The Best of Many Robustness Criteria in Decision Making: Formulation and Application to Robust Pricing

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Motivation

- robust decision making with non-Bayesian uncertainty
- well-studied: robustness against *environment* (uncertainty sets)
- less studied: robustness against optimization criteria
- maximin performance, minimax regret, maximin ratio – all well-motivated, but different
- This work: **systematic study of overfitting to robustness criteria**

Research questions:
How good is a prescription derived from one criterion when evaluated against another criterion?
Does there exist a prescription that performs well against all criteria of interest?

Problem Formulation

- **Pricing**: valuation distribution F unknown, in a given set $F \in \mathcal{F}$
  - $\mathcal{F}$: support, moments, quantiles
- Benchmark $\mathrm{OPT}(F) = \max_p p\bar{F}(p)$
- Price dist (mechanisms) Φ
- 3 focal objectives, focal mechanisms
- Revenue $\mathrm{Rev}(\Phi, F)$.
- $\mathrm{Regret}(\Phi, F) = \mathrm{OPT}(F) - \mathrm{Rev}(\Phi, F)$
- $\mathrm{Ratio}(\Phi, F) = \mathrm{Rev}(\Phi, F)/\mathrm{OPT}(F)$.

4 Focal Mechanisms From 3 Focal Objectives

$$\max_{\Phi\in\mathcal{M}}\min_{F\in\mathcal{F}} \mathrm{Revenue}(\Phi, F), \quad \min_{\Phi\in\mathcal{M}}\max_{F\in\mathcal{F}} \mathrm{Regret}(\Phi, F), \quad \max_{\Phi\in\mathcal{M}}\min_{F\in\mathcal{F}} \mathrm{Ratio}(\Phi, F)$$

Relative performance of a mechanism Φ over all criteria:

$$\mathrm{RelPerf}(\Phi, \mathrm{All}, \mathcal{F}) = \min_{F\in\mathcal{F}} \min \left\{ \frac{\mathrm{Revenue}(\Phi, F)}{\theta^*_{\mathrm{Revenue}}}, \frac{\theta^*_{\mathrm{Regret}}}{\mathrm{Regret}(\Phi, F)}, \frac{\mathrm{Ratio}(\Phi, F)}{\theta^*_{\mathrm{Ratio}}} \right\}$$

We evaluate RelPerf of the 3 focal mechanisms $\Phi^*_{\mathrm{Revenue}}, \Phi^*_{\mathrm{Regret}}, \Phi^*_{\mathrm{Ratio}}$ and the

$$\text{uniformly robust mechanism} \qquad \Phi^*_{\mathrm{All}} \in \arg\max_{\Phi\in\mathcal{M}} \mathrm{RelPerf}(\Phi, \mathrm{All}, \mathcal{F})$$

Main Result: Overfitting to criteria is real, but can be fixed!

Additional Information	Uniformly Robust Mechanism	Focal Mechanisms		
		revenue	regret	ratio
mean	**92%**	58%	44%	**68%**
mean and variance	**86%**	51%	49%	**71%**
median	**61%**	34%	0%	**41%**
lower bound	**58%**	**33%**	0%	31%

Table: Worst-case (across instances) relative performance across all criteria of the uniformly optimal mechanism, compared to that of the three focal mechanisms: maximin revenue mechanism ("revenue"), minimax regret mechanism ("regret") and maximin ratio mechanism ("ratio"). The performances of the uniformly robust optimal mechanism as well as the best among all focal mechanisms are bolded for emphasis.

Our Approach

- **LP duality**: $\mathrm{OPT}(F) = \max_p p\bar{F}(p)$ and take the dual for each p
- If the discretization grid is K, the LP has $\Theta(K)$ variables and $\Theta(K^2)$ constraints

RelPerf Across Instances