Battery Operations in Electricity Markets: Strategic Behavior and Distortions

Jerry Anunrojwong* Santiago R. Balseiro† Omar Besbes‡ Bolun Xu§

March 31, 2025

Abstract

Electric power systems are undergoing a major transformation as they integrate intermittent renewable energy sources, and batteries to smooth out variations in renewable energy production. As privately-owned batteries grow from their role as marginal "price-takers" to significant players in the market, a natural question arises: How do batteries operate in electricity markets, and how does the strategic behavior of decentralized batteries distort decisions compared to centralized batteries? We propose an analytically tractable model that captures salient features of the highly complex electricity market. We derive in closed form the resulting battery behavior and generation cost in three operating regimes: (i) no battery, (ii) centralized battery, and (ii) decentralized profit-maximizing battery. We establish that a decentralized battery distorts its discharge decisions in three ways. First, there is quantity withholding, i.e., discharging less than centrally optimal. Second, there is a shift in participation from day-ahead to real-time, i.e., postponing some of its discharge from day-ahead to real-time. Third, there is reduction in real-time responsiveness, or discharging less in response to smoothing real-time demand than centrally optimal. We also quantify the impact of the battery market power on total system cost via the Price of Anarchy metric, and prove that the it is always between 9/8 and 4/3. That is, incentive misalignment always exists, but it is bounded even in the worst case. We calibrate our model to real data from Los Angeles and Houston. Lastly, we show that competition is very effective at reducing distortions, but many market power mitigation mechanisms backfire, and lead to higher total cost.

*Columbia University, Graduate School of Business. Email: janunrojwong25@gsb.columbia.edu
†Columbia University, Graduate School of Business. Email: srb2155@columbia.edu
‡Columbia University, Graduate School of Business. Email: ob2105@columbia.edu
§Columbia University, Earth and Environmental Engineering. Email: bx2177@columbia.edu

1 Introduction

Climate change is the defining issue of our time [IPCC, 2023], and countries and regions have pledged to reduce their carbon emissions through international agreements and carbon neutrality pledges. For example, the United States and the European Union planned to reach net zero by 2050. The power sector in the United States is one of the largest emitting sectors, accounting for around 30 percent of total U.S. emissions [CBO, 2022].

To achieve this goal, the electric power systems are currently undergoing a major transformation by incorporating renewable energy resources, such as solar and wind. However, the availability of these renewable resources depends on exogenous factors that cannot be controlled. Because supply of power has to equal demand at all times on the electric grid, the system operator has to compensate for the real-time variability in one of two ways. The traditional way is to call up fast-responding "peaker" plants, but these plants are both expensive and have high emissions. Alternatively, the system can have enough energy storage resources, such as batteries, to smooth out fluctuations and variations in energy production and consumption over the course of a day. As costs fall and incentive schemes for renewables and batteries are enacted, battery storage capacity has been growing rapidly since 2021. California and Texas have emerged as front runners in the deployment of battery storage, with 8.6 GW and 4.1 GW of battery, respectively, as of April 2024, while other states are only starting to deploy batteries at scale [EIA, 2024b].[1] In deregulated electricity markets such as California and Texas, these grid-scale batteries, like other generation assets, are often privately owned by profit-driven investors. As batteries grow from their previous role as marginal "price-takers" to significant players in the market, a natural question arises:

> *How do batteries operate in electricity markets, and how does the strategic behavior of decentralized batteries distort decisions compared to centralized batteries?*

System operators have already observed strategic battery behaviors leading to negative outcomes. The Australian Energy Regulator reported strategic behavior from its (relatively small) 100MW/150MWh battery during tight market conditions on March 16-17, 2023 [AER, 2023, Parkinson, 2023]. After a generator outage (March 16) and a change in forecast price (March

[1] To put these numbers in context, California and Texas electricity demands on a typical day are around 20–40 GW and 40–70 GW, respectively, so California's batteries are already a substantial fraction of demand.

17), the battery rebid its capacity from the price floor up to $10,000/MWh and $15,000/MWh, respectively, setting the price. The report concludes, “This short-term strategic rebidding to capitalise on market conditions had the effect of exacerbating high prices. Again, while this behaviour may not be a breach of the rules, the ability of these participants to increase price through these rebidding strategies highlights the market power that participants may be able to exercise at certain times.” Batteries are also strategic in “normal” market conditions. California’s special report on battery storage [CAISO, 2023] suggests that batteries avoid being scheduled in day-ahead on average, preferring to participate in real-time markets. This strategic shift from day-ahead to real-time means the system operator has to commit additional more expensive generators in day-ahead.

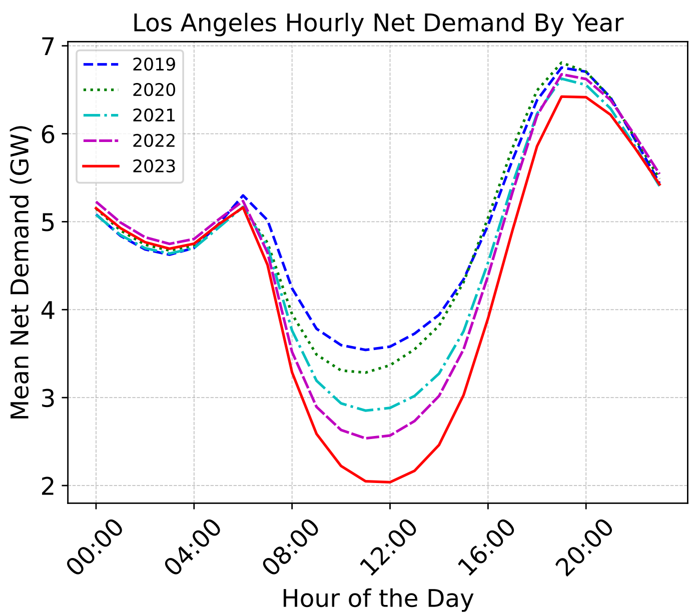

Figure 1: Los Angeles’ “duck curve,” hourly mean net demand by year, 2019–2023. The net demand is the energy demand minus renewable production. (Source: CAISO)

To understand the role of batteries, we must first understand the challenges from the mismatch between renewable energy production and demand. As an illustrative example, in Figure 1, we depict for the years 2019-2023 and for Los Angeles, the average hourly *net load* (or *net demand*), defined as the energy demand minus renewable production, which needs to be covered by conventional generators. The net demand admits a peak around 7–8PM when solar wanes and people come back from work, while during the off-peak around noon the net load is very low and has been getting lower every year due to increasing solar capacity. This leads to an increasingly steep *ramp* period during which energy generation needs to be increased quickly, and only a subset of expensive and emissions-heavy generators can do the task, negating part of the benefits of renewables. Due to its shape, this curve is often referred to as the “duck curve.”

Batteries are a natural complement to renewables in the electrical grid because they can charge during off-peak when energy is plentiful and price is low, and discharge during peak when energy is scarce and price is high. In doing so, batteries smooth out the demand by arbitraging between the off-peak and the peak periods during the day and make a profit from the price difference. As a price taker, such a battery is straightforwardly beneficial, but as discussed earlier, batteries are now a substantial fraction of demand in most regions in California and other markets are about to follow, and strategic behavior and market power become questions of interest.

Electricity markets are especially susceptible to the exercise of market power because demand and supply must be exactly equal at every *location* at any given time. Even though there are many grid-scale batteries in California, each can resemble a *local monopoly* in its region, because the transmission line infrastructure limits the amount of energy that can be transferred across regions. California regulators acknowledged that increasing volatility from renewables further exarcerbates the market fragmentation problem and approved a $7.3 billion plan to build additional transmission capacity [St. John, 2023]. The fragmented nature of the market is evident from the fact that wholesale electricity prices are very different across locations in California. For example, on May 27, 2024 at noon (off-peak), the real-time “base” price is $4/MWh but the *congestion* prices in some regions of California were as high as $120/MWh.

Starting from the deregulation of electricity markets in the 1990s, and learning from painful historical lessons along the way,[2] a large literature on detecting and mitigating market power has developed, and all US short-term wholesale markets have adopted various forms of market power mitigation procedures [Graf et al., 2021]. These measures primarily target conventional *generators*, because until recently the only feasible storage technology at scale was pumped hydro, which were relatively small. However, grid-scale battery systems are now projected to rise rapidly, and they are crucial for integrating renewables into the grid. These developments bring the questions around the potential for battery market power back to the forefront of electricity market design.

Batteries also pose an additional challenge to regulators more used to monitoring market power

[2] Most notoriously, the 2000–2001 California electricity crisis caused by flawed market design and market manipulation by energy companies, mainly Enron [Weare, 2003].

from generators. The cost bids of conventional generators are largely determined by known physical and operational constraints, which allows the system operators to ensure that generators' bids are "reasonable" most of the time based on such characteristics. The bids of batteries, however, are determined by (their predictions of) opportunity costs and not just physical marginal costs like generators. The questions of what *form of strategic behavior* a battery might take, and how it impacts *system performance*, are therefore crucial questions.

1.1 Summary of Main Contributions

At a high level, this paper identifies an important question – market power of batteries – and proposes a tractable model that is rich enough to analyze how batteries behave strategically in a two-settlement market, isolate the effects at play, and quantify their impact on system cost.

Modeling Contribution. We view the *formulation* of our model as one of our main contributions, because our model is fully microfounded and rich enough to incorporate salient institutional features yet simple enough to solve in closed form and isolate the main forces at play. In particular, we capture the two-settlement structure that is common in most markets, the duck curve demand trend (peak and off-peak), demand stochasticity and autocorrelation, and heterogeneity in generator ramp speeds. We can directly see from solutions to the model how each of these factors impact the types of strategic behaviors of batteries, and how they impact the resulting cost, under different operating regimes.

Main Features of the Model. We model a two-settlement centralized market with two periods in a day, a peak period and an off-peak period. The day-ahead market clears for each period at the beginning of the day with a day-ahead demand forecast. Then for each period, the real-time market clears after the demand for that period is realized. We assume that the demands in two periods are *random* and *correlated*, to investigate how batteries, as fast responders, react to demand stochasticity and temporal dependence. This form of modeling allows us to disentangle the two different kinds of demand-smoothing done by batteries: (i) intertemporal demand smoothing, or transferring *predictable* components of demand from peak to off-peak period, and (ii) smoothing *unpredictable* components of demand, by being responsive in real-time, discharging more when demand is unusually high, and vice versa, to reduce the cost impact of deviations from forecast.

We also assume that the battery's charge and discharge behavior has *price impact*. In other words, the price is endogenously determined.

Lastly, we microfound the model by assuming an exogenous set of generators. Only a subset of generators have fast enough ramping time to participate in real-time, and other generators can only participate in day-ahead. Given the supply curves of "slow" and "fast" generators, the prices in both day-ahead and real-time are set at a point where the supply from the generators equals the exogenous consumer demand net battery charge/discharge.

Battery Behavior. We analyze the behavior of a large battery in two regimes: the *centralized* regime, in which the system operator can control the battery charge/discharge to minimize system cost, and the *decentralized* regime, in which the battery independently makes charge/discharge decisions to maximize its profit. We find that in the centralized case, the battery perfectly smoothes the predictable demand in day-ahead, and unpredictable demand in real-time. Therefore, there is no economic withholding of any kind under this ideal baseline.

In the decentralized case, we show that the strategic battery distorts its discharge decisions relative to the centrally optimal in three ways. First is quantity withholding: expected total battery discharge is less than centrally optimal. Second is the shift from day-ahead to real-time: expected real-time battery discharge is positive, whereas it is zero in the centralized case, as the centrally optimal battery only responds to mean-zero demand fluctuations in real time. In other words, the battery "hides" part of its capacity in day-ahead, reducing its day-ahead participation, making the day-ahead planning more costly. Third is reduction in real-time responsiveness: the strategic battery discharges less to smooth real-time demand fluctuation, i.e., to reduce the cost impact of real-time deviations from forecast.

The first two types of distortion (quantity withholding and shift from day-ahead to real-time) are about arbitrage across time from peak to off-peak, and the relative weight between them depends on the *generation composition*. If most generators are fast (and can participate in real-time), then the shift from day-ahead to real-time dominates. If most generators are slow, then quantity withholding dominates. We elaborate on this in Section 5 (cf. Table 1).

Generation Cost Comparison. We compare the generation cost in the three regimes. Centralized cost is lower than decentralized cost, which is lower still than no-battery cost. Although there is incentive misalignment, having an independent battery is still better than not having one. We quantify the incentive misalignment by the ratio between the cost reduction achieved by a centrally controlled versus profit-maximizing battery, which we call the Price of Anarchy (PoA). We prove that PoA is between 9/8 and 4/3, and PoA is decreasing (i.e. the incentive alignment is better) in the share of fast generators and the steepness of the duck curve.

Numerical Illustration. We illustrate our results by calibrating our model with real data in two regions: Los Angeles and Houston. We find that, with a single monopoly battery, market power could lead to a nontrivial increase in generation cost, and all three types of distortion can be significant, but the effect of market power is relatively small compared to the gains from having enough battery capacity in the system.

1.2 Related Work

Our paper is related to several streams of literature.

Sequential Markets and Market Power Our work is most closely related to the literature on market power in sequential markets, starting with the seminal work of Allaz and Vila [1993]. The latter considers producers ("generators") rather than batteries, but some of their insights transfer to our setting. Just like in Allaz and Vila [1993], producers use the forward ("day-ahead") market even under complete certainty and perfect foresight because the forward market changes the marginal revenue on the spot ("real-time") market. Under market power, the forward market improves both producer profit and social welfare, because the producer can use the day-ahead market to reduce withholding. Ito and Reguant [2016] extends the static Cournot game of Bushnell et al. [2008] to sequential markets, and quantifies market power with limits to arbitrage in the Iberian electricity market. Borenstein et al. [2008] and Saravia [2003] also consider market power and arbitrage in electricity markets in California and New York, respectively. You et al. [2019] considers a fixed strategic load that can allocate to day-ahead or real-time. While these works focus on (perfect or imperfect) arbitrage between day-ahead and real-time by generators or purely financial "virtual" bidders and only assume one time period, our work assumes that generators are nonstrategic and focuses also on the arbitrage between peak and off-peak time periods by batteries.

Renewable Energy Operations. There is a vast literature on renewable energy in operations; for surveys, see e.g. Agrawal and Yücel [2021], Parker et al. [2019], Sunar and Swaminathan [2022]. Here, we highlight modeling works that are related to batteries, sequential markets, or market power. Sioshansi [2010] observes that price smoothing by batteries create welfare gains but the incentives may not be properly aligned for centrally optimal storage use. Sioshansi [2014] shows that the introduction of storage always increases welfare when generators are nonstrategic, but it can reduce welfare when generators are strategic. However, Sioshansi [2014] assumes that the demand is deterministic and clears in one stage, whereas we highlight the role that demand stochasticity and sequential market clearing play in different forms of distortion in battery behavior. Peura and Bunn [2021] uses a game-theoretic model to analyze how intermittent renewable production affect electricity prices in the presence of a forward market. While they do not consider batteries, the use of forward markets to improve welfare and reduce market power as in Allaz and Vila [1993] is related to our work. Acemoglu et al. [2017], Genc and Reynolds [2019] and Bahn et al. [2021] consider the impact of ownership models (similar to our centralized versus decentralized regimes) on competition and market power in renewables without batteries. Kaps et al. [2023] and Peng et al. [2021] develop models of joint investment in renewables, conventional generators, and storage. Wu et al. [2023] and Qi et al. [2015] consider investment in storage in different locations, but do not consider incentives. Zhou et al. [2016] analyzes storage operations and energy disposal in the presence of negative electricity prices.

There is also a nascent line of work on smoothing demand or shaving the peak beyond the use of batteries. Agrawal and Yücel [2022] analyzes the design of demand response programs, paying consumers to reduce consumption when the grid is under stress. Fattahi et al. [2023, 2024] analyze the use of direct load control contracts to smooth demand. Gao et al. [2024] studies different ways of aggregating distributed energy resources. Electric vehicles can also be used to shave the peak, although such uses are currently limited [Wu et al., 2022, Perakis and Thayaparan, 2023].

Market Power in Electricity Markets. There is a large empirical literature in economics measuring market power in electricity markets; see [Kellogg and Reguant, 2021, Section 4.2] for a survey, and Graf et al. [2021] for market power mitigation mechanisms. This literature is mostly focused on market power of generators; the exceptions are Karaduman [2023] and Butters et al. [2023]. Karaduman [2023] does not model sequential market clearing and focuses more on numerically computing a dynamic equilibrium between a monopoly storage and conventional generators calibrated to the South Australian electricity market. Like us, he documents the discrepancy between private and social incentives. In contrast, Butters et al. [2023] assumes that batteries do not have market power and focuses more on the impact of different incentive schemes on investments.

Battery Market Power in Power Systems There is a body of work in the power systems literature that study the market power of a monopoly battery with price impact [Mohsenian-Rad, 2015, Bjørndal et al., 2023, Hartwig and Kockar, 2016, Huang et al., 2018, Schill and Kemfert, 2011]. Other works design algorithms to maximize battery profit over time [Tómasson et al., 2020, Ward and Staffell, 2018, Cruise et al., 2019]. Whereas these papers propose detailed models in the form of large-scale mathematical programs that are numerically solved, our work gives a stylized model that can be directly analyzed and solved in closed form. The two lines of work are complementary; black box models can incorporate more details, while stylized models give sharp analytical insights and economic intuition. In particular, it is generally understood from this literature that batteries can strategically withhold capacity, but our work clarifies different *forms* of withholding and how they depend on market fundamentals.

2 Model

Two-Settlement Market. We consider a two-settlement centralized market clearing that is used by all wholesale electricity markets in the United States. The *day-ahead* (DA) market clears before the day begins, setting a price in each time period such that the amount of supply from all generators below the price equals the mean demand for that period. In other words, we assume that the system operator's day-ahead demand forecast is unbiased. Then, during the day, the *real-time* (RT) market clears the incremental real-time demand (henceforth just "RT demand"), which is the difference between the realized demand and the pre-committed DA demand. The RT demand can either be positive (higher demand than expected) or negative (lower demand than expected). If the RT demand is positive (resp. negative), a subset of generators that are fast enough to adjust in real-time are called on to increase (resp. decrease) production (cf. the generators section below). The battery makes the charge/discharge decision and amount after that period's actual demand is realized, affecting the net demand for that period (cf. the battery participation section below).

Generators. We model two types of generators: generators that have a slow ramp speed can only participate in DA (henceforth "slow generators"), whereas generators with a fast ramp speed can participate in both DA and RT (henceforth "fast generators"). We assume that we have a continuum of infinitesimally small generators, and each generator is specified by the cost. All conventional generators are assumed to be non-strategic, that is generators bid their true marginal cost. Each generator also follows the market operator's dispatch instructions on whether they produce. This is an intentional modeling choice to capture the shape of the supply curve and how it determines the clearing price and generation cost, while abstracting away non-convex elements such as start-up and no-load costs which we instead capture in a stylized way via the dichotomy between slow and fast generators. This model of generators is adapted from You et al. [2019]. (While thinking of generators as a continuum is convenient, it is not necessary: we get the same result with a finite set of generators with zero start-up and no-load costs whose combined cost functions correspond to the supply curve.)

Let $G_s(\lambda)$, respectively $G_f(\lambda)$, be the mass of slow, respectively fast generators, with cost less or equal than λ. The cost distributions $G_s(\cdot)$ and $G_f(\cdot)$ of slow and fast generators are primitives of the model, assumed to be strictly increasing.

Demand Process. There are two time periods in a day. Period 1 is the peak period, and period 2 is the off-peak period. Period 1 demand D_1 and period 2 demand D_2 are both random variables with a known joint distribution $(D_1, D_2) \sim \pi$.

For $t \in \{1, 2\}$, let $\mu_t \equiv \mathbb{E}[D_t]$ and $\sigma_t^2 \equiv \mathbb{E}[(D_t - \mu_t)^2]$ be the period-t marginal mean and variance, respectively. We will now define two notions of correlation that will later be important.

For each realization of D_1, we define the conditional mean and variance of D_2 as

$$\mu_{2|D_1} \equiv \mathbb{E}[D_2|D_1]\,, \quad \sigma_{2|D_1}^2 \equiv \mathrm{Var}(D_2|D_1). \tag{1}$$

Note that when D_1 is a random variable, so are $\mu_{2|D_1}$ and $\sigma_{2|D_1}^2$. By the law of total variance, we have $\sigma_2^2 = \mathrm{Var}(D_2) = \mathbb{E}(\mathrm{Var}(D_2|D_1)) + \mathrm{Var}(\mathbb{E}(D_2|D_1))$.

We define the *sequential correlation* ρ_s to be such that

$$\mathrm{Var}(\mathbb{E}(D_2|D_1)) = \mathbb{E}[(\mu_{2|D_1} - \mu_2)^2] = \rho_s^2\sigma_2^2\,, \quad \mathbb{E}[\sigma_{2|D_1}^2] = (1 - \rho_s^2)\sigma_2^2 \tag{2}$$

and $\rho_s^2 \in [0, 1]$. Note that if D_2 and D_1 are independent, then $\text{Var}(\mathbb{E}(D_2|D_1)) = \text{Var}(\mathbb{E}(D_2)) = 0$, so $\rho_s = 0$, whereas if D_2 is completely determined by D_1, then $\text{Var}(\mathbb{E}(D_2|D_1)) = \text{Var}(D_2) = \sigma_2^2$, so $\rho_s = 1$. Therefore, ρ_s is a "reasonable" normalized measure of dependence between D_2 and D_1.

Note that ρ_s does not need to be the same as the standard Pearson correlation ρ defined by

$$\rho \equiv \frac{\mathbb{E}[(D_1 - \mu_1)(D_2 - \mu_2)]}{\sigma_1 \sigma_2}. \tag{3}$$

However, if (D_1, D_2) are jointly normally distributed, then $\rho_s = \rho$. We will not make the normal assumption through most of our results; when we do, we will be explicit, and the normal assumption is made only for convenience or to highlight certain insights.

Recall that this demand process describes *net demand*, so uncertainties in the net demand comes from both consumer demand and renewable production, and they are both exogenous. In particular, we assume that electricity demand is perfectly inelastic. This is well-supported by empirical evidence [Joskow, 2006]. We model the demand process this way because we want to capture the fact that markets with high renewable and battery penetration such as Los Angeles exhibit a *duck curve* (cf. Figure 1): the price is highest in the evenings when solar generation wanes (the sun sets) and demand peaks as people come home from work, and the price is lowest in midday when solar production peaks. Period 1 and period 2 are meant to correspond to the peak (including the steep ramp leading to the peak) and the off-peak of the duck curve. Demands in both periods have a significant amount of variability, and they are also highly correlated, hence our modeling choice.

Battery Participation. There is a single battery. Before the day, the battery decides on the DA discharge amount z_1^{DA} and z_2^{DA} in period 1 and period 2 day-ahead. Then the real-time scenario materializes: the period 1 demand D_1 is realized, then depending on the demand realization, the battery decides on the RT discharge amount $z_1^{RT}(D_1)$. After period 2 demand D_2 is realized, the battery RT discharge amount in period 2 is $z_2^{RT}(D_1, D_2)$. Note that the battery charging is represented by *negative* discharge z.

The battery is assumed to have no state-of-charge constraint, no operating cost, and no efficiency losses. While our model features only one day (with multiple time periods in a day), we should interpret the model as a day in *steady state*. Equivalently, we have the same day that happens day after day. The battery cannot produce its own energy, so the total energy charged must equal the total energy discharged (plus the cycle inefficiency loss, which we assume to be zero for now).

Furthermore, we impose the condition that the net discharge throughout the day is zero for each demand realization: $z_1^{DA} + z_2^{DA} = 0$ and $z_1^{RT}(D_1) + z_2^{RT}(D_1, D_2) = 0$ for every (D_1, D_2) in the support. This assumption is made because (i) batteries cannot produce energy, only shift it across time, and (ii) batteries overwhelmingly arbitrage between peak and off-peak periods within a day rather than between days.

The fact that batteries typically have negligible net daily discharge, in agreement with (ii) above, is evident from the data. For each year in 2021–2023, we can calculate the mean absolute daily discharge over the year, which captures the average net daily position of batteries over the year. This net position is 1.0%, 2.9%, and 1.1% of total battery capacity in 2021, 2022, and 2023. The daily charge cycle of batteries is by design: less than 7% of installed storage have duration exceeding 4 hours [Denholm et al., 2023].

Note that while we model the battery as choosing a quantity in each scenario, the quantity discharged in practice depends on the specific market framework, which broadly falls into two categories. First is self-scheduling: the battery can decide the quantity to discharge in each period, which is the same as our model. Second is economic bidding, where the battery bids the charge and discharge curves as price-quantity pairs, and the quantity charged/discharged is determined from market clearing conditions. Given that the battery is the only strategic player in the environment, the battery can choose the bid curves to achieve any desired quantity level.

To conclude, the battery's decision variables are $z_1^{DA}, z_2^{DA}, z_1^{RT}(D_1), z_2^{RT}(D_1, D_2)$ subject to constraints $z_1^{DA} + z_2^{DA} = 0$ and $z_1^{RT}(D_1) + z_2^{RT}(D_1, D_2) = 0$ for all (D_1, D_2) in the support. Note that once we decide the discharge amount $z_1^{RT}(D_1)$ during peak period 1, then the discharge amount in off-peak period 2 is determined: during each off-peak, the battery charges back to full to be ready for discharge during peak the next day.

Net Demand and Price Formation Process. The DA demands in period 1 and period 2 are taken to be the mean μ_1 and μ_2 of D_1 and D_2, from the system operator's unbiased demand forecast. With the battery discharge, the DA *net demand* in period 1 and 2 are $d_1^{DA} = \mu_1 - z_1^{DA}$ and $d_1^{DA} = \mu_2 - z_2^{DA} = \mu_2 + z_1^{DA}$. The RT *net demand* in period 1 and 2 are $d_1^{RT}(D_1) = D_1 - \mu_1 - z_1^{RT}(D_1)$ and $d_2^{RT}(D_1, D_2) = D_2 - \mu_2 - z_2^{RT}(D_1, D_2) = D_2 - \mu_2 + z_1^{RT}(D_1)$. Note that the RT net demand is the *incremental* demand, i.e., the adjustment to the quantity cleared in day-ahead. (We slightly abuse the terminology here. The traditional definition of net demand is system demand minus renewable production, which is covered by conventional generators *and batteries*; this corresponds to D_1 and D_2 in the demand process section earlier. The "net demands" $d_1^{DA}, d_2^{DA}, d_1^{RT}, d_2^{RT}$ in this section are covered by conventional generators only.)

In each time t, the DA price λ_t^{DA} is set at the market clearing price, that is, the price such that the energy produced by generators with costs below the price exactly equals the net demand:

$$G_s(\lambda_t^{DA}) + G_f(\lambda_t^{DA}) = d_t^{DA}. \tag{4}$$

In RT, the price λ_t^{RT} is set such that the total energy produced equals the net demand (DA demand plus incremental RT demand). However, slow generators with total energy output $G_s(\lambda_t^{DA})$ can no longer be adjusted in real-time, so the system operator sets the price so that the RT generators adjust their output to match the realized net demand:

$$G_s(\lambda_t^{DA}) + G_f(\lambda_t^{RT}) = d_t^{DA} + d_t^{RT}\,. \tag{5}$$

Equations (4) and (5) relate the net DA and RT demands d_t^{DA}, d_t^{RT} to the DA and RT prices $\lambda_t^{DA}, \lambda_t^{RT}$. (Note that d_t^{RT} and λ_t^{RT} depend on $D = (D_1, D_2)$ which we omit for brevity.) We can invert these to get prices in terms of net demands.

We note that if RT demand is zero ($d_t^{RT} = 0$, no adjustment to demand), then DA and RT prices are equal: $\lambda_t^{DA} = \lambda_t^{RT}$. If RT demand is positive (resp. negative), then the RT price is higher (resp. lower) than the DA price.

Generation Cost. As the price formation process suggests, the slow generators are cleared in DA: they produce if and only if their costs are below λ_t^{DA}. The fast generators are cleared in RT: they produce if and only if their costs are below λ_t^{RT}. The total generation cost follows from integrating the mass of generators with cost less than the corresponding clearing price, λ_t^{DA} for slow generators and λ_t^{RT} for fast generators. Therefore, the total generation cost is given by

$$\sum_{t=1}^{2} \left(\int_{\lambda \le \lambda_t^{DA}} \lambda dG_s(\lambda) + \mathbb{E}_D \left[\int_{\lambda \le \lambda_t^{RT}} \lambda dG_f(\lambda) \right] \right) , \tag{6}$$

where the expectation is taken over the random demand.

Throughout, we consider the system cost to be generation cost from conventional generators. Because consumers are assumed to be price inelastic, maximizing welfare is equivalent to minimizing generation cost. This also matches the prevailing objective of independent system operators: the "unit commitment" and "economic dispatch" procedures in day-ahead and real-time both minimize generation cost [Kirschen and Strbac, 2018, Cretì and Fontini, 2019].

Battery Operation Models. We will compare three operating regimes.

- A first benchmark system is one without batteries.
- Centralized participation: In this system, the battery is directly controlled by the system operator and makes charge/discharge decisions to *minimize generation cost.*
- Decentralized participation: in this system, the battery is an independent entity that makes charge/discharge decisions to *maximize its own profit.*

Note that the DA and RT prices are endogenously determined by the battery's charge/discharge decisions, as outlined earlier as part of the price formation process.

Day-Ahead and Real-Time Supply Curves We have the relationships between DA and RT prices and demands in (4) and (5), which depend on G_s and G_f. G_s and G_f are demand functions for slow and fast generators, respectively, so we have a flexible way to define the relationship between demand and price via the specification of G_s and G_f.

We assume that at each price λ, a fraction k_f of generators are fast, and $k_s = 1 - k_f$ are slow. In other words, at any price point, there are fast generators that can adjust their production up and down. This assumption reflects the operating characteristics of the generators themselves: coal and nuclear plants are "slow," whereas natural gas and hydro plants are "fast." This assumption is also an implicit model of the system operator's *reserve requirement* in day-ahead scheduling, which ensure this property by committing some fast-responding generators in day-ahead (even when they are relatively expensive) for reliability. Let $G(\lambda) = G_s(\lambda) + G_f(\lambda)$ be the total supply function, then $G_s(\lambda) = k_s G(\lambda)$ and $G_f = k_f G(\lambda)$. Equations (4) and (5) imply

$$\lambda_t^{DA} = G^{-1}\left(d_t^{DA}\right) \tag{7}$$

$$\lambda_t^{RT} = G^{-1}\left(d_t^{DA} + \frac{1}{k_f} d_t^{RT}\right). \tag{8}$$

Note that while $G(\cdot)$ describes a supply curve that maps price to quantity, $G^{-1}(\cdot)$ *also* describes a supply curve, mapping quantity to price. We assume that the supply curve is linear:

$$G^{-1}(x) = \alpha + \beta x, \tag{9}$$

where $\alpha, \beta \geq 0$ are known constants. The parameter α is the "intercept" (minimum marginal cost for conventional generators), and the parameter β is the "slope." The linear supply curve assumption is commonly made in the literature, e.g., Sioshansi [2010, 2014], Ito and Reguant [2016], and we also make this assumption primarily for parsimony. We thus have price-demand relationships of the form $\lambda_t^{DA} = \alpha + \beta^{DA} d_t^{DA}, \lambda_t^{RT} = \lambda_t^{DA} + \beta^{RT} d_t^{RT}$ with $\beta^{DA} = \beta, \beta^{RT} = \beta / k_f$. These price-demand relationships are similar to those derived in [You et al., 2019, Equations (5) and (8)]. In this case, we can interpret β^{DA} and β^{RT} as price elasticities of day-ahead and real-time demand, respectively.[3]

3 No Battery Baseline

In the next three sections, we will characterize the optimal battery behavior and the corresponding generation cost in three regimes: no battery (§3), centralized cost-minimizing battery (§4), and decentralized profit-maximizing battery (§5). Our results hold for any given distribution π over (D_1, D_2).

Before we proceed, we first derive an expression for generation cost in terms of demands, which is used in all regimes we consider. Define the modified DA and RT demands as $\tilde{d}_t^{DA} = d_t^{DA}$, $\tilde{d}_t^{RT} = d_t^{DA} + d_t^{RT}/k_f$. (Both d_t^{RT} and $\tilde{d}_t^{RT}$ can depend on $D = (D_1, D_2)$, but we sometimes omit

[3] Strictly speaking, they are not price elasticities, because price elasticities are conventionally defined as the ratio of one *percentage (relative)* change against another percentage change, whereas our coefficient is the *slope*, or the ratio of the *absolute* price change against the absolute demand change. The intuition governing both is similar.

it for brevity.) The generation cost is then given by

$$\sum_{t=1}^{2} k_s \left(\alpha \tilde{d}_t^{DA} + \beta \frac{(\tilde{d}_t^{DA})^2}{2} \right) + k_f \mathbb{E}_D \left[\alpha \tilde{d}_t^{RT} + \beta \frac{(\tilde{d}_t^{RT})^2}{2} \right]. \tag{10}$$

In the no-battery case, the generation cost is computed from (10) with $\tilde{d}_1^{DA} = \mu_1, \tilde{d}_2^{DA} = \mu_2, \tilde{d}_1^{RT}(D_1) = \mu_1 + (D_1 - \mu_1)/k_f$ and $\tilde{d}_2^{DA}(D_1, D_2) = \mu_2 + (D_2 - \mu_2)/k_f$. The proof is given in Appendix A.

Theorem 1 (No Battery)**.** *The generation cost under no battery* Cost(NB) *is given by*

$$\text{Cost(NB)} = \alpha(\mu_1 + \mu_2) + \beta \left(\frac{\mu_1^2 + \mu_2^2}{2} + \frac{\sigma_1^2 + \sigma_2^2}{2k_f} \right).$$

This no-battery generation cost is a baseline to which we compare the other two regimes, centralized and decentralized. The generation cost depends only on the marginal mean μ_t and marginal variance σ_t^2, and not on the actual distribution beyond these moments, or the correlation of demands from the two periods. Correlation does not matter because there is no decision making (i.e., battery) linking the two periods, and the market clears in each period independently. The generation cost depends on the variance because the generation cost is quadratic in demand. Variability in demand therefore leads to higher cost. The generation cost is also decreasing in k_f, because a larger fraction of fast generators means that more generators can buffer the real-time variability of demand. In other words, with more fast generators, less expensive fast generators around the day-ahead point are enough to satisfy the real-time incremental demand, and the system does not need to use more expensive generators further away. This is also why k_f only appears in conjunction with the variance terms and not the mean terms: if the demands are deterministic $\sigma_1 = \sigma_2 = 0$, then the cost no longer depends on k_f.

4 Battery Operation: Centralized Solution

We now consider the case when there is a battery, and the system operator can directly control the battery to achieve the system goal, namely, minimize generation cost. The decision variables are the DA and RT discharges z_1^{DA} and $z_1^{RT}(D_1)$ for each realization of period-1 demand D_1, and the system operator solves

$$\min_{z_1^{DA}, z_1^{RT}(\cdot)} \sum_{t=1}^{2} \left[k_s \left(\alpha \tilde{d}_t^{DA} + \beta \frac{(\tilde{d}_t^{DA})^2}{2} \right) + k_f \mathbb{E}_D \left(\alpha \tilde{d}_t^{RT} + \beta \frac{(\tilde{d}_t^{RT})^2}{2} \right) \right],$$

where

$$\begin{aligned}
\tilde{d}_1^{DA} &= d_1^{DA} = \mu_1 - z_1^{DA} \\
\tilde{d}_2^{DA} &= d_2^{DA} = \mu_2 + z_1^{DA} \\
\tilde{d}_1^{RT} &= d_1^{DA} + \frac{d_1^{RT}}{k_f} = \mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f} \\
\tilde{d}_2^{RT} &= d_2^{DA} + \frac{d_2^{RT}}{k_f} = \mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}.
\end{aligned}$$

The following theorem gives the optimal battery decisions and the corresponding generation cost.

Theorem 2 (Centralized Battery)**.** *The centralized battery discharge decisions are given by*

$$\begin{aligned}
z_1^{DA} &= \frac{1}{2}(\mu_1 - \mu_2), \\
z_1^{RT}(D_1) &= \frac{1}{2}(D_1 - \mu_1) - \frac{1}{2}(\mu_{2|D_1} - \mu_2).
\end{aligned}$$

The corresponding system generation cost Cost(CN) *is*

$$\mathrm{Cost(CN)} = \alpha(\mu_1 + \mu_2) + \frac{\beta}{4}(\mu_1 + \mu_2)^2 + \frac{\beta}{4k_f}\left[\sigma_1^2 + (2 - \rho_s^2)\sigma_2^2 + 2\rho\sigma_1\sigma_2\right].$$

We can show that the centralized cost minimization problem is a convex quadratic optimization problem, whose unique optimal solution can be found from first-order conditions. The proof is given in Appendix A.

The DA demands in the two periods are μ_1 and μ_2 respectively, and the DA discharge is $z_1^{DA} = (\mu_1 - \mu_2)/2$, so the net demands of the two periods are equalized: $\mu_1 - z_1^{DA} = \mu_2 + z_2^{DA}$. The RT smoothing is more subtle, as we have to make the discharge decision $z_1^{RT}(D_1)$ knowing the realization of period 1 demand D_1 but not the period 2 demand D_2. If we replace the unknown period 2 demand with its conditional mean $\mu_{2|D_1}$, then the incremental RT demands of the two periods are $D_1 - \mu_1$ and $\mu_{2|D_1} - \mu_2$, so the RT discharge $z_1^{RT}(D_1) = \frac{1}{2}(D_1 - \mu_1) - \frac{1}{2}(\mu_{2|D_1} - \mu_2)$

is set such that the net RT demands of the two periods are equalized: $(D_1 - \mu_1) - z_1^{RT}(D_1) = (\mu_{2|D_1} - \mu_2) + z_1^{RT}(D_1)$.

Therefore, batteries improve social welfare by "smoothing" demand, in the sense that batteries shift demand from peak, where it is scarce and expensive, to off-peak, where it is plentiful and cheap, making the net demand profile over the day more equalized and smooth. Theorem 2 shows that in a centrally optimal world, the battery "perfectly" smooths demand *between* periods (peak versus off-peak) in DA stage, and perfectly smooths the "residual" random demand *within* each period in RT stage, in the sense that the net demands after battery between two periods are equal in both day-ahead and real-time: the battery does its "best" and does not withhold its capacity.

An alternative way to view $z_1^{RT}(D_1)$ is that it smooths the real-time component of demand followed by a correlation correction. The incremental demand in period 1 is $D_1 - \mu_1$ so the battery shifts half of it at $\frac{1}{2}(D_1 - \mu_1)$ to period 2. The extra term $-\frac{1}{2}(\mu_{2|D_1} - \mu_2)$ captures the effect of the demand dependence of period 2 that influences the decision in period 1. On one end of the spectrum, if D_1 and D_2 are independent, then $\mu_{2|D_1} = \mu_2$, so this term is zero, in agreement with the intuition that if demands are independent, then the future should not influence the current period's decision. On the other end, if $D_1 = D_2$ always, then $\mu_{2|D_1} = D_1$, and $\mu_{2|D_1} - \mu_2 = D_1 - \mu_1$, so the correlation correction term exactly cancels out the main term, and the battery discharges zero. This is also in agreement with the intuition that if the two periods are always the same, then there is no smoothing for the battery to do. The correlation correction intuition also shows that the real-time battery discharge *does* depend on the correlation between two demand periods, albeit implicitly, and that higher demand correlation reduces battery discharge.

The perfect smoothing of demand also implies that the prices in two periods are equal for both day-ahead and real-time, so battery profit is zero. This is centrally optimal but clearly not aligned with the goal of battery profit. If the battery is instead operated by an independent profit maximizer, then the battery will notice that the social optimum discharges "too much" and withholds some of its discharge. This is the source of incentive misalignment that we will quantify in §5.

Generation cost depends on the distribution π of (D_1, D_2) through marginal means μ_t, marginal variances σ_t^2, the sequential correlation ρ_s, and Pearson correlation ρ. The cost depends on the variances for the same reason as the no-battery case: because the cost is convex, demand variability increases cost. The cost is also decreasing in k_f, and k_f appears only in the variance terms for the same reason: more fast generators buffer the impact of demand variability. However, whereas the no-battery cost does not depend on the correlation, the centralized battery cost depends on the correlation because the battery shifts demand across different periods.

5 Battery Operation: Decentralized Solution

We now consider the regime when there is an independently operated battery that chooses its discharge/charge decisions $z_1^{DA}, z_1^{RT}(D_1)$ to maximize its profit. The battery solves

$$\max_{z_1^{DA}, z_1^{RT}(\cdot)} \lambda_1^{DA} z_1^{DA} + \lambda_2^{DA} z_2^{DA} + \mathbb{E}_D \left[\lambda_1^{RT} z_1^{RT} + \lambda_2^{RT} z_2^{RT} \right],$$

where the DA and RT prices are given by (7), (8), and the supply curve is given by (9). We derive the optimal battery behavior and the corresponding cost in the following theorem.

Theorem 3 (Decentralized Battery)**.** *The decentralized battery discharge decisions are given by*

$$\begin{aligned} z_1^{DA} &= \frac{(2-k_f)}{2(4-k_f)}(\mu_1 - \mu_2), \\ z_1^{RT}(D_1) &= \frac{k_f}{2(4-k_f)}(\mu_1 - \mu_2) + \frac{1}{4}(D_1 - \mu_1) - \frac{1}{4}(\mu_{2|D_1} - \mu_2). \end{aligned}$$

The corresponding generation cost Cost(DCN) *is*

$$\begin{aligned} \text{Cost(DCN)} =& \alpha(\mu_1 + \mu_2) \\ &+ \beta \left\{ \frac{(20 - 11k_f + k_f^2)}{4(4-k_f)^2}(\mu_1^2 + \mu_2^2) + \frac{(12 - 5k_f + k_f^2)}{2(4-k_f)^2}\mu_1\mu_2 + \frac{5\sigma_1^2 + (8 - 3\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2}{16k_f} \right\}. \end{aligned}$$

We can show that the profit maximization problem is also a convex quadratic optimization problem, whose unique solution can be found from first-order conditions. The proof is given in Appendix A. As in the centralized case, the discharge has a "non-random" component smoothing predictable demand fluctuations between time periods, and a "random" component (a factor of $D_1 - \mu_1$ and $\mu_{2|D_1} - \mu_2$) smoothing demand fluctuation within each period. We discuss each of these components in turn.

Non-random component of discharge. Let $\Delta\mu \equiv \mu_1 - \mu_2$ be the difference in mean net demands of the two periods, i.e., the steepness of the duck curve. The total expected discharge in period 1, $z_1^{DA} + \mathbb{E}_D[z_1^{RT}] = \frac{1}{(4-k_f)}\Delta\mu$ is strictly less than the centrally optimal discharge $\frac{1}{2}\Delta\mu$. We call this distortion *quantity withholding.* This form of distortion is familiar from the standard account of market power. As we observed in §4, centrally optimal battery discharges "too much," perfectly smoothing demand resulting in zero profit. The independent battery exercises market power by withholding capacity, resulting in less total discharge. Quantity withholding increases generation cost because it makes peak demand higher and off-peak demand lower, which nets out to higher cost because cost is quadratic in demand. Note that quantity withholding occurs even without demand randomness. We quantify the extent of quantity withholding by computing one minus the ratio between the decentralized and centralized expected battery discharge, which we define as

$$\text{quantity withholding} \equiv 1 - \frac{(z_1^{DA})_{DCN} + (\mathbb{E}_D[z_1^{RT}])_{DCN}}{(z_1^{DA})_{CN} + (\mathbb{E}_D[z_1^{RT}])_{CN}} = \frac{2-k_f}{4-k_f}. \tag{11}$$

Note that quantity withholding is defined such that if the decentralized battery (DCN) discharges as much as the centralized battery (CN), then withholding will be zero, whereas if the decentralized battery has zero discharge, then withholding will be one. Therefore, our definition captures the percentage of quantity withholding. Our computation shows that quantity withholding $(2-k_f)/(4-k_f)$ is decreasing in k_f. Intuitively, this is because more fast generators mean that the battery can discharge more with less price impact, so the battery needs to withhold less to maximize profit. The quantity withholding percentage is $1/2 = 50\%$ when generators are mostly slow ($k_f \approx 0$) and is $1/3 \approx 33.3\%$ when generators are mostly fast ($k_f \approx 1$).

Furthermore, the battery shifts a positive amount of the expected discharge $\frac{k_f}{2(4-k_f)}\Delta\mu$ to real time. In contrast, the centrally optimal battery has zero real-time discharge in expectation. We call this distortion the *shift from day-ahead to real-time.* Once the monopolist battery already commits the cleared quantity in the first (day-ahead) market, the battery's marginal revenue changes, enabling the battery to discharge more in total. In other words, the shift to real-time, while undesirable in itself, enables the battery to do less quantity withholding. This intuition is similar to how a monopolist sells to a population of nonstrategic consumers over two stages: with a higher-price in the first stage to capture higher-value consumers, and a lower price in the second stage. The quantity sold is split over two stages, but the total quantity sold is higher than a single-stage monopoly quantity. This effect is analogous to the forward market equilibrium in Allaz and Vila [1993], but we are the first to analyze this effect for batteries in electricity markets. A more informal way to think about this effect is this: because the two markets clear separately each with exogenous demand, the battery splits its quantity into two markets to reduce the quantity in each market and thus reduce the adverse price impact, which is increasing in each market's quantity.

Notably, This shift to real-time is a structural consequence of sequential market clearing by itself: it still exists even without different elasticities in two markets or demand randomness. Nevertheless, the relative elasticities of the two markets, as determined by the share of fast generators k_f, determines the *extent* of the shift to real-time. We quantify the extent of shift from day-ahead to real-time by computing the share of expected discharge in real-time as a fraction of total discharge, and compare this share between decentralized and centralized regime. In other words, we define shift from day-ahead to real-time as

$$\text{shift from day-ahead to real-time} \equiv \frac{(\mathbb{E}_D[z_1^{RT}])_{DCN}}{(z_1^{DA})_{DCN} + (\mathbb{E}_D[z_1^{RT}])_{DCN}} = \frac{k_f}{2} \tag{12}$$

Therefore, the shift from day-ahead to real-time is increasing in k_f. The shift percentage is 0% when generators are mostly slow ($k_f \approx 0$) and 50% when generators are mostly fast ($k_f \approx 1$). Intuitively, more fast generators mean that the real-time price impact is less so real-time participation is more attractive and the battery discharges more in real time. If (almost) all generators are slow, then the price impact is so large that it is not worth discharging in real time at all. Instead, the battery exercises market power by quantity withholding; we have seen earlier that quantity withholding is highest in this slow generator case.

The upshot of our discussion is that the battery strategically distorts the discharge via quantity withholding and shift from day-ahead to real-time, independent of demand randomness. Both types of distortion increase generation cost and the relative weight of each type depends on generator composition. More fast (resp. slow) generators mean more shift from day-ahead to real-time (resp. quantity withholding). We summarize the expected discharge in day-ahead, real-time, and total, as well as the extent of quantity withholding and shift from day-ahead to real-time in Table 1.

regime	generator composition	DA discharge	RT discharge	total discharge	quantity withholding	shift from DA to RT
decentralized	slow gen. dominate ($k_f \approx 0$)	$\Delta\mu/4$	0	$\Delta\mu/4$	50%	0%
	fast gen. dominate ($k_f \approx 1$)	$\Delta\mu/6$	$\Delta\mu/6$	$\Delta\mu/3$	33.3%	50%
centralized	centrally optimal	$\Delta\mu/2$	0	$\Delta\mu/2$	0%	0%

Table 1: Non-random component of battery discharge as a function of generation composition.

Random component of discharge. The random component of $z_1^{RT}(D_1)$ is $\frac{1}{4}(D_1 - \mu_1) - \frac{1}{4}(\mu_{2|D_1} - \mu_2)$, which is half of the centrally optimal perfect smoothing. We define the one minus the ratio of the random component of decentralized versus centralized discharge as *reduction in real-time responsiveness*:

$$\text{reduction in real-time responsiveness} \equiv 1 - \frac{\frac{1}{4}(D_1 - \mu_1) - \frac{1}{4}(\mu_{2|D_1} - \mu_2)}{\frac{1}{2}(D_1 - \mu_1) - \frac{1}{2}(\mu_{2|D_1} - \mu_2)} = \frac{1}{2} \tag{13}$$

Intuitively, the reduction in real-time responsiveness can also be viewed as a form of battery exercising market power via quantity withholding, but on the real-time mean-zero component of demand. This reduction in real-time responsiveness is always exactly 50% on both the realized demand and the correlation correction components. Just as we argued in the centralized case, the term $(\mu_{2|D_1} - \mu_2)$ can be viewed as "correlation correction" because it is zero when D_1 and D_2 are independent, and it exactly cancels out the main term when $D_1 = D_2$.

5.1 Comparing Generation Costs Across Different Regimes

We have derived battery discharge decisions and the corresponding generation costs under three regimes: no battery (Theorem 1), centralized battery (Theorem 2), and decentralized battery (Theorem 3). We can now compare generation costs (which are the system operator's objectives) between the three regimes, which we denote by Cost(NB), Cost(CN), and Cost(DCN), respectively.

We define the price of anarchy (PoA) as the relative cost reduction from the no-battery default of the centralized versus decentralized battery:

$$\text{PoA} \equiv \frac{\text{Cost(NB)} - \text{Cost(CN)}}{\text{Cost(NB)} - \text{Cost(DCN)}}. \tag{PoA}$$

While not obvious, having an independent battery always yields a cost reduction relative to the no-battery default (cf. Theorem 4). Given this, (PoA) is well-defined, and PoA ≥ 1.

The PoA metric captures the fact that there is a part of the generation cost that is "unavoidable" in the sense that even the centrally controlled battery cannot avoid it. Therefore, the PoA is defined to compare the relative cost reduction relative to the no-battery benchmark, which is the status quo before the introduction of battery.

Throughout this subsection, we will assume that (D_1, D_2) is jointly normal with correlation ρ; this is to highlight the qualitative dependence of different cost metrics on market fundamentals and to enable us to meaningfully discuss the correlation ρ.

Theorem 4 (Cost Comparisons). *For all market parameters $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$, we have*

(a) Cost(NB) $\geq$ Cost(DCN) *and* Cost(DCN) $\geq$ Cost(CN)*. Both inequalities become equalities if and only if $\mu_1 = \mu_2$ and $\sigma_1 = \rho\sigma_2$.*

(b) $9/8 \leq$ PoA $\leq 4/3$*. The lower bound is achieved when $k_f = 1$ and $\sigma_1 = \rho\sigma_2$. The upper bound is achieved when $k_f = 0$.* PoA *is decreasing in $|\mu_1 - \mu_2|$, increasing in $|\sigma_1 - \rho\sigma_2|$, and decreasing in k_f.*

The result that Cost(NB) $\geq$ Cost(DCN) is in the spirit of Sioshansi [2014], while Cost(DCN) $\geq$ Cost(CN) is necessarily true by definition. Taken together, these cost comparisons show that the three costs are ranked, so the PoA metric is well-defined and meaningful. We also derive a lower bound of 9/8 and an upper bound of 4/3 on PoA, and both bounds are the best possible. In other words, the incentive misalignment can increase the generation cost from 12.5% to 33.3% relative to the no-battery benchmark. Both bounds are attainable and are independent of the market parameters. On the one hand, the existence of an absolute lower bound strictly away from 1 means that in *any* market, there is always incentive misalignment (increasing cost by at least 12.5%). On the other hand, the existence of an absolute upper bound of 4/3 also shows that in *any* market, the incentive misalignment can be at most 33.3%, and this is even we assume the starkest conditions to make battery market power starkest: there is a single monopoly battery that is perfectly efficient. In §8, we show that PoA remains bounded even after relaxing assumptions.

The bounds and comparative statics of PoA follow from the following argument. The cost reductions of both centralized and decentralized regimes have two components: the contribution from the differences in means, and from the differences in variance, as shown by

$$\text{Cost(NB)} - \text{Cost(CN)} = \beta\frac{1}{4}(\mu_1 - \mu_2)^2 + \frac{\beta}{4k_f}(\sigma_1 - \rho\sigma_2)^2$$
$$\text{Cost(NB)} - \text{Cost(DCN)} = \beta\frac{(12 - 5k_f + k_f^2)}{4(4 - k_f)^2}(\mu_1 - \mu_2)^2 + \frac{3\beta}{16k_f}(\sigma_1 - \rho\sigma_2)^2$$

The variance component of the two cost gaps are always a factor of 4/3 from each other, whereas the mean component of the two cost gaps are a factor of $\frac{12-5k_f+k_f^2}{(4-k_f)^2}$ from each other. This factor reaches a minimum of 9/8 at $k_f = 1$ and a maximum of 4/3 at $k_f = 0$. Therefore, the ratio PoA is between 9/8 and 4/3. Because the mean factor component is less than the variance factor component of 4/3, if $|\mu_1 - \mu_2|$ increases, the mean component becomes more important and the PoA decreases, whereas if $|\sigma_1 - \rho\sigma_2|$ increases, the variance component becomes more important and PoA decreases. Lastly, because the mean factor is decreasing in k_f, the ratio PoA is also decreasing in k_f.

Because PoA is decreasing in $|\mu_1 - \mu_2|$, as renewable energy (especially solar) increases, widening the gap between peak mean net demand μ_1 and off-peak mean net demand μ_2, incentive alignment *improves*! A more severe duck curve increases the cost gaps of both centralized and decentralized regimes, but the decentralized cost reduction grows at a faster rate. Moreover, the fact that PoA is decreasing in k_f means that more fast generators improve incentive alignment. Intuitively, this is because non-strategic fast generators are ready to "step in" and cushion the price impact of real-time battery withholding. Lastly, because PoA is increasing in $|\sigma_1 - \rho\sigma_2|$, it is not necessarily monotonic in the correlation ρ. On the one hand, if $\sigma_1 \leq \sigma_2$ then PoA is always increasing in ρ. On the other hand, if $\sigma_1 \geq \sigma_2$ then PoA is decreasing in ρ when $\rho \leq \sigma_1/\sigma_2$ and increasing in ρ when $\rho \geq \sigma_1/\sigma_2$.

6 Numerical Illustrations

In this section, we illustrate the battery behavior and incentive misalignment numerically. To anchor ideas, and for illustration purposes, we use data from Los Angeles and Houston. We choose these regions as examples of markets that are well on the way in terms of renewable and battery adoption, and markets that are in transition, respectively. We also observe local monopoly effects in these regions. For example, in Los Angeles, as of April 2024, AES is the biggest player with 4 batteries with total capacity of 355 MW, the second biggest is VESI with two batteries, 20 MW each, and the rest are very small batteries with total capacity 27.2 MW [EIA, 2024a].

We emphasize that the results we present here are illustrative of the forces at play, but also do not account for how submarkets are connected in the electricity market. A full network analysis is beyond the scope of the current work.

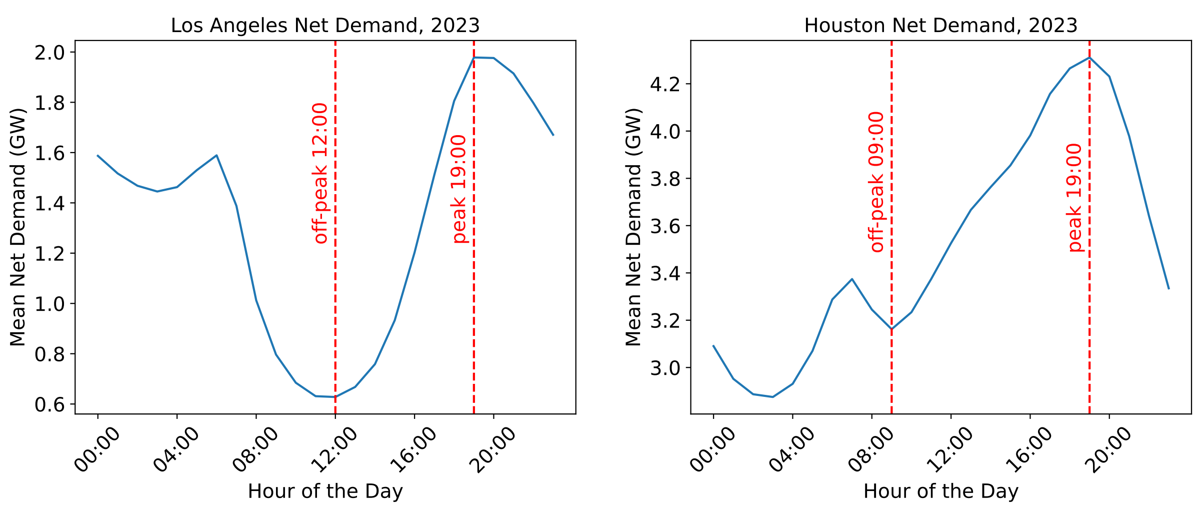

Figure 2: Mean net demand for each hour of the day in 2023 in Los Angeles and Houston, and the corresponding peak and off-peak hours.

We calibrate the supply curve parameters α, β from public market data. We estimate k_f from the share of energy produced from "fast" sources. If we assume that natural gas and hydro are fast, while nuclear and coal are slow, then we have $k_f = 0.93$ for Los Angeles and $k_f = 0.66$ for Houston.

In contrast to the generator parameters α, β, k_f which can be assumed to be constant throughout the year, the net demand has a significant seasonality component. We assume that the peak and off-peak net demands are jointly normal, and calibrate the marginal means μ_1, μ_2, marginal variances σ_1^2, σ_2^2, and the correlation ρ for each quarter of 2023. For Los Angeles, we take the peak hour to be 19:00-20:00 and the off-peak hour to be 12:00-13:00. For Houston, we take the peak hour to be 19:00-20:00 and the off-peak hour to be 09:00-10:00 (cf. Figure 2).

For each quarter, we calculate the price of anarchy. In Los Angeles, we have 1.15 for all four quarters. In Houston, we have Q1: 1.26, Q2: 1.24, Q3: 1.22, Q4: 1.26 with an average of 1.25. Therefore, even though the demand has high seasonality, the price of anarchy is fairly stable across seasons.

We can see that, if the battery achieves local monopoly in a region, the price of anarchy as well as all three types of distortion are practically significant. Nevertheless, the low values of the PoA shows that the impact on cost reduction is limited. The Los Angeles PoA of 1.15 (resp. Houston PoA of 1.25) means that the cost reduction from having decentralized batteries is already 87% (resp. 80%) of the best possible cost reduction. We should think of the best possible cost reduction not necessarily as the benchmark that could be achieved if only the central planner takes total control because in practice, the central planner also do not have perfect information and is not necessarily as nimble as private actors. Rather than viewing this result as saying that we could do even better with a benevolent monopoly in charge, we should view this result instead as saying that the liberalized electricity market status quo can give reasonable performance.

7 Competition and Market Power Mitigation Mechanisms

So far, we have assumed that there is a single battery monopoly which operates without restrictions to highlight the fundamental features of battery market power. In this section, we will analyze the equilibrium of a game with n competing batteries and shows that competition is quite effective at reducing strategic distortions. However, we will show that a few reasonable-sounding market mitigation measures that the system operator might deploy can backfire.

7.1 Competition

Theorem 5. *Consider n batteries deciding discharge quantities in each period in a Cournot equilibrium. Then, there is a unique equilibrium given by*

$$z_1^{DA} = \frac{(n+1-k_f)}{2((n+1)^2 - nk_f)}(\mu_1 - \mu_2)$$

$$z_1^{RT}(D_1) = \frac{k_f}{2((n+1)^2 - nk_f)}(\mu_1 - \mu_2) + \frac{(D_1 - \mu_1) - \left(\mu_{2|D_1} - \mu_2\right)}{2(n+1)}$$

For every market, we have the PoA bounds

$$1 + \frac{1}{n(n+1)(n^2+n+2)} \leq \text{PoA} \leq 1 + \frac{1}{n(n+2)}.$$

Note that Theorem 5 reduces to the decentralized monopoly case (Theorem 3) with $n = 1$. We can quantify the three types of distortions by comparing the total discharge in equilibrium $n(z_1)_{DCN}$ with the socially optimal discharge $(z_1)_{CN}$ which is the same as the one-battery case (because the battery has no capacity constraint). They are: quantity withholding $= (n+1-nk_f)/((n+1)^2-nk_f)$, shift from day-ahead to real-time $= k_f/(n+1)$, reduction in real-time responsiveness $= 1/(n+1)$.

In particular, the three types of distortions all go to zero at a rate of $1/n$ as n increases. Meanwhile, the PoA bounds show that PoA decreases to 1 at a rate of $1/n^2$ in the worst case. So just a moderate amount of competition can substantially reduce distortions and incentive misalignment. The caveat that competition also substantially reduces battery profit, which might deter battery market entry.

7.2 Market Power Mitigation Mechanisms

7.2.1 Regulating Day-Ahead versus Real-Time Discrepancy

Out of the three types of distortions that we have identified in our model, the shift from day-ahead to real-time is easiest to observe in the data. California's special report on battery storage [CAISO, 2023] shows the hourly average battery bids and market prices in day-ahead and real-time (Figure 3 and Figure 4 in Appendix D). In day-ahead, battery bids are significantly higher than market clearing prices, whereas in real-time, battery bids are comparable to market clearing prices. This means that batteries avoid being scheduled in day-ahead, preferring to be scheduled in real-time instead. This corresponds to the shift from day-ahead to real-time that we have identified. We consider two market policies that the regulator could enact to mitigate the day-ahead versus real-time discrepancy:

(P1) The regulator can require that the battery discharges zero in expectation in real-time. This policy basically "bans" the shift from DA to RT, which is observable. Equivalently, this policy imitates the socially optimal (centralized) behavior.

(P2) The regulator can introduce *virtual bidders*, which are purely financial bidders that arbitrage between day-ahead and real-time prices in each period, using market dynamics.

Unfortunately, both of these policies will backfire.

Theorem 6. *Both (P1) and (P2) lead to more quantity withholding, lower battery profit, and higher system cost.*

Intuitively, the monopoly battery splits the exercise of market power "optimally" between quantity withholding and shift from DA to RT. If the regulator bans shift from DA to RT, the battery will have to do more quantity withholding to compensate. Whereas the shift from DA to RT will still make the battery capacity available (only later than optimal), quantity withholding will make the battery capacity not available at all, which further increases system cost. This also lowers battery profit because the profit-maximizing battery would not choose this quantity withholding allocation without the external constraint.

7.2.2 Battery Discharge Subsidy

If we think the charge and discharge behavior of batteries gives positive externalities to the system, we can subsidize battery discharge, analogous to renewable production tax credits for renewables but for batteries. The regulator selects a subsidy price s such that the regulator pays sz if the battery discharges z during peak period.

Theorem 7. *For any subsidy level $s > 0$, total financial cost, defined as a sum of subsidy cost and system generation cost, increases compared to no subsidy.*

We note that even though subsidy increases total financial cost, it might still be a good policy if having lower carbon emissions is desirable in its own right, e.g., via the social cost of carbon.

8 Extensions

So far, we have made assumptions to get the most parsimonious model that shows the main economic and operational drivers of strategic battery behavior. In this section, we will show that the main insights continue to hold in more complex settings.

8.1 Multiple Time Periods

In this subsection, we extend our results to $T \geq 2$ periods, indexed by $t \in \{1, \ldots, T\} \equiv [T]$. The demands of all periods are random variables drawn from a known joint distribution $D \equiv (D_1, \ldots, D_T) \sim \pi$. As before, we consider the centralized case where the battery minimizes generation cost, and the decentralized case where the battery maximizes its profit. The battery decides the discharge quantities z_t^{DA} for each period t in day-ahead, and $z_t^{RT}(D_{1:t})$ for each period t in real-time as a function of the realized history $D_{1:t}$ subject to $\sum_t z_t^{DA} = 0$ and $\sum_t z_t^{RT}(D_{1:t}) = 0$ for all D. The market prices are determined by (4) and (5), and the generation cost is also given by (6) (but summing over all $t \in [T]$), and the price of anarchy is defined by (PoA) as before.

Theorem 9 in Appendix E gives explicit formulas for the centralized and decentralized battery decisions z_t^{DA} and $z_t^{RT}(D_{1:t})$. This theorem reduces to previous results with $T = 2$. We can see from the explicit formulas that the three types of distortions (quantity withholding, shift from day-ahead to real-time, reduction in real-time responsiveness) continue to hold with the same amount as in (11), (12), (13). We can prove that the bounds $9/8 \leq \text{PoA} \leq 4/3$ on the Price of Anarchy also continue to hold for independent normal demand for every T, and we believe that the bounds will hold more broadly under reasonable assumptions on the correlation matrix.

8.2 Battery Inefficiency

In this subsection, we are back to the two-period model, but we now assume that the battery is not perfectly efficient. Rather, it has a round trip efficiency of $\eta \in (0, 1]$. That is, the energy the battery discharges is η times the amount it charges. In practice, η is typically between 0.85 and 0.98 for lithium-ion batteries [Koohi-Fayegh and Rosen, 2020]. If we assume that period 1 is a peak period and the battery charges, while period 2 is an off-peak period and the battery discharges, then we have $z_1^{DA} + \eta z_2^{DA} = z_1^{RT} + \eta z_2^{RT} = 0$. Theorem 10 in Appendix E gives explicit formulas for centralized and decentralized battery decisions. We also have the bounds $9/8 \leq \text{PoA} \leq 4/3$ as before.

8.3 Non-Parallel Supply Curves

We have assumed that the same fraction k_f of generators are fast at every price λ, that is $G_s(\lambda) = k_s G(\lambda)$ and $G_f(\lambda) = k_f G(\lambda)$ and the supply curve is linear: $G^{-1}(x) = \alpha + \beta x$. We now relax the former assumption and assume that the fast and slow supply curves are linear but not necessarily a multiple of each other: $G_s^{-1}(x) = \alpha_s + \beta_s x$ and $G_f^{-1}(x) = \alpha_f + \beta_f x$. Theorem 11 shows that we can still characterize the battery strategies in closed form, and the bounds $9/8 \leq \text{PoA} \leq 4/3$ still hold.

8.4 Convex Supply Curves

We have assumed that the total supply curve is linear: $G^{-1}(x) = \alpha + \beta x$. This is a good first-order approximation for most hours in the market, but we might want to also take into account that prices might increase super-linearly when the demand is unusually high. Here, we assume that $G^{-1}(x) = \alpha + \beta x + \gamma x^2$ for $\gamma \geq 0$. Theorem 12 derives the centralized and decentralized battery strategies as linearized convexity corrections in γ.

8.5 Battery Capacity

We have assumed that the battery has unlimited capacity, so the only constraints are that the total net discharge is zero. Now we assume that the monopoly battery has a given capacity C. We do not have closed form solutions, but they can be numerically approximated. Figure 5 shows the Price of Anarchy values, calibrated with Los Angeles and Houston data, are comparable to before.

8.6 Battery Investments and Operations

Given our model's focus on daily cycles of the market, taking market participants as fixed, it naturally fits with the *system operator*'s goal of ensuring proper market functioning. However, it can also be used to understand higher-level decisions such as investment in battery capacity. Here, we assume that the battery capacity is endogenous. There is an investment cost c_{inv} per unit of battery capacity. The decision maker first decides the battery capacity C to invest in, then decides

$z_1^{DA}, z_1^{RT}(D_1)$ to operate the battery with this capacity. The centralized case minimizes total cost, which is a sum of investment cost $c_{\text{inv}}C$ and generation cost. The decentralized case maximizes net profit, which is the arbitrage profit minus investment cost. Figure 6 shows that the Price of Anarchy values, calibrated with Los Angeles and Houston data, are comparable to before.

9 Conclusion

We formulate and solve an analytical model of market power of batteries in electricity markets. We find that profit-maximizing batteries strategically distort their decisions by quantity withholding, shifting participation from day-ahead to real-time, and reducing real-time responsiveness, and quantify the extent of each form of distortion. The larger the share of fast generators, the more batteries do shift to real-time rather than quantity withholding, and vice versa. Battery distortion due to incentive misalignment leads to an increase in generation cost between 12.5% and 33.3%, and the misalignment is largest in relative terms when generators are slow, and and the duck curve is shallow. Numerical illustrations with Los Angeles and Houston data suggest that, if a battery achieves local monopoly, these effects could be practically significant, but the loss from market power is bounded even in the worst case. A moderate amount of competition is very effective at reducing distortions, with the caveat that it also substantially reduces battery profits, which might deter battery market entry. However, market power mitigation mechanisms can backfire. While our base model is intentionally parsimonious to most clearly highlight the main drivers of battery incentive misalignment, the insights and quantitative bounds continue in more general settings.

There are many avenues for future work. Our model considers each region separately, which can be a good first-order approximation for highly fragmented markets. For moderately fragmented markets, the network structure and locational marginal pricing market clearing should be modeled explicitly, and the question of market power over a network is worth investigating. Our model also assumes that the environment is probabilistically known. Arguably, however, battery behavior is also partly shaped by uncertainty and robustness considerations. For example, if the market price occasionally spikes, then part of battery withholding behavior might simply be contingency preparation rather than market power. Understanding the role of price and system forecast, Bayesian and non-Bayesian uncertainty, and distinguishing between strategic behavior and standard operating procedures is an important future direction.

References

Daron Acemoglu, Ali Kakhbod, and Asuman Ozdaglar. Competition in electricity markets with renewable energy sources. *The Energy Journal*, 38(1):137–155, 2017.

Australian Energy Regulator AER. Electricity prices above $5,000/MWh - January to March 2023, 2023.

Vishal Agrawal and Şafak Yücel. *Renewable Energy Sourcing*. Springer Series in Supply Chain Management. 2021.

Vishal V. Agrawal and Şafak Yücel. Design of electricity demand-response programs. *Management Science*, 68(10):7441–7456, 2022. doi: 10.1287/mnsc.2021.4278.

Blaise Allaz and Jean-Luc Vila. Cournot competition, forward markets and efficiency. *Journal of Economic Theory*, 59(1):1–16, 1993. ISSN 0022-0531.

Olivier Bahn, Mario Samano, and Paul Sarkis. Market power and renewables: The effects of ownership transfers. *The Energy Journal*, 42(4):195–225, 2021.

Endre Bjørndal, Mette Helene Bjørndal, Stefano Coniglio, Marc-Fabian Körner, Christina Leinauer, and Martin Weibelzahl. Energy storage operation and electricity market design: On the market power of monopolistic storage operators. *European Journal of Operational Research*, 307(2): 887–909, 2023. ISSN 0377-2217.

Severin Borenstein, James Bushnell, Christopher R. Knittel, and Catherine Wolfram. Inefficiencies and market power in financial arbitrage: A study of california's electricity markets. *The Journal of Industrial Economics*, 56(2):347–378, 2008.

James B. Bushnell, Erin T. Mansur, and Celeste Saravia. Vertical arrangements, market structure, and competition: An analysis of restructured us electricity markets. *American Economic Review*, 98(1):237–266, 2008.

R. Andrew Butters, Jackson Dorsey, and Gautam Gowrisankaran. Soaking up the sun: Battery investment, renewable energy, and market equilibrium. 2023.

CAISO. CAISO Special Report on Battery Storage. Technical report, 2023. URL https://www.caiso.com/Documents/2022-Special-Report-on-Battery-Storage-Jul-7-2023.pdf.

Congressional Budget Office CBO. Emissions of carbon dioxide in the electric power sector, 2022. URL https://www.cbo.gov/publication/58419.

Anna Cretì and Fulvio Fontini. *Economics of Electricity: Markets, Competition and Rules*. Cambridge University Press, 2019.

James Cruise, Lisa Flatley, Richard Gibbens, and Stan Zachary. Control of energy storage with market impact: Lagrangian approach and horizons. *Operations Research*, 67(1):1–9, 2019.

Paul Denholm, Wesley Cole, and Nate Blair. Moving beyond 4-hour li-ion batteries: Challenges and opportunities for long(er)-duration energy storage. Technical report, National Renewable Energy Laboratory, 2023.

U.S. Energy Information Administration EIA. Form EIA-860M, Monthly Update to the Annual Electric Generator Report, April 2024a. URL https://www.eia.gov/electricity/data/eia860m/. Accessed: 2024-05-27.

U.S. Energy Information Administration EIA. U.S. battery storage capacity expected to nearly double in 2024, 2024b. URL https://www.eia.gov/todayinenergy/detail.php?id=61202.

Ali Fattahi, Sriram Dasu, and Reza Ahmadi. Peak-load energy management by direct load control contracts. *Management Science*, 69(5):2788–2813, 2023. doi: 10.1287/mnsc.2022.4493.

Ali Fattahi, Saeed Ghodsi, Sriram Dasu, and Reza Ahmadi. Flattening energy-consumption curves by monthly constrained direct load control contracts. *Operations Research*, 72(2):570–590, 2024. doi: 10.1287/opre.2021.0638.

Zuguang Gao, Khaled Alshehri, and John R. Birge. Aggregating distributed energy resources: Efficiency and market power. *Manufacturing & Service Operations Management*, 26(3):834–852, 2024.

Talat S. Genc and Stanley S. Reynolds. Who should own a renewable technology? ownership theory and an application. *International Journal of Industrial Organization*, 63:213–238, 2019.

Christoph Graf, Emilio La Pera, Federico Quaglia, and Frank A. Wolak. Market power mitigation mechanisms for wholesale electricity markets: Status quo and challenges. 2021.

Karl Hartwig and Ivana Kockar. Impact of strategic behavior and ownership of energy storage on provision of flexibility. *IEEE Transactions on Sustainable Energy*, 7(2):744–754, 2016.

Qisheng Huang, Yunjian Xu, Tao Wang, and Costas A. Courcoubetis. Market mechanisms for cooperative operation of price-maker energy storage in a power network. *IEEE Transactions on Power Systems*, 33(3):3013–3028, 2018.

IPCC. Summary for policymakers. In H. Lee Core Writing Team and J. Romero (eds.), editors, *Climate Change 2023: Synthesis Report. Contribution of Working Groups I, II and III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change*, pages 1–34. IPCC, Geneva, Switzerland, 2023. doi: 10.59327/IPCC/AR6-9789291691647.001.

Koichiro Ito and Mar Reguant. Sequential markets, market power, and arbitrage. *American Economic Review*, 106(7):1921–57, July 2016.

Paul L Joskow. Competitive electricity markets and investment in new generating capacity. Working paper, Center for Energy and Environmental Policy Research, Massachusetts Institute of Technology, Cambridge, MA, 2006.

Christian Kaps, Simone Marinesi, and Serguei Netessine. When should the off-grid sun shine at night? optimum renewable generation and energy storage investments. *Management Science*, 69 (12):7633–7650, 2023.

Ömer Karaduman. Economics of grid-scale energy storage in wholesale electricity markets. Technical report, 2023.

Ryan Kellogg and Mar Reguant. Energy and environmental markets, industrial organization, and regulation. In *Handbook of Industrial Organization*, volume 5, pages 615–742. 2021.

Daniel S. Kirschen and Goran Strbac. *Fundamentals of Power System Economics*. Wiley, 2nd edition, 2018.

S. Koohi-Fayegh and M.A. Rosen. A review of energy storage types, applications and recent developments. *Journal of Energy Storage*, 27:101047, 2020. ISSN 2352-152X.

Hamed Mohsenian-Rad. Coordinated price-maker operation of large energy storage units in nodal energy markets. *IEEE Transactions on Power Systems*, 31(1):786–797, 2015.

Geoffrey G. Parker, Burcu Tan, and Osman Kazan. Electric power industry: Operational and public policy challenges and opportunities. *Production and Operations Management*, 28, 2019.

Giles Parkinson. The big battery being used to push electricity prices to the market cap, Jul 3 2023. URL https://reneweconomy.com.au/the-big-battery-being-used-to-push-electricity-prices-to-the-market-cap/. Accessed: Jan 29, 2024.

Xiaoshan Peng, Owen Q. Wu, and Gilvan Souza. Renewable, flexible, and storage capacities: Friends or foes? Working paper, SSRN, 2021.

Georgia Perakis and Leann Thayaparan. The role of driver behavior in moving the electric grid to zero emissions. Technical report, 2023.

Heikki Peura and Derek W. Bunn. Renewable power and electricity prices: The impact of forward markets. *Management Science*, 67(8):4772–4788, 2021.

Wei Qi, Yong Liang, and Zuo-Jun Max Shen. Joint planning of energy storage and transmission for wind energy generation. *Operations Research*, 63(6):1280–1293, 2015.

Celeste Saravia. Speculative trading and market performance: The effect of arbitrageurs on efficiency and market power in the new york electricity market. University of California Berkeley CSEM Working Paper 121, 2003.

Wolf-Peter Schill and Claudia Kemfert. Modeling strategic electricity storage: The case of pumped hydro storage in germany. *The Energy Journal*, 32(3):59–87, 2011.

Ramteen Sioshansi. Welfare impacts of electricity storage and the implications of ownership structure. *The Energy Journal*, 31(2):173–198, 2010.

Ramteen Sioshansi. When energy storage reduces social welfare. *Energy Economics*, 41:106–116, 2014.

Jeff St. John. California has a new $7.3b plan to fix its transmission problems, May 2023. URL https://www.canarymedia.com/articles/transmission/california-has-a-new-7-3b-plan-to-fix-its-transmission-problems. Accessed May 27, 2024.

Nur Sunar and Jayashankar M. Swaminathan. Socially relevant and inclusive operations management. *Production and Operations Management*, 31(12):4379–4392, 2022.

Egill Tómasson, Mohammad Reza Hesamzadeh, and Frank A. Wolak. Optimal offer-bid strategy of an energy storage portfolio: A linear quasi-relaxation approach. *Applied Energy*, 260:114251, 2020.

K.R. Ward and I. Staffell. Simulating price-aware electricity storage without linear optimisation. *Journal of Energy Storage*, 20:78–91, 2018.

Christopher Weare. *The California Electricity Crisis: Causes and Policy Options*. Public Policy Institute of California, San Francisco, 2003. ISBN 1-58213-064-7. Available at: https://www.ppic.org/publication/the-california-electricity-crisis-causes-and-policy-options/.

Owen Q. Wu, Şafak Yücel, and Yangfang (Helen) Zhou. Smart charging of electric vehicles: An innovative business model for utility firms. *Manufacturing & Service Operations Management*, 24(5):2481–2499, 2022.

Owen Q. Wu, Roman Kapuscinski, and Santhosh Suresh. On the distributed energy storage investment and operations. *Manufacturing & Service Operations Management*, 25(6):2277–2297, 2023.

Pengcheng You, Dennice F. Gayme, and Enrique Mallada. The role of strategic load participants in two-stage settlement electricity markets. In *2019 IEEE 58th Conference on Decision and Control (CDC)*, pages 8416–8422, 2019. doi: 10.1109/CDC40024.2019.9029514.

Yangfang (Helen) Zhou, Alan Scheller-Wolf, Nicola Secomandi, and Stephen Smith. Electricity trading and negative prices: Storage vs. disposal. *Management Science*, 62(3):880–898, 2016.

A Proofs for Section 3

Proposition 1. *We have*

$$
\begin{aligned}
\mathbb{E}[(\mu_{2|D_1} - \mu_2)^2] &= \rho_s^2 \sigma_2^2 \\
\mathbb{E}[\sigma_{2|D_1}^2] &= (1 - \rho_s^2)\sigma_2^2 \\
\mathbb{E}[D_2 - \mu_{2|D_1}] &= 0 \\
\mathbb{E}[(D_2 - \mu_{2|D_1})(D_1 - \mu_1)] &= 0 \\
\mathbb{E}[(D_2 - \mu_{2|D_1})(D_2 - \mu_2)] &= (1 - \rho_s^2)\sigma_2^2 \\
\mathbb{E}[(D_1 - \mu_1)(\mu_{2|D_1} - \mu_2)] &= \rho\sigma_1\sigma_2 \\
\mathbb{E}[(D_2 - \mu_2)(\mu_{2|D_1} - \mu_2)] &= \rho_s^2 \sigma_2^2
\end{aligned}
$$

Also, for each constant c, we have

$$
\mathbb{E}[(D_2 - c)^2 | d_1] = \sigma_{2|d_1}^2 + (\mu_{2|d_1} - c)^2
$$

Proof. Proof of Proposition 1. The first two equations hold by definition of ρ_s. For the third equation, $bE[D_2 - \mu_{2|D_1}] = \mathbb{E}[\mathbb{E}[D_2 - \mu_{2|D_1}|D_1]] = \mathbb{E}[\mu_{2|D_1} - \mu_{2|D_1}] = 0$. For the fourth equation, $\mathbb{E}[(D_2 - \mu_{2|D_1})(D_1 - \mu_1)] = \mathbb{E}[\mathbb{E}[(D_2 - \mu_{2|D_1})(D_1 - \mu_1)|D_1]] = \mathbb{E}[(D_1 - \mu_1)(\mu_{2|D_1} - \mu_{2|D_1})] = 0$. For the fifth equation,

$$
\begin{aligned}
\mathbb{E}[(D_2 - \mu_{2|D_1})(D_2 - \mu_2)] &= \mathbb{E}[(D_2 - \mu_{2|D_1})^2] + \mathbb{E}[(D_2 - \mu_{2|D_1})(\mu_{2|D_1} - \mu_2)] \\
&= \mathbb{E}[\mathbb{E}[(D_2 - \mu_{2|D_1})^2|D_1]] + \mathbb{E}[\mathbb{E}[(D_2 - \mu_{2|D_1})(\mu_{2|D_1} - \mu_2)|D_1]] \\
&= \mathbb{E}[\sigma_{2|D_1}^2] + \mathbb{E}[(\mu_{2|D_1} - \mu_{2|D_1})(\mu_{2|D_1} - \mu_2)] \\
&= (1 - \rho_s^2)\sigma_2^2 + 0 = (1 - \rho_s^2)\sigma_2^2
\end{aligned}
$$

For the sixth equation, $\mathbb{E}[(D_1 - \mu_1)(\mu_{2|D_1} - \mu_2)] = \mathbb{E}[\mathbb{E}[(D_1 - \mu_1)(D_2 - \mu_2)|D_1]] = \mathbb{E}[(D_1 - \mu_1)(D_2 - \mu_2)] = \rho\sigma_1\sigma_2$. For the seventh equation, $\mathbb{E}[(D_2 - \mu_2)(\mu_{2|D_1} - \mu_2)] = \mathbb{E}[\mathbb{E}[(D_2 - \mu_2)(\mu_{2|D_1} - \mu_2)|D_1]] = \mathbb{E}[(\mu_{2|D_1} - \mu_2)^2] = \rho_s^2\sigma_2^2$

Lastly, we have

$$
\begin{aligned}
\mathbb{E}[(D_2 - c)^2 | d_1] &= \mathbb{E}[(D_2 - \mu_{2|d_1} + \mu_{2|d_1} - c)^2 | d_1] \\
&= \mathbb{E}[(D_2 - \mu_{2|d_1})^2 | d_1] + (\mu_{2|d_1} - c)^2 + 2(\mu_{2|d_1} - c)\mathbb{E}[(D_2 - \mu_{2|d_1} | d_1] \\
&= \sigma^2_{2|d_1} + (\mu_{2|d_1} - c)^2 + 2(\mu_{2|d_1} - c) \cdot 0 \\
&= \sigma^2_{2|d_1} + (\mu_{2|d_1} - c)^2
\end{aligned}
$$

□

Proof. Proof of Theorem 1.

For $t \in \{1, 2\}$, we compute

$$
\begin{aligned}
\mathbb{E}[(\tilde{d}_t^{RT})] &= \mathbb{E}\left[\mu_t + \frac{D_t - \mu_t}{k_f}\right] = \mu_t \\
\mathbb{E}[(\tilde{d}_t^{RT})^2] &= \mathbb{E}\left[\left(\mu_t + \frac{D_t - \mu_t}{k_f}\right)^2\right] = \mu_t^2 + \frac{2\mu_t}{k_f}\mathbb{E}[(D_t - \mu_t)] + \frac{1}{k_f^2}\mathbb{E}[(D_t - \mu_t)^2] = \mu_t^2 + \frac{\sigma_t^2}{k_f^2}
\end{aligned}
$$

Therefore, the generation cost is

$$
\sum_{t=1}^{2} k_s \left[\alpha\mu_t + \frac{\beta}{2}\mu_t^2\right] + k_f \left[\alpha\mu_t + \frac{\beta}{2}\left(\mu_t^2 + \frac{\sigma_t^2}{k_f^2}\right)\right]
$$

which simplifies to the given expression.

□

B Proofs for Section 4

Proof. Proof of Theorem 2.

Generation cost is

$$
\begin{aligned}
&\alpha(\mu_1 + \mu_2) \\
&+ k_s \left[\frac{\beta}{2}\left[(\mu_1 - z_1^{DA})^2 + (\mu_2 + z_1^{DA})^2\right]\right] \\
&+ k_f \mathbb{E}_{D_1, D_2}\left\{\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 + \left(\mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}\right)^2\right]\right\}
\end{aligned}
$$

We first note that the generation cost is strictly convex (quadratic) in the decision variables. So, the global minimum is achieved where first-order conditions hold with equality.

For each fixed $D_1 = d_1$, we take the derivative w.r.t $z_1^{RT}(d_1)$. By the law of iterated expectations, the expectation $\mathbb{E}_{D_1,D_2}$ can be viewed as taking expectation $\mathbb{E}_{D_1}$ followed by the conditional expectation $\mathbb{E}_{D_2|D_1}$, by focusing on $z_1^{RT}(d_1)$ we fix the value $D_1 = d_1$ while the inner expectation becomes an expectation over $D_2 \sim \pi(\cdot|D_1 = d_1)$. We therefore get

$$\mathbb{E}_{D_2\sim\pi(\cdot|D_1=d_1)}\left\{\beta\left[-\frac{1}{k_f}\left(\mu_1 - z_1^{DA} + \frac{d_1-\mu_1-z_1^{RT}(d_1)}{k_f}\right) + \frac{1}{k_f}\left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(D_1)}{k_f}\right)\right]\right\} = 0.$$

The expression is linear in D_2, and the expectation is $\mathbb{E}[D_2|D_1 = d_1] = \mu_{2|d_1}$. Therefore,

$$-k_f(\mu_1-\mu_2) + 2k_f z_1^{DA} - (d_1-\mu_1) + (\mu_{2|d_1}-\mu_2) + 2z_1^{RT}(d_1) = 0$$

This allows us to write $z_1^{RT}(d_1)$ in terms of z_1^{DA}:

$$z_1^{RT}(d_1) = -k_f z_1^{DA} + \frac{k_f}{2}(\mu_1-\mu_2) + \frac{1}{2}(d_1-\mu_1) - \frac{1}{2}(\mu_{2|d_1}-\mu_2)$$

In particular, this implies

$$\mathbb{E}[z_1^{RT}(D_1)] = -k_f z_1^{DA} + \frac{k_f}{2}(\mu_1-\mu_2)$$

Taking the derivative w.r.t. z_1^{DA} gives

$$\begin{aligned}
&k_s\left[\beta\left[-(\mu_1 - z_1^{DA}) + (\mu_2 + z_1^{DA})\right]\right] \\
&+ k_f\mathbb{E}_{D_1,D_2}\left\{\beta\left[-\left(\mu_1 - z_1^{DA} + \frac{D_1-\mu_1-z_1^{RT}(D_1)}{k_f}\right) + \left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(D_1)}{k_f}\right)\right]\right\} = 0
\end{aligned}$$

or

$$(\mu_2 - \mu_1 + 2z_1^{DA}) + \mathbb{E}\left[2z_1^{RT}(D_1)\right] = 0$$

Using the expression for $\mathbb{E}[z_1^{RT}(D_1)]$ derived earlier, we get

$$
\begin{aligned}
z_1^{DA} &= \frac{\mu_1 - \mu_2}{2} \\
z_1^{RT}(d_1) &= \frac{1}{2}(d_1 - \mu_1) - \frac{1}{2}(\mu_{2|d_1} - \mu_2).
\end{aligned}
$$

We now compute the generation cost. We have

$$
\begin{aligned}
&\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 \\
&= \frac{1}{k_f^2}\left(k_f\mu_1 + (D_1 - \mu_1) - k_f z_1^{DA} - z_1^{RT}(D_1)\right)^2 \\
&= \frac{1}{4k_f^2}\left[k_f(\mu_1 + \mu_2) + (D_1 - \mu_1) + (\mu_{2|D_1} - \mu_2)\right]^2
\end{aligned}
$$

Using Proposition 1, we can evaluate

$$
\mathbb{E}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2\right] = \frac{1}{4k_f^2}\left[k_f^2(\mu_1 + \mu_2)^2 + \sigma_1^2 + \rho_s^2\sigma_2^2 + 2\rho\sigma_1\sigma_2\right]
$$

Similarly,

$$
\begin{aligned}
&\mathbb{E}\left[\left(\mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}\right)^2\right] \\
&= \frac{1}{4k_f^2}\mathbb{E}\left[\left(2(D_2 - \mu_{2|D_1}) + (\mu_{2|D_1} - \mu_2) + (D_1 - \mu_1) + k_f(\mu_1 + \mu_2)\right)^2\right] \\
&= \frac{1}{4k_f^2}\mathbb{E}\left[4\sigma_{2|D_1}^2 + \left((\mu_{2|D_1} - \mu_2) + (D_1 - \mu_1) + k_f(\mu_1 + \mu_2)\right)^2\right] \\
&= \frac{1}{4k_f^2}\left[4(1 - \rho_s^2)\sigma_2^2 + \rho_s^2\sigma_2^2 + \sigma_1^2 + k_f^2(\mu_1 + \mu_2)^2 + 2\rho\sigma_1\sigma_2\right]
\end{aligned}
$$

So

$$
\begin{aligned}
&\mathbb{E}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2\right] + \mathbb{E}\left[\left(\mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}\right)^2\right] \\
&= \frac{1}{2k_f^2}\left[\sigma_1^2 + (2 - \rho_s^2)\sigma_2^2 + k_f^2(\mu_1 + \mu_2)^2 + 2\rho\sigma_1\sigma_2\right].
\end{aligned}
$$

We also have

$$(\mu_1 - z_1^{DA})^2 + (\mu_2 + z_1^{DA})^2 = \left(\frac{\mu_1 + \mu_2}{2}\right)^2 + \left(\frac{\mu_1 + \mu_2}{2}\right)^2 = \frac{1}{2}(\mu_1 + \mu_2)^2$$

Therefore, the total generation cost is

$$\alpha(\mu_1 + \mu_2) + k_s \left[\frac{\beta}{4}(\mu_1 + \mu_2)^2\right] + k_f \left[\frac{\beta}{2}\frac{1}{2k_f^2}\left[\sigma_1^2 + (2 - \rho_s^2)\sigma_2^2 + k_f(\mu_1 + \mu_2)^2 + 2\rho\sigma_1\sigma_2\right]\right]$$

which simplifies to the generation cost expression in the theorem. □

C Proofs for Section 5

Proof. Proof of Theorem 3

The prices are given by

$$\begin{aligned}
\lambda_1^{DA} &= \alpha + \beta d_1^{DA} \\
\lambda_2^{DA} &= \alpha + \beta d_2^{DA} \\
\lambda_1^{RT} &= \alpha + \beta \left(d_1^{DA} + \frac{d_1^{RT}}{k_f}\right) \\
\lambda_2^{RT} &= \alpha + \beta \left(d_2^{DA} + \frac{d_2^{RT}}{k_f}\right)
\end{aligned}$$

with

$$\begin{aligned}
d_1^{DA} &= \mu_1 - z_1^{DA} \\
d_2^{DA} &= \mu_2 + z_1^{DA} \\
d_1^{RT} &= D_1 - \mu_1 - z_1^{RT}(D_1) \\
d_2^{RT} &= D_2 - \mu_2 + z_1^{RT}(D_1)
\end{aligned}$$

The battery maximizes profit:

$$\Pi = (\lambda_1^{DA} - \lambda_2^{DA})z_1^{DA} + \mathbb{E}\left[(\lambda_1^{RT} - \lambda_2^{RT})z_1^{RT}(D_1)\right]$$

We can write

$$\begin{aligned}\Pi &= \beta(\mu_1 - \mu_2 - 2z_1^{DA})z_1^{DA} \\ &+ \mathbb{E}\left[\beta\left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((D_1-\mu_1)-(D_2-\mu_2)-2z_1^{RT}(D_1))}{k_f}\right)z_1^{RT}(D_1)\right]\end{aligned}$$

Taking derivative w.r.t. $z_1^{RT}(d_1)$ for a given fixed d_1 gives, for each d_1,

$$\mathbb{E}_{D_2\sim\pi(\cdot|d_1)}\left[\beta\left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((d_1-\mu_1)-(D_2-\mu_2)-4z_1^{RT}(d_1))}{k_f}\right)\right] = 0$$

This reduces to

$$z_1^{RT}(d_1) = \frac{1}{4}(d_1-\mu_1) - \frac{1}{4}(\mu_{2|d_1}-\mu_2) + \frac{k_f}{4}(\mu_1-\mu_2-2z_1^{DA})$$

In particular,

$$\mathbb{E}[z_1^{RT}(D_1)] = \frac{k_f}{4}(\mu_1-\mu_2-2\bar{z}_1^{DA})$$

Now we take derivative w.r.t z_1^{DA}:

$$\beta(\mu_1-\mu_2-4z_1^{DA}) + \mathbb{E}\left[\beta(-2)z_1^{RT}(D_1)\right] = 0$$

Using the expression for $\mathbb{E}[z_1^{RT}(D_1)]$ derived earlier gives

$$\begin{aligned}\bar{z}_1^{DA} &= \frac{(2-k_f)}{2(4-k_f)}(\mu_1-\mu_2) \\ \bar{z}_1^{RT}(d_1) &= \frac{1}{4}(d_1-\mu_1) - \frac{1}{4}(\mu_{2|d_1}-\mu_2) + \frac{k_f}{2(4-k_f)}(\mu_1-\mu_2)\end{aligned}$$

Now we compute the generation cost. The demands are given by

$$
\begin{aligned}
d_1^{DA} &= \mu_1 - z_1^{DA} = \frac{(6-k_f)}{2(4-k_f)}\mu_1 + \frac{(2-k_f)}{2(4-k_f)}\mu_2 \\
d_2^{DA} &= \mu_2 + z_1^{DA} = \frac{(2-k_f)}{2(4-k_f)}\mu_1 + \frac{(6-k_f)}{2(4-k_f)}\mu_2 \\
d_1^{RT} &= \frac{3}{4}(d_1-\mu_1) + \frac{1}{4}(\mu_{2|d_1}-\mu_2) - \frac{k_f}{2(4-k_f)}(\mu_1-\mu_2) \\
d_2^{RT} &= (d_2-\mu_2) + \frac{1}{4}(d_1-\mu_1) - \frac{1}{4}(\mu_{2|d_1}-\mu_2) + \frac{k_f}{2(4-k_f)}(\mu_1-\mu_2)
\end{aligned}
$$

We calculate the modified real-time demand

$$
\begin{aligned}
\tilde{d_1^{RT}} &= d_1^{DA} + \frac{d_1^{RT}}{k_f} = \frac{3}{4k_f}(d_1-\mu_1) + \frac{1}{4k_f}(\mu_{2|d_1}-\mu_2) + \frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2 \\
\tilde{d_2^{RT}} &= d_2^{DA} + \frac{d_2^{RT}}{k_f} = \frac{1}{k_f}(d_2-\mu_2) + \frac{1}{4k_f}(d_1-\mu_1) - \frac{1}{4k_f}(\mu_{2|d_1}-\mu_2) + \frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2
\end{aligned}
$$

We calculate, using Proposition 1:

$$
\begin{aligned}
\mathbb{E}[(\tilde{d_1^{RT}})^2] &= \mathbb{E}\left(\frac{3}{4k_f}(D_1-\mu_1) + \frac{1}{4k_f}(\mu_{2|D_1}-\mu_2) + \frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2\right)^2 \\
&= \frac{9\sigma_1^2 + \rho_s^2\sigma_2^2 + 6\rho\sigma_1\sigma_2}{16k_f^2} + \left(\frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2\right)^2 \\
\mathbb{E}[(\tilde{d_2^{RT}})^2] &= \mathbb{E}\left(\frac{4(D_2-\mu_2) + (D_1-\mu_1) - (\mu_{2|D_1}-\mu_2)}{4k_f} + \frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2\right)^2 \\
&= \frac{\sigma_1^2 + (16-7\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2}{16k_f^2} + \left(\frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2\right)^2
\end{aligned}
$$

The generation cost is

$$
\alpha(\mu_1+\mu_2) + k_s\left[\frac{\beta}{2}\left[(d_1^{DA})^2 + (d_2^{DA})^2\right]\right] + k_f\mathbb{E}\left[\frac{\beta}{2}\left[(\tilde{d_1^{RT}})^2 + (\tilde{d_2^{RT}})^2\right]\right]
$$

Using the expressions we have previously computed, this generation cost simplifies to the one given in the theorem. $\square$

Figure 2.3.1 Hourly average day-ahead bids and nodal prices (by quarter)

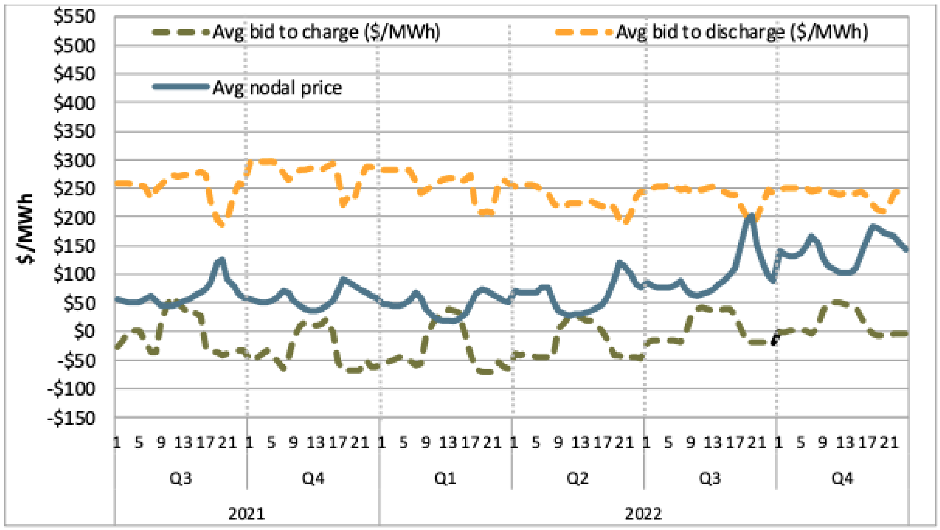

Figure 3: Day-Ahead discharge bid $\gg$ price (avoid DA scheduling)

Figure 2.3.2 Hourly average real-time battery bids and nodal prices (by quarter)

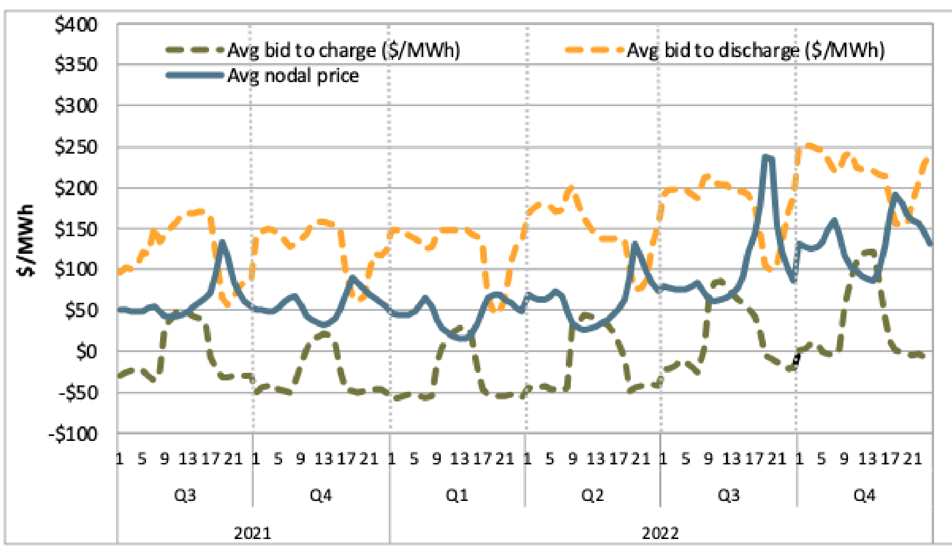

Figure 4: Real-time discharge bid $\approx$ price (batteries suddenly show up in RT)

D Proofs for Section 7

Proof. Proof of Theorem 5.

Assume that there are n batteries indexed by $b \in [n] \equiv \{1, 2, \ldots, n\}$. Assume that battery b

discharges $z_{b,t}^{DA}$ in day-ahead in time $t \in \{1,2\}$ and $z_{b,1}^{RT}(D_1)$ in real-time in time 1, and $z_{b,2}^{RT}(D_1, D_2)$ in RT in time 2, with $z_{b,1}^{DA} + z_{b,2}^{DA} = z_{b,1}^{RT}(D_1) + z_{b,2}^{RT}(D_1, D_2) = 0$.

We have

$$\begin{aligned}
\tilde{d}_1^{DA} &= \mu_1 - \sum_b z_{b,1}^{DA} \\
\tilde{d}_2^{DA} &= \mu_2 + \sum_b z_{b,1}^{DA} \\
\tilde{d}_1^{RT}(d_1) &= \mu_1 - \sum_b z_{b,1}^{DA} + \frac{1}{k_f}\left(d_1 - \mu_1 - \sum_b z_{b,1}^{RT}(d_1)\right) \\
\tilde{d}_2^{RT}(d_1, d_2) &= \mu_2 + \sum_b z_{b,1}^{DA} + \frac{1}{k_f}\left(d_2 - \mu_2 + \sum_b z_{b,1}^{RT}(d_1)\right)
\end{aligned}$$

and

$$\begin{aligned}
\lambda_1^{DA} &= \alpha + \beta \tilde{d}_1^{DA} \\
\lambda_2^{DA} &= \alpha + \beta \tilde{d}_2^{DA} \\
\lambda_1^{RT} &= \alpha + \beta \tilde{d}_1^{RT} \\
\lambda_2^{RT} &= \alpha + \beta \tilde{d}_2^{RT}
\end{aligned}$$

Battery b's profit is given by

$$\Pi_b = (\lambda_1^{DA} - \lambda_2^{DA}) z_{b,1}^{DA} + \mathbb{E}[(\lambda_1^{RT} - \lambda_2^{RT}) z_{b,1}^{RT}(D_1)]$$

We can write

$$\begin{aligned}
\Pi_b = \beta &\left(\mu_1 - \mu_2 - 2\sum_b z_{b,1}^{DA}\right) z_{b,1}^{DA} \\
&+ \mathbb{E}\left[\beta\left\{\mu_1 - \mu_2 - 2\sum_b z_{b,1}^{DA} + \frac{1}{k_f}\Big((D_1 - \mu_1) - (D_2 - \mu_2) - 2\sum_b z_{b,1}^{RT}(D_1)\Big)\right\} z_{b,1}^{RT}(D_1)\right]
\end{aligned}$$

Fix $D_1 = d_1$. We take derivative w.r.t. $z_1^{RT}(d_1)$ to get

$$\mathbb{E}\left[\beta\left\{\mu_1 - \mu_2 - 2\sum_b z_{b,1}^{DA} + \frac{1}{k_f}\Big((d_1 - \mu_1) - (D_2 - \mu_2) - 2\sum_{b'\neq b} z_{b',1}^{RT}(d_1) - 4z_{b,1}^{RT}(d_1)\Big)\right\}\middle| D_1 = d_1\right] = 0$$

or

$$k_f^2\left(\mu_1 - \mu_2 - 2\sum_b z_{b,1}^{DA}\right) + k_f\Big((d_1 - \mu_1) - (\mu_{2|d_1} - \mu_2) - 2\sum_{b'\neq b} z_{b',1}^{RT}(d_1) - 4z_{b,1}^{RT}(d_1)\Big) = 0 \quad (14)$$

We take derivative w.r.t $z_{b,1}^{DA}$ to get

$$\left(\mu_1 - \mu_2 - 2\sum_{b'\neq b} z_{b',1}^{DA} - 4z_{b,1}^{DA}\right) + \mathbb{E}\left[\{-2\}z_{b,1}^{RT}(D_1)\right] = 0 \quad (15)$$

We will assume that the equilibrium is symmetric. (We can show directly that any equilibrium is symmetric, because the main terms are determined by linear equations with a unique solution which we will derive below.)

The main term of (14) is

$$k_f(\mu_1 - \mu_2 - 2nz_{b,1}^{DA}) + (d_1 - \mu_1) - (\mu_{2|d_1} - \mu_2) - (2n+2)z_{b,1}^{RT}(d_1) = 0$$

or

$$z_{b,1}^{RT}(d_1) = \frac{1}{2(n+1)}\left(k_f(\mu_1 - \mu_2) - 2k_f n z_{b,1}^{DA} + (d_1 - \mu_1) - (\mu_{2|d_1} - \mu_2)\right)$$

The main term of (15) is

$$\mu_1 - \mu_2 - (2n+2)z_{b,1}^{DA} - 2\mathbb{E}[z_{b,1}^{RT}(D_1)] = 0$$

From the expression for $z_{b,1}^{RT}(d_1)$, we have

$$\mathbb{E}[z_{b,1}^{RT}(D_1)] = \frac{1}{2(n+1)}\left(k_f(\mu_1 - \mu_2) - 2k_f n z_{b,1}^{DA}\right)$$

Substituting this in gives

$$\mu_1 - \mu_2 - (2n+2)z_{b,1}^{DA} - 2 \cdot \frac{1}{2(n+1)} \left(k_f(\mu_1 - \mu_2) - 2k_f n z_{b,1}^{DA}\right) = 0$$

or

$$z_{b,1}^{DA} = \frac{(n+1-k_f)}{2((n+1)^2 - nk_f)}(\mu_1 - \mu_2)$$

which gives

$$z_{b,1}^{RT}(d_1) = \frac{k_f}{2((n+1)^2 - nk_f)}(\mu_1 - \mu_2) + \frac{1}{2(n+1)}(d_1 - \mu_1) - \frac{1}{2(n+1)}(\mu_{2|d_1} - \mu_2)$$

We now compute the generation cost. The generation cost is given by

$$\alpha(\mu_1 + \mu_2) + k_s \frac{\beta}{2}\left[(\tilde{d}_1^{DA})^2 + (\tilde{d}_2^{DA})^2\right] + k_f \frac{\beta}{2}\mathbb{E}\left[(\tilde{d}_1^{RT})^2 + (\tilde{d}_2^{RT})^2\right]$$

which evaluates to

$$\begin{aligned}
&\alpha(\mu_1 + \mu_2) \\
&+ \beta\Bigg\{\frac{(2 + 6n + 7n^2 + 4n^3 + n^4 - (4n + 5n^2 + 2n^3)k_f + n^2 k_f^2)}{4((n+1)^2 - nk_f)^2}(\mu_1^2 + \mu_2^2) \\
&+ \frac{n(2 + 5n + 4n^2 + n^3 - (3n + 2n^2)k_f + nk_f^2)}{2((n+1)^2 - nk_f)^2}\mu_1\mu_2 \\
&+ \frac{(2 + 2n + n^2)\sigma_1^2 + (2 + 4n + 2n^2 - (2n + n^2)\rho_s^2)\sigma_2^2 + 2n(2+n)\rho\sigma_1\sigma_2}{4(n+1)^2 k_f}\Bigg\}
\end{aligned}$$

This gives

$$\begin{aligned}
\mathrm{Cost(NB)} - \mathrm{Cost(CN)} &= \beta\frac{1}{4}(\mu_1 - \mu_2)^2 + \frac{\beta}{4k_f}(\sigma_1 - \rho\sigma_2)^2 \\
\mathrm{Cost(NB)} - \mathrm{Cost(DCN)} &= \beta\frac{(n(2 + 5n + 4n^2 + n^3) - n^2(3 + 2n)k_f + n^2k_f^2)}{4((n+1)^2 - nk_f)^2}(\mu_1 - \mu_2)^2 + \frac{n(n+2)\beta}{4(n+1)^2 k_f}(\sigma_1 - \rho\sigma_2)^2
\end{aligned}$$

Therefore,

$$1+\frac{1}{n(n+1)(n^2+n+2)}=\frac{(n^2+n+1)^2}{n(n+1)(n^2+n+2)}\leq \text{PoA}\leq \frac{(n+1)^2}{n(n+2)}=1+\frac{1}{n(n+2)}.$$

□

Proof. Proof of Theorem 6. We first consider (P1). We want to maximize profit with the extra constraint that $\mathbb{E}[z_1^{RT}]=0$.

The profit is

$$\begin{aligned}\Pi &= (\lambda_1^{DA}-\lambda_2^{DA})z_1^{DA}+\mathbb{E}\left[(\lambda_1^{RT}-\lambda_2^{RT})z_1^{RT}(D_1)\right]+\theta\mathbb{E}\left[z_1^{RT}\right]\\ &= \beta(\mu_1-\mu_2-2z_1^{DA})z_1^{DA}\\ &+\mathbb{E}\left[\beta\left(\mu_1-\mu_2-2z_1^{DA}+\frac{((D_1-\mu_1)-(D_2-\mu_2)-2z_1^{RT}(D_1))}{k_f}\right)z_1^{RT}(D_1)\right]\end{aligned}$$

We can ignore the factor of β in maximizing profit.

We can write the Lagrangian

$$\begin{aligned}\mathcal{L} &= (\mu_1-\mu_2-2z_1^{DA})z_1^{DA}\\ &+\mathbb{E}\left[\left(\mu_1-\mu_2-2z_1^{DA}+\frac{((D_1-\mu_1)-(D_2-\mu_2)-2z_1^{RT}(D_1))}{k_f}\right)z_1^{RT}(D_1)\right]+\theta\mathbb{E}\left[z_1^{RT}\right]\end{aligned}$$

Taking derivative w.r.t. $z_1^{RT}(d_1)$ for a given fixed d_1 gives, for each d_1,

$$\mathbb{E}_{D_2\sim\pi(\cdot|d_1)}\left[\left(\mu_1-\mu_2-2z_1^{DA}+\frac{((d_1-\mu_1)-(D_2-\mu_2)-4z_1^{RT}(d_1))}{k_f}\right)\right]+\theta=0$$

This reduces to

$$z_1^{RT}(d_1)=\frac{1}{4}k_f\theta+\frac{1}{4}(d_1-\mu_1)-\frac{1}{4}(\mu_{2|d_1}-\mu_2)+\frac{k_f}{4}(\mu_1-\mu_2-2z_1^{DA})$$

In particular,

$$\mathbb{E}[z_1^{RT}(D_1)]=\frac{1}{4}k_f\theta+\frac{k_f}{4}(\mu_1-\mu_2-2\bar{z}_1^{DA})$$

This should be zero. Therefore,

$$\theta = -(\mu_1 - \mu_2 - 2z_1^{DA})$$

Now we take derivative w.r.t z_1^{DA}:

$$(\mu_1 - \mu_2 - 4z_1^{DA}) + \mathbb{E}\left[(-2)z_1^{RT}(D_1)\right] = 0$$

Because $\mathbb{E}[z_1^{RT}] = 0$, we have

$$\begin{aligned}
\bar{z}_1^{DA} &= \frac{1}{4}(\mu_1 - \mu_2) \\
\bar{z}_1^{RT}(d_1) &= \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2)
\end{aligned}$$

Now we compute the generation cost. The demands are given by

$$\begin{aligned}
d_1^{DA} &= \mu_1 - z_1^{DA} = \frac{3}{4}\mu_1 + \frac{1}{4}\mu_2 \\
d_2^{DA} &= \mu_2 + z_1^{DA} = \frac{1}{4}\mu_1 + \frac{3}{4}\mu_2 \\
d_1^{RT} &= \frac{3}{4}(d_1 - \mu_1) + \frac{1}{4}(\mu_{2|d_1} - \mu_2) \\
d_2^{RT} &= (d_2 - \mu_2) + \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2)
\end{aligned}$$

We calculate the modified real-time demand

$$\begin{aligned}
\tilde{d}_1^{RT} &= d_1^{DA} + \frac{d_1^{RT}}{k_f} = \frac{3}{4k_f}(d_1 - \mu_1) + \frac{1}{4k_f}(\mu_{2|d_1} - \mu_2) + \frac{3}{4}\mu_1 + \frac{1}{4}\mu_2 \\
\tilde{d}_2^{RT} &= d_2^{DA} + \frac{d_2^{RT}}{k_f} = \frac{1}{k_f}(d_2 - \mu_2) + \frac{1}{4k_f}(d_1 - \mu_1) - \frac{1}{4k_f}(\mu_{2|d_1} - \mu_2) + \frac{1}{4}\mu_1 + \frac{3}{4}\mu_2
\end{aligned}$$

We calculate, using Proposition 1:

$$\begin{aligned}
\mathbb{E}[(\tilde{d}_1^{RT})^2] &= \mathbb{E}\left(\frac{3}{4k_f}(D_1-\mu_1)+\frac{1}{4k_f}(\mu_{2|D_1}-\mu_2)+\frac{3}{4}\mu_1+\frac{1}{4}\mu_2\right)^2 \\
&= \frac{9\sigma_1^2+\rho_s^2\sigma_2^2+6\rho\sigma_1\sigma_2}{16k_f^2}+\left(\frac{3}{4}\mu_1+\frac{1}{4}\mu_2\right)^2 \\
\mathbb{E}[(\tilde{d}_2^{RT})^2] &= \mathbb{E}\left(\frac{4(D_2-\mu_2)+(D_1-\mu_1)-(\mu_{2|D_1}-\mu_2)}{4k_f}+\frac{1}{4}\mu_1+\frac{3}{4}\mu_2\right)^2 \\
&= \frac{\sigma_1^2+(16-7\rho_s^2)\sigma_2^2+6\rho\sigma_1\sigma_2}{16k_f^2}+\left(\frac{1}{4}\mu_1+\frac{3}{4}\mu_2\right)^2
\end{aligned}$$

The generation cost is

$$\alpha(\mu_1+\mu_2)+k_s\left[\frac{\beta}{2}\left[(d_1^{DA})^2+(d_2^{DA})^2\right]\right]+k_f\mathbb{E}\left[\frac{\beta}{2}\left[(\tilde{d}_1^{RT})^2+(\tilde{d}_2^{RT})^2\right]\right]$$

which simplifies to

$$\text{Cost(DCN-Reg)}=\alpha(\mu_1+\mu_2)+\beta\left[\frac{5\mu_1^2+6\mu_1\mu_2+5\mu_2^2}{16}+\frac{5\sigma_1^2+(8-3\rho_s^2)\sigma_2^2+6\rho\sigma_1\sigma_2}{16k_f}\right]$$

Here, DCN-Reg means decentralized but "regulated".

We can compute

$$\text{Cost(DCN-Reg)}-\text{Cost(DCN)}=\beta\frac{k_f(4+k_f)}{16(4-k_f)^2}(\mu_1-\mu_2)^2\geq 0$$

so Cost(DCN-Reg) $\geq$ Cost(DCN). So this regulation (requiring that the battery discharges zero in expectation in real time) always increases cost!

Under this regulation, the three types of distortions are

$$\begin{aligned}
\text{quantity withholding} &= \frac{1}{2} \\
\text{shift from DA to RT} &= 0 \\
\text{reduction in RT responsiveness} &= \frac{1}{2}
\end{aligned}$$

So the reduction in RT responsiveness is the same. The quantity withholding is worse, and the shift from DA to RT is better (obviously! because it is designed specifically to combat this). So stamping down on withholding of the second kind (shift from DA to RT) means spillover to withholding of the first kind (quantity withholding). As we have seen above, the net effect on the system is to increase cost. And the battery profit decreases by definition (profit under constrained maximization is lower than profit under unconstrained maximization).

Now we consider (P2). This is a consequence of a more general theorem, Theorem 8, that characterizes the equilibrium with any number of batteries and virtual bidders.

□

Theorem 8 (Battery Competition and Virtual Bidders)**.** *Let there be B batteries and V virtual bidders in a Cournot competition. Then each battery b's DA and RT discharges in period 1 are*

$$
\begin{aligned}
z_{b,1}^{DA} &= \frac{(B+V+1)-(V+1)k_f}{2((B+V+1)(B+1)-Bk_f)}(\mu_1-\mu_2) \\
z_{b,1}^{RT}(D_1) &= \frac{(D_1-\mu_1)-(\mu_{2|D_1}-\mu_2)}{2(B+1)} + \frac{(V+1)k_f}{2((B+V+1)(B+1)-Bk_f)}(\mu_1-\mu_2)
\end{aligned}
$$

Each virtual bidder v's discharges in period 1 are

$$
y_{v,1} = \frac{Bk_f}{2((B+V+1)(B+1)-Bk_f)}(\mu_1-\mu_2)
$$

The generation cost is

$$
\frac{(B^2+2B+2)(B+V+1)^2+B^2k_f^2-B(2B^2+5B+4+2(B+2)V)k_f}{4((B+V+1)(B+1)-Bk_f)^2}\mu^2+\frac{1}{4k_f}\left(1+\frac{1}{(B+1)^2}\right)\sigma^2
$$

Proof. Proof of Theorem 7.

$$
\begin{aligned}
\Pi &= \beta(\mu_1-\mu_2-2z_1^{DA})z_1^{DA} \\
&+ \mathbb{E}\left[\beta\left(\mu_1-\mu_2-2z_1^{DA}+\frac{((D_1-\mu_1)-(D_2-\mu_2)-2z_1^{RT}(D_1))}{k_f}\right)z_1^{RT}(D_1)\right] \\
&+ s\left(z_1^{DA}+\mathbb{E}[z_1^{RT}]\right)
\end{aligned}
$$

Taking derivative w.r.t. $z_1^{RT}(d_1)$ for a given fixed d_1 gives, for each d_1,

$$\beta \mathbb{E}_{D_2 \sim \pi(\cdot|d_1)} \left[\left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((d_1 - \mu_1) - (D_2 - \mu_2) - 4z_1^{RT}(d_1))}{k_f} \right) \right] + s = 0$$

This reduces to

$$z_1^{RT}(d_1) = \frac{k_f s}{4\beta} + \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2) + \frac{k_f}{4}(\mu_1 - \mu_2 - 2z_1^{DA})$$

In particular,

$$\mathbb{E}[z_1^{RT}(D_1)] = \frac{k_f s}{4\beta} + \frac{k_f}{4}(\mu_1 - \mu_2 - 2z_1^{DA})$$

Now we take derivative w.r.t z_1^{DA}:

$$\beta(\mu_1 - \mu_2 - 4z_1^{DA}) + \beta \mathbb{E}\left[(-2)z_1^{RT}(D_1)\right] + s = 0$$

Substituting the expression for $\mathbb{E}[z_1^{RT}]$, we have

$$\begin{aligned} z_1^{DA} &= \frac{(2 - k_f)}{2(4 - k_f)}(\mu_1 - \mu_2) - \frac{k_f s}{2(4 - k_f)\beta} \\ z_1^{RT}(d_1) &= \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2) + \frac{k_f}{2(4 - k_f)}(\mu_1 - \mu_2) + \frac{k_f s}{(4 - k_f)\beta} \end{aligned}$$

Note that with positive s, the battery does LESS quantity withholding and MORE shift from day-ahead to real-time. This is a good sign.

The total discharge is

$$z_1^{DA} + \mathbb{E}[z_1^{RT}] = \frac{1}{4 - k_f}(\mu_1 - \mu_2) + \frac{k_f s}{2(4 - k_f)\beta}$$

Now we compute the generation cost.

Now we compute the generation cost. The demands are given by

$$
\begin{aligned}
d_1^{DA} &= \mu_1 - z_1^{DA} = \frac{k_f s}{2(4-k_f)\beta} + \frac{(6-k_f)}{2(4-k_f)}\mu_1 + \frac{(2-k_f)}{2(4-k_f)}\mu_2 \\
d_2^{DA} &= \mu_2 + z_1^{DA} = -\frac{k_f s}{2(4-k_f)\beta} + \frac{(2-k_f)}{2(4-k_f)}\mu_1 + \frac{(6-k_f)}{2(4-k_f)}\mu_2 \\
d_1^{RT} &= -\frac{k_f s}{(4-k_f)\beta} - \frac{k_f}{2(4-k_f)}(\mu_1 - \mu_2) + \frac{3}{4}(d_1 - \mu_1) + \frac{1}{4}(\mu_{2|d_1} - \mu_2) \\
d_2^{RT} &= \frac{k_f s}{(4-k_f)\beta} + \frac{k_f}{2(4-k_f)}(\mu_1 - \mu_2) + (d_2 - \mu_2) + \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2)
\end{aligned}
$$

We calculate the modified real-time demand

$$
\begin{aligned}
\tilde{d}_1^{RT} &= d_1^{DA} + \frac{d_1^{RT}}{k_f} = \frac{3}{4k_f}(d_1 - \mu_1) + \frac{1}{4k_f}(\mu_{2|d_1} - \mu_2) + \frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2 - \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta} \\
\tilde{d}_2^{RT} &= d_2^{DA} + \frac{d_2^{RT}}{k_f} = \frac{1}{k_f}(d_2 - \mu_2) + \frac{1}{4k_f}(d_1 - \mu_1) - \frac{1}{4k_f}(\mu_{2|d_1} - \mu_2) + \frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2 + \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta}
\end{aligned}
$$

We calculate, using Proposition 1:

$$
\begin{aligned}
\mathbb{E}[(\tilde{d}_1^{RT})^2] &= \mathbb{E}\left(\frac{3}{4k_f}(D_1 - \mu_1) + \frac{1}{4k_f}(\mu_{2|D_1} - \mu_2) + \frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2 - \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta}\right)^2 \\
&= \frac{9\sigma_1^2 + \rho_s^2\sigma_2^2 + 6\rho\sigma_1\sigma_2}{16k_f^2} + \left(\frac{(5-k_f)}{2(4-k_f)}\mu_1 + \frac{(3-k_f)}{2(4-k_f)}\mu_2 - \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta}\right)^2 \\
\mathbb{E}[(\tilde{d}_2^{RT})^2] &= \mathbb{E}\left(\frac{4(D_2 - \mu_2) + (D_1 - \mu_1) - (\mu_{2|D_1} - \mu_2)}{4k_f} + \frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2 + \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta}\right)^2 \\
&= \frac{\sigma_1^2 + (16 - 7\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2}{16k_f^2} + \left(\frac{(3-k_f)}{2(4-k_f)}\mu_1 + \frac{(5-k_f)}{2(4-k_f)}\mu_2 + \frac{(2-k_f)}{2(4-k_f)}\frac{s}{\beta}\right)^2
\end{aligned}
$$

The total cost (generation cost plus subsidy cost) is

$$
\text{Cost(DCN-}s) \equiv \alpha(\mu_1 + \mu_2) + k_s\left[\frac{\beta}{2}\left[(d_1^{DA})^2 + (d_2^{DA})^2\right]\right] + k_f\mathbb{E}\left[\frac{\beta}{2}\left[(\tilde{d}_1^{RT})^2 + (\tilde{d}_2^{RT})^2\right]\right] + s\left(\frac{\mu_1 - \mu_2}{4-k_f} + \frac{k_f}{2(4-k_f)}\right.
$$

We can then calculate

$$
\text{Cost(DCN-}s) - \text{Cost(DCN-0)} = \frac{(2-k_f)(4+k_f)}{2(4-k_f)^2}(\mu_1 - \mu_2)s + \frac{(12 - 5k_f)k_f}{4(4-k_f)^2\beta}s^2 \geq 0
$$

So for $s \geq 0$, Cost(DCN-s) is minimized at $s = 0$.

$\square$

E Proofs for Section 8

Theorem 9 (Multiple Time Periods)**.** *For each $t' < t$, define $D_{1:t} \equiv (D_1, D_2, \ldots, D_t)$, $\mu_t \equiv \mathbb{E}[D_t]$, $\bar{\mu} = (\mu_1 + \cdots + \mu_T)/T$, and $\mu_{t|d_{1:t'}} = \mathbb{E}[D_t|D_{1:t'} = d_{1:t'}]$.*

The centralized battery discharge decisions are given by, for each period t,

$$
\begin{aligned}
z_t^{DA} &= \mu_t - \bar{\mu} \\
z_t^{RT}(D_{1:t}) &= \frac{(T-t)}{(T-t+1)}(D_t - \mu_t) - \sum_{t'=1}^{t-1} \frac{1}{(T-t'+1)}(D_{t'} - \mu_{t'}) \\
&\quad - \frac{1}{(T-t+1)} \sum_{i=t+1}^{T} (\mu_{i|D_{1:t}} - \mu_i) + \sum_{t'=1}^{t-1} \sum_{i=t'+1}^{T} \frac{1}{(T-t')(T-t'+1)}(\mu_{i|D_{1:t'}} - \mu_i)
\end{aligned}
$$

The decentralized battery discharge decisions are given by, for each period t,

$$
\begin{aligned}
z_t^{DA} &= \frac{(2-k_f)}{(4-k_f)}(\mu_t - \bar{\mu}) \\
z_t^{RT}(D_{1:t}) &= \frac{k_f}{(4-k_f)}(\mu_t - \bar{\mu}) + \frac{(T-t)}{2(T-t+1)}(D_t - \mu_t) - \sum_{t'=1}^{t-1} \frac{1}{2(T-t'+1)}(D_{t'} - \mu_{t'}) \\
&\quad - \frac{1}{2(T-t+1)} \sum_{i=t+1}^{T} (\mu_{i|D_{1:t}} - \mu_i) + \sum_{t'=1}^{t-1} \sum_{i=t'+1}^{T} \frac{1}{2(T-t')(T-t'+1)}(\mu_{i|D_{1:t'}} - \mu_i)
\end{aligned}
$$

If we further assume that each D_t is normal and independent, the bounds $9/8 \leq \mathrm{PoA} \leq 4/3$ always hold.

Remark. Even though Theorem 9 proves the bounds $9/8 \leq \mathrm{PoA} \leq 4/3$ only in the special case when $(D_1, \ldots, D_T)$ are independent and normal, we conjecture that the same bound still holds under a reasonable assumption on the covariance matrix, such as when each two periods are positively correlated: $\mathrm{Cov}(D_{t_1}, D_{t_2}) \geq 0$ for each t_1, t_2.

Proof of Theorem 9. We first solve the centralized case.

We want to minimize generation cost. The generation cost is

$$\sum_{t=1}^{T}\Bigg[(1-k_f)\left(\alpha(\mu_t - z_t^{DA}) + \frac{\beta}{2}(\mu_t - z_t^{DA})^2\right)$$

$$+k_f\mathbb{E}\left[\alpha\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right) + \frac{\beta}{2}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right)^2\right]\Bigg]$$

$$= \alpha T\bar{\mu} + \frac{\beta}{2}\Bigg\{(1-k_f)\sum_{t=1}^{T-1}(\mu_t - z_t^{DA})^2 + (1-k_f)\left(\mu_T + \sum_{t=1}^{T-1} z_t^{DA}\right)^2$$

$$+k_f\sum_{t=1}^{T-1}\mathbb{E}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right)^2 + k_f\mathbb{E}\left(\mu_T + \sum_{t=1}^{T-1} z_t^{DA} + \frac{D_T - \mu_T + \sum_{t=1}^{T-1} z_t^{RT}}{k_f}\right)^2\Bigg\}$$

Therefore, we want to minimize the expression in $\{\cdots\}$.

Fix $1 \leq t \leq T-1$. For each i, $t \leq i \leq T-1$, take the derivative with respect to $z_i^{RT}(d_{1:i})$ and take expectation over $D_{(t+1):i}$ (so the equation depends only on $d_{1:t}$ gives

$$k_f(\mu_T - \mu_i) - (\mu_{i|d_{1:t}} - \mu_i) + (\mu_{T|d_{1:t}} - \mu_T) + k_f z_i^{DA} + k_f\sum_{t'=1}^{T-1} z_{t'}^{DA} + \sum_{t'=1}^{t-1} z_{t'}^{RT} + \sum_{t'=t}^{T-1}(1+\mathbf{1}(t=i))\mathbb{E}[z_{t'}^{RT}|d_{1:t}] = 0 \tag{16}$$

where $\mu_{i|d_{1:t}} = d_t$ for $i = t$, and $\mathbb{E}[z_{t'}^{RT}|d_{1:t}] = z_t^{RT}(d_{1:t})$ for $t' = t$.

We will first solve for the DA variables. So we will take expectation of (16) for $i = t$ over $D_{1:t}$ to get

$$k_f\sum_{t'=1}^{T} z_{t'}^{DA} + \mathbb{E}[z_t^{RT}] + \sum_{t'=1}^{T-1}\mathbb{E}[z_{t'}^{RT}] = 0$$

Because this holds for every t, and only $\mathbb{E}[z_t^{RT}]$ depends on t in the above equation, we get that $\mathbb{E}[z_t^{RT}]$ must be equal for every t: $\mathbb{E}[z_t^{RT}] = \mathbb{E}[z_1^{RT}]$ and

$$k_f\sum_{t'=1}^{T} z_{t'}^{DA} + T\mathbb{E}[z_1^{RT}] = 0$$

We now take the derivative with respect to z_t^{DA}:

$$-2(1-k_f)(\mu_t - z_t^{DA}) + (1-k_f)(2)\left(\mu_T + \sum_{t'=1}^{T-1} z_{t'}^{DA}\right)$$

$$+k_f(2)\left(-\frac{1}{k_f}\right)\mathbb{E}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right) + k_f\left(\frac{2}{k_f}\right)\mathbb{E}\left(\mu_T + \sum_{t'=1}^{T-1} z_{t'}^{DA} + \frac{D_T - \mu_T + \sum_{t'=1}^{T-1} z_{t'}^{RT}}{k_f}\right)$$

Replacing $\mathbb{E}[z_t^{RT}] = \mathbb{E}[z_1^{RT}]$, we get

$$2(2-k_f)\left(\mu_T - \mu_t + z_t^{DA} + \sum_{t'=1}^{T-1} z_{t'}^{DA}\right) + \frac{2}{k_f}T\mathbb{E}[z_1^{RT}] = 0$$

This holds for every t. So there is a constant c such that $z_t^{DA} = \mu_t - \mu_T + c$ This gives

$$2(2-k_f)T(c + \bar{\mu} - \mu_T) + \frac{2T}{k_f}\mathbb{E}[z_1^{RT}] = 0 \text{ and } 0 = k_f \sum_{t'=1}^{T} z_{t'}^{DA} + T\mathbb{E}[z_1^{RT}] = k_f T(\bar{\mu} - \mu_T + c) + T\mathbb{E}[z_1^{RT}]$$

Therefore, $\mathbb{E}[z_1^{RT}] = 0$ and $c = \mu_T - \bar{\mu}$, so $z_t^{DA} = \mu_t - \bar{\mu}$

Now we solve for $z_t^{RT}(d_{1:t})$.

Substituting $z_t^{DA} = \mu_t - \bar{\mu}$ in (16) gives

$$-(\mu_{i|d_{1:t}} - \mu_i) + (\mu_{T|d_{1:t}} - \mu_T) + \sum_{t'=1}^{t-1} z_{t'}^{RT} + \sum_{t'=t}^{T-1}(1 + \mathbf{1}(t=i))\mathbb{E}[z_{t'}^{RT}|d_{1:t}] = 0 \tag{17}$$

Summing (17) over all $t \le i \le T-1$ gives

$$-\sum_{i=t}^{T-1}(\mu_{i|d_{1:t}} - \mu_i) + (T-t)(\mu_{T|d_{1:t}} - \mu_T) + (T-t)\sum_{t'=1}^{t-1} z_{t'}^{RT} + (T-t+1)\sum_{t'=t}^{T-1}\mathbb{E}[z_{t'}^{RT}|d_{1:t}] = 0$$

This gives $\sum_{t'=t}^{T-1}\mathbb{E}[z_{t'}^{RT}|d_{1:t}]$ in terms of $z_{t'}^{RT}$, $t' \le t-1$. We substitute this into (17) with $i = t$:

$$-(d_t - \mu_t) + (\mu_{T|d_{1:t}} - \mu_T) + \sum_{t'=1}^{t-1} z_{t'}^{RT} + z_t^{RT} + \sum_{t'=t}^{T-1}\mathbb{E}[z_{t'}^{RT}|d_{1:t}] = 0$$

$$(T-t+1)\left(-(d_t-\mu_t)+(\mu_{T|d_{1:t}}-\mu_T)+\sum_{t'=1}^{t-1} z_{t'}^{RT}+z_t^{RT}\right)$$
$$=-\sum_{i=t}^{T-1}(\mu_{i|d_{1:t}}-\mu_i)+(T-t)(\mu_{T|d_{1:t}}-\mu_T)+(T-t)\sum_{t'=1}^{t-1} z_{t'}^{RT}$$

$$-(T-t)(d_t-\mu_t)+\sum_{i=t+1}^{T-1}(\mu_{i|d_{1:t}}-\mu_i)+(\mu_{T|d_{1:t}}-\mu_T)+\sum_{t'=1}^{t-1} z_{t'}^{RT}+(T-t+1)z_t^{RT}=0$$

This recursion has the form

$$(T-t+1)z_t^{RT}+\sum_{t'=1}^{t-1} z_{t'}^{RT}=a_t$$

with

$$a_t=(T-t)(d_t-\mu_t)-\sum_{i=t+1}^{T}(\mu_{i|d_{1:t}}-\mu_i)$$

This gives $Tz_1^{RT}=a_1$ so $z_1^{RT}=a_1/T$ and

$$\begin{aligned}
(T-t)z_{t+1}^{RT}-(T-t+1)z_t^{RT}+z_t^{RT}&=a_{t+1}-a_t\\
z_{t+1}^{RT}-z_t^{RT}&=\frac{1}{(T-t)}(a_{t+1}-a_t)\\
z_t^{RT}=\frac{a_1}{T}+\sum_{t'=1}^{t-1}\frac{1}{(T-t')}(a_{t'+1}-a_{t'})&=\frac{1}{(T-t+1)}a_t-\sum_{t'=1}^{t-1}\frac{1}{(T-t')(T-t'+1)}a_{t'}\\
&=\frac{1}{(T-t+1)}\left((T-t)(d_t-\mu_t)-\sum_{i=t+1}^{T}(\mu_{i|d_{1:t}}-\mu_i)\right)\\
&-\sum_{t'=1}^{t-1}\frac{1}{(T-t')(T-t'+1)}\left((T-t')(d_{t'}-\mu_{t'})-\sum_{i=t'+1}^{T}(\mu_{i|d_{1:t'}}-\mu_i)\right)
\end{aligned}$$

Therefore

$$z_t^{RT} = \frac{(T-t)}{(T-t+1)}(d_t - \mu_t) - \sum_{t'=1}^{t-1} \frac{1}{(T-t'+1)}(d_{t'} - \mu_{t'}) - \frac{1}{(T-t+1)} \sum_{i=t+1}^{T} (\mu_{i|d_{1:t}} - \mu_i)$$
$$+ \sum_{t'=1}^{t=1} \sum_{i=t'+1}^{T} \frac{1}{(T-t')(T-t'+1)} (\mu_{i|d_{1:t'}} - \mu_i)$$

We now solve the decentralized case. The battery profit is

$$\Pi = \sum_{t=1}^{T} \lambda_t^{DA} z_t^{DA} + \mathbb{E}\left[\sum_{t=1}^{T} \lambda_t^{RT} z_t^{RT}\right]$$
$$= \sum_{t=1}^{T-1} (\lambda_t^{DA} - \lambda_T^{DA}) z_t^{DA} + \mathbb{E}\left[\sum_{t=1}^{T-1} (\lambda_t^{RT} - \lambda_T^{RT}) z_t^{RT}\right]$$
$$= \beta \sum_{t=1}^{T-1} \left(\mu_t - \mu_T - 2z_t^{DA} - \sum_{k\neq t} z_k^{DA}\right) z_t^{DA} + \beta \sum_{t=1}^{T-1} \mathbb{E}\left[\left(\mu_t - \mu_T - 2z_t^{DA} - \sum_{k\neq t} z_k^{DA} + \frac{(D_t - \mu_t) - (D_T - \mu_T) -}{k_f}\right.\right.$$

For a given k, consider the derivative of the profit w.r.t $z_k^{RT}(d_{1:k})$. We get

$$\sum_{t\neq k} \left(-\frac{1}{k_f}\right) \mathbb{E}[z_t^{RT} | d_{1:k}] + \Big(\mu_k - \mu_T - 2z_k^{DA} - \sum_{t\neq k} z_t^{DA}$$
$$+ \frac{1}{k_f}\Big((d_k - \mu_k) - (\mu_{T|d_{1:k}} - \mu_T) - \sum_{t\neq k} \mathbb{E}[z_t^{RT} | d_{1:k}] - 4z_k^{RT}(d_{1:k})\Big)\Big) = 0$$

where, of course, if $t \leq k$, then $\mathbb{E}[z_t^{RT} | d_{1:k}] = z_t^{RT}(d_{1:t})$.

or

$$+k_f \left(\mu_k - \mu_T - 2z_k^{DA} - \sum_{t\neq k} z_t^{DA}\right) + (d_k - \mu_k) - (\mu_{T|d_{1:k}} - \mu_T) - 2\sum_{t\neq k} \mathbb{E}[z_t^{RT}] - 4z_k^{RT} = 0$$

We will first take the expectations to eliminate all randomness (and solve for the DA variables first):

We have

$$\frac{\Pi}{\beta} = \sum_t \left(\mu_t - \mu_T - 2z_t^{DA} - \sum_{t' \neq t} z_{t'}^{DA} \right) z_t^{DA}$$

$$+ \sum_t \mathbb{E}\left[\left(\mu_t - \mu_T - 2z_t^{DA} - \sum_{t' \neq t} z_{t'}^{DA} + \frac{(D_t - \mu_t) - (D_T - \mu_T) - 2z_t^{RT} - \sum_{t' \neq t} z_{t'}^{RT}}{k_f} \right) z_t^{RT} \right]$$

Derivative w.r.t. $z_k^{RT}(d_{1:k})$ gives

$$\mathbb{E}\left[\sum_{t' \neq k} \left(-\frac{1}{k_f} \right) z_{t'}^{RT} + \left(\mu_k - \mu_T - 2z_k^{DA} - \sum_{t' \neq k} z_{t'}^{DA} + \frac{(D_k - \mu_k) - (D_T - \mu_T) - 4z_k^{RT} - \sum_{t' \neq k} z_{t'}^{RT}}{k_f} \right) \middle| d_{1:k} \right]$$

or

$$k_f(\mu_k - \mu_T) - k_f z_k^{DA} - k_f \sum_t z_t^{DA} - 2\sum_t \mathbb{E}[z_t^{RT} | d_{1:k}] - 2z_k^{RT} + (d_k - \mu_k) - (\mu_{T|d_{1:k}} - \mu_T) = 0$$

(Without further specification, $\sum_t$ means sum over $t = 1, \ldots, T-1$.) Of course, when $t \leq k$, we have $\mathbb{E}[z_t^{RT} | d_{1:k}] = z_t^{RT}(d_{1:t})$.

We will use this equation later to solve for individual $z_k^{RT}(d_{1:k})$. For now, we take the expectation over all randomness to get

$$k_f(\mu_k - \mu_T) - k_f z_k^{DA} - k_f \sum_t z_t^{DA} - 2\sum_t \mathbb{E}[z_t^{RT}] - 2\mathbb{E}[z_k^{RT}] = 0$$

We want to calculate $\sum_t \mathbb{E}[z_t^{RT}]$. Summing the above for all $k \in [T-1]$ gives

$$k_f \sum_t (\mu_t - \mu_T) - k_f T \sum_t z_t^{DA} - 2T \sum_t \mathbb{E}[z_t^{RT}] = 0$$

So

$$\sum_t \mathbb{E}[z_t^{RT}] = -\frac{k_f}{2} \sum_t z_t^{DA} + \frac{k_f}{2T} \sum_t (\mu_t - \mu_T)$$

Substituting this back in gives

$$\mathbb{E}[z_k^{RT}] = -\frac{k_f}{2} z_k^{DA} + \frac{k_f}{2}(\mu_k - \mu_T) - \frac{k_f}{2T}\sum_t (\mu_t - \mu_T)$$

Now take the derivative w.r.t. z_k^{DA}:

$$\sum_{t \neq k} (-1) z_t^{DA} + \left(\mu_k - \mu_T - 4z_k^{DA} - \sum_{t' \neq k} z_{t'}^{DA}\right) + \sum_{t \neq k} (-1)\mathbb{E}[z_t^{RT}] + (-2)\mathbb{E}[z_k^{RT}] = 0$$

or

$$-2z_k^{DA} - 2\sum_t z_t^{DA} + (\mu_k - \mu_T) - \mathbb{E}[z_k^{RT}] - \sum_t \mathbb{E}[z_t^{RT}] = 0$$

Substituting the value of $\mathbb{E}[z_k^{RT}]$ and $\sum_t \mathbb{E}[z_t^{RT}]$ gives

$$z_k^{DA} = -\sum_t z_t^{DA} + \frac{(2-k_f)}{(4-k_f)}(\mu_k - \mu_T)$$

Summing over $k \in [T-1]$ gives

$$\sum_t z_t^{DA} = \frac{(2-k_f)}{(4-k_f)} \frac{1}{T} \sum_t (\mu_t - \mu_T)$$

Therefore,

$$z_k^{DA} = \frac{(2-k_f)}{(4-k_f)} \left((\mu_k - \mu_T) - \frac{1}{T}\sum_t (\mu_t - \mu_T) \right) = \frac{(2-k_f)}{(4-k_f)}(\mu_k - \bar{\mu})$$

where $\bar{\mu} = (\mu_1 + \cdots + \mu_{T-1} + \mu_T)/T$. Substituting this into the $\mathbb{E}[z_k^{RT}]$ expression gives

$$\mathbb{E}[z_k^{RT}] = \frac{k_f}{(4-k_f)} \left((\mu_k - \mu_T) - \frac{1}{T}\sum_t (\mu_t - \mu_T) \right) = \frac{k_f}{(4-k_f)}(\mu_k - \bar{\mu})$$

Now we will solve for each z_k^{RT}.

Recall

$$k_f(\mu_k - \mu_T) - k_f z_k^{DA} - k_f \sum_t z_t^{DA} - 2\sum_t \mathbb{E}[z_t^{RT}|d_{1:k}] - 2z_k^{RT} + (d_k - \mu_k) - (\mu_{T|d_{1:k}} - \mu_T) = 0$$

Substituting the z_k^{DA} expression gives

$$\frac{2k_f}{(4-k_f)}(\mu_k - \mu_T) - 2\sum_{t=1}^{k-1} z_t^{RT}(d_{1:t}) - 4z_k^{RT}(d_{1:k}) - 2\sum_{t=k+1}^{T-1} \mathbb{E}[z_t^{RT}|d_{1:k}] + (d_k - \mu_k) - (\mu_{T|d_{1:k}} - \mu_T) = 0 \tag{18}$$

This equation will be used a lot: (18)

Fix a t, $1 \le t \le T-1$. We will solve for $z_t^{RT}(d_{1:t})$ in terms of $d_{1:t}$.

The equation with $k = t$ is

$$\frac{2k_f}{(4-k_f)}(\mu_t - \mu_T) - 2\sum_{t'=1}^{t-1} z_{t'}^{RT} - 4z_t^{RT} - 2\sum_{t'=t+1}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}] + (d_t - \mu_t) - (\mu_{T|d_{1:t}} - \mu_T) = 0$$

Now take the equation with $t+1 \le k \le T-1$ and take expectation over $D_{(t+1):k}$ (so the equation depends only on $d_{1:t}$:

$$\frac{2k_f}{(4-k_f)}(\mu_k - \mu_T) - 2\sum_{t'=1}^{t-1} z_{t'}^{RT} - 2z_t^{RT} - 2\sum_{t'=t+1}^{T-1} (1 + \mathbf{1}(t' = k))\mathbb{E}[z_{t'}^{RT}|d_{1:t}] + (\mu_{k|d_{1:t}} - \mu_k) - (\mu_{T|d_{1:t}} - \mu_T) = 0$$

Summing these equations for all $t \le k \le T-1$ gives

$$\frac{2k_f}{(4-k_f)}\sum_{k=t}^{T-1}(\mu_k - \mu_T) - 2(T-t)\sum_{t'=1}^{t-1} z_{t'}^{RT} - 2(T-t+1)\sum_{t'=t}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}] - (T-t)(\mu_{T|d_{1:t}} - \mu_T)$$
$$+(d_t - \mu_t) + \sum_{k=t+1}^{T-1}(\mu_{k|d_{1:t}} - \mu_k) = 0$$

Therefore,

$$\sum_{t'=t}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}] = \frac{k_f}{(T-t+1)(4-k_f)} \sum_{k=t}^{T-1} (\mu_k - \mu_T) - \frac{(T-t)}{(T-t+1)} \sum_{t'=1}^{t-1} z_{t'}^{RT}$$
$$- \frac{(T-t)}{2(T-t+1)} (\mu_{T|d_{1:t}} - \mu_T) + \frac{1}{2(T-t+1)} (d_t - \mu_t) + \frac{1}{2(T-t+1)} \sum_{k=t+1}^{T-1} (\mu_{k|d_{1:t}} - \mu_k)$$

The equation with $k = t$ says:

$$\frac{2k_f}{(4-k_f)} (\mu_t - \mu_T) - 2 \sum_{t'=1}^{t-1} z_{t'}^{RT} - 2z_t^{RT} - 2 \sum_{t'=t}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}] + (d_t - \mu_t) - (\mu_{T|d_{1:t}} - \mu_T) = 0$$

Solving for z_t^{RT} gives

$$z_t^{RT} = \frac{k_f}{(4-k_f)} (\mu_t - \mu_T) - \sum_{t'=1}^{t-1} z_{t'}^{RT} - \sum_{t'=t}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}] + \frac{1}{2}(d_t - \mu_t) - \frac{1}{2}(\mu_{T|d_{1:t}} - \mu_T)$$

Substituting the expression for $\sum_{t'=t}^{T-1} \mathbb{E}[z_{t'}^{RT}|d_{1:t}]$ gives

$$z_t^{RT} = \frac{k_f}{(4-k_f)} \left(\mu_t - \frac{1}{(T-t+1)} \sum_{k=t}^{T} \mu_k \right) - \frac{1}{T-t+1} \sum_{t'=1}^{t-1} z_{t'}^{RT} + \frac{(T-t)}{2(T-t+1)} (d_t - \mu_t)$$
$$- \frac{1}{2(T-t+1)} (\mu_{T|d_{1:t}} - \mu_T) - \frac{1}{2(T-t+1)} \sum_{k=t+1}^{T-1} (\mu_{k|d_{1:t}} - \mu_k)$$

This gives a recursion that gives z_t^{RT} in terms of $z_{t'}^{RT}$ for $1 \le t' \le t-1$. The recursion has the form

$$z_t^{RT} = a_t - \frac{1}{(T-t+1)} \sum_{t'=1}^{t-1} z_{t'}^{RT}$$

This gives

$$
\begin{aligned}
(T-t+1)z_t^{RT} &= (T-t+1)a_t - \sum_{t'=1}^{t-1} z_{t'}^{RT} \\
(T-t)z_{t+1}^{RT} - (T-t+1)z_t^{RT} &= (T-t)a_{t+1} - (T-1+t)a_t - z_t^{RT} \\
z_{t+1}^{RT} - z_t^{RT} &= a_{t+1} - a_t - \frac{1}{(T-t)}a_t
\end{aligned}
$$

Summing these equations from 1 to $t-1$ gives

$$
z_t^{RT} - z_1^{RT} = a_t - a_1 - \sum_{t'=1}^{t-1} \frac{1}{(T-t')}a_{t'}
$$

We also have $z_1^{RT} = a_1$, so

$$
z_t^{RT} = a_t - \sum_{t'=1}^{t-1} \frac{1}{(T-t')}a_{t'}
$$

Note that we have

$$
a_t = \frac{k_f}{(4-k_f)}\left(\mu_t - \frac{1}{(T-t+1)}\sum_{k=t}^{T}\mu_k\right) + \frac{(T-t)}{2(T-t+1)}(d_t-\mu_t) - \frac{1}{2(T-t+1)}\sum_{k=t+1}^{T}(\mu_{k|d_{1:t}} - \mu_k)
$$

We have

$$
\begin{aligned}
z_1^{RT} &= a_1 \\
&= \frac{k_f}{(4-k_f)}(\mu_1 - \bar{\mu}) + \frac{(T-1)}{2T}(d_1-\mu_1) - \frac{1}{2T}\sum_{k=2}^{T}(\mu_{k|d_1} - \mu_k)
\end{aligned}
$$

and

$$
\begin{aligned}
z_2^{RT} &= a_1 - \frac{1}{(T-1)} z_1^{RT} \\
&= \frac{k_f}{(4-k_f)}\left(\mu_2 - \frac{1}{(T-1)}\sum_{k=2}^{T}\mu_k\right) + \frac{(T-2)}{2(T-1)}(d_2-\mu_2) - \frac{1}{2(T-1)}\sum_{k=3}^{T}(\mu_{k|d_{1:2}} - \mu_k) \\
&\quad - \frac{1}{(T-1)}\left(\frac{k_f}{(4-k_f)}(\mu_1-\bar{\mu}) + \frac{(T-1)}{2T}(d_1-\mu_1) - \frac{1}{2T}\sum_{k=2}^{T}(\mu_{k|d_1}-\mu_k)\right) \\
&= \frac{k_f}{(4-k_f)}(\mu_2-\bar{\mu}) + \frac{(T-2)}{2(T-1)}(d_2-\mu_2) - \frac{1}{2(T-1)}\sum_{k=3}^{T}(\mu_{k|d_{1:2}}-\mu_k) - \frac{1}{2T}(d_1-\mu_1) + \frac{1}{2T(T-1)}\sum_{k=2}^{T}(\mu_{k|d_1}-\mu_k)
\end{aligned}
$$

We substitute the expressions of a_t and $a_{t'}$ in the z_t^{RT} expression. The "constant" term is $k_f/(4-k_f)$ times

$$
\begin{aligned}
&\mu_t - \frac{1}{(T-t+1)}\left(T\bar{\mu} - \sum_{k=1}^{t-1}\mu_k\right) \\
&- \sum_{t'=1}^{t-1}\frac{1}{(T-t')}\left(\mu_{t'} - \frac{1}{(T-t'+1)}\left(T\bar{\mu} - \sum_{k=1}^{t'-1}\mu_k\right)\right) \\
&= \mu_t + \left(-\frac{T}{(T-t+1)} + \sum_{t'=1}^{t-1}\frac{T}{(T-t')(T-t'+1)}\right)\bar{\mu} \\
&+ \frac{1}{(T-t+1)}\sum_{k=1}^{t-1}\mu_k - \sum_{t'=1}^{t-1}\frac{1}{(T-t')}\mu_{t'} - \sum_{t'=1}^{t-1}\sum_{k=1}^{t'-1}\frac{1}{(T-t')(T-t'+1)}\mu_k
\end{aligned}
$$

The coefficient of $\bar{\mu}$ is

$$
-\frac{T}{(T-t+1)} + \sum_{t'=1}^{t-1}\left(\frac{T}{T-t'} - \frac{T}{T-t'+1}\right) = -\frac{T}{(T-t+1)} + \left(\frac{T}{T-t+1} - \frac{T}{T}\right) = -1
$$

We swap the order of the double summation (over t' and k, to over k and t') to get

$$
\sum_{k=1}^{t-1}\sum_{t'=k+1}^{t-1}\left(\frac{1}{(T-t')} - \frac{1}{(T-t'+1)}\right)\mu_k = \sum_{k=1}^{t-1}\left(\frac{1}{T-t+1} - \frac{1}{T-k}\right)\mu_k
$$

Therefore, all the terms cancel out to $\mu_t - \bar{\mu}$ and the constant term is

$$\frac{k_f}{(4-k_f)}(\mu_t - \bar{\mu})$$

The rest of z_t^{RT} is

$$\begin{aligned} &\frac{(T-t)}{2(T-t+1)}(d_t - \mu_t) - \frac{1}{2(T-t+1)}\sum_{k=t+1}^{T}(\mu_{k|d_{1:t}} - \mu_k) \\ -&\sum_{t'=1}^{t-1}\frac{1}{(T-t')}\left(\frac{(T-t')}{2(T-t'+1)}(d_{t'} - \mu_{t'}) - \frac{1}{2(T-t'+1)}\sum_{k=t'+1}^{T}(\mu_{k|d_{1:t'}} - \mu_k)\right) \end{aligned}$$

Therefore, we have

$$\begin{aligned} z_t^{RT}(d_{1:t}) = &\frac{k_f}{(4-k_f)}(\mu_t - \bar{\mu}) + \frac{(T-t)}{2(T-t+1)}(d_t - \mu_t) - \sum_{t'=1}^{t-1}\frac{1}{2(T-t'+1)}(d_{t'} - \mu_{t'}) \\ &- \frac{1}{2(T-t+1)}\sum_{k=t+1}^{T}(\mu_{k|d_{1:t}} - \mu_k) + \sum_{t'=1}^{t-1}\sum_{k=t'+1}^{T}\frac{1}{2(T-t')(T-t'+1)}(\mu_{k|d_{1:t'}} - \mu_k) \end{aligned}$$

We will now calculate Cost(CN) and Cost(DCN) when the demands in all periods are independent.

We have

$$\text{Cost} = \alpha T\bar{\mu} + \frac{\beta}{2}\text{Cost'}$$

with

$$\text{Cost'} = (1-k_f)\sum_{t=1}^{T}(\mu_t - z_t^{DA})^2 + k_f\sum_{t=1}^{T}\mathbb{E}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right)^2$$

First, we have

$$
\begin{aligned}
\text{Cost'(NB)} &= (1-k_f)\sum_{t=1}^{T}(\mu_t - 0)^2 + k_f \sum_{t=1}^{T} \mathbb{E}\left(\mu_t - 0 + \frac{D_t - \mu_t - 0}{k_f}\right)^2 \\
&= (1-k_f)\sum_{t=1}^{T}\mu_t^2 + k_f \sum_{t=1}^{T}\left(\mu_t^2 + \frac{\sigma_t^2}{k_f^2}\right) \\
&= \sum_{t=1}^{T}\mu_t^2 + \sum_{t=1}^{T}\frac{\sigma_t^2}{k_f}
\end{aligned}
$$

Now we compute Cost'(CN). We have $\mu_t - z_t^{DA} = \bar{\mu}$ for every $1 \leq t \leq T$. For $1 \leq t \leq T-1$, we have

$$
D_t - \mu_t - z_t^{RT} = \sum_{t'=1}^{t} \frac{1}{(T-t'+1)}(D_{t'} - \mu_{t'})
$$

For $t = T$, we have

$$
z_T^{RT} = -\sum_{t=1}^{T-1} z_t^{RT} = \sum_{t=1}^{T-1} \frac{1}{T-t+1}(D_t - \mu_t)
$$

so

$$
D_T - \mu_T - z_T^{RT} = (D_T - \mu_T) - \sum_{t=1}^{T-1} \frac{1}{T-t+1}(D_t - \mu_t)
$$

For both $1 \leq t \leq T-1$ and $t = T$, we have

$$
\mathbb{E}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right)^2 = (\bar{\mu})^2 + \sum_{t'=1}^{t} \frac{1}{(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f^2}
$$

Now we note that

Therefore,

$$
\begin{aligned}
\text{Cost'(CN)} &= (1-k_f)T(\bar{\mu})^2 + k_f\left(T(\bar{\mu})^2 + \sum_{t'=1}^{T} \frac{1}{(T-t'+1)}\frac{\sigma_{t'}^2}{k_f^2}\right) \\
&= T(\bar{\mu})^2 + \sum_{t=1}^{T} \frac{1}{(T-t+1)}\frac{\sigma_t^2}{k_f}
\end{aligned}
$$

Now we compute Cost'(DCN) We have

$$\mu_t - z_t^{DA} = \mu_t - \frac{(2-k_f)}{(4-k_f)}(\mu_t - \bar{\mu}) = \bar{\mu} + \frac{2}{(4-k_f)}(\mu_t - \bar{\mu})$$

For $1 \leq t \leq T-1$, we have

$$D_t - \mu_t - z_t^{RT} = -\frac{k_f}{(4-k_f)}(\mu_t - \bar{\mu}) + \frac{(T-t+2)}{2(T-t+1)}(D_t - \mu_t) + \sum_{t'=1}^{t-1} \frac{1}{2(T-t'+1)}(D_{t'} - \mu_{t'})$$

so

$$\mathbb{E}\left(\mu_t - z_t^{DA} + \frac{D_t - \mu_t - z_t^{RT}}{k_f}\right)^2 = \left(\bar{\mu} + \frac{3}{(4-k_f)}(\mu_t - \bar{\mu})\right)^2 + \left(\frac{(T-t+2)}{2(T-t+1)}\right)\frac{\sigma_t^2}{k_f^2} + \sum_{t'=1}^{T-1} \frac{1}{4(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f^2}$$

Now we compute

$$z_T^{RT} = -\sum_{t=1}^{T-1} z_t^{RT} = \frac{k_f}{(4-k_f)}(\mu_T - \bar{\mu}) - \sum_{t=1}^{T-1} \frac{1}{2(T-t+1)}(D_t - \mu_t)$$

so

$$D_T - \mu_T - z_T^{RT} = \frac{k_f}{(4-k_f)}(\mu_T - \bar{\mu}) + (D_T - \mu_T) + \sum_{t=1}^{T-1} \frac{1}{2(T-t+1)}(D_t - \mu_t)$$

$$\mathbb{E}\left(\mu_T - z_T^{DA} + \frac{D_T - \mu_T - z_T^{RT}}{k_f}\right)^2 = \left(\bar{\mu} + \frac{1}{(4-k_f)}(\mu_T - \bar{\mu})\right)^2 + \frac{\sigma_T^2}{k_f^2} + \sum_{t=1}^{T-1} \frac{1}{4(T-t+1)^2}\frac{\sigma_t^2}{k_f^2}$$

So the equation for $1 \leq t \leq T-1$ also holds for $t = T$ as well. Therefore,

$$\begin{aligned}
\text{Cost'(DCN)} = (1-k_f) &\sum_{t=1}^{T} \left(\bar{\mu} + \frac{2}{(4-k_f)}(\mu_t - \bar{\mu})\right)^2 \\
&+ k_f \sum_{t=1}^{T} \left(\frac{1}{(4-k_f)^2}(\mu_t - \bar{\mu})^2 + \frac{(T-t+2)^2}{4(T-t+1)^2}\frac{\sigma_t^2}{k_f} + \sum_{t'=1}^{t-1} \frac{1}{4(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f}\right)
\end{aligned}$$

Now we note that

$$\sum_{t=1}^{T}\sum_{t'=1}^{t-1}\frac{1}{4(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f}=\sum_{t'=1}^{T-1}\sum_{t=t'+1}^{T}\frac{1}{4(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f}=\sum_{t'=1}^{T-1}\frac{(T-t')}{4(T-t'+1)^2}\frac{\sigma_{t'}^2}{k_f}$$

and collecting the coefficients of σ_t^2/k_f:

$$\frac{(T-t+2)^2}{4(T-t+1)^2}+\frac{(T-t)}{4(T-t+1)^2}=\frac{(T-t+4)}{4(T-t+1)}$$

for $1 \leq t \leq T-1$ and $(T-T+2)^2/(4(T-T+1)^2)=1$, which is also equal to the above for $t=T$. Therefore,

$$\text{Cost'(DCN)}=T\bar{\mu}^2+\frac{4-3k_f}{(4-k_f)^2}\sum_{t=1}^{T}(\mu_t-\bar{\mu})^2+\sum_{t=1}^{T}\frac{(T-t+4)}{4(T-t+1)}\frac{\sigma_t^2}{k_f}$$

Now, we note that

$$\sum_{t=1}^{T}\mu_t^2-T\bar{\mu}^2=\sum_{t=1}^{T}(\mu_t-\bar{\mu})^2$$

Therefore,

$$\begin{aligned}\text{Cost'(NB)}-\text{Cost'(CN)}&=\sum_{t=1}^{T}(\mu_t-\bar{\mu})^2+\sum_{t=1}^{T}\frac{(T-t)}{(T-t+1)}\frac{\sigma_t^2}{k_f}\\ \text{Cost'(NB)}-\text{Cost'(DCN)}&=\frac{12-5k_f+k_f^2}{(4-k_f)^2}\sum_{t=1}^{T}(\mu_t-\bar{\mu})^2+\sum_{t=1}^{T}\frac{3(T-t)}{4(T-t+1)}\frac{\sigma_t^2}{k_f}\end{aligned}$$

Note that $\frac{12-5k_f+k_f^2}{(4-k_f)^2}\in\left[\frac{9}{8},\frac{4}{3}\right]$. Therefore,

$$\text{PoA}=\frac{\text{Cost(NB)}-\text{Cost(CN)}}{\text{Cost(NB)}-\text{Cost(DCN)}}=\frac{\text{Cost'(NB)}-\text{Cost'(CN)}}{\text{Cost'(NB)}-\text{Cost'(DCN)}}\in\left[\frac{9}{8},\frac{4}{3}\right].$$

We now calculate these quantities for a general multivariate normal distribution with $T=3$ periods:

$$(D_1, D_2, D_3) \sim N\left(\begin{pmatrix}\mu_1\\ \mu_2\\ \mu_3\end{pmatrix}, \begin{pmatrix}\sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3\\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3\\ \rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2\end{pmatrix}\right).$$

We have

$$\begin{aligned}
\text{Cost'(NB)} - \text{Cost'(CN)} &= \sum_{t=1}^{3}(\mu_t - \bar{\mu})^2 + \frac{2\sigma_1^2}{3k_f} + \frac{\sigma_2^2}{2k_f} - \frac{2\rho_{12}\sigma_1\sigma_2}{3k_f} - \frac{2\rho_{13}\sigma_1\sigma_3}{3k_f} - \frac{\rho_{23}\sigma_2\sigma_3}{k_f}\\
\text{Cost'(NB)} - \text{Cost'(DCN)} &= \frac{12 - 5k_f + k_f^2}{(4-k_f)^2}\sum_{t=1}^{3}(\mu_t - \bar{\mu})^2\\
&\quad + \frac{\sigma_1^2}{2k_f} + \frac{(27 + 2\rho_{12}^2)\sigma_2^2}{72k_f} + \frac{5\rho_{13}^2\sigma_3^2}{18k_f} - \frac{\rho_{12}\sigma_1\sigma_2}{2k_f} - \frac{\rho_{13}\sigma_1\sigma_3}{2k_f} + \frac{(22\rho_{12}\rho_{13} - 36\rho_{23})\sigma_2\sigma_3}{72k_f}
\end{aligned}$$

Therefore,

$$\begin{aligned}
&4(\text{Cost'(NB)} - \text{Cost'(DCN)}) - 3(\text{Cost'(NB)} - \text{Cost'(CN)})\\
&= \frac{k_f(4+k_f)}{(4-k_f)^2}\sum_{t=1}^{3}(\mu_t - \bar{\mu})^2 + \frac{\rho_{12}^2\sigma_2^2 + 10\rho_{13}^2\sigma_3^2 + (11\rho_{12}\rho_{13} + 9\rho_{23})\sigma_2\sigma_3}{9k_f}
\end{aligned}$$

We can see that if all correlations are positive $\rho_{12}, \rho_{13}, \rho_{23} \geq 0$, then the expression above will be ≥ 0, so we still have PoA $\leq 4/3$. However, the above expression is not always negative. The constraints on the covariance matrix is that the matrix must be positive semidefinite, that is $\rho_{12}, \rho_{13}, \rho_{23} \in [-1, 1]$ and $1 - \rho_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\rho_{12}\rho_{13}\rho_{23} \geq 0$. In fact, numerical optimization suggests that the maximum value of PoA is around 2, achieved at, for example, $\sigma_1 = \sigma_2 \downarrow 0, \sigma_3 = 1, \rho_{12} = \rho_{13} = 0, \rho_{23} = -1$. However, the parameter values where PoA is this high are expected to be quite pathological, and we believe that for reasonable parameter values, PoA should be much smaller than this, and is likely below 4/3.

□

Theorem 10 (Round-trip Inefficiency)**.** *Let the battery's round-trip efficiency be* $\eta \in (0, 1]$ *such that the battery discharges in period 1 (peak) and charges in period 2 (off-peak).*

The centralized battery discharge decisions are given by

$$z_1^{DA} = \frac{\eta^2}{1+\eta^2}\mu_1 - \frac{\eta}{1+\eta^2}\mu_2$$
$$z_1^{RT}(D_1) = \frac{\eta^2}{1+\eta^2}(D_1-\mu_1) - \frac{\eta}{1+\eta^2}(\mu_{2|D_1}-\mu_2)$$

The decentralized battery discharge decisions are given by, for each period t,

$$z_1^{DA} = \frac{(2-k_f)}{(4-k_f)}\left(\frac{\eta^2}{1+\eta^2}\mu_1 - \frac{\eta}{1+\eta^2}\mu_2\right)$$
$$z_1^{RT}(D_1) = \frac{k_f}{(4-k_f)}\left(\frac{\eta^2}{1+\eta^2}\mu_1 - \frac{\eta}{1+\eta^2}\mu_2\right)\frac{\eta^2}{2(1+\eta^2)}(D_1-\mu_1) - \frac{\eta}{2(1+\eta^2)}(\mu_{2|D_1}-\mu_2)$$

If we further assume that $(D_1, D_2) \sim \pi$ is jointly multivariate normal, then the bounds $9/8 \leq \mathrm{PoA} \leq 4/3$ always hold.

Proof. Proof of Theorem 10.

The calculations are similar to those in the proof of Theorem CN and Theorem DCN, but with $z_2^{DA} = -z_1^{DA}/\eta$ and $z_2^{RT} = -z_1^{RT}/\eta$ instead.

We first compute the optimal strategy in the centralized case (CN).

Generation cost is

$$\begin{aligned}
&\alpha(\mu_1+\mu_2)\\
&+k_s\left[\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA}\right)^2 + \left(\mu_2 + \frac{z_1^{DA}}{\eta}\right)^2\right]\right]\\
&+k_f\mathbb{E}\left\{\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1-\mu_1-z_1^{RT}(D_1)}{k_f}\right)^2 + \left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2-\mu_2+z_1^{RT}(D_1)/\eta}{k_f}\right)^2\right]\right\}
\end{aligned}$$

For each fixed $D_1 = d_1$, we take the derivative w.r.t $z_1^{RT}(d_1)$:

$$\mathbb{E}_{D_2\sim\pi(\cdot|D_1=d_1)}\left\{-\frac{1}{k_f}\left(\mu_1 - z_1^{DA} + \frac{d_1-\mu_1-z_1^{RT}(d_1)}{k_f}\right) + \frac{1}{\eta k_f}\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2-\mu_2+z_1^{RT}(d_1)/\eta}{k_f}\right)\right\}$$

We now evaluate the expectations over $D_2 \sim \pi(\cdot|d_1)$:

$$-\frac{1}{k_f}\left(\mu_1 - z_1^{DA} + \frac{d_1 - \mu_1 - z_1^{RT}(d_1)}{k_f}\right) + \frac{1}{\eta k_f}\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{\mu_{2|d_1} - \mu_2 + z_1^{RT}(d_1)/\eta}{k_f}\right) = 0$$

The above simplifies to

$$\eta(\mu_{2|d_1} - \mu_2) - \eta^2(d_1 - \mu_1) - \eta^2 k_f \mu_1 + \eta k_f \mu_2 + (1+\eta^2)k_f z_1^{DA} + (1+\eta^2) z_1^{RT}(d_1) = 0$$

or

$$z_1^{RT}(d_1) = -k_f z_1^{DA} + \frac{-\eta(\mu_{2|d_1} - \mu_2) + \eta^2(d_1 - \mu_1) + \eta^2 k_f \mu_1 - \eta k_f \mu_2}{1+\eta^2}$$

Therefore,

$$\mathbb{E}[\bar{z}_1^{RT}(D_1)] = -k_f \bar{z}_1^{DA} + \frac{\eta k_f(\eta\mu_1 - \mu_2)}{1+\eta^2}$$

We take the derivative w.r.t z_1^{DA}:

$$\begin{aligned} &(1-k_f)\left[-(\mu_1 - z_1^{DA}) + \frac{1}{\eta}\left(\mu_2 + \frac{z_1^{DA}}{\eta}\right)\right] \\ &+k_f \mathbb{E}\left[-\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right) + \frac{1}{\eta}\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)/\eta}{k_f}\right)\right] \end{aligned}$$

or

$$\begin{aligned} &(1-k_f)\left[-\eta(\eta\mu_1 - \mu_2) + (1+\eta^2) z_1^{DA}\right] \\ &+k_f\left[-\eta(\eta\mu_1 - \mu_2) + (1+\eta^2) z_1^{DA} + \frac{1}{k_f}(1+\eta^2)\mathbb{E}[z_1^{RT}(D_1)]\right] \end{aligned}$$

or

$$-\eta(\eta\mu_1 - \mu_2) + (1+\eta^2)\bar{z}_1^{DA} + (1+\eta^2)\mathbb{E}[\bar{z}_1^{RT}(D_1)] = 0$$

Substituting the expression for $\mathbb{E}[\bar{z}_1^{RT}(D_1)]$ gives

$$-\eta(\eta\mu_1 - \mu_2) + (1+\eta^2)\bar{z}_1^{DA} - k_f(1+\eta^2)\bar{z}_1^{DA} + \eta k_f(\eta\mu_1 - \mu_2) = 0$$

or

$$\bar{z}_1^{DA} = \frac{\eta(\eta\mu_1 - \mu_2)}{1+\eta^2}$$

This gives

$$\bar{z}_1^{RT}(d_1) = \frac{\eta^2}{1+\eta^2}(d_1 - \mu_1) - \frac{\eta}{1+\eta^2}(\mu_{2|d_1} - \mu_2)$$

We now compute the generation cost Cost(CN). We have

$$(\mu_1 - z_1^{DA})^2 + \left(\mu_2 + \frac{z_1^{DA}}{\eta}\right)^2 = \frac{(\mu_1 + \eta\mu_2)^2}{1+\eta^2}$$

and

$$\begin{aligned}
&\mathbb{E}\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\mathbb{E}\left(k_f(\mu_1 + \eta\mu_2) + (D_1 - \mu_1) + \eta(\mu_{2|D_1} - \mu_2)\right)^2 \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\left(k_f^2(\mu_1 + \eta\mu_2)^2 + \sigma_1^2 + \eta^2\rho_s^2\sigma_2^2 + 2\eta\rho\sigma_1\sigma_2\right)
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)/\eta}{k_f}\right)^2 \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\mathbb{E}\left(k_f\eta(\mu_1 + \eta\mu_2) + \eta(D_1 - \mu_1) + (1+\eta^2)(D_2 - \mu_2) - (\mu_{2|D_1} - \mu_2)\right)^2 \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\left(k_f^2\eta^2(\mu_1 + \eta\mu_2)^2 + \eta^2\sigma_1^2 + (1+\eta^2)^2\sigma_2^2 + \rho_s^2\sigma_2^2 + 2\eta(1+\eta^2)\rho\sigma_1\sigma_2 - 2\eta\rho\sigma_1\sigma_2 - 2(1+\eta^2)\rho_s^2\sigma_2^2\right) \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\left(k_f^2\eta^2(\mu_1 + \eta\mu_2)^2 + \eta^2\sigma_1^2 + ((1+\eta^2)^2 - (1+2\eta^2)\rho_s^2)\sigma_2^2 + 2\eta^3\rho\sigma_1\sigma_2\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathbb{E}\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 + \mathbb{E}\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)/\eta}{k_f}\right)^2 \\
&= \frac{1}{k_f^2(1+\eta^2)^2}\Big(k_f^2(1+\eta^2)(\mu_1+\eta\mu_2)^2 + (1+\eta^2)\sigma_1^2 + (1+\eta^2)(1+\eta^2-\rho_s^2)\sigma_2^2 + 2\eta(1+\eta^2)\rho\sigma_1\sigma_2\Big) \\
&= \frac{1}{k_f^2(1+\eta^2)}\Big(k_f^2(\mu_1+\eta\mu_2)^2 + \sigma_1^2 + (1+\eta^2-\rho_s^2)\sigma_2^2 + 2\eta\rho\sigma_1\sigma_2\Big)
\end{aligned}$$

Therefore, the generation cost is

$$\begin{aligned}
&\alpha(\mu_1+\mu_2) \\
&+ k_s\left[\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA}\right)^2 + \left(\mu_2 + \frac{z_1^{DA}}{\eta}\right)^2\right]\right] \\
&+ k_f\mathbb{E}\left\{\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 + \left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)/\eta}{k_f}\right)^2\right]\right\}
\end{aligned}$$

so

$$\text{Cost(CN)} = \alpha(\mu_1+\mu_2) + \beta\frac{(\mu_1+\eta\mu_2)^2}{2(1+\eta^2)} + \frac{\beta}{2(1+\eta^2)k_f}\left[\sigma_1^2 + (1+\eta^2-\rho_s^2)\sigma_2^2 + 2\eta\rho\sigma_1\sigma_2\right]$$

We now consider the decentralized case.

The prices are given by

$$\begin{aligned}
\lambda_1^{DA} &= \alpha + \beta(\mu_1 - z_1^{DA}) \\
\lambda_2^{DA} &= \alpha + \beta\left(\mu_2 + \frac{z_1^{DA}}{\eta}\right) \\
\lambda_1^{RT} &= \alpha + \beta\left(\mu_1 - z_1^{DA} + \frac{d_1 - \mu_1 - z_1^{RT}}{k_f}\right) \\
\lambda_2^{RT} &= \alpha + \beta\left(\mu_2 + \frac{z_1^{DA}}{\eta} + \frac{d_2 - \mu_2 + z_1^{RT}/\eta}{k_f}\right)
\end{aligned}$$

Battery profit is

$$
\begin{aligned}
&\lambda_1^{DA} z_1^{DA} + \lambda_2^{DA} z_2^{DA} + \mathbb{E}[\lambda_1^{RT} z_1^{RT} + \lambda_2^{RT} z_2^{RT}] \\
&= \left(\lambda_1^{DA} - \frac{\lambda_2^{DA}}{\eta}\right) z_1^{DA} + \mathbb{E}\left[\left(\lambda_1^{RT} - \frac{\lambda_2^{RT}}{\eta}\right) z_1^{RT}\right] \\
&= \beta\left(\mu_1 - \frac{\mu_2}{\eta} - \frac{1+\eta^2}{\eta^2} z_1^{DA}\right) z_1^{DA} \\
&+ \beta\mathbb{E}\left[\left(\mu_1 - \frac{\mu_2}{\eta} - \frac{1+\eta^2}{\eta^2} z_1^{DA} + \frac{1}{k_f}\left(D_1 - \mu_1 - \frac{D_2-\mu_2}{\eta} - \frac{1+\eta^2}{\eta^2} z_1^{RT}(D_1)\right)\right) z_1^{RT}(D_1)\right]
\end{aligned}
$$

Taking the derivative with respect to $z_1^{RT}(d_1)$:

$$
\mu_1 - \frac{\mu_2}{\eta} - \frac{1+\eta^2}{\eta^2} z_1^{DA} + \frac{1}{k_f}\left(d_1 - \mu_1 - \frac{\mu_{2|d_1} - \mu_2}{\eta} - \frac{1+\eta^2}{\eta^2} 2 z_1^{RT}(d_1)\right) = 0
$$

or

$$
z_1^{RT}(d_1) = -\frac{k_f}{2} z_1^{DA} + \frac{\eta k_f}{2(1+\eta^2)}(\eta\mu_1 - \mu_2) + \frac{\eta^2}{2(1+\eta^2)}(d_1 - \mu_1) - \frac{\eta}{2(1+\eta^2)}(\mu_{2|d_1} - \mu_2)
$$

This gives

$$
\mathbb{E}[z_1^{RT}(D_1)] = -\frac{k_f}{2} z_1^{DA} + \frac{\eta k_f}{2(1+\eta^2)}(\eta\mu_1 - \mu_2)
$$

Taking the derivative with respect to z_1^{DA}

$$
\mu_1 - \frac{\mu_2}{\eta} - \frac{2(1+\eta^2)}{\eta^2} z_1^{DA} + \mathbb{E}\left[\left(-\frac{1+\eta^2}{\eta^2}\right) z_1^{RT}(D_1)\right] = 0
$$

Substituting the expression for $\mathbb{E}[z_1^{RT}(D_1)]$ gives

$$
z_1^{DA} = \frac{(2-k_f)}{(4-k_f)} \frac{\eta}{(1+\eta^2)}(\eta\mu_1 - \mu_2)
$$

This gives

$$
z_1^{RT}(d_1) = \frac{k_f}{(4-k_f)} \frac{\eta}{1+\eta^2}(\eta\mu_1 - \mu_2) + \frac{\eta^2}{2(1+\eta^2)}(d_1 - \mu_1) - \frac{\eta}{2(1+\eta^2)}(\mu_{2|d_1} - \mu_2)
$$

Substituting these into the generation cost expression and using Proposition 1 yields the generation cost

$$
\begin{aligned}
&\mathrm{Cost(DCN)} = \alpha(\mu_1 + \mu_2) \\
&+ \beta\left\{ \frac{16 + 4\eta^2 - (8 + 3\eta^2)k_f + k_f^2}{2(1+\eta^2)(4-k_f)^2}\mu_1^2 + \frac{4 + 16\eta^2 - (3+8\eta^2)k_f + \eta^2 k_f^2}{2(1+\eta^2)(4-k_f)^2}\mu_2^2 + \frac{\eta(12 - 5k_f + k_f^2)}{(1+\eta^2)(4-k_f)^2}\mu_1\mu_2 \right. \\
&\left. + \frac{(4+\eta^2)\sigma_1^2 + (4 + 4\eta^2 - 3\rho_s^2)\sigma_2^2 + 6\eta\rho\sigma_1\sigma_2}{8(1+\eta^2)k_f} \right\}
\end{aligned}
$$

The cost under a no-battery regime is exactly the same as before in Theorem 1 (no η is involved, because there is no charging and discharging):

$$
\mathrm{Cost(NB)} = \alpha(\mu_1 + \mu_2) + \beta\left(\frac{\mu_1^2 + \mu_2^2}{2} + \frac{\sigma_1^2 + \sigma_2^2}{2k_f}\right)
$$

Assuming $\rho_s = \rho$, we therefore have

$$
\begin{aligned}
\mathrm{Cost(NB)} - \mathrm{Cost(CN)} &= \beta\left(\frac{(\eta\mu_1 - \mu_2)^2}{2(1+\eta^2)} + \frac{(\eta\sigma_1 - \rho\sigma_2)^2}{2k_f(1+\eta^2)}\right) \\
\mathrm{Cost(NB)} - \mathrm{Cost(DCN)} &= \beta\left(\frac{(12 - 5k_f + k_f^2)}{(4-k_f)^2}\frac{(\eta\mu_1 - \mu_2)^2}{2(1+\eta^2)} + \frac{3(\eta\sigma_1 - \rho\sigma_2)^2}{8k_f(1+\eta^2)}\right)
\end{aligned}
$$

Because $\frac{(12-5k_f+k_f^2)}{(4-k_f)^2} \in \left[\frac{3}{4}, \frac{8}{9}\right]$ for $k_f \in [0,1]$, we have PoA $\in \left[\frac{9}{8}, \frac{4}{3}\right]$ as desired.

□

Theorem 11 (Non-Parallel Supply Curves)**.** *Here, we assume*

$$
\begin{aligned}
G_s^{-1}(x) &= \alpha_s + \beta_s x \\
G_f^{-1}(x) &= \alpha_f + \beta_f x
\end{aligned}
$$

(Note that if we want to match the previous assumption, we will have $\alpha_s = \alpha, \beta_s = \beta/k_s, \alpha_f = \alpha, \beta_f = \beta/k_f$ with $k_s = 1 - k_f$.) The centralized battery discharge decisions are given by

$$
\begin{aligned}
z_1^{DA} &= \frac{1}{2}(\mu_1 - \mu_2) \\
z_1^{RT}(D_1) &= \frac{1}{2}(D_1 - \mu_1) - \frac{1}{2}(\mu_{2|D_1} - \mu_2)
\end{aligned}
$$

The decentralized battery discharge decisions are given by

$$
\begin{aligned}
z_1^{DA} &= \frac{2\beta_f + \beta_s}{2(4\beta_f + 3\beta_s)}(\mu_1 - \mu_2) \\
z_1^{RT}(D_1) &= \frac{1}{4}(D_1 - \mu_1) - \frac{1}{4}(\mu_{2|D_1} - \mu_2) + \frac{\beta_s}{2(4\beta_f + 3\beta_s)}(\mu_1 - \mu_2)
\end{aligned}
$$

The system generation costs in each regime (no battery NB, centralized CN, decentralized DCN) are given by

$$
\begin{aligned}
\text{Cost(NB)} &= -\frac{(\alpha_f - \alpha_s)^2}{\beta_f + \beta_s} + \frac{(\alpha_f\beta_s + \alpha_s\beta_f)}{(\beta_s + \beta_f)}(\mu_1 + \mu_2) + \frac{\beta_f\beta_s}{2(\beta_f + \beta_s)}(\mu_1^2 + \mu_2^2) + \frac{\beta_f}{2}(\sigma_1^2 + \sigma_2^2) \\
\text{Cost(CN)} &= -\frac{(\alpha_f - \alpha_s)^2}{(\beta_s + \beta_f)} + \frac{(\alpha_f\beta_s + \alpha_s\beta_f)}{(\beta_s + \beta_f)}(\mu_1 + \mu_2) + \frac{\beta_s\beta_f}{4(\beta_s + \beta_f)}(\mu_1 + \mu_2)^2 + \frac{\beta_f}{4}(\sigma_1^2 + (2 - \rho_s^2)\sigma_2^2 + 2\rho\sigma_1\sigma_2) \\
\text{Cost(DCN)} &= -\frac{(\alpha_f - \alpha_s)^2}{\beta_f + \beta_s} + \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f}(\mu_1 + \mu_2) \\
&+ \frac{\beta_f\beta_s(20\beta_f^2 + 29\beta_f\beta_s + 10\beta_s^2)}{4(\beta_f + \beta_s)(4\beta_f + 3\beta_s)^2}(\mu_1^2 + \mu_2^2) + \frac{\beta_f\beta_s(12\beta_f^2 + 19\beta_f\beta_s + 8\beta_s^2)}{2(\beta_f + \beta_s)(4\beta_f + 3\beta_s)^2}\mu_1\mu_2 \\
&+ \frac{\beta_f}{16}\left(5\sigma_1^2 + (8 - 3\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2\right)
\end{aligned}
$$

Assuming $\rho_s = \rho$, we have

$$\frac{9}{8} \leq \text{PoA} \leq \frac{4}{3}$$

for every parameter. The lower bound 9/8 is achieved when $\beta_f \ll \beta_s$. The upper bound 4/3 is achieved when $\beta_s \ll \beta_f$.

Proof. Proof of Theorem 11. We then have

$$G_s(\lambda) = \frac{\lambda - \alpha_s}{\beta_s}, \quad G_f(\lambda) = \frac{\lambda - \alpha_f}{\beta_f}$$

From

$$
\begin{aligned}
G_s(\lambda_t^{DA}) + G_f(\lambda_t^{DA}) &= d_t^{DA} \\
G_s(\lambda_t^{DA}) + G_f(\lambda_t^{RT}) &= d_t^{DA} + d_t^{RT}
\end{aligned}
$$

with $d_t^{DA} = \mu_t - z_t^{DA}, d_t^{RT} = D_t - \mu_t - z_t^{RT}$. Solving for $\lambda_t^{DA}, \lambda_t^{RT}$ gives

$$\lambda_t^{DA} = \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{\beta_s + \beta_f} d_t^{DA}$$
$$\lambda_t^{RT} = \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{\beta_s + \beta_f} d_t^{DA} + \beta_f d_t^{RT}$$

The generation cost is given by

$$\sum_{t=1}^{2}\left[\int_{\lambda\le\lambda_t^{DA}} \lambda dG_s(\lambda) + \mathbb{E}\left[\int_{\lambda\le\lambda_t^{RT}} \lambda dG_f(\lambda)\right]\right] = \sum_{t=1}^{2}\left[\frac{(\lambda_t^{DA})^2 - \alpha_s^2}{2\beta_s} + \mathbb{E}\left[\frac{(\lambda_t^{RT})^2 - \alpha_f^2}{2\beta_f}\right]\right]$$

We first calculate the no-battery case.

Let $\alpha = \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_f + \beta_s}$. WE have

$$\lambda_1^{DA} = \alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_1$$
$$\lambda_2^{DA} = \alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_2$$
$$\lambda_1^{RT} = \alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_1 + \beta_f(D_1 - \mu_1)$$
$$\lambda_2^{RT} = \alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_2 + \beta_f(D_2 - \mu_2)$$
$$\mathbb{E}(\lambda_1^{RT})^2 = \left(\alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_1\right)^2 + \beta_f^2\sigma_1^2$$
$$\mathbb{E}(\lambda_2^{RT})^2 = \left(\alpha + \frac{\beta_f\beta_s}{\beta_f + \beta_s}\mu_2\right)^2 + \beta_f^2\sigma_2^2$$

So

$$\begin{aligned}
\text{Cost(NB)} &= \left(\frac{1}{2\beta_s} + \frac{1}{2\beta_f}\right)((\lambda_1^{DA})^2 + (\lambda_2^{DA})^2) + \frac{\beta_f^2\sigma_1^2 + \beta_f^2\sigma_2^2}{2\beta_f} - \frac{\alpha_s^2}{\beta_s} - \frac{\alpha_f^2}{\beta_f} \\
&= \left(\frac{(\beta_f + \beta_s)}{\beta_f\beta_s}\alpha^2 - \frac{\alpha_s^2}{\beta_s} - \frac{\alpha_f^2}{\beta_f}\right) + \alpha(\mu_1 + \mu_2) + \frac{\beta_f\beta_s}{2(\beta_f + \beta_s)}(\mu_1^2 + \mu_2^2) + \frac{\beta_f}{2}\sigma_1^2 + \frac{\beta_f}{2}\sigma_2^2 \\
&= -\frac{(\alpha_f - \alpha_s)^2}{\beta_f + \beta_s} + \alpha(\mu_1 + \mu_2) + \frac{\beta_f\beta_s}{2(\beta_f + \beta_s)}(\mu_1^2 + \mu_2^2) + \frac{\beta_f}{2}\sigma_1^2 + \frac{\beta_f}{2}\sigma_2^2
\end{aligned}$$

Now we calculate the centralized case.

Remember that $d_1^{DA} = \mu_1 - z_1^{DA}, d_2^{DA} = \mu_2 + z_1^{DA}, d_1^{RT} = D_1 - \mu_1 - z_1^{RT}(D_1), d_2^{RT} = D_2 - \mu_2 +$

$z_1^{RT}(D_1)$

For each fixed $D_1 = d_1$, we take the derivative w.r.t. $z_1^{RT}(d_1)$. We get

$$\mathbb{E}\left[2\lambda_1^{RT}(-\beta_f) + 2\lambda_2^{RT}(\beta_f)|D_1 = d_1\right] = 0$$

or

$$\mathbb{E}\left[\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_2 - \mu_1 + 2z_1^{DA}) + \beta_f((D_2-\mu_2) - (D_1-\mu_1) + 2z_1^{RT}(D_1))|D_1 = d_1\right] = 0$$

or $\mathbb{E}[\lambda_2^{RT} - \lambda_1^{RT}|D_1 = d_1] = 0$, which becomes

$$\beta_s(\mu_2 - \mu_1 + 2z_1^{DA}) + (\beta_s+\beta_f)((\mu_{2|d_1} - \mu_2) - (d_1 - \mu_1) + 2z_1^{RT}(d_1)) = 0$$

this allows us to write $z_1^{RT}(d_1)$ in terms of z_1^{DA}:

$$z_1^{RT}(d_1) = -\frac{\beta_s}{\beta_s+\beta_f}z_1^{DA} + \frac{\beta_s}{2(\beta_s+\beta_f)}(\mu_1-\mu_2) + \frac{1}{2}(d_1-\mu_1) - \frac{1}{2}(\mu_{2|d_1} - \mu_2)$$

In particular, this implies

$$\mathbb{E}[z_1^{RT}(D_1)] = -\frac{\beta_s}{\beta_s+\beta_f}z_1^{DA} + \frac{\beta_s}{2(\beta_s+\beta_f)}(\mu_1-\mu_2)$$

Taking the derivative w.r.t. z_1^{DA} gives

$$\frac{1}{2\beta_s}\left[2\lambda_1^{DA}\left(-\frac{\beta_s\beta_f}{\beta_s+\beta_f}\right) + 2\lambda_2^{DA}\left(\frac{\beta_s\beta_f}{\beta_s+\beta_f}\right)\right] + \frac{1}{2\beta_f}\mathbb{E}\left[2\lambda_1^{RT}\left(-\frac{\beta_s\beta_f}{\beta_s+\beta_f}\right) + 2\lambda_2^{RT}\left(\frac{\beta_s\beta_f}{\beta_s+\beta_f}\right)\right] = 0$$

or

$$\frac{\lambda_2^{DA} - \lambda_1^{DA}}{\beta_s} + \frac{\mathbb{E}[\lambda_2^{RT} - \lambda_1^{RT}]}{\beta_f} = 0$$

or

$$\frac{1}{\beta_s}\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_2-\mu_1+2z_1^{DA}) + \frac{1}{\beta_f}\left(\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_2-\mu_1+2z_1^{DA}) + \beta_f\mathbb{E}[(D_2-\mu_2) - (D_1-\mu_1) + 2z_1^{RT}(D_1)]\right) = 0$$

or

$$(\mu_2 - \mu_1 + 2z_1^{DA}) + 2\mathbb{E}[z_1^{RT}(D_1)] = 0$$

Using the expression for $\mathbb{E}[z_1^{RT}(D_1)]$ derived earlier, we get

$$\begin{aligned}
z_1^{DA} &= \frac{1}{2}(\mu_1 - \mu_2) \\
z_1^{RT}(d_1) &= \frac{1}{2}(d_1 - \mu_1) - \frac{1}{2}(\mu_{2|d_1} - \mu_2)
\end{aligned}$$

We now compute generation cost. We have

$$\begin{aligned}
\lambda_1^{DA} = \lambda_2^{DA} &= \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{2(\beta_s + \beta_f)}(\mu_1 + \mu_2) \\
\lambda_1^{RT} &= \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{2(\beta_s + \beta_f)}(\mu_1 + \mu_2) + \frac{\beta_f}{2}((D_1 - \mu_1) + (\mu_{2|D_1} - \mu_2)) \\
\lambda_2^{RT} &= \frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{2(\beta_s + \beta_f)}(\mu_1 + \mu_2) + \frac{\beta_f}{2}(2(D_2 - \mu_2) + (D_1 - \mu_1) - (\mu_{2|D_1} - \mu_2))
\end{aligned}$$

We have

$$\mathbb{E}[(\lambda_1^{RT})^2] = \left(\frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{2(\beta_s + \beta_f)}(\mu_1 + \mu_2)\right)^2 + \frac{\beta_f^2}{4}(\sigma_1^2 + \rho_s^2\sigma_2^2 + 2\rho\sigma_1\sigma_2)$$

and

$$\mathbb{E}[(\lambda_2^{RT})^2] = \left(\frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s + \beta_f} + \frac{\beta_s\beta_f}{2(\beta_s + \beta_f)}(\mu_1 + \mu_2)\right)^2 + \frac{\beta_f^2}{4}(\sigma_1^2 + (4 - 3\rho_s^2)\sigma_2^2 + 2\rho\sigma_1\sigma_2)$$

The generation cost is therefore

$$
\begin{aligned}
\text{Cost(CN)} &= \sum_{t=1}^{2}\left[\frac{(\lambda_t^{DA})^2-\alpha_s^2}{2\beta_s}+\mathbb{E}\left[\frac{(\lambda_t^{RT})^2-\alpha_f^2}{2\beta_f}\right]\right] \\
&= \frac{(\lambda_1^{DA})^2-\alpha_s^2}{\beta_s}+\frac{1}{2\beta_f}\left[2(\lambda_1^{DA})^2+\frac{\beta_f^2}{4}(2\sigma_1^2+(4-2\rho_s^2)\sigma_2^2+4\rho\sigma_1\sigma_2)-2\alpha_f^2\right] \\
&= \left(\frac{1}{\beta_s}+\frac{1}{\beta_f}\right)(\lambda_1^{DA})^2-\frac{\alpha_s^2}{\beta_s}-\frac{\alpha_f^2}{\beta_f}+\frac{\beta_f}{4}(\sigma_1^2+(2-\rho_s^2)\sigma_2^2+2\rho\sigma_1\sigma_2) \\
&= \frac{\beta_s+\beta_f}{\beta_s\beta_f}\left(\frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}+\frac{\beta_s\beta_f}{2(\beta_s+\beta_f)}(\mu_1+\mu_2)\right)^2-\frac{\alpha_s^2}{\beta_s}-\frac{\alpha_f^2}{\beta_f}+\frac{\beta_f}{4}(\sigma_1^2+(2-\rho_s^2)\sigma_2^2+2\rho\sigma_1\sigma_2) \\
&= -\frac{(\alpha_f-\alpha_s)^2}{(\beta_s+\beta_f)}+\frac{(\alpha_f\beta_s+\alpha_s\beta_f)}{(\beta_s+\beta_f)}(\mu_1+\mu_2)+\frac{\beta_s\beta_f}{4(\beta_s+\beta_f)}(\mu_1+\mu_2)^2+\frac{\beta_f}{4}(\sigma_1^2+(2-\rho_s^2)\sigma_2^2+2\rho\sigma_1\sigma_2)
\end{aligned}
$$

Lastly, we calculate the decentralized case.

The battery maximizes profit

$$
\begin{aligned}
\Pi &= (\lambda_1^{DA}-\lambda_2^{DA})z_1^{DA}+\mathbb{E}[(\lambda_1^{RT}-\lambda_2^{RT})z_1^{RT}(D_1)] \\
&= \frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_1-\mu_2-2z_1^{DA})z_1^{DA} \\
&+\mathbb{E}\left[\left(\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_1-\mu_2-2z_1^{DA})+\beta_f((D_1-\mu_1)-(D_2-\mu_2)-2z_1^{RT}(D_1))\right)z_1^{RT}(D_1)\right]
\end{aligned}
$$

Taking the derivative w.r.t. $z_1^{RT}(d_1)$ for a given fixed d_1 gives, for each d_1,

$$
\mathbb{E}\left[\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_1-\mu_2-2z_1^{DA})+\beta_f(D_1-\mu_1)-\beta_f(D_2-\mu_2)-4\beta_f z_1^{RT}(D_1)|D_1=d_1\right]=0
$$

This reduces to

$$
z_1^{RT}(d_1)=\frac{1}{4}(d_1-\mu_1)-\frac{1}{4}(\mu_{2|d_1}-\mu_2)+\frac{\beta_s}{4(\beta_s+\beta_f)}(\mu_1-\mu_2-2z_1^{DA})
$$

In particular,

$$
\mathbb{E}[z_1^{RT}(D_1)]=\frac{\beta_s}{4(\beta_s+\beta_f)}(\mu_1-\mu_2-2z_1^{DA})
$$

Now we take derivative w.r.t z_1^{DA}:

$$\frac{\beta_s\beta_f}{\beta_s+\beta_f}(\mu_1-\mu_2-4z_1^{DA})+\mathbb{E}\left[\frac{\beta_s\beta_f}{\beta_s+\beta_f}(-2)z_1^{RT}(D_1)\right]=0$$

Using the expression for $\mathbb{E}[z_1^{RT}(D_1)]$ derived earlier gives

$$\begin{aligned}
z_1^{DA} &= \frac{(\beta_s+2\beta_f)}{2(3\beta_s+4\beta_f)}(\mu_1-\mu_2)\\
z_1^{RT}(d_1) &= \frac{1}{4}(d_1-\mu_1)-\frac{1}{4}(\mu_{2|d_1}-\mu_2)+\frac{\beta_s}{2(3\beta_s+4\beta_f)}(\mu_1-\mu_2)
\end{aligned}$$

We then have

$$\begin{aligned}
\lambda_1^{DA} &= \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}+\frac{\beta_s\beta_f(5\beta_s+6\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1+\frac{\beta_s\beta_f(\beta_s+2\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\\
\lambda_2^{DA} &= \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}+\frac{\beta_s\beta_f(\beta_s+2\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1+\frac{\beta_s\beta_f(5\beta_s+6\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\\
\lambda_1^{RT} &= \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}+\frac{\beta_s\beta_f(4\beta_s+5\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1+\frac{\beta_s\beta_f(2\beta_s+3\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\\
&\quad+\frac{3\beta_f}{4}(D_1-\mu_1)+\frac{\beta_f}{4}(\mu_{2|D_1}-\mu_2)\\
\lambda_2^{RT} &= \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}+\frac{\beta_s\beta_f(2\beta_s+3\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1+\frac{\beta_s\beta_f(4\beta_s+5\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\\
&\quad+\frac{\beta_f}{4}(D_1-\mu_1)-\frac{\beta_f}{4}(\mu_{2|D_1}-\mu_2)+\beta_f(D_2-\mu_2)
\end{aligned}$$

We have

$$\mathrm{Cost(DCN)}=\sum_{t=1}^{2}\left[\frac{(\lambda_t^{DA})^2-\alpha_s^2}{2\beta_s}+\mathbb{E}\left[\frac{(\lambda_t^{RT})^2-\alpha_f^2}{2\beta_f}\right]\right]$$

Note that

$$\mathbb{E}[(\lambda_1^{RT})^2] = \left(\frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s+\beta_f} + \frac{\beta_s\beta_f(4\beta_s+5\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1 + \frac{\beta_s\beta_f(2\beta_s+3\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\right)^2 + \frac{\beta_f^2}{16}\left(9\sigma_1^2 + \rho_s^2\sigma_2^2 + 6\rho\sigma_1\sigma_2\right)$$

$$\mathbb{E}[(\lambda_2^{RT})^2] = \left(\frac{\alpha_f\beta_s + \alpha_s\beta_f}{\beta_s+\beta_f} + \frac{\beta_s\beta_f(2\beta_s+3\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_1 + \frac{\beta_s\beta_f(4\beta_s+5\beta_f)}{2(\beta_s+\beta_f)(3\beta_s+4\beta_f)}\mu_2\right)^2 + \frac{\beta_f^2}{16}\left(\sigma_1^2 + (16-7\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2\right)$$

Let $\alpha = \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}$.

The cost becomes

$$\frac{\alpha^2(\beta_f+\beta_s)}{\beta_f\beta_s} + \alpha(\mu_1+\mu_2) + \frac{\beta_f\beta_s(20\beta_f^2+29\beta_f\beta_s+10\beta_s^2)}{4(\beta_f+\beta_s)(4\beta_f+3\beta_s)^2}(\mu_1^2+\mu_2^2) + \frac{\beta_f\beta_s(12\beta_f^2+19\beta_f\beta_s+8\beta_s^2)}{2(\beta_f+\beta_s)(4\beta_f+3\beta_s)^2}\mu_1\mu_2 - \frac{2\alpha_s^2}{2\beta_s} - \frac{2\alpha_f^2}{2\beta_f} + \frac{\beta_f^2/16}{2\beta_f}(10\sigma_1^2 + (16-6\rho_s^2)\sigma_2^2 + 12\rho\sigma_1\sigma_2)$$

Now we note that

$$\frac{\alpha^2(\beta_f+\beta_s)}{\beta_f\beta_s} - \frac{\alpha_s^2}{\beta_s} - \frac{\alpha_f^2}{\beta_f} = -\frac{(\alpha_f-\alpha_s)^2}{\beta_f+\beta_s}$$

Therefore,

$$\mathrm{Cost(DCN)} = -\frac{(\alpha_f-\alpha_s)^2}{\beta_f+\beta_s} + \frac{\alpha_f\beta_s+\alpha_s\beta_f}{\beta_s+\beta_f}(\mu_1+\mu_2) + \frac{\beta_f\beta_s(20\beta_f^2+29\beta_f\beta_s+10\beta_s^2)}{4(\beta_f+\beta_s)(4\beta_f+3\beta_s)^2}(\mu_1^2+\mu_2^2) + \frac{\beta_f\beta_s(12\beta_f^2+19\beta_f\beta_s+8\beta_s^2)}{2(\beta_f+\beta_s)(4\beta_f+3\beta_s)^2}\mu_1\mu_2 + \frac{\beta_f}{16}\left(5\sigma_1^2 + (8-3\rho_s^2)\sigma_2^2 + 6\rho\sigma_1\sigma_2\right)$$

We can now prove the bounds on the Price of Anarchy. Assuming $\rho_s = \rho$, we have

$$\mathrm{Cost(NB)} - \mathrm{Cost(CN)} = \frac{\beta_f\beta_s}{4(\beta_f+\beta_s)}(\mu_1-\mu_2)^2 + \frac{\beta_f}{4}(\sigma_1-\rho\sigma_2)^2$$

$$\mathrm{Cost(NB)} - \mathrm{Cost(DCN)} = \frac{\beta_f\beta_s(12\beta_f^2+19\beta_f\beta_s+8\beta_s^2)}{4(\beta_f+\beta_s)(4\beta_f+3\beta_s)^2}(\mu_1-\mu_2)^2 + \frac{3\beta_f}{16}(\sigma_1-\rho\sigma_2)^2$$

This implies $9/8 \leq \text{PoA} \leq 4/3$ as desired.

$\square$

Theorem 12 (Convex Supply Curves)**.** *Assume $G^{-1}(x) = \alpha + \beta x + \gamma x^2$ with $\alpha, \beta, \gamma \geq 0$.*

The centralized battery discharge decisions are given by

$$\begin{aligned} z_1^{DA} &= \frac{1}{2}(\mu_1 - \mu_2) + O(\gamma^2) \\ z_1^{RT}(D_1) &= \frac{1}{2}(D_1 - \mu_1) - \frac{1}{2}(\mu_{2|D_1} - \mu_2) - \frac{\sigma^2_{2|D_1}}{2k_f}\frac{\gamma}{\beta} + O(\gamma^2), \end{aligned}$$

The decentralized battery discharge decisions are given by

$$\begin{aligned} z_1^{DA} &= \frac{(2-k_f)}{2(4-k_f)}(\mu_1 - \mu_2) - \frac{(\sigma_1^2 - \sigma_2^2)}{2k_f(4-k_f)}\frac{\gamma}{\beta} + O(\gamma^2) \\ z_1^{RT}(D_1) &= \frac{k_f}{2(4-k_f)}(\mu_1 - \mu_2) + \frac{1}{4}(D_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2) + \left(\frac{\sigma_1^2 - \sigma_2^2}{4(4-k_f)} - \frac{\sigma^2_{2|D_1}}{4k_f}\right)\frac{\gamma}{\beta} + O(\gamma^2) \end{aligned}$$

Proof. Proof of Theorem 12. We first solve the centralized case.

Generation cost is

$$\begin{aligned} &\alpha(\mu_1 + \mu_2) \\ &+ k_s\left[\frac{\beta}{2}\left[(\mu_1 - z_1^{DA})^2 + (\mu_2 + z_1^{DA})^2\right] + \frac{1}{3}\left[(\mu_1 - z_1^{DA})^3 + (\mu_2 + z_1^{DA})^3\right]\gamma\right] \\ &+ k_f\mathbb{E}_{D_1,D_2}\left\{\frac{\beta}{2}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^2 + \left(\mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}\right)^2\right]\right. \\ &\left.+ \frac{1}{3}\left[\left(\mu_1 - z_1^{DA} + \frac{D_1 - \mu_1 - z_1^{RT}(D_1)}{k_f}\right)^3 + \left(\mu_2 + z_1^{DA} + \frac{D_2 - \mu_2 + z_1^{RT}(D_1)}{k_f}\right)^3\right]\gamma\right\} \end{aligned}$$

We first note that if $\gamma = 0$, then the generation cost is strictly convex (quadratic) in the decision variables. So, for γ sufficiently close to zero, the global minimum is achieved where first-order conditions hold with equality.

For each fixed $D_1 = d_1$, we take the derivative w.r.t $z_1^{RT}(d_1)$. By the law of iterated expectations, the expectation $\mathbb{E}_{D_1,D_2}$ can be viewed as taking expectation $\mathbb{E}_{D_1}$ followed by the conditional expectation $\mathbb{E}_{D_2|D_1}$, by focusing on $z_1^{RT}(d_1)$ we fix the value $D_1 = d_1$ while the inner expectation becomes an expectation over $D_2 \sim \pi(\cdot|D_1 = d_1)$. We therefore get

$$\mathbb{E}_{D_2\sim\pi(\cdot|D_1=d_1)}\Bigg\{\beta\left[-\frac{1}{k_f}\left(\mu_1 - z_1^{DA} + \frac{d_1-\mu_1-z_1^{RT}(d_1)}{k_f}\right) + \frac{1}{k_f}\left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(D_1)}{k_f}\right)\right]$$
$$+\left[-\frac{1}{k_f}\left(\mu_1 - z_1^{DA} + \frac{d_1-\mu_1-z_1^{RT}(d_1)}{k_f}\right)^2 + \frac{1}{k_f}\left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(d_1)}{k_f}\right)^2\right]\gamma\Bigg\} = 0$$

We will calculate the expectations in terms of $\mu_{2|d_1}$ and $\sigma^2_{2|d_1}$. First we evaluate the expectation, using the last equation of Proposition 1 to evaluate the quadratic term in D_2, then multiply across by k_f:

$$\frac{\beta}{k_f}\left[-k_f(\mu_1-\mu_2) + 2k_f z_1^{DA} - (d_1-\mu_1) + (\mu_{2|d_1}-\mu_2) + 2z_1^{RT}(d_1)\right]$$
$$+\frac{\gamma}{k_f^2}\left[-\left(k_f(\mu_1 - z_1^{DA}) + d_1 - \mu_1 - z_1^{RT}(d_1)\right)^2 + \sigma^2_{2|d_1} + \left(\mu_{2|d_1} - \mu_2 + z_1^{RT}(d_1) + k_f(\mu_2 + z_1^{DA})\right)^2\right] = 0$$

Let $\tilde{\gamma} = \gamma/\beta$. We get

$$\begin{aligned}
&k_f\left[(\mu_{2|d_1}-\mu_2) - (d_1-\mu_1) - k_f(\mu_1-\mu_2) + 2k_f z_1^{DA} + 2z_1^{RT}(d_1)\right] \\
&+\tilde{\gamma}\Big\{\sigma^2_{2|d_1} + \left[(\mu_{2|d_1}-\mu_2) + (d_1-\mu_1) + k_f(\mu_1+\mu_2)\right]\times \\
&\left[(\mu_{2|d_1}-\mu_2) - (d_1-\mu_1) - k_f(\mu_1-\mu_2) + 2k_f z_1^{DA} + 2z_1^{RT}(d_1)\right]\Big\} = 0
\end{aligned} \tag{19}$$

Taking the derivative w.r.t. z_1^{DA} gives

$$k_s\left[\beta\left[-(\mu_1 - z_1^{DA}) + (\mu_2 + z_1^{DA})\right] + \gamma\left[-(\mu_1 - z_1^{DA})^2 + (\mu_2 + z_1^{DA})^2\right]\right]$$
$$+k_f\mathbb{E}_{D_1,D_2}\Bigg\{\beta\left[-\left(\mu_1 - z_1^{DA} + \frac{D_1-\mu_1-z_1^{RT}(D_1)}{k_f}\right) + \left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(D_1)}{k_f}\right)\right]$$
$$+\gamma\left[-\left(\mu_1 - z_1^{DA} + \frac{D_1-\mu_1-z_1^{RT}(D_1)}{k_f}\right)^2 + \left(\mu_2 + z_1^{DA} + \frac{D_2-\mu_2+z_1^{RT}(D_1)}{k_f}\right)^2\right]\Bigg\} = 0$$

or (with $\tilde{\gamma} = \gamma/\beta$):

$$
\begin{aligned}
&(k_s + k_f)(\mu_2 - \mu_1 + 2z_1^{DA}) + k_s(\mu_2 + \mu_1)(\mu_2 - \mu_1 + 2z_1^{DA})\tilde{\gamma} \\
&+ k_f \mathbb{E}_{D_1,D_2}\Bigg\{ \frac{1}{k_f}\left[(D_2 - \mu_2) - (D_1 - \mu_1) + 2z_1^{RT}(D_1)\right] \\
&+ \frac{\tilde{\gamma}}{k_f^2}\left[k_f(\mu_2 - \mu_1 + 2z_1^{DA}) + (D_2 - \mu_2) - (D_1 - \mu_1) + 2z_1^{RT}(D_1)\right] \times \left[(D_1 - \mu_1) + (D_2 - \mu_2) + k_f(\mu_1 + \mu_2)\right] \Bigg\} = 0
\end{aligned}
$$

or

$$
\begin{aligned}
&(\mu_2 - \mu_1 + 2z_1^{DA}) + k_s(\mu_2 + \mu_1)(\mu_2 - \mu_1 + 2z_1^{DA})\tilde{\gamma} \\
&+ k_f \mathbb{E}_{D_1,D_2}\Bigg\{ \frac{1}{k_f}\left[2z_1^{RT}(D_1)\right] \\
&+ \frac{\tilde{\gamma}}{k_f^2}\left[k_f(\mu_2 - \mu_1 + 2z_1^{DA}) + (D_2 - \mu_2) - (D_1 - \mu_1) + 2z_1^{RT}(D_1)\right] \\
&\times \left[(D_1 - \mu_1) + (D_2 - \mu_2) + k_f(\mu_1 + \mu_2)\right] \Bigg\} = 0
\end{aligned} \tag{20}
$$

We now have equations (19) and (20) for z_1^{DA} and $z_1^{RT}(d_1)$ from first-order conditions on $z_1^{RT}(d_1)$ and z_1^{DA}, respectively, which we want to solve.

We now write

$$
\begin{aligned}
z_1^{DA} &:= \bar{z}_1^{DA} + \hat{z}_1^{DA}\tilde{\gamma} + O(\tilde{\gamma}^2) \\
z_1^{RT}(d_0) &:= \bar{z}_1^{RT}(d_0) + \hat{z}_1^{RT}(d_0)\tilde{\gamma} + O(\tilde{\gamma}^2).
\end{aligned}
$$

The main term of the $z_1^{RT}(d_1)$ derivative equation is, for every d_1,

$$
(\mu_{2|d_1} - \mu_2) - (d_1 - \mu_1) - k_f(\mu_1 - \mu_2) + 2k_f\bar{z}_1^{DA} + 2\bar{z}_1^{RT}(d_1) = 0
$$

or

$$
\bar{z}_1^{RT}(d_1) = -k_f\bar{z}_1^{DA} + \frac{k_f}{2}(\mu_1 - \mu_2) + \frac{1}{2}(d_1 - \mu_1) - \frac{1}{2}(\mu_{2|d_1} - \mu_2)
$$

Therefore,

$$\mathbb{E}[\bar{z}_1^{RT}(D_1)] = -k_f \bar{z}_1^{DA} + \frac{k_f}{2}(\mu_1 - \mu_2)$$

The main term of the z_1^{DA} derivative equation (20) gives

$$\mu_2 - \mu_1 + 2\bar{z}_1^{DA} + 2\mathbb{E}[\bar{z}_1^{RT}(D_1)] = 0$$

Substituting the expression for $\mathbb{E}[\bar{z}_1^{RT}(D_1)]$ gives

$$\bar{z}_1^{DA} = \frac{\mu_1 - \mu_2}{2}$$

Putting $\bar{z}_1^{DA}$ into the expression for $\bar{z}_1^{RT}(d_1)$ gives

$$\bar{z}_1^{RT}(d_1) = \frac{1}{2}(d_1 - \mu_1) - \frac{1}{2}(\mu_{2|d_1} - \mu_2).$$

The curvature correction term of the $z_1^{RT}(d_1)$ derivative equation (19) gives

$$\begin{aligned} 2k_f^2 \hat{z}_1^{DA} + 2k_f \hat{z}_1^{RT}(d_1) + \Big\{ \sigma_{2|d_1}^2 + \big[(\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) + k_f(\mu_1 + \mu_2)\big] \times \\ \big[(\mu_{2|d_1} - \mu_2) - (d_1 - \mu_1) - k_f(\mu_1 - \mu_2) + 2k_f \bar{z}_1^{DA} + 2\bar{z}_1^{RT}(d_1)\big] \Big\} = 0 \end{aligned}$$

Note that $\big[(\mu_{2|d_1} - \mu_2) - (d_1 - \mu_1) - k_f(\mu_1 - \mu_2) + 2k_f \bar{z}_1^{DA} + 2\bar{z}_1^{RT}(d_1)\big] = 0$ for every d_1 by the defining equation for $z_1^{RT}(d_1)$. Therefore,

$$2k_f^2 \hat{z}_1^{DA} + 2k_f \hat{z}_1^{RT}(d_1) + \sigma_{2|d_1}^2 = 0$$

or

$$\hat{z}_1^{RT}(d_1) = -k_f \hat{z}_1^{DA} - \frac{\sigma_{2|d_1}^2}{2k_f}$$

Taking the expectation over d_1 and using Proposition 1 gives

$$\mathbb{E}[\hat{z}_1^{RT}(D_1)] = -k_f\hat{z}_1^{DA} - \frac{(1-\rho_s^2)\sigma_2^2}{2k_f}$$

The curvature correction term of the z_1^{DA} derivative equation (20) gives

$$\begin{aligned}
&2\hat{z}_1^{DA} + k_s(\mu_2+\mu_1)(\mu_2-\mu_1+2\bar{z}_1^{DA}) \\
+\mathbb{E}\Bigg[&2\hat{z}_1^{RT}(D_1) + \frac{1}{k_f^2}\left[k_f(\mu_2-\mu_1+2\bar{z}_1^{DA}) + (D_2-\mu_2) - (D_1-\mu_1) + 2\bar{z}_1^{RT}(D_1)\right] \\
&\times\left[(D_1-\mu_1)+(D_2-\mu_2)+k_f(\mu_1+\mu_2)\right]\Bigg] = 0
\end{aligned}$$

Using Proposition 1, the last expectation term evaluates to $0 + (1-\rho_s^2)\sigma_2^2 + 0 = (1-\rho_s^2)\sigma_2^2$. Also substitute the expression for $\mathbb{E}[\hat{z}_1^{RT}(D_1)]$ gives

$$2\hat{z}_1^{DA} + 2\left(-k_f\hat{z}_1^{DA} - \frac{(1-\rho_s^2)\sigma_2^2}{2k_f}\right) + \frac{1}{k_f}(1-\rho_s^2)\sigma_2^2 = 0$$

This yields

$$\hat{z}_1^{DA} = 0.$$

Therefore,

$$\hat{z}_1^{RT}(d_1) = -\frac{\sigma_{2|d_1}^2}{2k_f}.$$

We conclude that the battery discharges are given by

$$\begin{aligned}
z_1^{DA} &= \frac{1}{2}(\mu_1-\mu_2) + O(\tilde{\gamma}^2) \\
z_1^{RT}(d_1) &= \frac{1}{2}(d_1-\mu_1) - \frac{1}{2}(\mu_{2|d_1}-\mu_2) - \frac{\sigma_{2|d_1}^2}{2k_f}\tilde{\gamma} + O(\tilde{\gamma}^2),
\end{aligned}$$

as claimed in the theorem.

We now solve the decentralized case.

The prices are given by

$$
\begin{aligned}
\lambda_1^{DA} &= \alpha + \beta d_1^{DA} + \gamma (d_1^{DA})^2 \\
\lambda_2^{DA} &= \alpha + \beta d_2^{DA} + \gamma (d_2^{DA})^2 \\
\lambda_1^{RT} &= \alpha + \beta \left(d_1^{DA} + \frac{d_1^{RT}}{k_f} \right) + \gamma \left(d_1^{DA} + \frac{d_1^{RT}}{k_f} \right)^2 \\
\lambda_2^{RT} &= \alpha + \beta \left(d_2^{DA} + \frac{d_2^{RT}}{k_f} \right) + \gamma \left(d_2^{DA} + \frac{d_2^{RT}}{k_f} \right)^2
\end{aligned}
$$

with

$$
\begin{aligned}
d_1^{DA} &= \mu_1 - z_1^{DA} \\
d_2^{DA} &= \mu_2 + z_1^{DA} \\
d_1^{RT} &= D_1 - \mu_1 - z_1^{RT}(D_1) \\
d_2^{RT} &= D_2 - \mu_2 + z_1^{RT}(D_1)
\end{aligned}
$$

The battery maximizes profit:

$$
\Pi = (\lambda_1^{DA} - \lambda_2^{DA}) z_1^{DA} + \mathbb{E}\left[(\lambda_1^{RT} - \lambda_2^{RT}) z_1^{RT}(D_1) \right]
$$

We can write

$$
\begin{aligned}
\Pi &= \beta(\mu_1 - \mu_2 - 2z_1^{DA}) z_1^{DA} + \gamma(\mu_1 + \mu_2)(\mu_1 - \mu_2 - 2z_1^{DA}) z_1^{DA} \\
&+ \mathbb{E}\Big[\beta \left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((D_1 - \mu_1) - (D_2 - \mu_2) - 2z_1^{RT}(D_1))}{k_f} \right) z_1^{RT}(D_1) \\
&+ \gamma \left(\mu_1 + \mu_2 + \frac{(D_1 - \mu_1) + (D_2 - \mu_2)}{k_f} \right) \left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((D_1 - \mu_1) - (D_2 - \mu_2) - 2z_1^{RT}(D_1))}{k_f} \right) z_1^{RT}(D_1) \Big]
\end{aligned}
$$

Taking derivative w.r.t. $z_1^{RT}(d_1)$ for a given fixed d_1 gives, for each d_1,

$$
\begin{aligned}
&\mathbb{E}_{D_2 \sim \pi(\cdot|d_1)} \Bigg[\beta \left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((d_1 - \mu_1) - (D_2 - \mu_2) - 4z_1^{RT}(d_1))}{k_f} \right) \\
&+ \gamma \left(\mu_1 + \mu_2 + \frac{(d_1 - \mu_1) + (D_2 - \mu_2)}{k_f} \right) \left(\mu_1 - \mu_2 - 2z_1^{DA} + \frac{((d_1 - \mu_1) - (D_2 - \mu_2) - 4z_1^{RT}(d_1))}{k_f} \right) \Bigg] = 0
\end{aligned}
$$

This reduces to

$$
\begin{aligned}
&k_f^2(\mu_1 - \mu_2 - 2z_1^{DA}) + k_f(d_1 - \mu_1) - k_f(\mu_{2|d_1} - \mu_2) - 4k_f z_1^{RT}(d_1) \\
&+ \gamma \Bigg\{ - \sigma^2_{2|d_1} \\
&+ \big((\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) + k_f\mu_1 + k_f\mu_2\big) \times \big(-(\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) - 4z_1^{RT}(d_1) + k_f\mu_1 - k_f\mu_2 - 2k_f z_1^{DA}\big) \Bigg\}
\end{aligned} \tag{21}
$$

Now we take derivative w.r.t z_1^{DA}:

$$
\begin{aligned}
&\beta(\mu_1 - \mu_2 - 4z_1^{DA}) + \gamma(\mu_1 + \mu_2)(\mu_1 - \mu_2 - 4z_1^{DA}) \\
&+ \mathbb{E}\left[\beta(-2)z_1^{RT}(D_1) + \gamma\left(\mu_1 + \mu_2 + \frac{(D_1 - \mu_1) + (D_2 - \mu_2)}{k_f}\right)(-2)z_1^{RT}(D_1)\right] = 0
\end{aligned} \tag{22}
$$

We now have (21) and (22) from the first order conditions over $z_1^{RT}(D_1)$ and z_1^{DA}. Now we look at the main and curvature correction terms in turn.

Let $\tilde{\gamma} = \gamma/\beta$. We write

$$
\begin{aligned}
z_1^{DA} &= \bar{z}_1^{DA} + \hat{z}_1^{DA}\tilde{\gamma} + O(\tilde{\gamma}^2) \\
z_1^{RT}(D_1) &= \bar{z}_1^{RT}(D_1) + \hat{z}_1^{RT}(D_1)\tilde{\gamma} + O(\tilde{\gamma}^2)
\end{aligned}
$$

The main term of (21) gives

$$
k_f(\mu_1 - \mu_2 - 2\bar{z}_1^{DA}) + (d_1 - \mu_1) - (\mu_{2|d_1} - \mu_2) - 4z_1^{RT}(d_1) = 0
$$

or

$$
\bar{z}_1^{RT}(d_1) = \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2) + \frac{k_f}{4}(\mu_1 - \mu_2 - 2\bar{z}_1^{DA})
$$

This gives

$$\mathbb{E}[\bar{z}_1^{RT}(D_1)] = \frac{k_f}{4}(\mu_1 - \mu_2 - 2\bar{z}_1^{DA})$$

The main term of (22) gives

$$\beta(\mu_1 - \mu_2 - 4\bar{z}_1^{DA}) + \mathbb{E}\left[\beta(-2)\bar{z}_1^{RT}(D_1)\right] = 0$$

Using the expression of $\mathbb{E}[\bar{z}_1^{RT}(D_1)]$ gives

$$\mu_1 - \mu_2 - 4\bar{z}_1^{DA} - \frac{k_f}{2}(\mu_1 - \mu_2 - 2\bar{z}_1^{DA}) = 0$$

which gives

$$\bar{z}_1^{DA} = \frac{(2 - k_f)}{2(4 - k_f)}(\mu_1 - \mu_2)$$

Plugging this value of $\bar{z}_1^{DA}$ into the equation for $\bar{z}_1^{RT}(D_1)$ gives

$$\bar{z}_1^{RT}(d_1) = \frac{1}{4}(d_1 - \mu_1) - \frac{1}{4}(\mu_{2|d_1} - \mu_2) + \frac{k_f}{2(4 - k_f)}(\mu_1 - \mu_2)$$

The curvature correction term of (21) gives

$$\begin{aligned}
& - 2k_f^2\hat{z}_1^{DA} - 4k_f\hat{z}_1^{RT}(d_1) \\
& + \Bigg\{ - \sigma^2_{2|d_1} \\
& + \big((\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) + k_f\mu_1 + k_f\mu_2\big) \times \big(-(\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) - 4\bar{z}_1^{RT}(d_1) + k_f\mu_1 - k_f\mu_2 - 2k_f\bar{z}_1^{DA}\big) \Bigg\}
\end{aligned}$$

Using the expressions for $\bar{z}_1^{DA}$ and $\bar{z}_1^{RT}(d_1)$, we calculate

$$\big(-(\mu_{2|d_1} - \mu_2) + (d_1 - \mu_1) - 4\bar{z}_1^{RT}(d_1) + k_f\mu_1 - k_f\mu_2 - 2k_f\bar{z}_1^{DA}\big) = 0$$

Therefore, the last product term is zero. We therefore have

$$-2k_f^2\hat{z}_1^{DA} - 4k_f\hat{z}_1^{RT}(d_1) - \sigma_{2|d_1}^2 = 0$$

or

$$\hat{z}_1^{RT}(d_1) = -\frac{k_f}{2}\hat{z}_1^{DA} - \frac{\sigma_{2|d_1}^2}{4k_f}$$

This also gives

$$\mathbb{E}[\hat{z}_1^{RT}(D_1)] = -\frac{k_f}{2}\hat{z}_1^{DA} - \frac{(1-\rho_s^2)\sigma_2^2}{4k_f}$$

The curvature correction term of (22) gives

$$\begin{aligned}
&-4\hat{z}_1^{DA} + (\mu_1+\mu_2)(\mu_1-\mu_2-4\bar{z}_1^{DA}) \\
&+\mathbb{E}\left[(-2)\hat{z}_1^{RT}(D_1) + \left(\mu_1+\mu_2+\frac{(D_1-\mu_1)+(D_2-\mu_2)}{k_f}\right)(-2)\bar{z}_1^{RT}(D_1)\right] = 0
\end{aligned}$$

We have calculated $\mathbb{E}[\hat{z}_1^{RT}(D_1)]$ above. Using the expression for $\bar{z}_1^{RT}(D_1)$ and Proposition 1, we can also calculate

$$\begin{aligned}
&\mathbb{E}[4((D_1-\mu_1)+(D_2-\mu_2))\bar{z}_1^{RT}(D_1)] \\
&=\mathbb{E}\left[((D_1-\mu_1)+(D_2-\mu_2))\left((D_1-\mu_1)-(\mu_{2|D_1}-\mu_2)+\frac{2k_f}{(4-k_f)}(\mu_1-\mu_2)\right)\right] \\
&=\mathbb{E}[(D_1-\mu_1)^2-(D_1-\mu_1)(\mu_{2|D_1}-\mu_2)+(D_2-\mu_2)(D_1-\mu_1)-(D_2-\mu_2)(\mu_{2|D_1}-\mu_2)] \\
&\quad+\frac{2k_f}{(4-k_f)}(\mu_1-\mu_2)\mathbb{E}[(D_1-\mu_1)+(D_2-\mu_2)] \\
&=\sigma_1^2-\rho\sigma_1\sigma_2+\rho\sigma_1\sigma_2-\rho_s^2\sigma_2^2+0 \\
&=\sigma_1^2-\rho_s^2\sigma_2^2
\end{aligned}$$

Therefore,

$$-4\hat{z}_1^{DA} + (\mu_1+\mu_2)\left(1-4\cdot\frac{(2-k_f)}{2(4-k_f)}\right)(\mu_1-\mu_2)$$
$$-2\left(-\frac{k_f}{2}\hat{z}_1^{DA} - \frac{(1-\rho_s^2)\sigma_2^2}{4k_f}\right) - 2(\mu_1+\mu_2)\left(\frac{k_f}{2(4-k_f)}(\mu_1-\mu_2)\right) - \frac{2}{k_f}\cdot\frac{\sigma_1^2-\rho_s^2\sigma_2^2}{4} = 0$$

or

$$-4\hat{z}_1^{DA} + \frac{k_f}{4-k_f}(\mu_1^2-\mu_2^2) + k_f\hat{z}_1^{DA} + \frac{(1-\rho_s^2)\sigma_2^2}{2k_f} - \frac{k_f}{4-k_f}(\mu_1^2-\mu_2^2) - \frac{\sigma_1^2-\rho_s^2\sigma_2^2}{2k_f} = 0$$

or

$$\hat{z}_1^{DA} = -\frac{\sigma_1^2-\sigma_2^2}{2k_f(4-k_f)}$$

Substituting the expression of $\hat{z}_1^{DA}$ into that of $\hat{z}_1^{RT}(d_1)$ gives

$$\hat{z}_1^{RT}(d_1) = \frac{\sigma_1^2-\sigma_2^2}{4(4-k_f)} - \frac{\sigma_{2|d_1}^2}{4k_f}$$

This leads to the decentralized battery strategies as desired.

$\square$

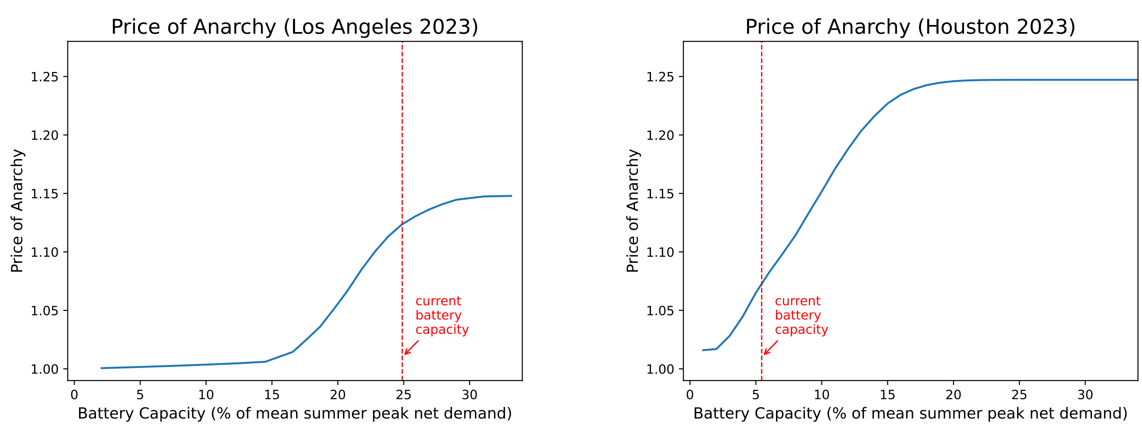

Figure 5: Price of Anarchy with finite battery capacity.

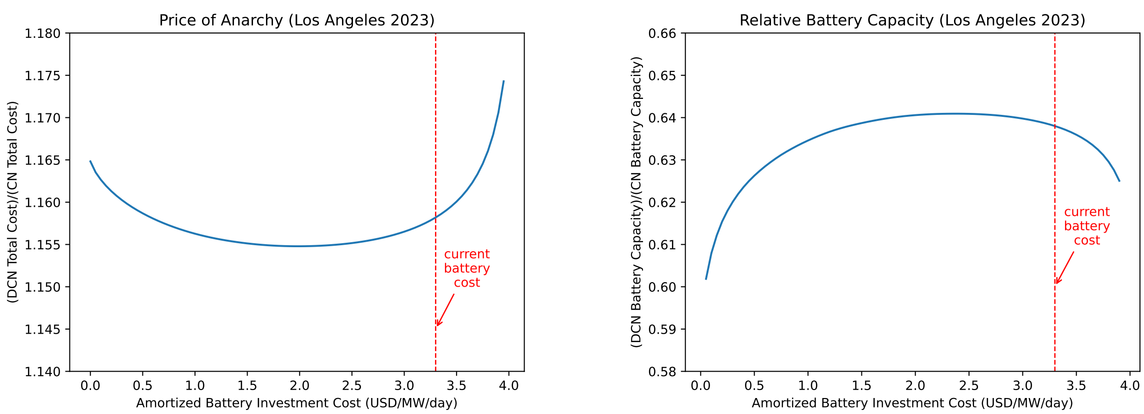

Figure 6: Price of Anarchy with endogenous battery capacity, considering battery investments and operations.