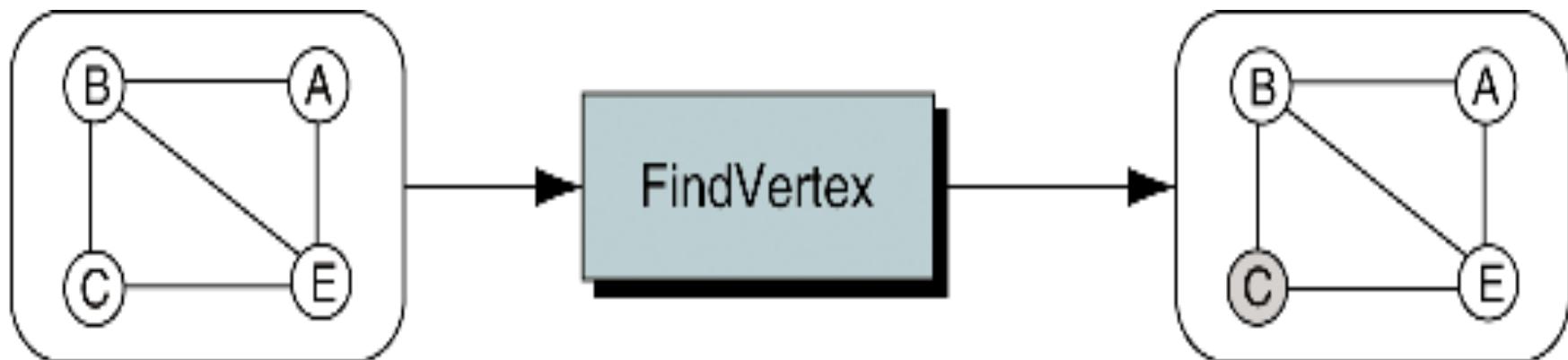
Insert and Delete Vertex



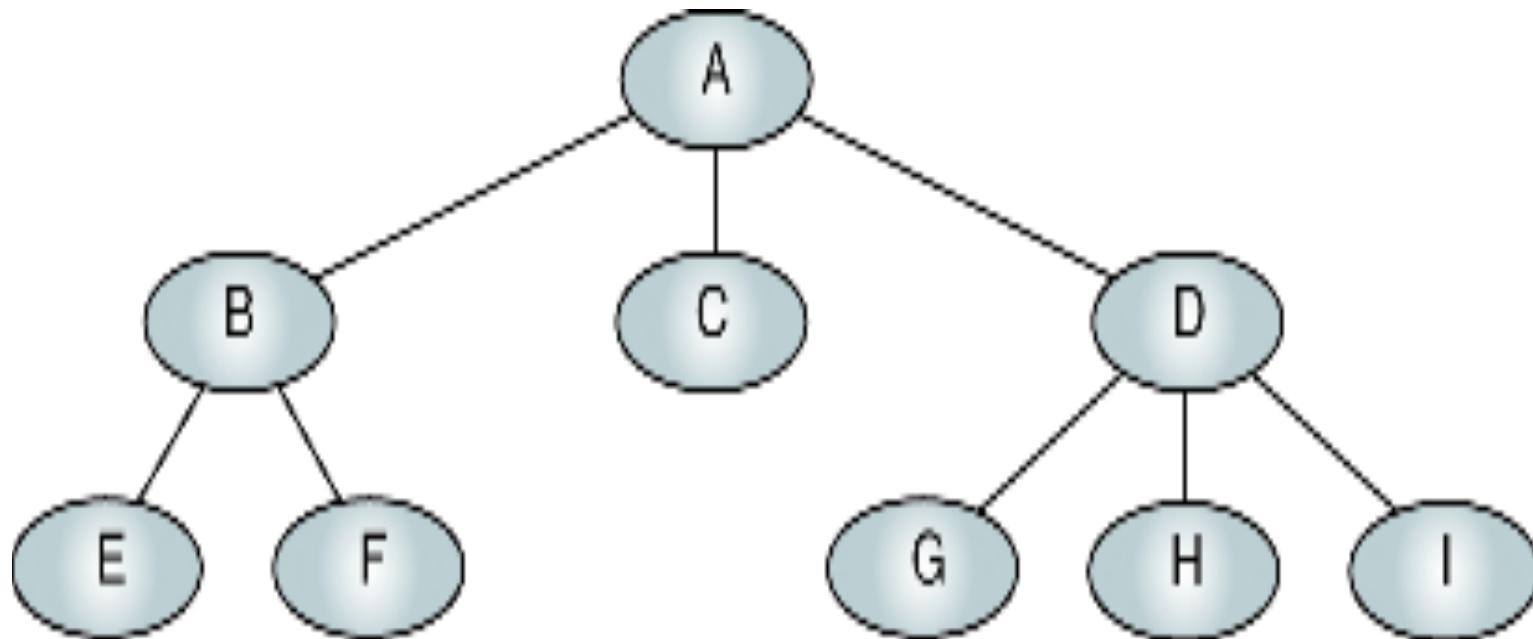
Add and Delete Edge



Find Vertex

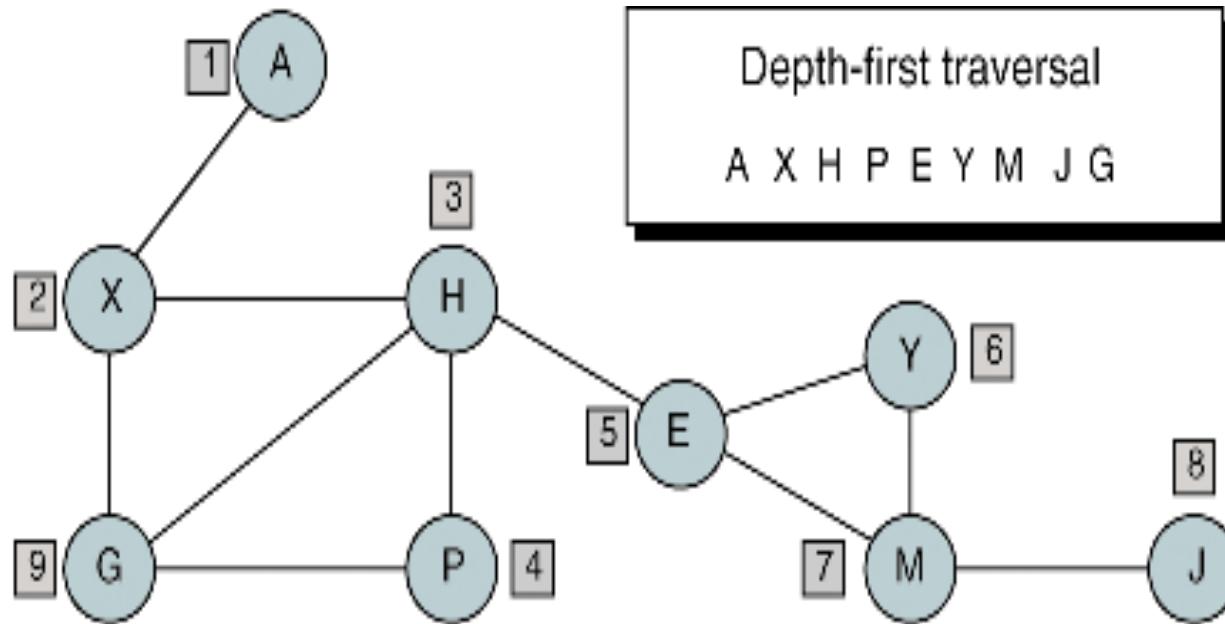


Depth-first Traversal of Tree

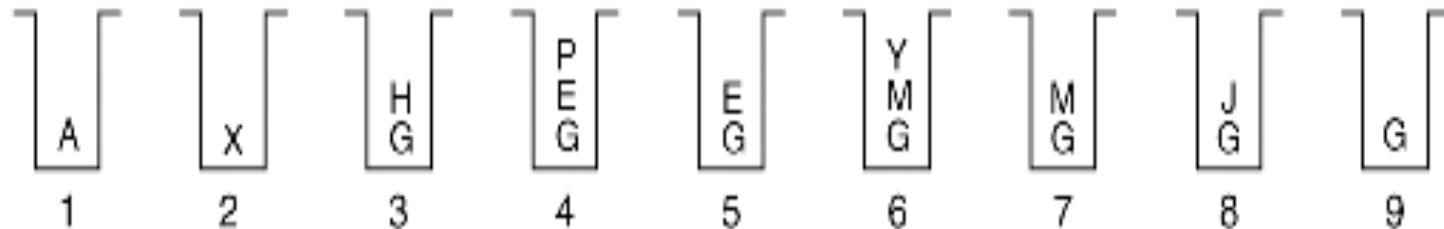


Depth-first traversal: A B E F C D G H I

Depth-first Traversal of a Graph

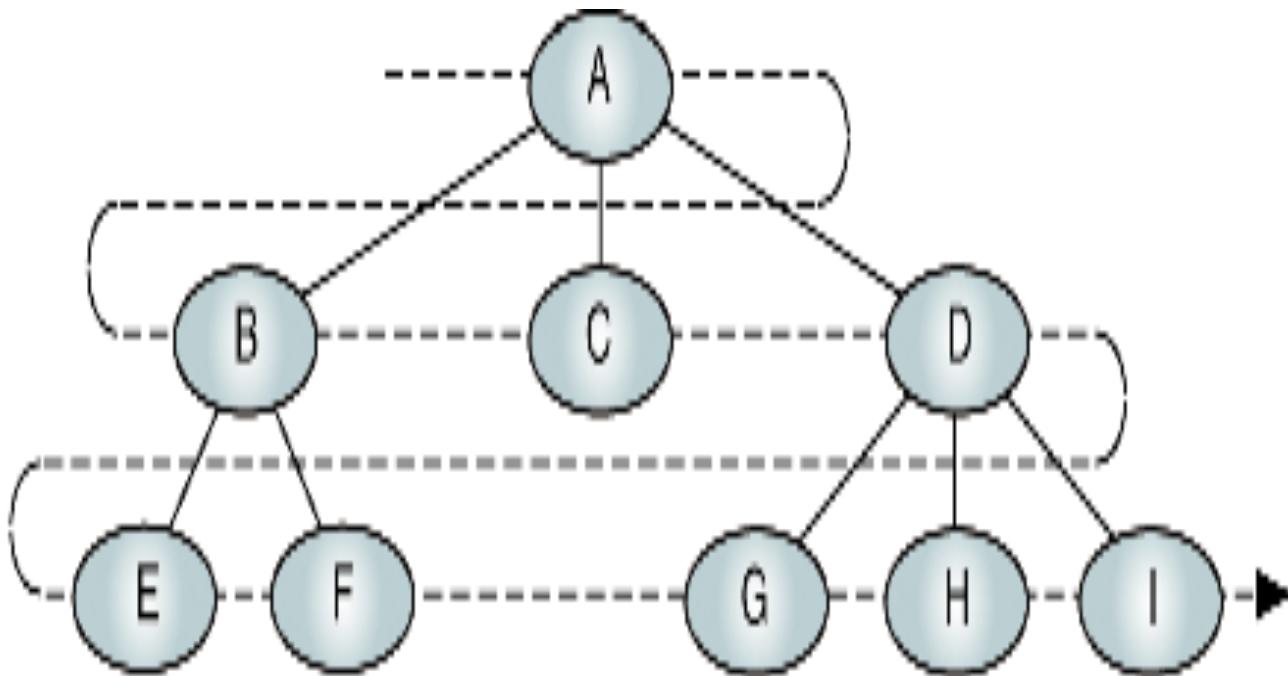


(a) Graph



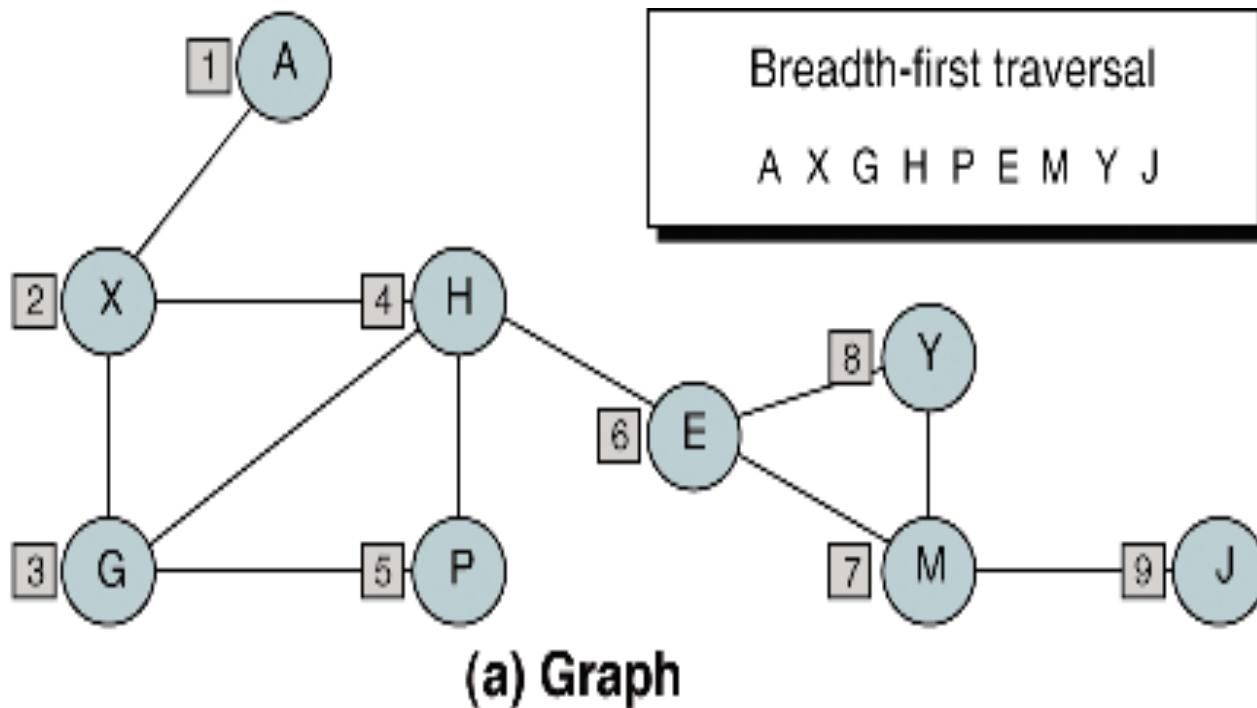
(b) Stack contents

Breadth-first Traversal of a Tree



Breadth-first traversal: A B C D E F G H I

Breadth-first Traversal of a Graph



11-3 Graph Storage Structures

To represent a graph, we need to store two sets. The first set represents the vertices of the graph, and the second set represents the edges or arcs. The two most common structures used to store these sets are arrays and linked lists.

- **Adjacency Matrix**
- **Adjacency List**

Adjacency Matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	1	0	1	0	1	0
C	0	1	0	1	1	0
D	0	0	1	0	1	0
E	0	1	1	1	0	1
F	0	0	0	0	1	0

Vertex vector Adjacency matrix

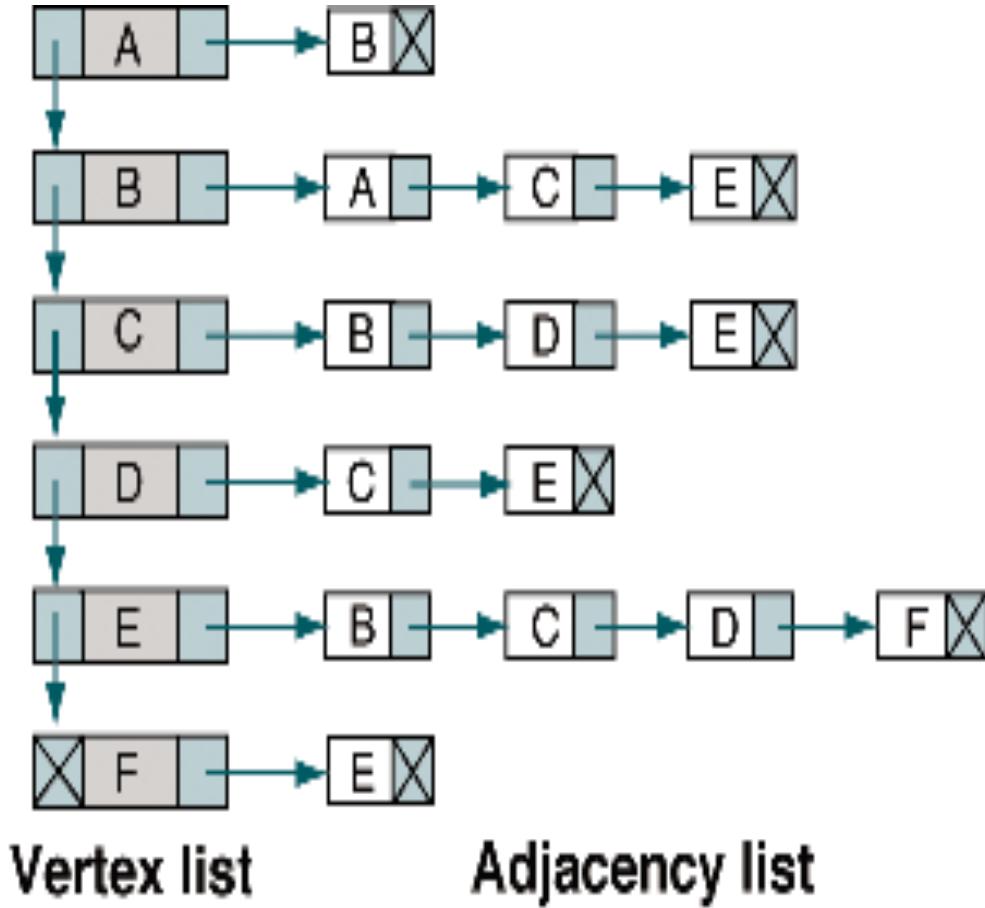
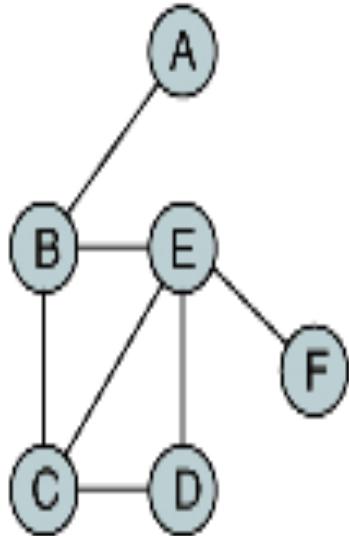
(a) Adjacency matrix for nondirected graph

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	1	0
C	0	0	0	1	1	0
D	0	0	0	0	0	0
E	0	0	0	1	0	1
F	0	0	0	0	0	0

Vertex vector Adjacency matrix

(b) Adjacency matrix for directed graph

Adjacency List

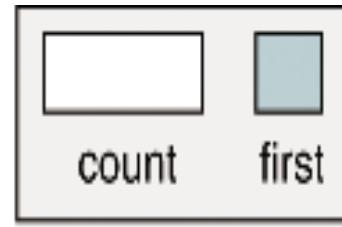


11-4 Graph Algorithms

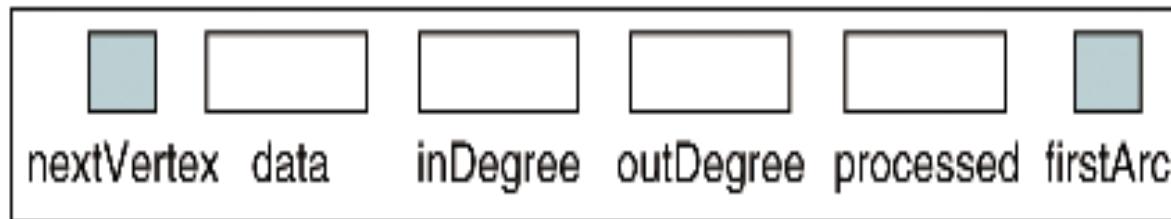
In this section we develop a minimum set of algorithms that are needed to create and maintain a directed graph.

- Create Graph
- Insert Vertex
- Delete Vertex
- Insert Arc
- Delete Arc
- Retrieve Vertex
- Depth-first Traversal
- Breadth-first Traversal
- Destroy Graph

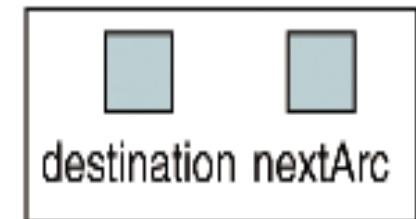
Graph Data Structure



graphHead



graphVertex



graphArc

Create Graph

```
Algorithm createGraph
```

Initializes the metadata elements of a graph structure.

Pre graph is a reference to metadata structure

Post metadata elements have been initialized

```
1 set count to 0
```

```
2 set first to null
```

```
3 return graph head
```

```
end createGraph
```

Insert Vertex

```
Algorithm insertVertex (graph, dataIn)
Allocates memory for a new vertex and copies the data to it.
    Pre    graph is a reference to graph head structure
           dataIn contains data to be inserted into vertex
    Post   new vertex allocated and data copied
1 allocate memory for new vertex
2 store dataIn in new vertex
3 initialize metadata elements in newNode
4 increment graph count
    Now find insertion point
5 if (empty graph)
    1 set graph first to newNode
6 else
    1 search for insertion point
    2 if (inserting before first vertex)
        1 set graph first to new vertex
    3 else
        1 insert new vertex in sequence
7 end if
end insertVertex
```

Delete Vertex

```
Algorithm deleteVertex (graph, key)
Deletes an existing vertex only if its degree is 0.
    Pre    graph is reference pointer to graph head
           key is the key of the vertex to be deleted
    Post   vertex deleted (if degree zero)
    Return +1 if successful
           -1 if degree not zero
           -2 if key not found
1 if (empty graph)
    1 return -2
2 end if
    Locate vertex to be deleted
3 search for vertex to be deleted
4 if (not found)
    1 return -2
5 end if
    Found vertex to be deleted. Test degree.
6 if (vertex inDegree > 0 OR outDegree > 0)
    1 return -1
7 end if
    Okay to delete vertex
8 delete vertex
9 decrement graph count
10 return 1
end deleteVertex
```

Insert Arc

```
Algorithm insertArc (graph, fromKey, toKey)
```

Adds an arc between two vertices.

Pre graph is reference to graph head structure
fromKey is the key of the originating vertex
toKey is the key of the destination vertex

Post arc added to adjacency list

Return +1 if successful

-2 if fromKey not found

-3 if toKey not found

1 allocate memory for new arc

Locate source vertex

2 search and set fromVertex

3 if (from vertex not found)

1 return -2

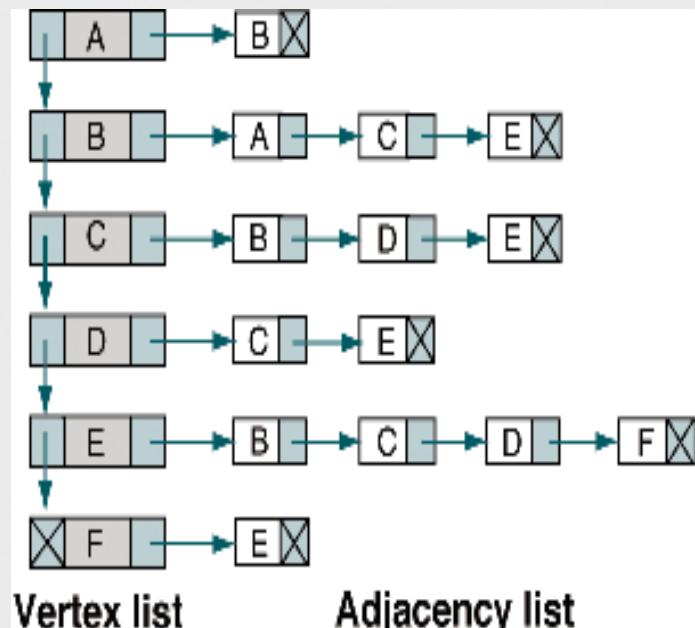
4 end if

Now locate to vertex

5 search and set toVertex

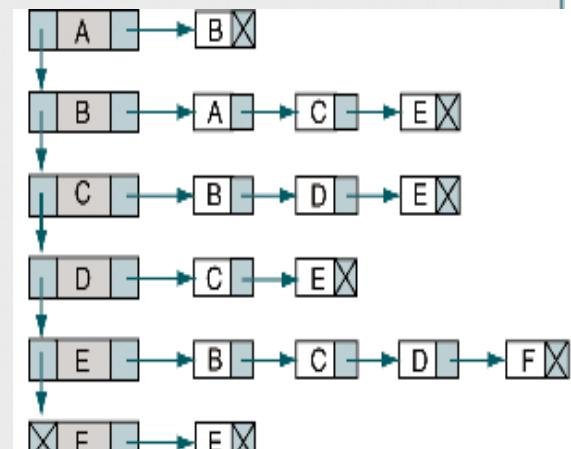
6 if (to vertex not found)

1 return -3



Insert Arc (cont.)

```
7 end if
    From and to vertices located. Insert new arc.
8 increment fromVertex outDegree
9 increment toVertex inDegree
10 set arc destination to toVertex
11 if (fromVertex arc list empty)
    Inserting first arc
1   set fromVertex firstArc to new arc
2   set new arc nextArc to null
3   return 1
12 end if
    Find insertion point in adjacency (arc) list
13 find insertion point in arc list
14 if (insert at beginning of arc list)
    Insertion before first arc
    1   set fromVertix firstArc to new arc
15 else
    1   insert in arc list
16 end if
17 return 1
end insertArc
```



Vertex list

Adjacency list

Delete Arc

```
Algorithm deleteArc (graph, fromKey, toKey)
```

Deletes an arc between two vertices.

Pre graph is reference to a graph head structure
fromKey is the key of the originating vertex
toKey is the key of the destination vertex

Post vertex deleted

Return +1 if successful

-2 if fromKey not found

-3 if toKey not found

1 if (empty graph)

 1 return -2

2 end if

 Locate source vertex

3 search and set fromVertex to vertex with key equal to
 fromKey

4 if (fromVertex not found)

 1 return -2

5 end if

 Locate destination vertex in adjacency list

Delete Arc (cont.)

```
6 if (fromVertex arc list null)
    1   return -3
7 end if
8 search and find arc with key equal to toKey
9 if (toKey not found)
    1   return -3
10 end if
      fromVertex, toVertex, and arc all located. Delete arc.
11 set toVertex to arc destination
12 delete arc
13 decrement fromVertex outDegree
14 decrement toVertex  inDegree
15 return 1
end deleteArc
```

Retrieve Vertex

```
Algorithm retrieveVertex (graph, key, dataOut)
Data contained in vertex identified by key passed to caller.
    Pre    graph is reference to a graph head structure
          key is the key of the vertex data
          dataOut is reference to data variable
    Post   vertex data copied to dataOut
    Return +1 if successful
          -2 if key not found

1 if (empty graph)
  1 return -2
2 end if
3 search for vertex
4 if (vertex found)
  1 move locnPtr data to dataOut
  2 return 1
5 else
  1 return -2
6 end if
end retrieveVertex
```

Depth-first Traversal

```
Algorithm depthFirst (graph)
```

Process the keys of the graph in depth-first order.

Pre graph is a pointer to a graph head structure

Post vertices "processed"

```
1 if (empty graph)
```

```
1 return
```

Set processed flags to not processed

```
2 set walkPtr to graph first
```

```
3 loop (through all vertices)
```

```
1 set processed to 0
```

```
4 end loop
```

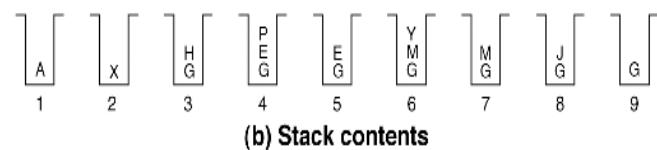
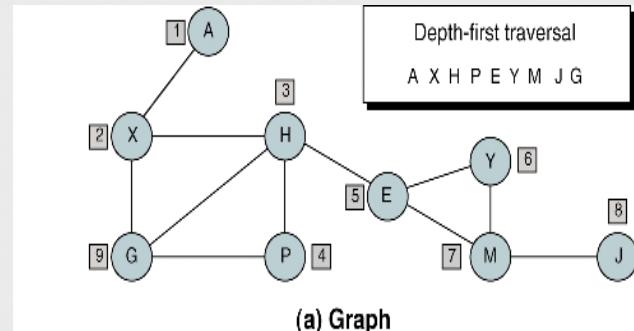
Process each vertex in list

```
5 createStack (stack)
```

Depth-first Traversal (cont.)

? → 6 loop (through vertex list)
1 if (vertex not processed and not in stack)
 Push and set flag to stack
1 pushStack (stack, walkPtr)
2 set walkPtr processed to 1
3 end if
 Process vertex at stack top
4 loop (not emptyStack(stack))
1 set vertex to popStack(stack)
2 process (vertex)
3 set vertex to processed
 Push non-processed vertices from adjacency list
4 loop (through arc list)
1 if (arc destination not in stack)
 1 pushStack(stack, destination)
 2 set destination to in stack
2 end if
3 get next destination
5 end loop
5 end loop
2 end if
3 get next vertex
7 end loop
8 destroyStack(stack)
end depthFirst

{ } { }



Breadth-first Traversal

```
Algorithm breadthFirst (graph)
Processes the keys of the graph in breadth-first order.
  Pre  graph is pointer to graph head structure
  Post vertices processed
1 if (empty graph)
  1 return
2 end if
  First set all processed flags to not processed
3 createQueue (queue)
4 loop (through all vertices)
  1 set vertix to not processed
5 end loop
  Process each vertex in vertex list
6 loop (through all vertices)
  1 if (vertix not processed)
    1 if (vertix not in queue)
      Enqueue and set process flag to queued (1)
      1 enqueue (queue, walkPtr)
      2 set vertix to enqueueued
    2 end if
    Now process descendants of vertex at queue front
  3 loop (not emptyQueue (queue))
    1 set vertex to dequeue (queue)
      Process vertex and flag as processed
```

Breadth-first Traversal (cont.)

```
2 process (vertex)
3 set vertix to processed
    Enqueue non-processed vertices from adjacency list
4 loop (through adjacency list)
    1 if (destination not enqueued or processed)
        1 enqueue (queue, destination)
        2 set destination to enqueued
    2 end if
    3 get next destination
    5 end loop
4 end loop
2 end if
3 get next vertix
7 end loop
8 destroyQueue (queue)
end breadthFirst
```

Destroy Graph

```
Algorithm destroyGraph (graph)
Traverses graph deleting all vertices and arcs.

Pre Nothing
Post All vertices and arcs deleted

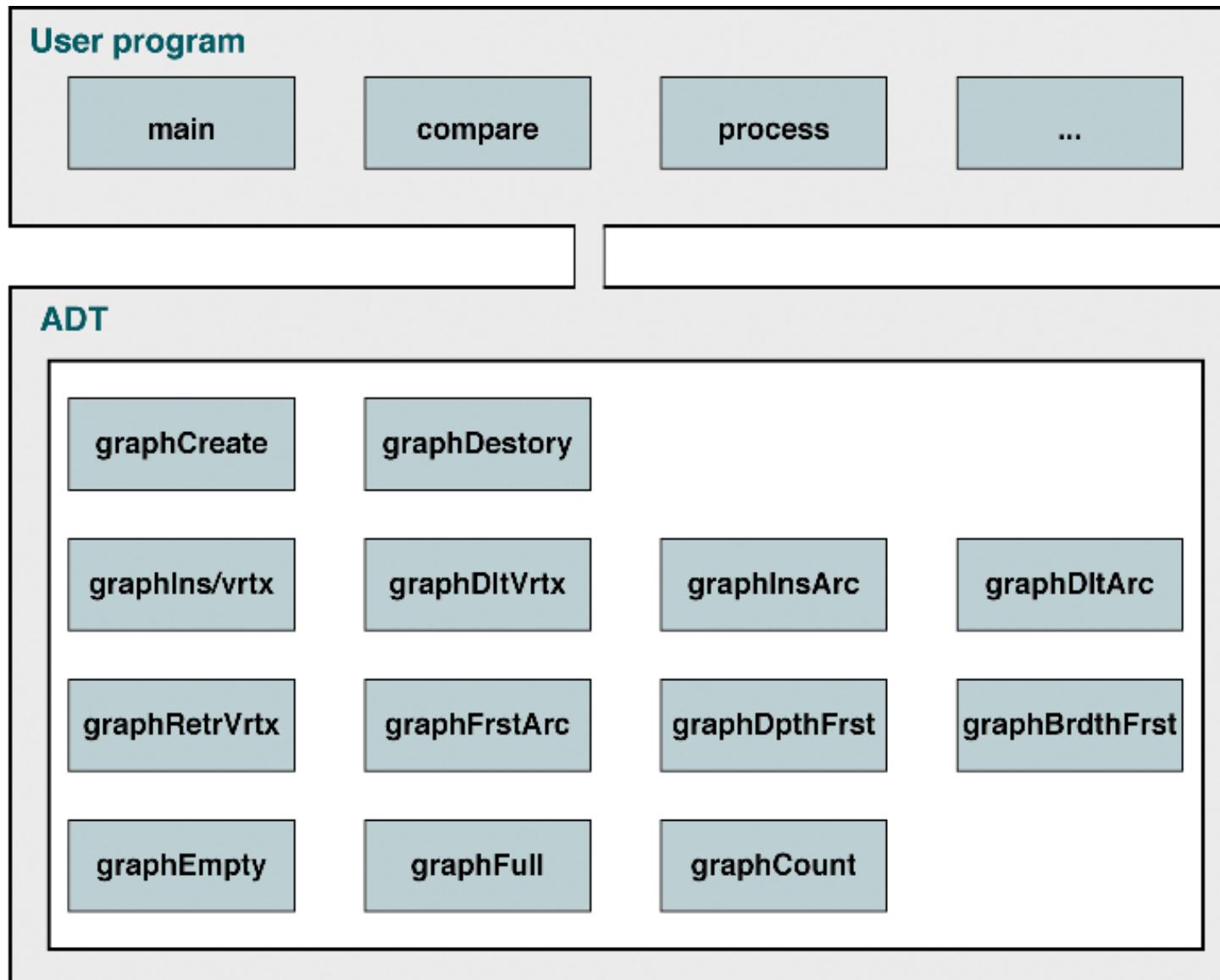
1 if (empty graph)
    1 return
2 loop (more vertices)
    1 loop (vertex outDegree > 0)
        1 delete vertex firstArc
        2 subtract 1 from vertex outDegree
    2 end loop
3 end loop
end destroyGraph
```

11-5 Graph ADT

*We begin by developing the code for the three graph structures.
We then develop the code for the ADT functions.*

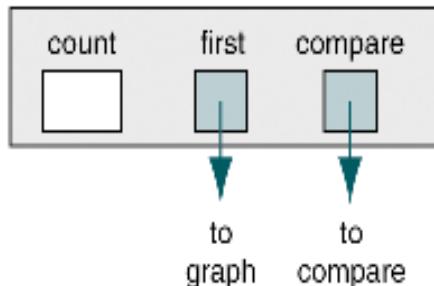
- **Data Structure**
- **Functions**

Graph ADT Design

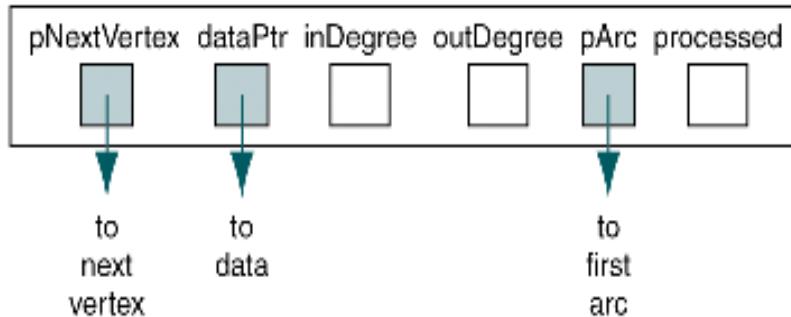


Graph Data Structure

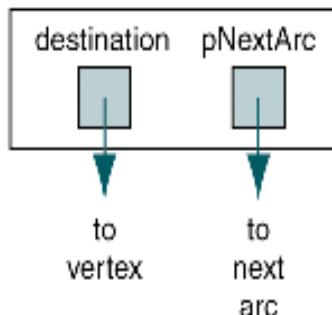
GRAPH



VERTEX



ARC



```
typedef struct
{
    int             count;
    struct vertex* first;
    int (*compare)
        (void* arg1,
         void* arg2);
} GRAPH;

typedef struct vertex
{
    struct vertex* pNextVertex;
    void*          dataPtr;
    int            inDegree;
    int            outDegree;
    short          processed;
    struct arc*    pArc;
} VERTEX;

typedef struct arc
{
    struct vertex* destination;
    struct arc*    pNextArc;
} ARC;
```

Graph Declarations

```
1  /* ===== graphs.h =====
2   This header file contains the declarations and
3   prototype functions for graphs as developed in
4   this text.
5
6   Written by:
7   Date:
8 */
9 #include "queueADT.h"
10 #include "stackADT.h"
11 #include "stdbool.h"
```

Graph Declarations (cont.)

```
12 // ===== STRUCTURES =====
13 typedef struct
14 {
15     int             count;
16     struct vertex* first;
17     int (*compare) (void* arg1, void* arg2);
18 } GRAPH;
19
20 typedef struct vertex
21 {
22     struct vertex* pNextVertex;
23     void*          dataPtr;
24     int            inDegree;
25     int            outDegree;
26     short          processed;
27     struct arc*    pArc;
28 } VERTEX;
```

Graph Declarations (cont.)

```
29
30 typedef struct arc
31 {
32     struct vertex* destination;
33     struct arc* pNextArc;
34 } ARC;
35
36 // ===== Prototype Declarations =====
37
38 GRAPH* graphCreate
39             (int (*compare) (void* arg1, void* arg2));
40 GRAPH* graphDestroy (GRAPH* graph);
41
```

Graph Declarations (cont.)

```
42 void graphInsVrtx    (GRAPH* graph, void*   dataInPtr);
43 int graphDltVrtx     (GRAPH* graph, void*   dltKey);
44 int graphInsArc      (GRAPH* graph, void*   pFromKey,
45                           void*   pToKey);
46 int graphDltArc      (GRAPH* graph, void*   pFromKey,
47                           void*   pToKey);
48
49 int graphRetrVrtx    (GRAPH* graph, void*   pKey,
50                           void**  pDataOut);
51 int graphFrstArc     (GRAPH* graph, void*   pKey,
52                           void**  pDataOut);
53
54 void graphDpthFrst   (GRAPH* graph,
55                           void (*process) (void* dataPtr));
56 void graphBrdthFrst  (GRAPH* graph,
57                           void (*process) (void* dataPtr));
58
59 bool graphEmpty (GRAPH* graph);
60 bool graphFull  (GRAPH* graph);
61 int  graphCount (GRAPH* graph);
```

Graph Insert Vertex

```
1  /* ===== graphInsVrtx =====
2   This function inserts new data into the graph.
3     Pre graph is pointer to valid graph structure
4     Post data inserted or abort if memory O/F
5 */
6 void graphInsVrtx (GRAPH* graph, void* dataInPtr)
7 {
8 // Local Definitions
9   VERTEX* newPtr;
10  VERTEX* locPtr;
11  VERTEX* predPtr;
12
13 // Statements
14  newPtr = (VERTEX*)malloc(sizeof (VERTEX));
15  if (newPtr)
16  {
17    newPtr->pNextVertex = NULL;
18    newPtr->dataPtr = dataInPtr;
19    newPtr->inDegree = 0;
20    newPtr->outDegree = 0;
21    newPtr->processed = 0;
22    newPtr->pArc = NULL;
23    (graph->count)++;
24 } // if malloc
```

Graph Insert Vertex (cont.)

```
25     else
26         printf("Overflow error 100\a\n"),
27         exit (100);
28
29 // Now find insertion point
30 locPtr = graph->first;
31 if (!locPtr)
32     // Empty graph. Insert at beginning
33     graph->first = newPtr;
34 else
35 {
36     predPtr = NULL;
37     while (locPtr && (graph->compare
38                         (dataInPtr, locPtr->dataPtr) > 0))
39     {
40         predPtr = locPtr;
41         locPtr = locPtr->pNextVertex;
42     } // while
43     if (!predPtr)
44         // Insert before first vertex
45         graph->first = newPtr;
46     else
47         predPtr->pNextVertex = newPtr;
48         newPtr->pNextVertex = locPtr;
49 } // else
50 return;
51 } // graphInsVrtx
```

Graph Delete Vertex

```
1  /* ===== graphDltVrtx =====
2   Deletes existing vertex only if its degree is zero.
3     Pre    graph is pointer to graph head structure
4           dltKey is key of vertex to be deleted
5     Post   Vertex deleted if degree zero
6           -or- An error code is returned
7     Return Success +1 if successful
8           -1 if degree not zero
9           -2 if dltKey not found
10 */
11 int graphDltVrtx (GRAPH* graph, void* dltKey)
12 {
13 // Local Definitions
14     VERTEX* predPtr;
15     VERTEX* walkPtr;
```

Graph Delete Vertex (cont.)

```
16 // Statements
17     if (!graph->first)
18         return -2;
19
20
21     // Locate vertex to be deleted
22     predPtr = NULL;
23     walkPtr = graph->first;
24     while (walkPtr
25         && (graph->compare(dltKey, walkPtr->dataPtr) > 0))
26     {
27         predPtr = walkPtr;
28         walkPtr = walkPtr->pNextVertex;
29     } // walkPtr &&
30     if (!walkPtr
31         || graph->compare(dltKey, walkPtr->dataPtr) != 0)
32         return -2;
33
34     // Found vertex. Test degree
35     if ((walkPtr->inDegree > 0)
36         || (walkPtr->outDegree > 0))
37         return -1;
38
39     // OK to delete
40     if (!predPtr)
41         graph->first = walkPtr->pNextVertex;
42     else
43         predPtr->pNextVertex = walkPtr->pNextVertex;
44     --graph->count;
45     free(walkPtr);
46     return 1;
47 } // graphDltVrtx
```

Graph Insert Arc

```
1  /* ===== graphInsArc =====
2   Adds an arc vertex between two vertices.
3     Pre    graph is a pointer to a graph
4             fromKey is pointer to start vertex key
5             toKey   is pointer to dest'n vertex key
6     Post   Arc added to adjacency list
7     Return success +1 if successful
8             -1 if memory overflow
9             -2 if fromKey not found
10            -3 if toKey not found
11 */
12 int graphInsArc (GRAPH* graph, void* pFromKey,
13                   void* pToKey)
14 {
```

Graph Insert Arc (cont.)

```
15 // Local Definitions
16     ARC*      newPtr;
17     ARC*      arcPredPtr;
18     ARC*      arcWalkPtr;
19     VERTEX*   vertFromPtr;
20     VERTEX*   vertToPtr;
21
22 // Statements
23     newPtr = (ARC*)malloc(sizeof(ARC));
24     if (!newPtr)
25         return (-1);
26
27 // Locate source vertex
28     vertFromPtr = graph->first;
29     while (vertFromPtr && (graph->compare(pFromKey,
30                                         vertFromPtr->dataPtr) > 0))
```

Graph Insert Arc (cont.)

```
31      {
32          vertFromPtr = vertFromPtr->pNextVertex;
33      } // while vertFromPtr &&
34      if (!vertFromPtr || (graph->compare(pFromKey,
35                                              vertFromPtr->dataPtr) != 0))
36          return (-2);
37
38      // Now locate to vertex
39      vertToPtr = graph->first;
40      while (vertToPtr
41              && graph->compare(pToKey, vertToPtr->dataPtr) > 0)
42      {
43          vertToPtr = vertToPtr->pNextVertex;
44      } // while vertToPtr &&
45      if (!vertToPtr ||
46          (graph->compare(pToKey, vertToPtr->dataPtr) != 0))
47          return (-3);
48
```

Graph Insert Arc (cont.)

```
49 // From and to vertices located. Insert new arc
50     ++vertFromPtr->outDegree;
51     ++vertToPtr ->inDegree;
52     newPtr->destination = vertToPtr;
53     if (!vertFromPtr->pArc)
54     {
55         // Inserting first arc for this vertex
56         vertFromPtr->pArc = newPtr;
57         newPtr->pNextArc = NULL;
58         return 1;
59     } // if new arc
60 }
```

Graph Insert Arc (cont.)

```
61 // Find insertion point in adjacency (arc) list
62 arcPredPtr = NULL;
63 arcWalkPtr = vertFromPtr->pArc;
64 while (arcWalkPtr
65     && graph->compare(pToKey,
66                         arcWalkPtr->destination->dataPtr) >= 0)
67 {
68     arcPredPtr = arcWalkPtr;
69     arcWalkPtr = arcWalkPtr->pNextArc;
70 } // arcWalkPtr &&
71
72 if (!arcPredPtr)
73     // Insertion before first arc
74     vertFromPtr->pArc      = newPtr;
75 else
76     arcPredPtr->pNextArc = newPtr;
77 newPtr->pNextArc = arcWalkPtr;
78     return 1;
79 } // graphInsArc
```

Graph Delete Arc

```
1  /* ===== graphDltArc =====
2   Deletes an existing arc.
3     Pre    graph is pointer to graph head structure
4           fromKey is key of start vertex; toKey is
5           toKey is key of dest'n of delete vertex
6     Post   Arc deleted
7     Return Success +1 if successful
8             -2 if fromKey not found
9             -3 if toKey not found
10 */
11 int graphDltArc (GRAPH* graph,
12                   void* fromKey, void* toKey)
13 {
14 // Local Definitions
15     VERTEX* fromVertexPtr;
16     VERTEX* toVertexPtr;
17     ARC*    preArcPtr;
18     ARC*    arcWalkPtr;
19 }
```

Graph Delete Arc (cont.)

```
20 // Statements
21     if (!graph->first)
22         return -2;
23
24     // Locate source vertex
25     fromVertexPtr = graph->first;
26     while (fromVertexPtr && (graph->compare(fromKey,
27                                     fromVertexPtr->dataPtr) > 0))
28         fromVertexPtr = fromVertexPtr->pNextVertex;
29
30     if (!fromVertexPtr || graph->compare(fromKey,
31                                     fromVertexPtr->dataPtr) != 0)
32         return -2;
33
34     // Locate destination vertex in adjacency list
35     if (!fromVertexPtr->pArc)
36         return -3;
37
38     preArcPtr = NULL;
39     arcWalkPtr = fromVertexPtr->pArc;
```

Graph Delete Arc (cont.)

```
40     while (arcWalkPtr && (graph->compare(toKey,
41                                     arcWalkPtr->destination->dataPtr) > 0))
42     {
43         preArcPtr = arcWalkPtr;
44         arcWalkPtr = arcWalkPtr->pNextArc;
45     } // while arcWalkPtr &&
46     if (!arcWalkPtr || (graph->compare(toKey,
47                                     arcWalkPtr->destination->dataPtr) != 0))
48         return -3;
49     toVertexPtr = arcWalkPtr->destination;
50
51     // from, toVertex & arcPtr located. Delete arc
52     --fromVertexPtr->outDegree;
53     --toVertexPtr -> inDegree;
54     if (!preArcPtr)
55         // Deleting first arc
56         fromVertexPtr->pArc = arcWalkPtr->pNextArc;
57     else
58         preArcPtr->pNextArc = arcWalkPtr->pNextArc;
59     free (arcWalkPtr);
60     return 1;
61 } // graphDltArc
```

Graph Depth-first Traversal

```
1  /* ===== graphDpthFrst =====
2   Process data in graph in depth-first order.
3   Pre graph is the a pointer to graph head
4   Post vertices "processed".
5
6   Processed Flag: 0 = not processed
7           1 = pushed into stack
8           2 = processed
9 */
10 void graphDpthFrst (GRAPH* graph,
11                      void (*process) (void* dataPtr))
12 {
```

Graph Depth-first Traversal (cont.)

```
13 // Local Definitions
14     bool      success;
15     VERTEX*  walkPtr;
16     VERTEX*  vertexPtr;
17     VERTEX*  vertToPtr;
18     STACK * stack;
19     ARC*    arcWalkPtr;
20
21 // Statements
22 if (!graph->first)
23     return;
24
25 // Set processed flags to not processed
26 walkPtr = graph->first;
27 while (walkPtr)
28 {
29     walkPtr->processed = 0;
30     walkPtr           = walkPtr->pNextVertex;
31 } // while
```

Graph Depth-first Traversal (cont.)

```
32
33     // Process each vertex in list
34     stack = createStack ();
35     walkPtr = graph->first;
36     while (walkPtr)
37     {
38         if (walkPtr->processed < 2) ?
39         {
40             if (walkPtr->processed < 1)
41             {
42                 // Push & set flag to pushed
43                 success = pushStack (stack, walkPtr);
44                 if (!success)
45                     printf("\aStack overflow 100\a\n"),
46                         exit (100);
47                 walkPtr->processed = 1;
48             } // if processed < 1
49         } // if processed < 2
```

0 = not processed
1 = pushed into stack
2 = processed

Graph Depth-first Traversal (cont.)

```
50     // Process descendants of vertex at stack top
51     while (!emptyStack (stack))
52     {
53         vertexPtr = popStack(stack);
54         process (vertexPtr->dataPtr);
55         vertexPtr->processed = 2;
56
57         // Push all vertices from adjacency list
58         arcWalkPtr = vertexPtr->pArc;
59         while (arcWalkPtr)
60         {
```

Graph Breadth-first Traversal

```
1  /* ===== graphBrdthFrst =====
2   Process the data of the graph in breadth-first order.
3     Pre  graph is pointer to graph head structure
4     Post graph has been processed
5     Processed Flag: 0 = not processed
6                   1 = enqueue
7                   2 = processed
8 */
9  void graphBrdthFrst (GRAPH* graph,
10                      void (*process) (void* dataPtr))
11 {
12 // Local Definitions
13     bool      success;
14     VERTEX*  walkPtr;
15     VERTEX*  vertexPtr;
16     VERTEX*  vertToPtr;
17     QUEUE*   queue;
18     ARC*    arcWalkPtr;
19 }
```

Graph Breadth-first Traversal (cont.)

```
20 // Statements
21     if (!graph->first)
22         return;
23
24     // Set processed flags to not processed
25     walkPtr = graph->first;
26     while (walkPtr)
27     {
28         walkPtr->processed = 0;
29         walkPtr             = walkPtr->pNextVertex;
30     } // while
31
32     // Process each vertex in list
33     queue = createQueue ();
34     walkPtr = graph->first;
35     while (walkPtr)
36     {
37         if (walkPtr->processed < 2)
38         {
39             if (walkPtr->processed < 1)
40                 {
```

Graph Breadth-first Traversal (cont.)

```
41          // Enqueue & set flag to queue
42          success = enqueue(queue, walkPtr);
43          if (!success)
44              printf("\aQueue overflow 100\a\n"),
45              exit (100);
46          walkPtr->processed = 1;
47      } // if processed < 1
48  } // if processed < 2
49 // Process descendants of vertex at que front
50 while (!emptyQueue (queue))
51 {
52     dequeue(queue, (void**)&vertexPtr);
53     process (vertexPtr->dataPtr);
54     vertexPtr->processed = 2;

55
56     // Enqueue vertices from adjacency list
57     arcWalkPtr = vertexPtr->pArc;
58     while (arcWalkPtr)
59     {
60         vertToPtr = arcWalkPtr->destination;
61         if (vertToPtr->processed == 0)
62         {
63             success = enqueue(queue, vertToPtr);
64             if (!success)
65                 printf("\aQueue overflow 101\a\n"),
66                 exit (101);
67             vertToPtr->processed = 1;
```

Graph Breadth-first Traversal (cont.)

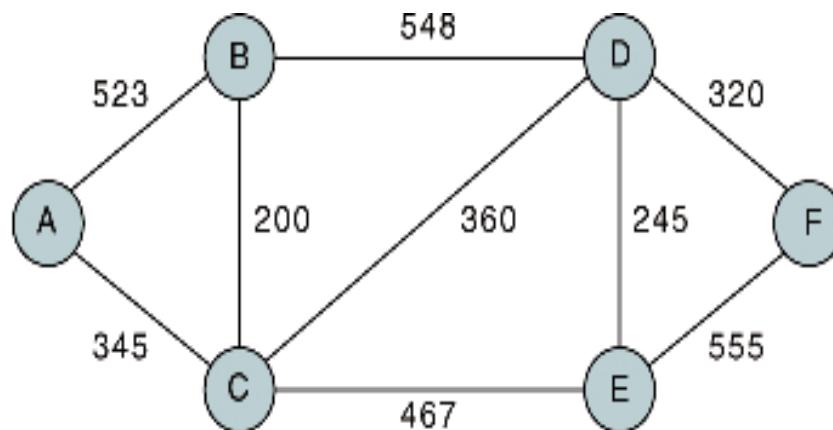
```
68          } // if vertToPtr
69          arcWalkPtr = arcWalkPtr->pNextArc;
70      } // while pWalkArc
71  } // while !emptyQueue
72  walkPtr = walkPtr->pNextVertex;
73 } // while walkPtr
74 destroyQueue(queue);
75 return;
76 } // graphBrdthFrst
```

11-6 Networks

A network is a graph whose lines are weighted. It is also known as a weighted graph. Included in this section are two graph applications that process networks.

- Minimum Spanning Tree
- Shortest Path Algorithm

Storing Weights in Graph Structures

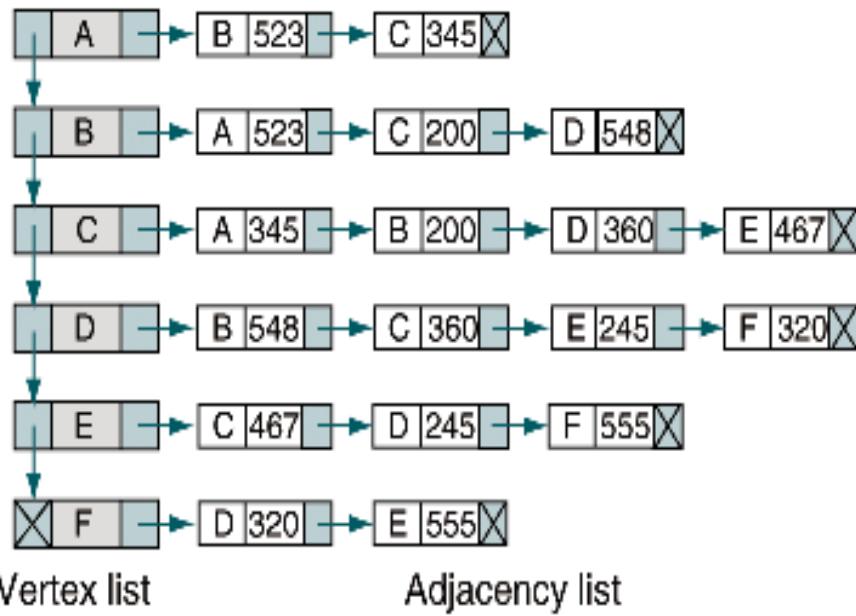


City Network

	A	B	C	D	E	F
A	0	523	345	0	0	0
B	523	0	200	548	0	0
C	345	200	0	360	467	0
D	0	548	360	0	245	320
E	0	0	467	245	0	555
F	0	0	0	320	555	0

Vertex
vector

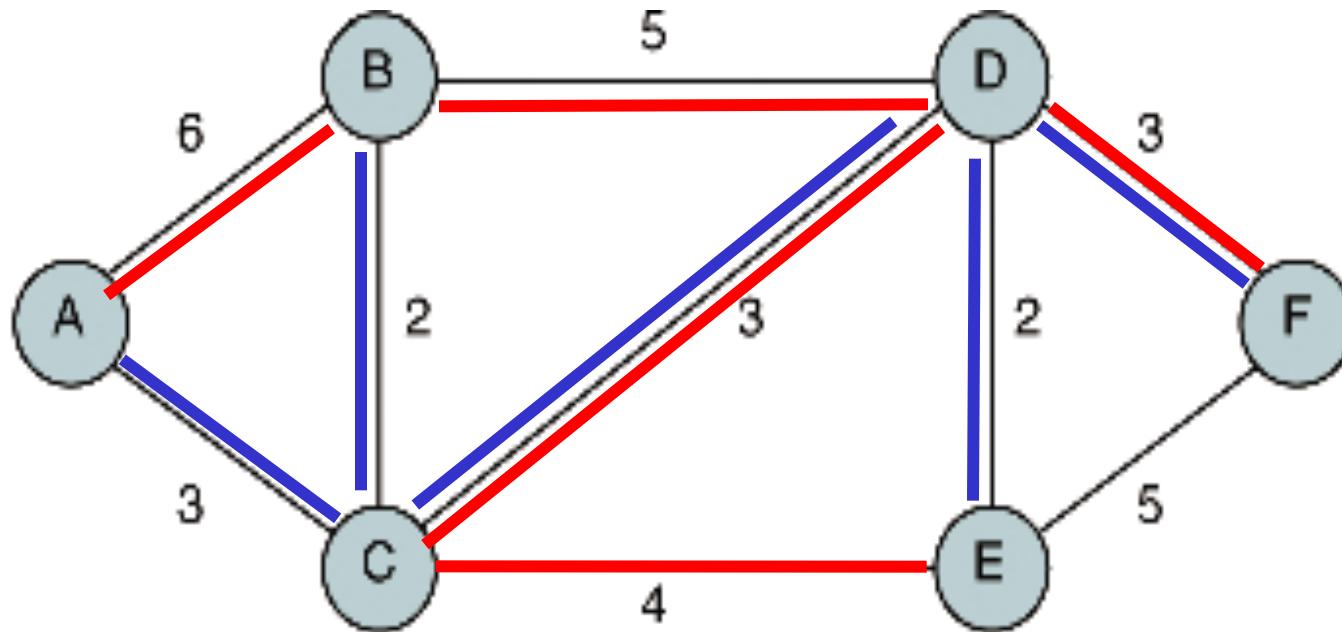
Adjacency matrix



Spanning Tree of a Graph

- A spanning tree contains all of the vertices in a graph
- A minimum spanning tree is a spanning tree in which the total weight of the lines is guaranteed to be the minimum of all possible trees in the graph

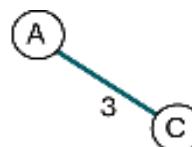
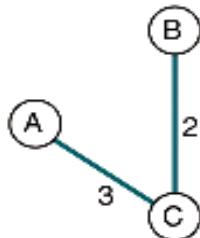
— Total weight = 21
— Total weight = 13



Develop Minimum Spanning Tree of Graph

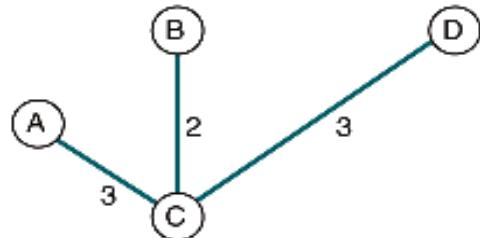
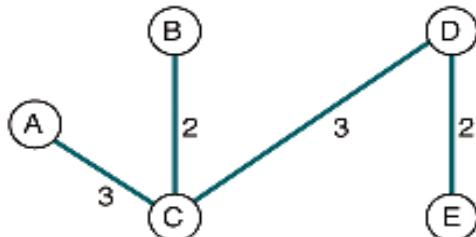
(A)

(a) Insert first vertex

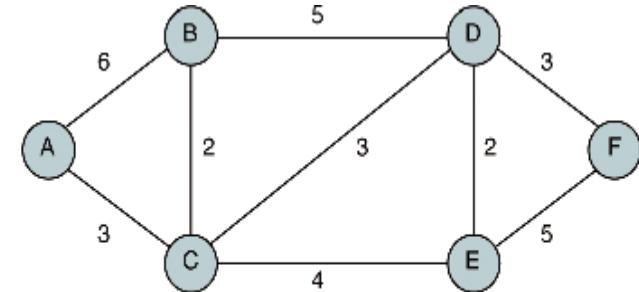


(b) Insert edge AC

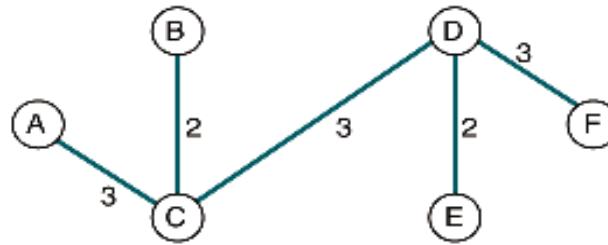
(c) Insert edge BC



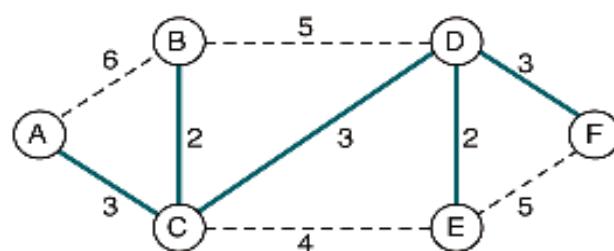
(d) Insert edge CD



(e) Insert edge DE



(f) Insert edge DF



(g) Final tree in the graph

Data Structure for Spanning Tree

```
graphHead
    count
    first
end graphHead
```

```
graphVertex
    nextVertex
    data
    inDegree
    outDegree
    inTree
    firstEdge
end graphVertex
```

```
graphEdge
    destination
    weight
    nextEdge
    inTree
end graphEdge
```

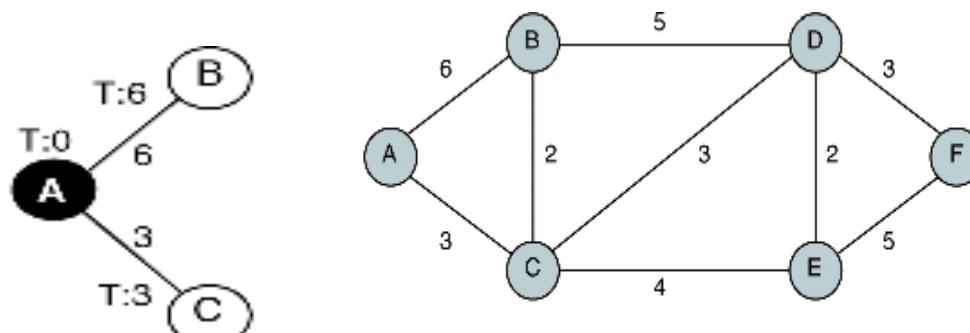
Minimum Spanning Tree of a Graph

```
Algorithm spanningTree (graph)
Determine the minimum spanning tree of a network.
    Pre graph contains a network
    Post spanning tree determined
1 if (empty graph)
1   return
2 end if
3 loop (through all vertices)
    Set inTree flags false.
    1 set vertex inTree flag to false
    2 loop (through all edges)
        1 set edge inTree flag to false
        2 get next edge
    3 end loop
    4 get next vertex
4 end loop
Now derive spanning tree.
5 set first vertex to in tree
6 set treeComplete to false
7 loop (not treeComplete)
    1 set treeComplete to true
    2 set minEdge to maximum integer
    3 set minEdgeLoc to null
    4 loop (through all vertices)
        Walk through graph checking vertices in tree.
```

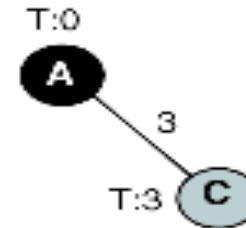
Minimum Spanning Tree of a Graph (cont.)

```
1 if (vertex in tree AND vertex outDegree > 0)
1 loop (through all edges)
1 if (destination not in tree)
    set destination inTree flag to false)
1 set treeComplete to false
2 if (edge weight < minEdge)
    1 set minEdge to edge weight
    2 set minEdgeLoc to edge
    3 end if
2 end if
3 get next edge
2 end loop
2 end if
3 get next vertex
5 end loop
6 if (minEdgeLoc not null)
    Found edge to insert into tree.
    1 set minEdgeLoc inTree flag to true
    2 set destination inTree flag to true
7 end if
8 end loop
end spanningTree
```

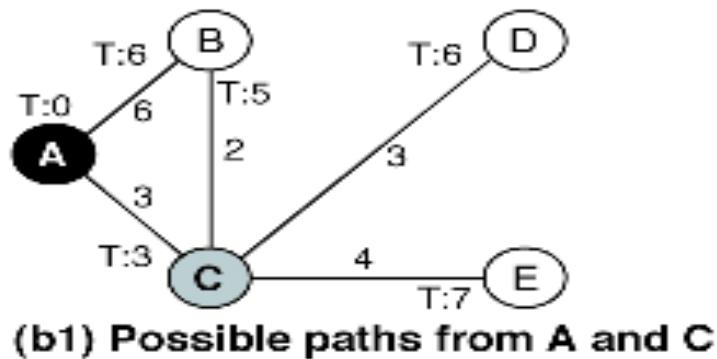
Determining Shortest Path (Dijkstra)



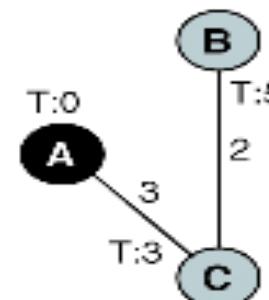
(a1) Possible paths from A



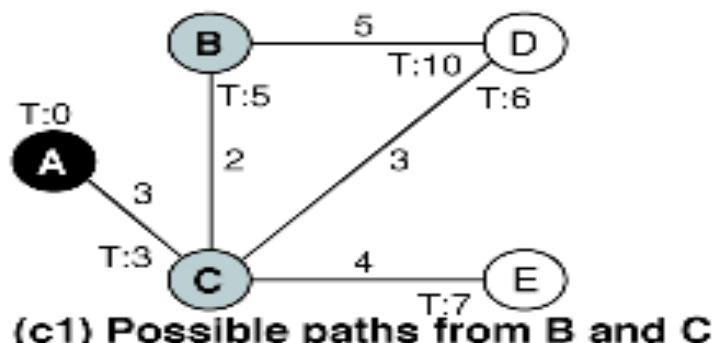
(a2) Tree after insertion of C



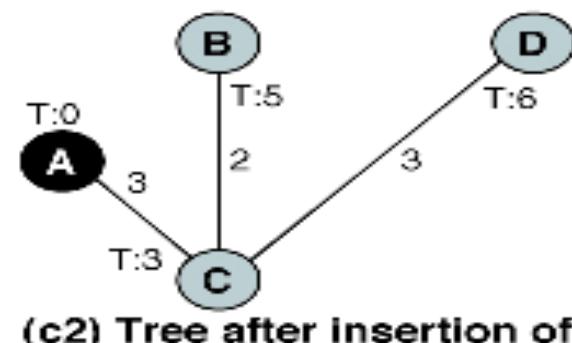
(b1) Possible paths from A and C



(b2) Tree after insertion of B

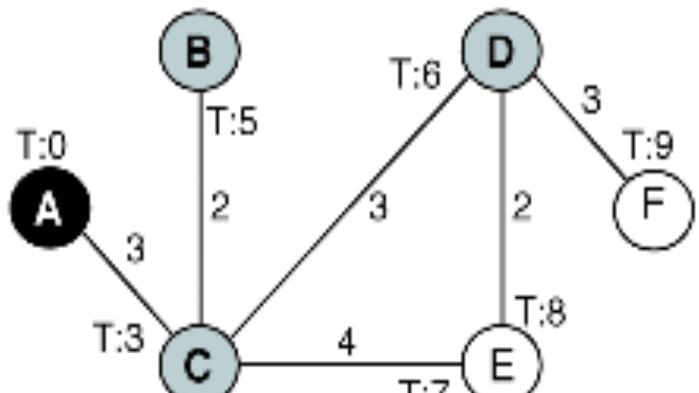


(c1) Possible paths from B and C

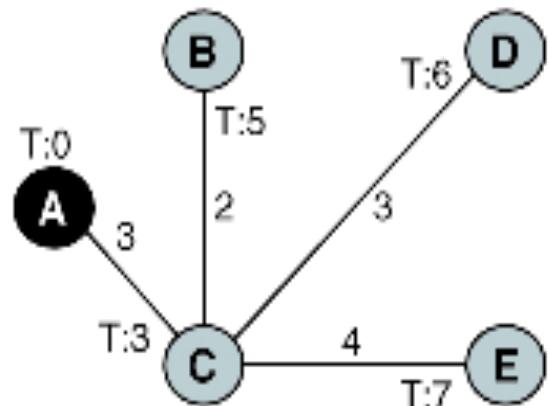


(c2) Tree after insertion of D

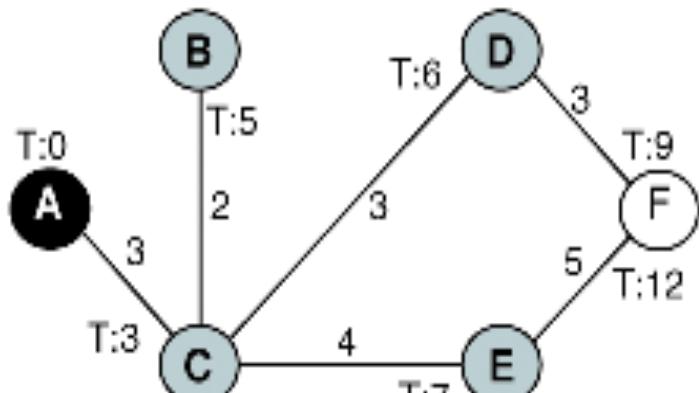
Determining Shortest Path (cont.)



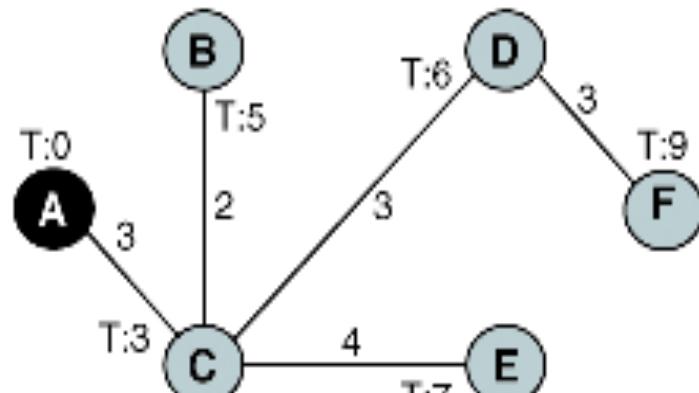
(d1) Possible paths from C and D



(d2) Tree after insertion of E



(e1) Possible paths from D and E



(e2) Tree after insertion of F

T: n Total path length from A to node

Shortest Path

```
Algorithm shortestPath (graph)
Determine shortest path from a network vertex to other
vertices.

    Pre    graph is pointer to network
    Post   minimum path tree determined

1 if (empty graph)
1   return
2 end if
3 loop (through all vertices)
    Initialize inTree flags and path length.
    1 set vertex inTree flag to false
    2 set vertex pathLength to maximum integer
    3 loop (through all edges)
        1 set edge inTree flag to false
        2 get next edge
    4 end loop
    5 get next vertex
4 end loop
Now derive minimum path tree.
5 set first vertex inTree flag to true
6 set vertex pathLength to 0
7 set treeComplete to false
```

Shortest Path (cont.)

```
8 loop (not treeComplete)
1   set treeComplete to true
2   set minEdgeLoc to null
3   set pathLoc    to null
4   set newPathLen to maximum integer
5   loop (through all vertices)
      Walk through graph checking vertices in tree.
      1 if (vertex in tree AND outDegree > 0)
          1 set edgeLoc to firstEdge
          2 set minPath to pathLength
          3 set minEdge to maximum integer
          4 loop (through all edges)
              Locate smallest path from this vertex.
              1 if (destination not in tree)
                  1 set treeComplete to false
                  2 if (edge weight < minEdge)
                      1 set minEdge to edge weight
                      2 set minEdgeLoc to edgeLoc
                      3 end if
                  2 end if
                  3 set edgeLoc to edgeLoc nextEdge
              5 end loop
              Test for shortest path.
              6 if (minPath + minEdge < newPathLen)
                  1 set newPathLen to minPath + minEdge
                  2 set pathLoc    to minEdgeLoc
                  7 end if
              2 end if
              3 get next vertex
      6 end loop
```

Shortest Path (cont.)

```
7 if (pathLoc not null)
    Found edge to insert into tree.
    1 set pathLoc           inTree flag to true
    2 set pathLoc destination inTree flag to true
    3 set pathLoc destination pathLength to newPathLen
    8 end if
    9 end loop
end shortestPath
```