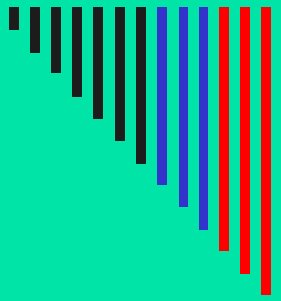


Chapter 9

Heap

Objectives

Upon completion you will be able to:

- **Define and implement heap structures**
- **Understand the operation and use of the heap ADT**
- **Design and implement selection applications using a heap**
- **Design and implement priority queues using a heap**

9-1 Basic Concepts

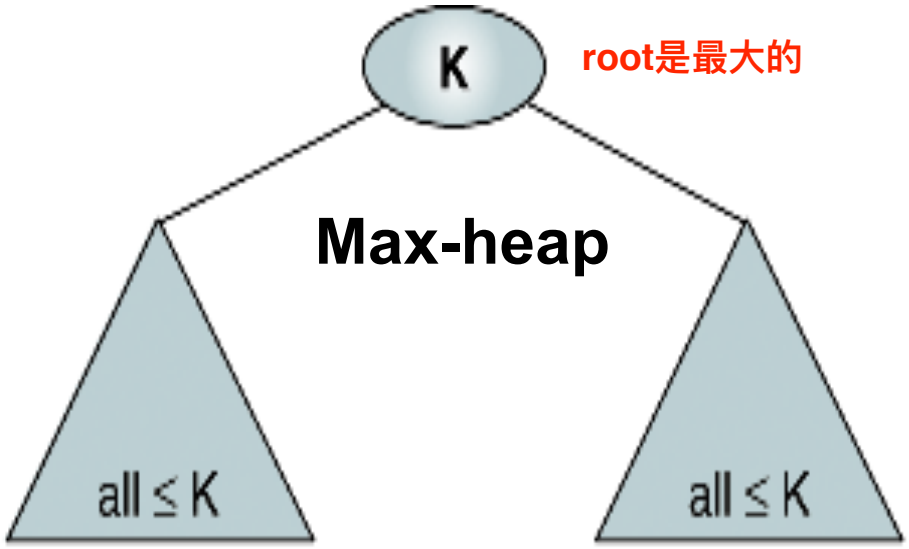
A heap is a binary tree whose left and right subtrees have values less than their parents. We begin with a discussion of the basic heap structure and its two primary operations, reheap up and reheap down.

- Definition
- Maintenance Operations

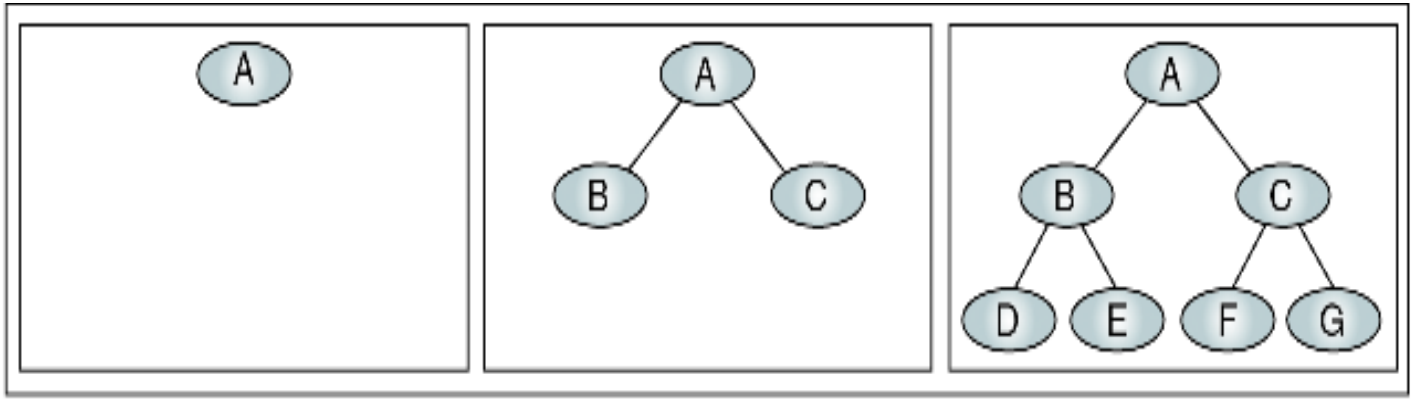
Heap

- Rule 1: the binary tree is complete or nearly complete
- Rule 2: the key value of each node is greater than or equal to the key value in each of its descendents
- A heap tree is often implemented in an array rather than a linked list which makes for very efficient processing
- It's meaningless to traverse the heap tree

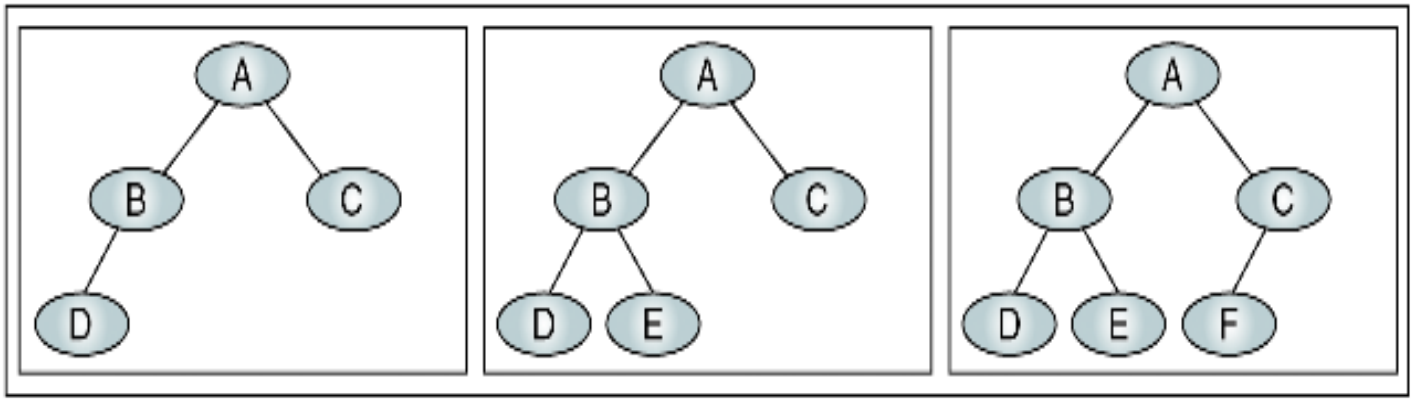
也有Min-heap
也就是root是最小的



Complete and Nearly Complete Trees

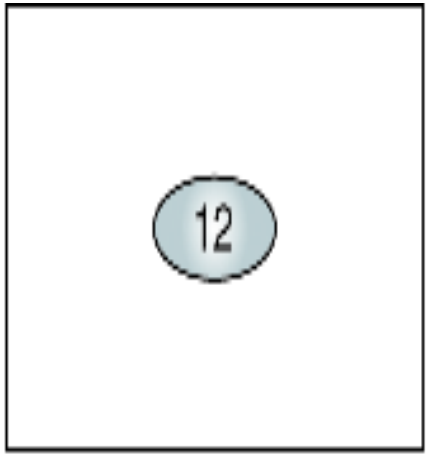


(a) Complete trees (at levels 0, 1, and 2)

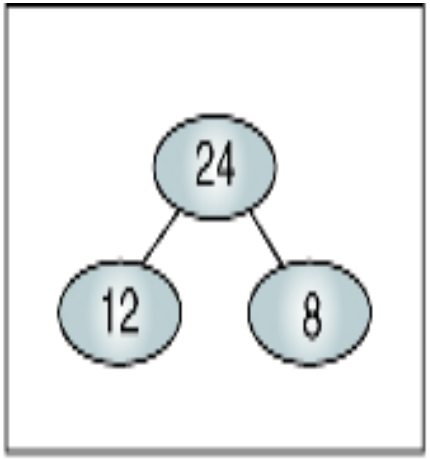


(b) Nearly complete trees (at level 2)

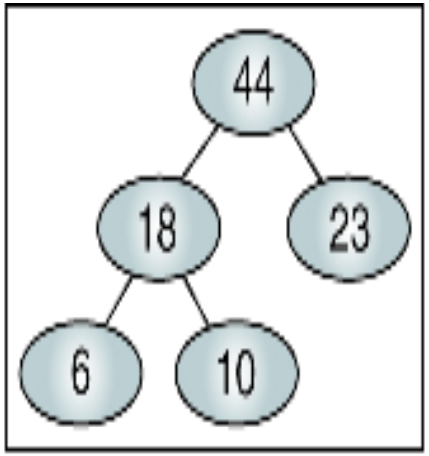
Heap Trees



(a) Root-only heap



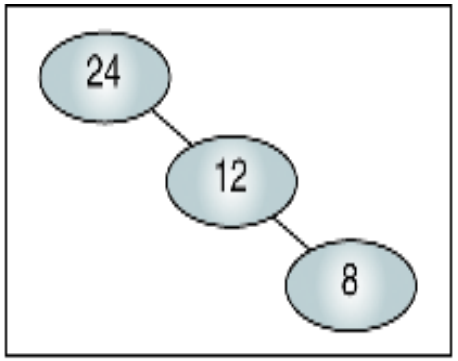
(b) Two-level heap



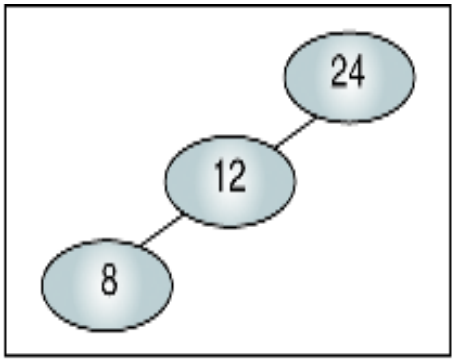
(c) Three-level heap

nearly complete

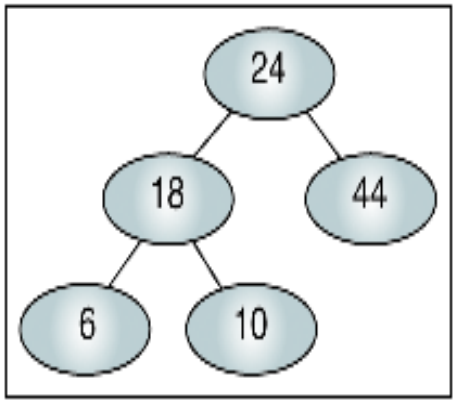
Invalid Heaps



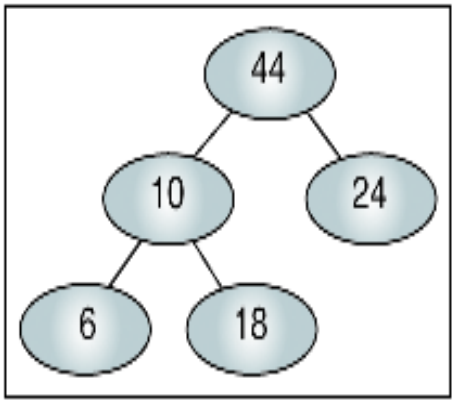
(a) Not nearly complete (rule 1)



(b) Not nearly complete (rule 1)



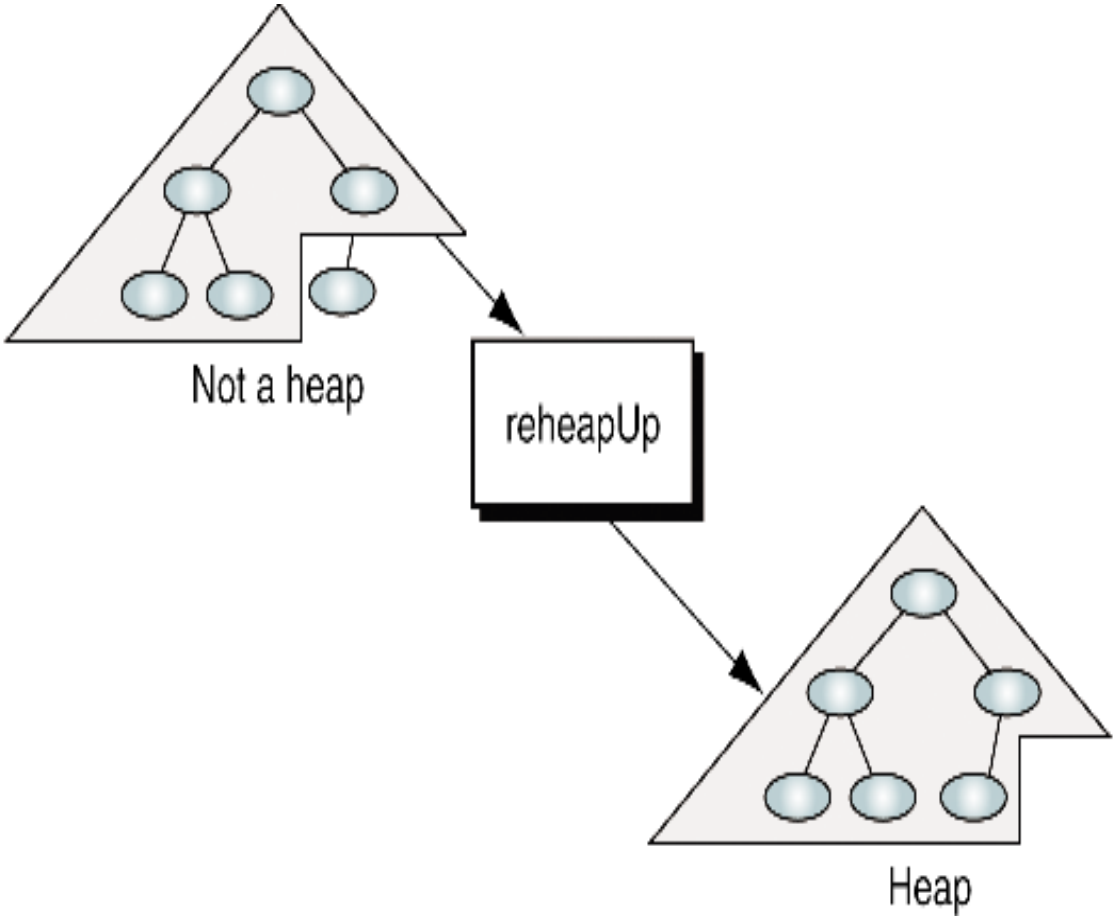
(c) Root not largest (rule 2)



(d) Subtree 10 not a heap (rule 2)

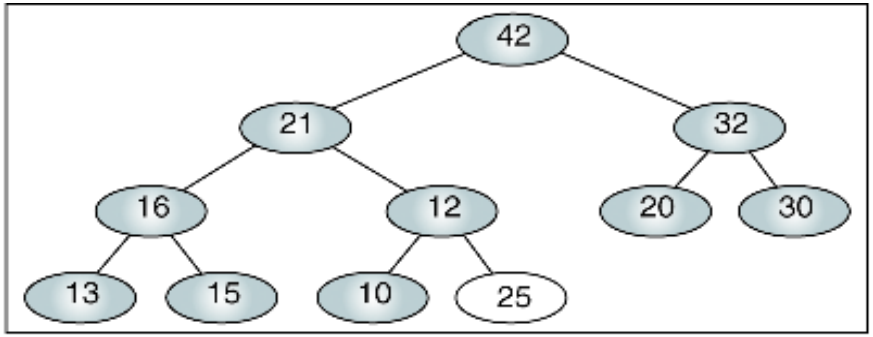
Rehaep Up Operation

- After inserting a node into a heap, the reheap up operation may be needed

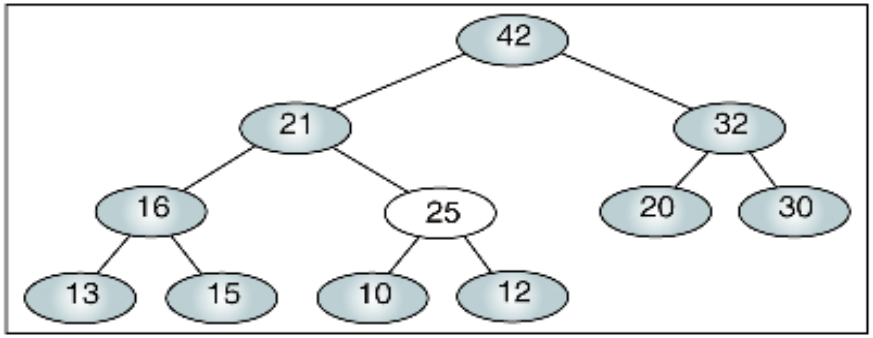


Reheap up Example

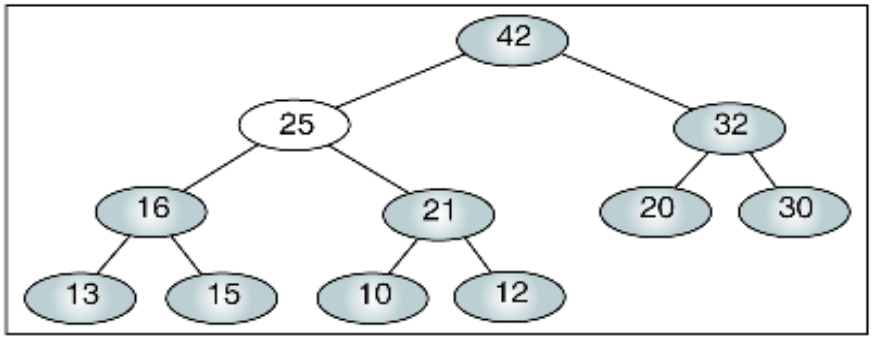
worst case: O(logn)



(a) Original tree: not a heap



(b) Last element (25) moved up

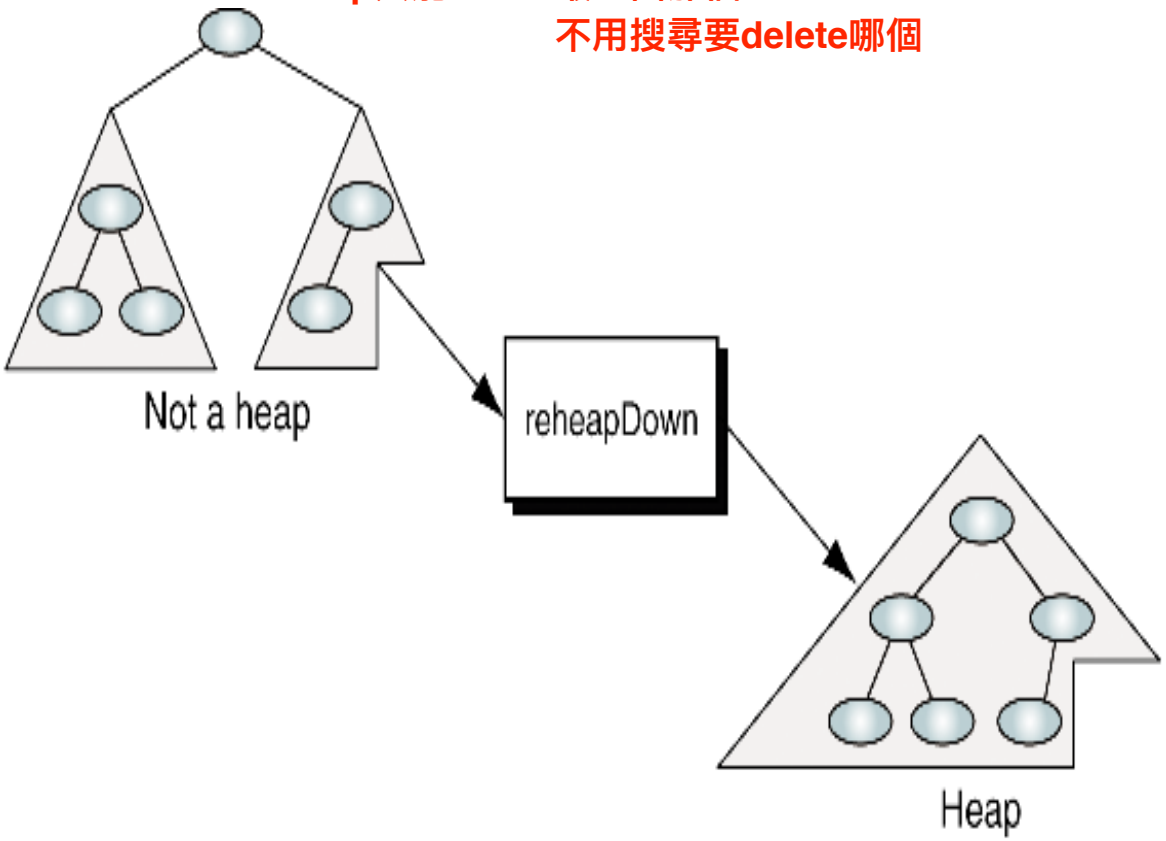


(c) Moved up again: tree is a heap

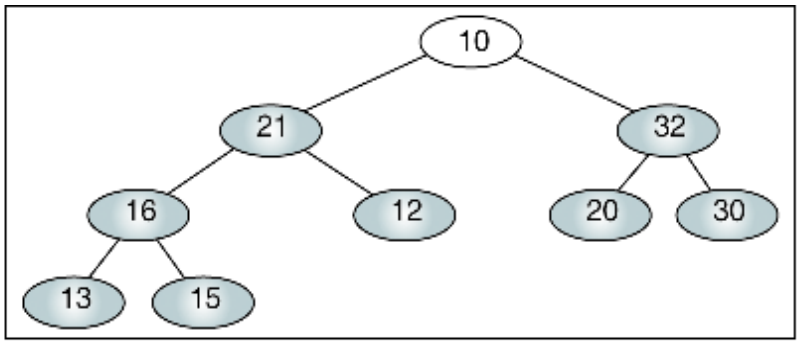
Reheap Down Operation

- After deleting a node (root), the reheap down is needed

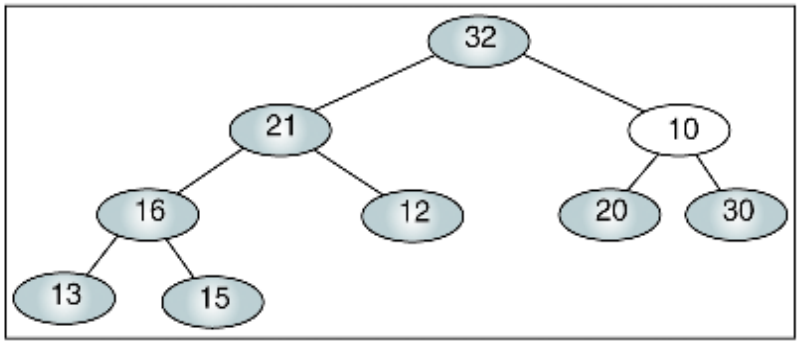
heap只能delete最上面那個node
不用搜尋要delete哪個



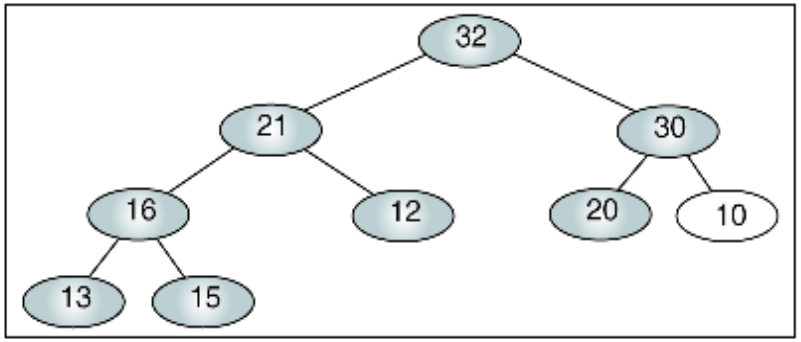
Reheap Down Example



(a) Original tree: not a heap



(b) Root moved down (right)



(c) Moved down again: tree is a heap

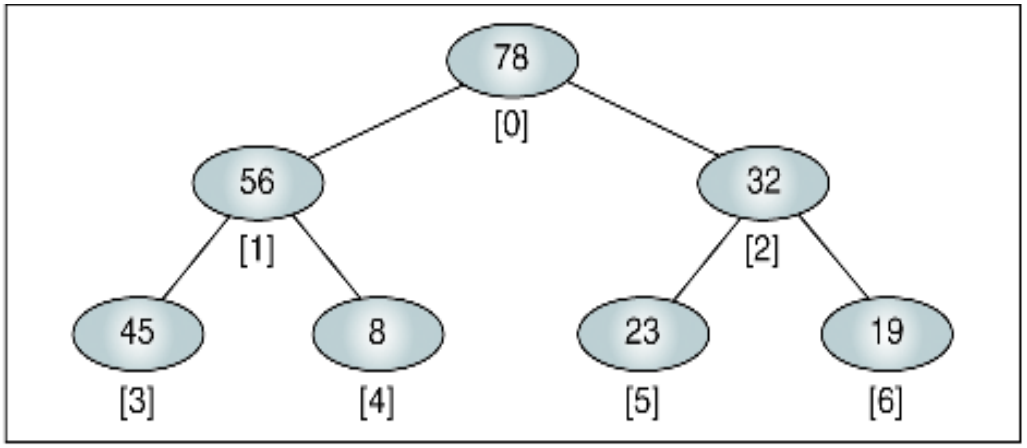
9-2 Heap Implementation

Heaps are usually implemented in an array structure. In this section we discuss and develop five heap algorithms.

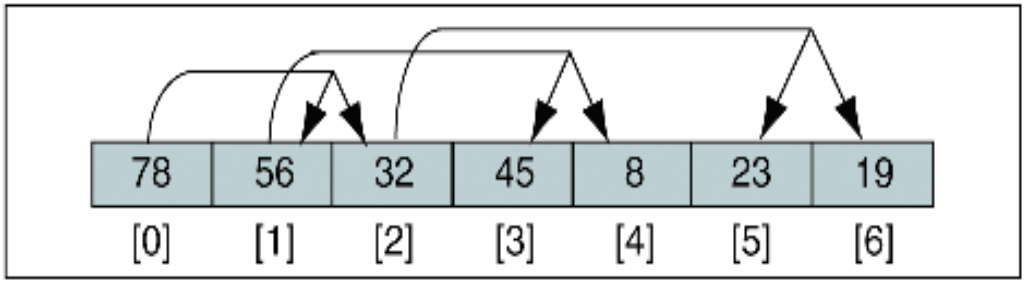
- Reheap Up
- Reheap Down
- Build a Heap
- Insert a Node into a Heap
- Delete a Node from a Heap

Heaps in Arrays

- For a node located at index *i*, its children are found at
  - Left child: 2 *i* + 1; right child: 2 *i* + 2; parent: $\lfloor (i-1)/2 \rfloor$ (i-1)/2
- A heap can be implemented in an array because it must be a complete or nearly complete binary tree, which allows a fixed relationship between each node and its children



(a) Heap in its logical form



(b) Heap in an array

Reheap Up

```
Algorithm reheapUp (heap, newNode)
Reestablishes heap by moving data in child up to its
correct location in the heap array.
   Pre  heap is array containing an invalid heap
        ¬newNode is index location to new data in heap
   Post heap has been reordered
1  if (newNode not the root)
   1   set parent to parent of newNode
   2   if (newNode key > parent key)
       1   exchange newNode and parent)
       2   reheapUp (heap, parent)
   3   end if
2  end if
end reheapUp
```

Reheap Down

```
Algorithm reheapDown (heap, root, last)
Reestablishes heap by moving data in root down to its
correct location in the heap.
   Pre    heap is an array of data
          root is root of heap or subheap
          last is an index to the last element in heap
   Post   heap has been restored
   Determine which child has larger key
1  if (there is a left subtree)
   1   set leftKey to left subtree key
   2   if (there is a right subtree)
       1   set rightKey to right subtree key
   3   else
       1   set rightKey to null key
```

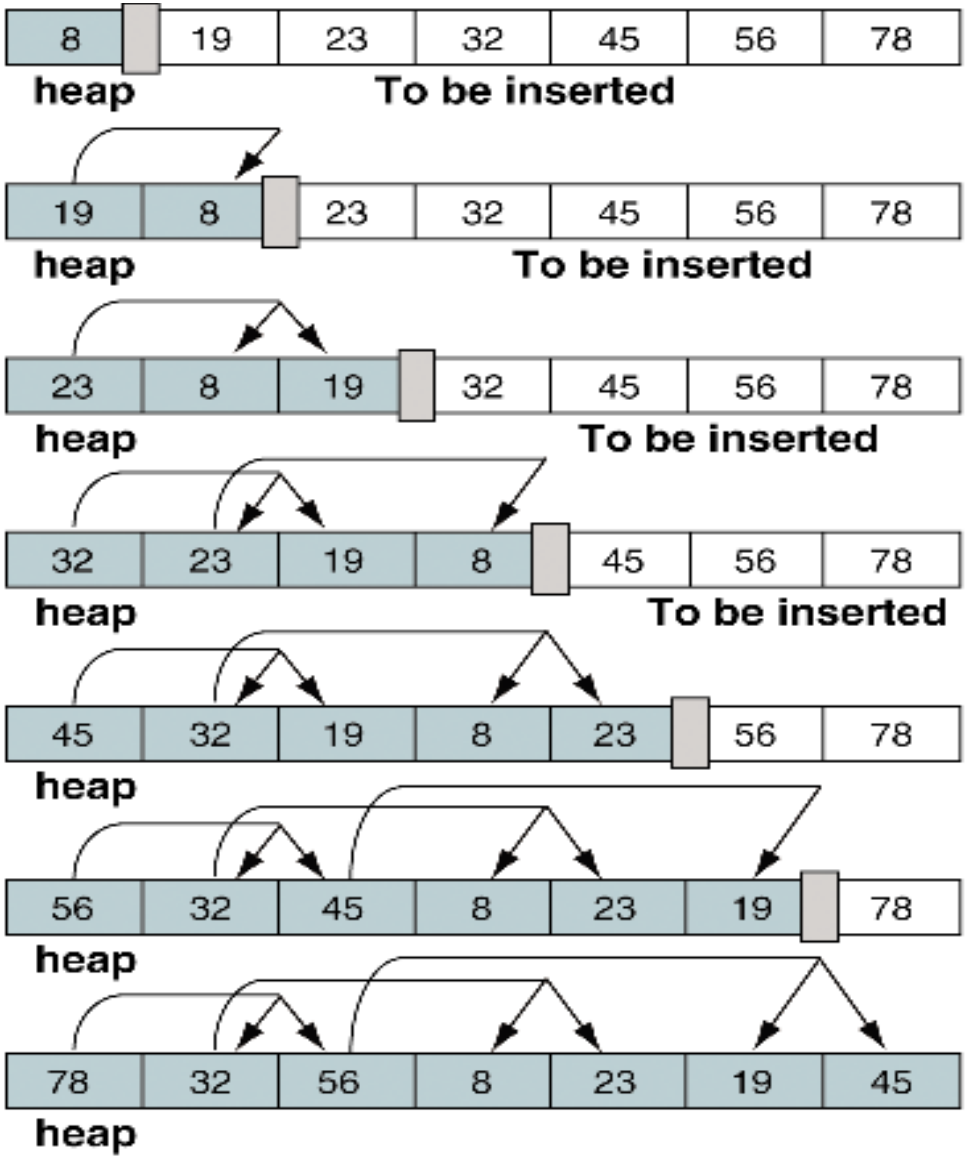
Reheap Down (cont.)

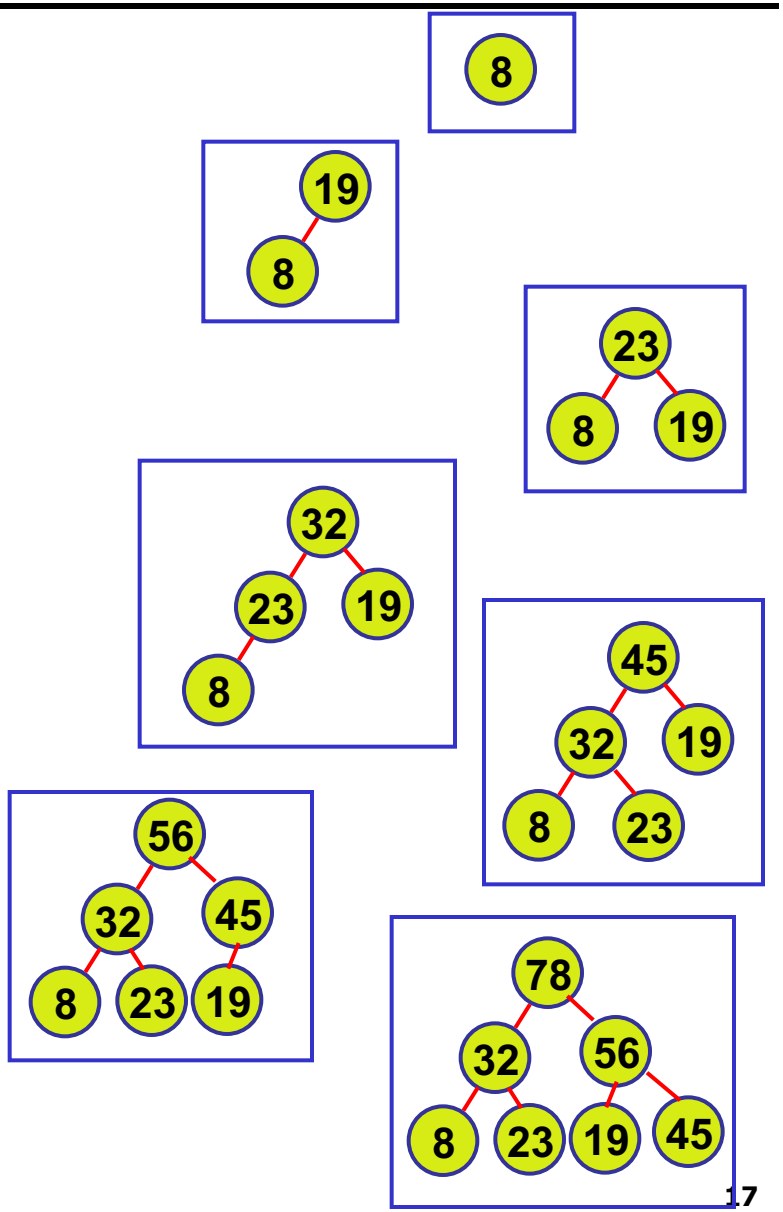
```
    4   end if
    5   if (leftKey > rightKey)
        1   set largeSubtree to left subtree
    6   else
        1   set largeSubtree to right subtree
    7   end if
        Test if root > larger subtree
    8   if (root key < largeSubtree key)
        1   exchange root and largeSubtree
        2   reheapDown (heap, largeSubtree, last)
    9   end if
 2  end if
end reheapDown
```

Build Heap

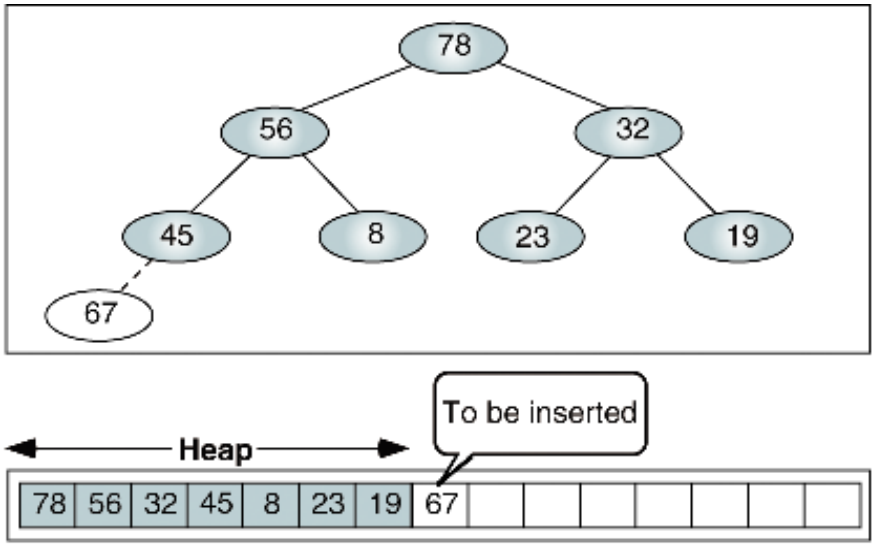
```
Algorithm buildHeap (heap, size)
Given an array, rearrange data so that they form a heap.
   Pre    heap is array containing data in nonheap order
          size is number of elements in array
   Post   array is now a heap
1  set walker to 1
2  loop (walker < size)
   1   reheapUp(heap, walker)
   2   increment walker
3  end loop
end buildHeap
```

Building a Heap

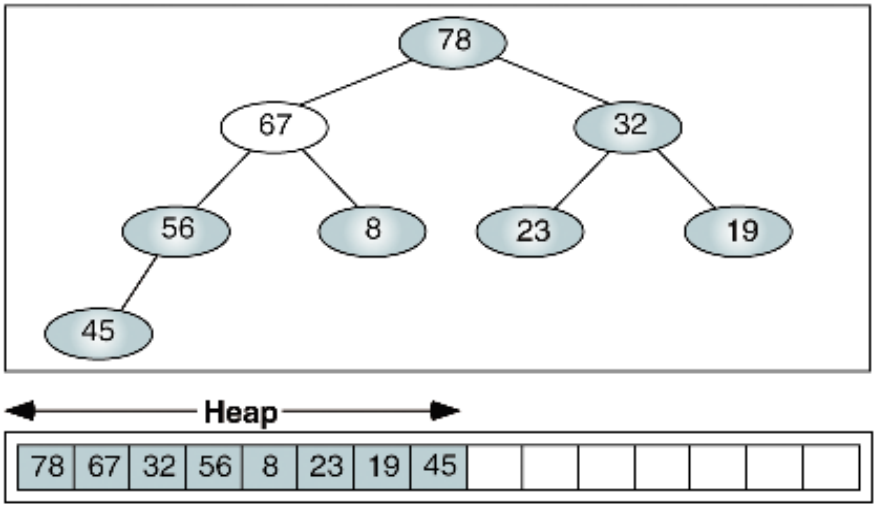




Insert Node



(a) Before reheap up



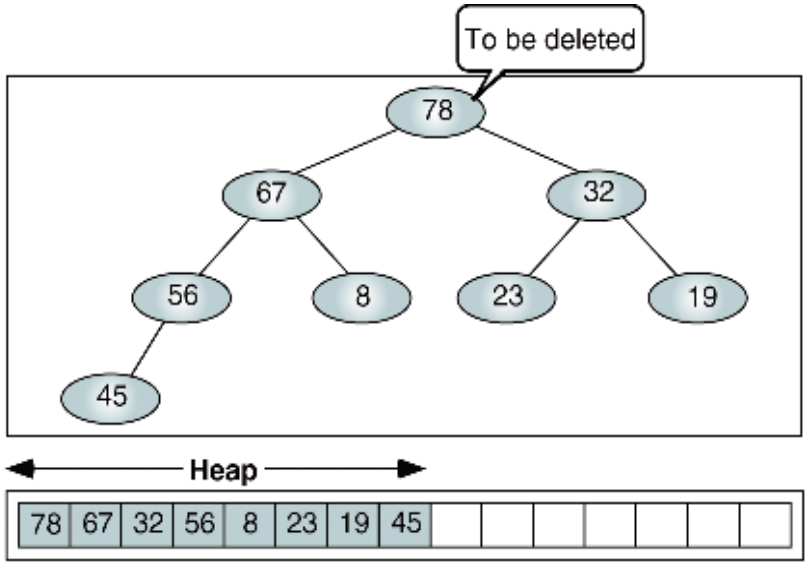
(b) After reheap up

Insert Heap

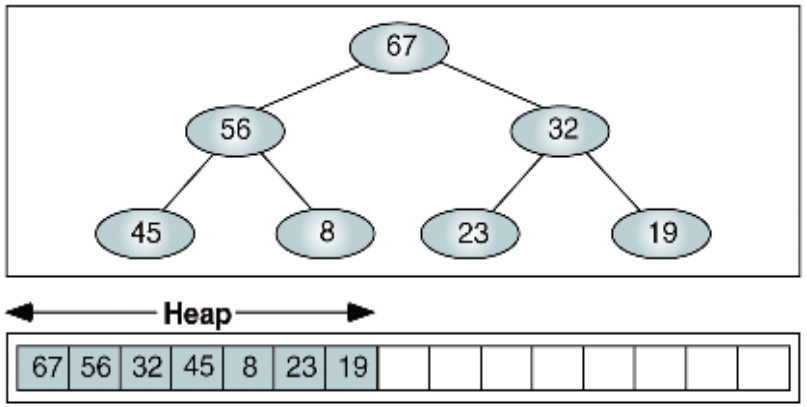
```
Algorithm insertHeap (heap, last, data)
Inserts data into heap.
   Pre    heap is a valid heap structure
          last is reference parameter to last node in heap
          data contains data to be inserted
   Post   data have been inserted into heap
   Return true if successful; false if array full
1  if (heap full)
   1   return false
2  end if
3  increment last
4  move data to last node
5  reheapUp (heap, last)
6  return true
end insertHeap
```

Delete Heap Node

- The most common and meaningful logic is to delete the root



(a) Before delete



(b) After delete

Delete Heap Node

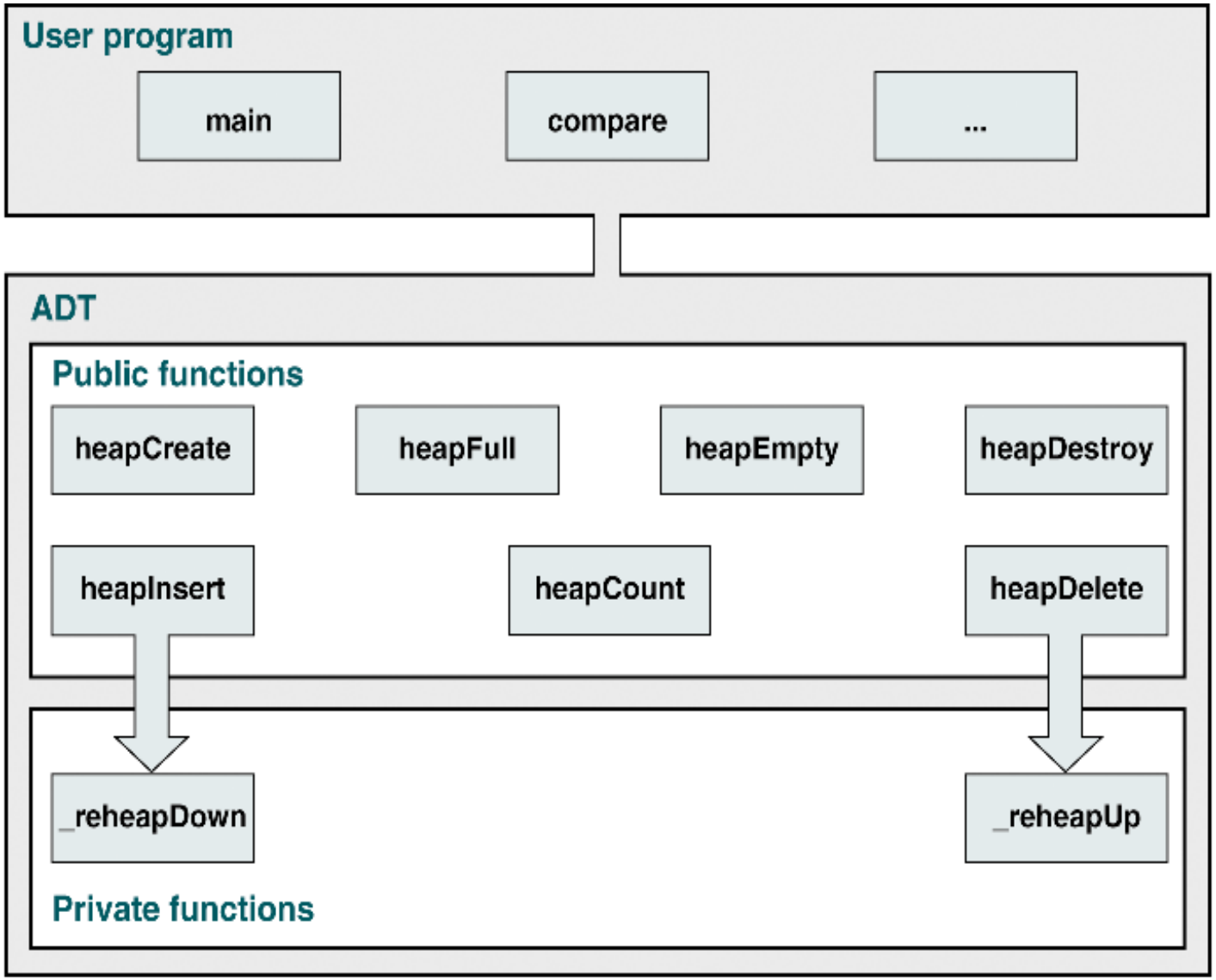
```
Algorithm deleteHeap (heap, last, dataOut)
Deletes root of heap and passes data back to caller.
   Pre    heap is a valid heap structure
          last is reference parameter to last node in heap
          dataOut is reference parameter for output area
   Post   root deleted and heap rebuilt
          root data placed in dataOut
   Return true if successful; false if array empty
1  if (heap empty)
   1   return false
2  end if
3  set dataOut to root data
4  move last data to root
5  decrement last
6  reheapDown (heap, 0, last)
7  return true
end deleteHeap
```

9-3 Heap ADT

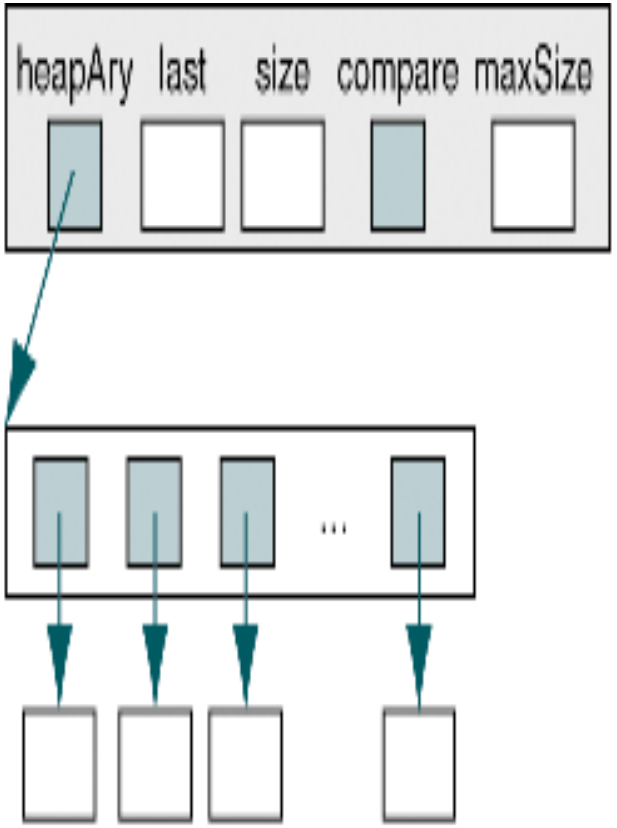
We begin with a discussion of the heap ADT design and then develop the C code for the five major functions developed in Section 9.2. Basic functions, such as heap full, are not provided.

- Heap Structure
- Heap Algorithms

Heap ADT Design



Heap ADT Structure




```
typedef struct
{
   void** heapAry;
   int    last;
   int    size;
   int   (*compare)(void* argu1, void* argu2);
   int    maxSize;
} HEAP;
```

Heap Declaration

```
 5  #include <stdbool.h>
 6
 7  typedef struct
 8  {
 9     void** heapAry;
10     int    last;
11     int    size;
12     int    (*compare) (void* argu1, void* argu2);
13     int    maxSize;
14  } HEAP;
15
16  // Prototype Definitions
17  HEAP* heapCreate (int maxSize,
18                    int (*compare) (void* arg1, void* arg2));
19  bool  heapInsert  (HEAP* heap, void*  dataPtr);
20  bool  heapDelete  (HEAP* heap, void** dataOutPtr);
21  int   heapCount   (HEAP* heap);
22  bool  heapFull    (HEAP* heap);
23  bool  heapEmpty   (HEAP* heap);
24  void  heapDestroy (HEAP* heap);
25
26  static void _reheapUp   (HEAP* heap, int childLoc);
27  static void _reheapDown (HEAP* heap, int root);
```

Create Heap Application interface

```
1  /* ==================== heapCreate ====================
2     Allocates memory for heap and returns address of
3     heap head structure.
4        Pre  Nothing
5        Post heap created and address returned
6                  if memory overflow, NULL returned
7  */
8  #include <math.h>
9
10 HEAP* heapCreate (int maxSize,
11                   int  (*compare) (void* argu1, void* argu2))
12 {
13 // Local Definitions
14    HEAP* heap;
15
```

Create Heap Application interface (cont.)

```
16  // Statements
17     heap = (HEAP*)malloc(sizeof (HEAP));
18     if (!heap)
19        return NULL;
20
21     heap->last    = -1;
22     heap->compare = compare;
23
24     // Force heap size to power of 2 -1
25     heap->maxSize =
26              (int) pow (2, ceil(log2(maxSize))) - 1;
27     heap->heapAry = (void*)
28                     calloc(heap->maxSize, sizeof(void*));
29     return heap;
30  }  // createHeap
```

Insert Heap Application interface

```
1  /* ================== heapInsert ====================
2     Inserts data into heap.
3        Pre    Heap is a valid heap structure
4               last is pointer to index for last element
5               data is data to be inserted
6        Post   data have been inserted into heap
7        Return true if successful; false if array full
8  */
9  bool heapInsert (HEAP* heap, void* dataPtr)
10 {
11 // Statements
12    if (heap->size == 0)                    // Heap empty
13       {
14        heap->size                 = 1;
15        heap->last                 = 0;
16        heap->heapAry[heap->last] = dataPtr;
17        return true;
18       } // if
19    if (heap->last == heap->maxSize - 1)
20      return false;
21    ++(heap->last);
22    ++(heap->size);
23    heap->heapAry[heap->last] = dataPtr;
24    _reheapUp (heap, heap->last);
25    return true;
26 }  // heapInsert
```

Insert ReHeap Up Function

```
1  /* ==================== reheapUp ====================
2     Reestablishes heap by moving data in child up to
3     correct location heap array.
4        Pre  heap is array containing an invalid heap
5             newNode is index to new data in heap
6        Post newNode inserted into heap
7  */
8  void  _reheapUp  (HEAP* heap, int childLoc)
9  {
10 // Local Definitions
11    int    parent;
12    void** heapAry;
13    void*  hold;
14
```

Insert ReHeap Up Function (cont.)

```
15  // Statements
16     // if not at root of heap -- index 0
17     if (childLoc)
18        {
19         heapAry = heap->heapAry;
20         parent = (childLoc - 1)/ 2;
21         if (heap->compare(heapAry[childLoc],
22                           heapAry[parent]) > 0)
23             // child is greater than parent -- swap
24             {
25              hold               = heapAry[parent];
26              heapAry[parent]    = heapAry[childLoc];
27              heapAry[childLoc] = hold;
28             _reheapUp (heap, parent);
29             } // if heap[]
30        } // if newNode
31     return;
32  }  // reheapUp
```

Delete Heap Application Interface

```
1  /*  ================== heapDelete ==================
2     Deletes root of heap and passes data back to caller.
3        Pre    heap is a valid heap structure
4               last is reference to last node in heap
5               dataOut is reference to output area
6        Post   last deleted and heap rebuilt
7               deleted data passed back to user
8        Return true if successful; false if array empty
9  */
10 bool heapDelete (HEAP* heap, void** dataOutPtr)
11 {
12 // Statements
13    if (heap->size == 0)
14        // heap empty
15        return false;
16    *dataOutPtr = heap->heapAry[0];
17    heap->heapAry[0]  = heap->heapAry[heap->last];
18    (heap->last)--;
19    (heap->size)--;
20    _reheapDown (heap, 0);
21    return true;
22 }  // heapDelete
```

Internal Reheap Down Function

```
1  /* ================== reheapDown ==================
2     Reestablishes heap by moving data in root down to its
3     correct location in the heap.
4        Pre  heap is array of data
5             root is root of heap or subheap
6             last is an index to last element in heap
7        Post heap has been restored
8  */
9  void _reheapDown (HEAP* heap, int root)
10 {
11 // Local Definitions
```

Internal Reheap Down Function (cont.)

```
12      void* hold;
13      void* leftData;
14      void* rightData;
15      int   largeLoc;
16      int   last;
17
18  // Statements
19      last = heap->last;
20      if ((root * 2 + 1) <= last)          // left subtree
21          // There is at least one child
22          {
```

Internal Reheap Down Function (cont.)

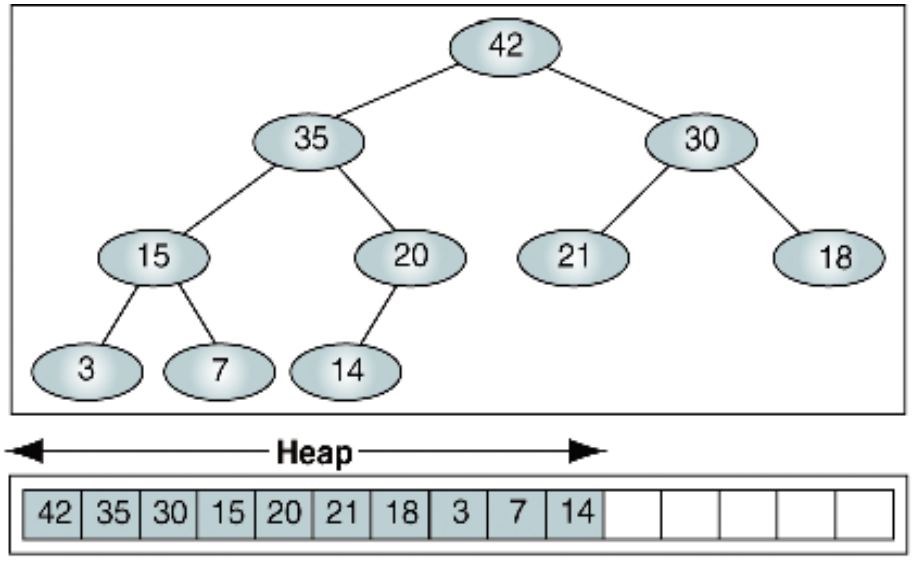
```
23          leftData    = heap->heapAry[root * 2 + 1];
24          if ((root * 2 + 2) <= last)  // right subtree
25             rightData = heap->heapAry[root * 2 + 2];
26          else
27             rightData = NULL;
28
29          // Determine which child is larger
30          if ((!rightData)
31                || heap->compare (leftData, rightData) > 0)
32             {
33              largeLoc = root * 2 + 1;
34             } // if no right key or leftKey greater
35          else
36             {
37              largeLoc = root * 2 + 2;
38             } // else
39          // Test if root > larger subtree
40          if (heap->compare (heap->heapAry[root],
41              heap->heapAry[largeLoc]) < 0)
42              {
43               // parent < children
44               hold = heap->heapAry[root];
45               heap->heapAry[root] =
46                     heap->heapAry[largeLoc];
47               heap->heapAry[largeLoc] = hold;
48               _reheapDown (heap, largeLoc);
49              } // if root <
50       } // if root
51       return;
52    }  // reheapDown
```

9-4 Heap Applications

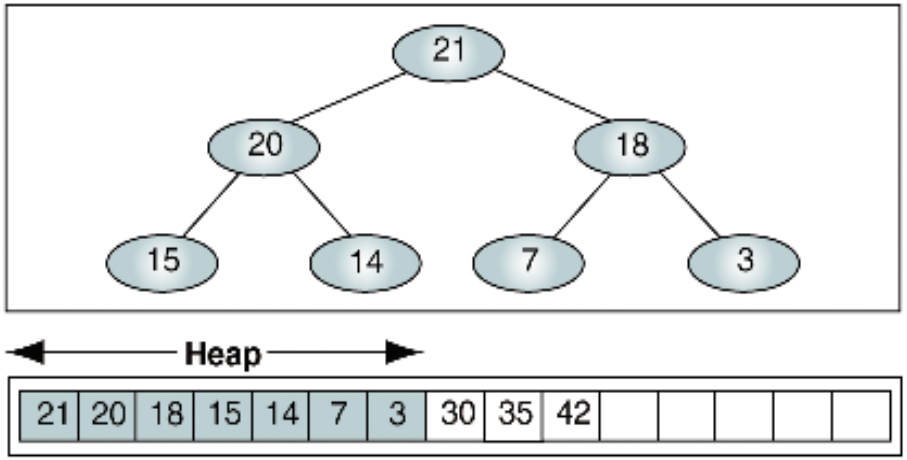
We discuss two of the three common heap applications-selection and priority queues. For selection applications, we develop a high-level algorithm. For priority queues, we develop the C code.

- Selection Algorithms
- Priority Queues
- Sorting (Chapter 12)

Heap Selection



(a) Original heap



(b) After three deletions

Heap Selection

```
Algorithm selectK (heap, k, heapLast)
Select the k-th largest element from a list
   Pre    heap is an array implementation of a heap
          k is the ordinal of the element desired
          heapLast is reference parameter to last element
   Post   k-th largest value returned
1  if (k > heap size)
   1   return false
2  end if
3  set origHeapSize to heapLast + 1
4  loop (k times)
   1   set tempData to root data
   2   deleteHeap (heap, heapLast, dataOut)
   3   move tempData to heapLast + 1
5  end loop
   Desired element is now at top of heap
6  move root data to holdOut
   Reconstruct heap
7  loop (while heapLast < origHeapSize)
   1   increment heapLast
   2   reheapUp (heap, heapLast)
8  end loop
9  return holdOut
end selectK
```

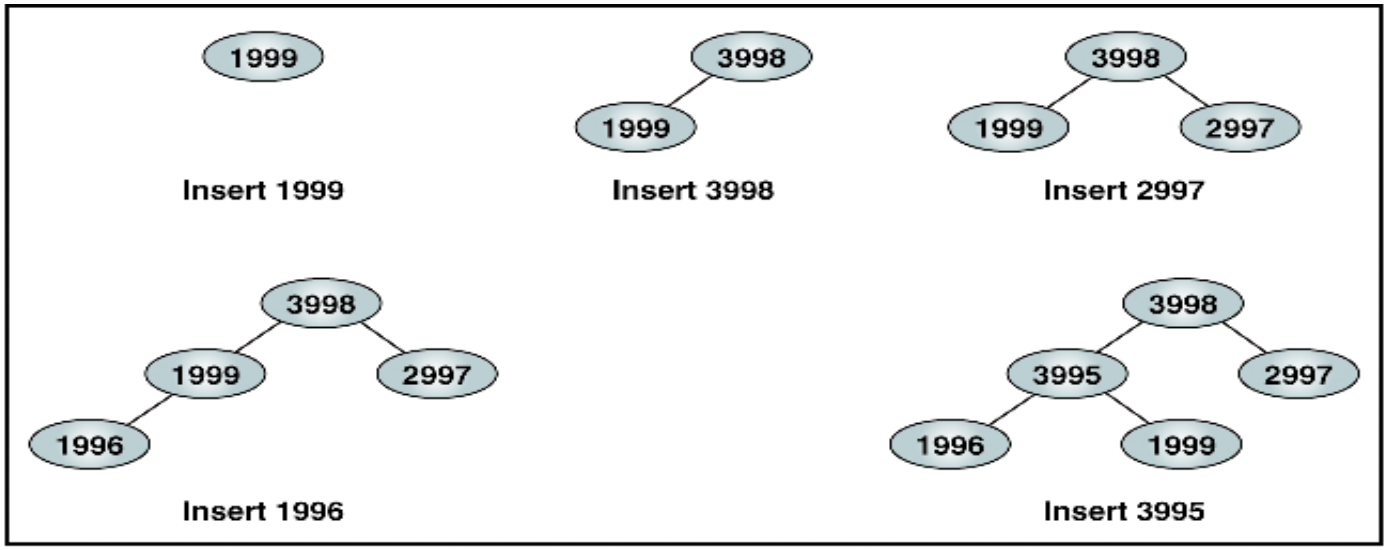
Priority Queue Priority Numbers

Priority	Serial	Priority	Serial	Priority	Serial
1	999	3	999	5	999
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
1	000	3	000	5	000
2	999	4	999		
.	.	.	.		
.	.	.	.		
.	.	.	.		
2	000	4	000		

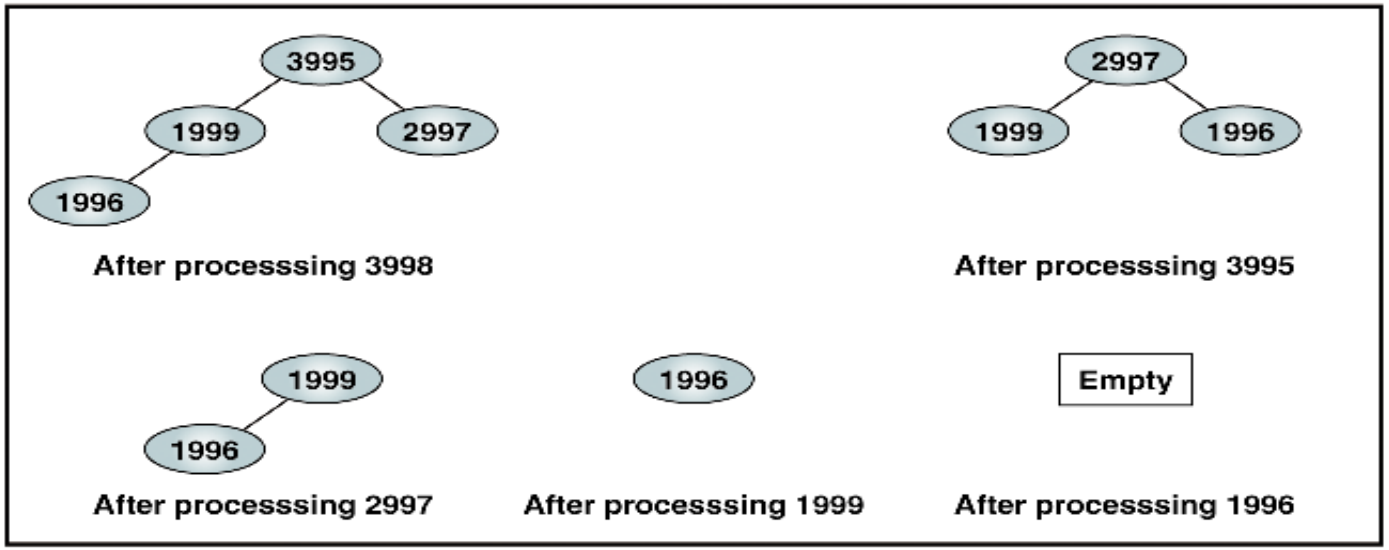
Priority Number Assignments

Arrival	Priority	Priority
1	low	1999 (1 & (1000 - 1)
2	high	3998 (3 & (1000 - 2)
3	medium	2997 (2 & (1000 - 3)
4	low	1996 (1 & (1000 - 4)
5	high	3995 (3 & (1000 - 5)

Priority Queue Example

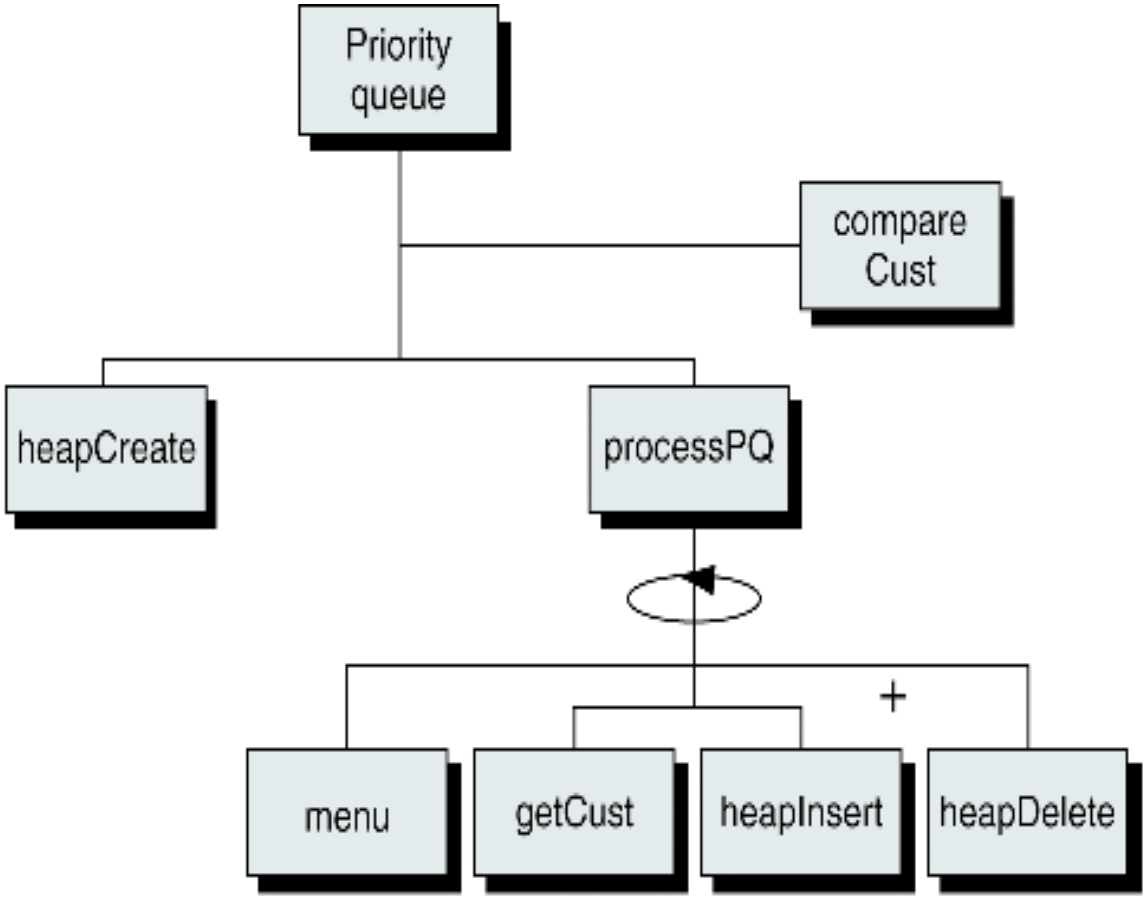


(a) Insert customers



(b) Process customers

Priority Queue Design



Priority Queue Implementation

```
 1  /* Implement priority queue using heap.
 2        Written by:
 3        Date:
 4  */
 5  #include <stdio.h>
 6  #include <ctype.h>
 7  #include <stdbool.h>
 8
 9  #include "P9-heap.h"
10
11  // Constant Definitions
12     const int maxQueue = 20;
13
14  // Structure Declarations
15     typedef struct
16        {
17         int  id;
18         int  priority;
19         int  serial;
20        } CUST;
```

Priority Queue Implementation (cont.)

```
21
22  // Prototype Declarations
23  int   compareCust (void* cust1, void* cust2);
24  void  processPQ   (HEAP* heap);
25  char  menu        (void);
26  CUST* getCust     (void);
27
28  int main (void)
29  {
30  // Local Definitions
31     HEAP* prQueue;
32
33  // Statements
34     printf("Begin Priority Queue Demonstration\n");
35
36     prQueue = heapCreate(maxQueue, compareCust);
37     processPQ (prQueue);
38
39     printf("End Priority Queue Demonstration\n");
40     return 0;
41  }  // main
```

Priority Queue Implementation (cont.)

```
42
43  /* ====================== compare ======================
44     Compare priority of two customers to determine
45     who has higher priority.
46        Pre  Given two customer structures
47        Post if cust1 >  cust2 return +1
48             if cust1 == cust2 return 0
49             if cust1 <  cust2 return -1
50  */
51  int compareCust (void* cust1, void* cust2)
52  {
53  // Local Definitions
54     CUST c1;
55     CUST c2;
56
57  // Statements
58     c1 = *(CUST*)cust1;
59     c2 = *(CUST*)cust2;
60
61     if (c1.serial < c2.serial)
62         return -1;
63     else if (c1.serial == c2.serial)
64         return 0;
65     return +1;
66  }  // compareCust
67
```

Priority Queue Implementation (cont.)

```
68  /* ==================== processPQ ====================
69     Compare priority of two customers to determine
70     who has higher priority.
71        Pre  Given two customer structures
72        Post if cust1 >  cust2 return +1
73             if cust1 == cust2 return 0
74             if cust1 <  cust2 return -1
75  */
76  void processPQ (HEAP* prQueue)
77  {
78  // Local Definitions
79     CUST* cust;
80     bool  result;
81     char  option;
82     int   numCusts = 0;
83
```

Priority Queue Implementation (cont.)

```
84  // Statements
85     do
86        {
87         option = menu ();
88         switch (option)
89            {
90             case 'e':
91             cust = getCust ();
92             numCusts++;
93             cust->serial =
94                cust->priority * 1000 + (1000 - numCusts);
95                result = heapInsert (prQueue, cust);
96                if (!result)
97                    printf("Error inserting into heap\n"),
98                        exit (101);
99                break;
100            case 'd':
101             result = heapDelete (prQueue, (void**)&cust);
102               if (!result)
103                  printf("Error: customer not found\n");
```

Priority Queue Implementation (cont.)

```
104                 else
105                   {
106                    printf("Customer %4d deleted\n",
107                           cust->id);
108                    numCusts--;
109                   } // else
110               } // switch
111           } while (option != 'q');
112       return;
113   }  // processPQ
114
```

Priority Queue Implementation (cont.)

```
115 /* ==================== menu ====================
116    Display menu and get action.
117       Pre  nothing
118       Post action read and validated
119 */
120 char menu (void)
121 {
122 // Local Declarations
123    char option;
124    bool valid;
125
126 // Statements
127    printf( "\n============ Menu =============\n" );
128    printf( " e :     Enter Customer Flight   \n" );
129    printf( " d :     Delete Customer Flight  \n" );
130    printf( " q :     Quit.                   \n" );
131    printf( "================================\n" );
132    printf( "Please enter your choice:  " );
```

Priority Queue Implementation (cont.)

```
133
134     do
135        {
136         scanf(" %c", &option);
137         option = tolower (option);
138         switch (option)
139            {
140             case 'e':
141             case 'd':
142             case 'q': valid = true;
143                       break;
144
145            default:  printf("Invalid choice. Re-Enter: ");
146                       valid = false;
147                       break;
148             } // switch
149        }  while (!valid);
150     return option;
151  }  // menu
```

Priority Queue Implementation (cont.)

```
152
153 /* ==================== getCust ====================
154    Reads customer data from keyboard.
155       Pre  nothing
156       Post data read and returned in structure
157 */
158 CUST* getCust (void)
159 {
160 // Local Definitions
161    CUST* cust;
```

Priority Queue Implementation (cont.)

```
162
163 // Statements
164    cust = (CUST*)malloc(sizeof (CUST));
165    if (!cust)
166       printf("Memory overflow in getCust\n"),
167           exit (200);
168
169    printf("Enter customer id:       ");
170    scanf ("%d", &cust->id);
171    printf("Enter customer priority: ");
172    scanf ("%d", &cust->priority);
173    return cust;
174 }  // getCust
```