# Battery Operations in Electricity Markets: Strategic Behavior and Distortions

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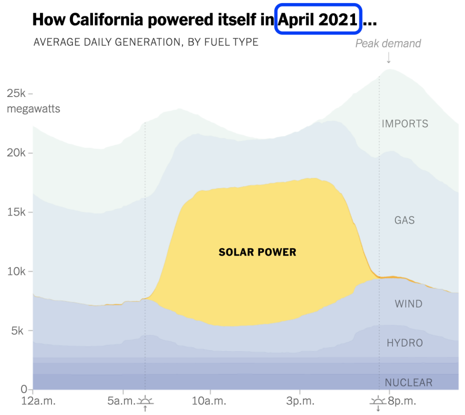
# The Growth of Batteries in California ⏩ NUMBERS




New York Times (May 7, 2024)

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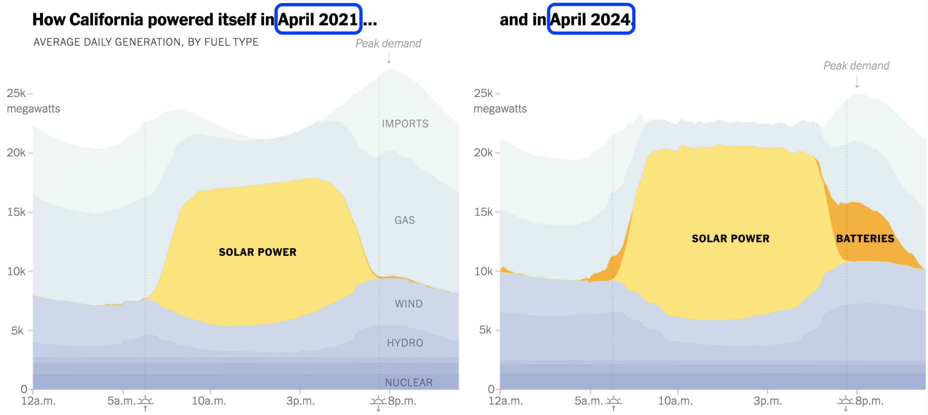


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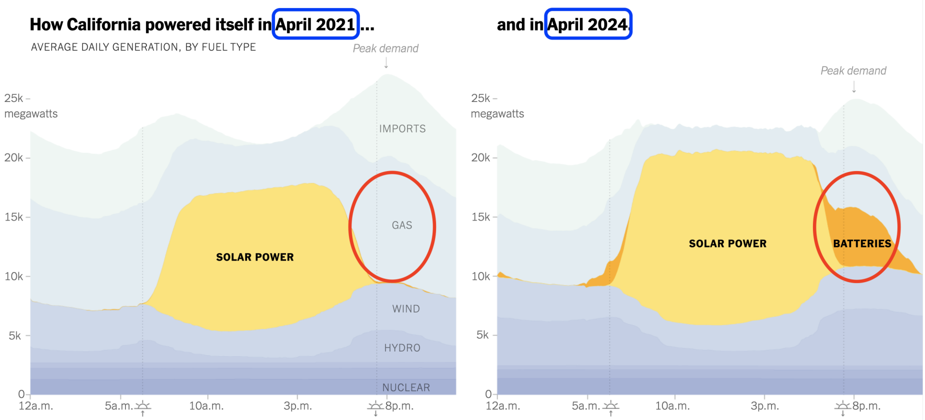


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# Grid-Scale Battery Storage ⏩ LOCATIONS


Figure: Tesla's 750 MW/3000 MWh battery storage in Moss Landing, California

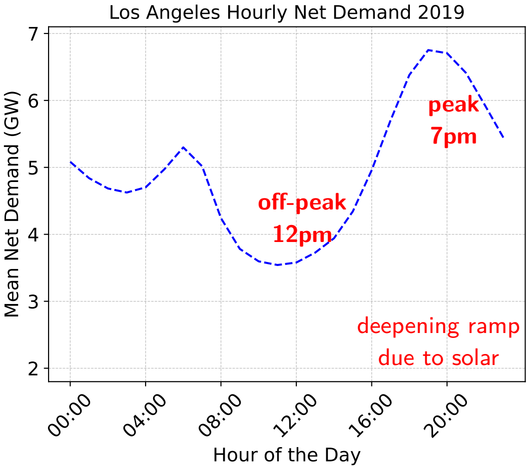
# California's market power mitigation attempt

> *"None of these storage resources are currently subject to market power mitigation, and the CAISO believes that it is important to develop mitigation measures to manage market power given the rapidly growing number and influence of energy storage resources."*
>
> **— California's Storage Default Energy Bid Initiative**

▸▸ California Report ▸▸ California Battery Bids ▸▸ 2000-01 Crisis

# Understanding demand dynamics over a day

$$\text{net demand} = \underbrace{\text{system demand}}_{\substack{\text{constant}\\ \text{year-on-year}}} - \underbrace{\text{renewables}}_{\substack{\text{increasing}\\ \text{year-on-year}}} = \text{dispatchable resources}$$



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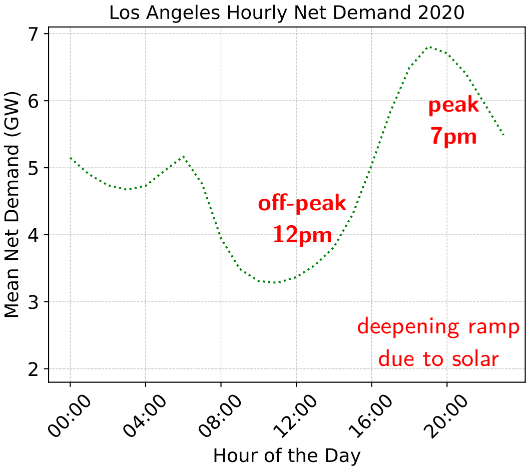

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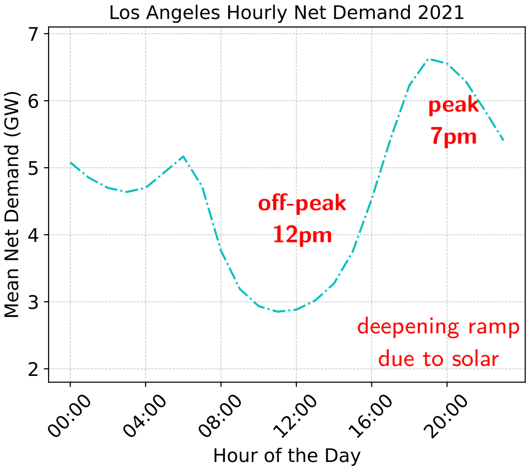

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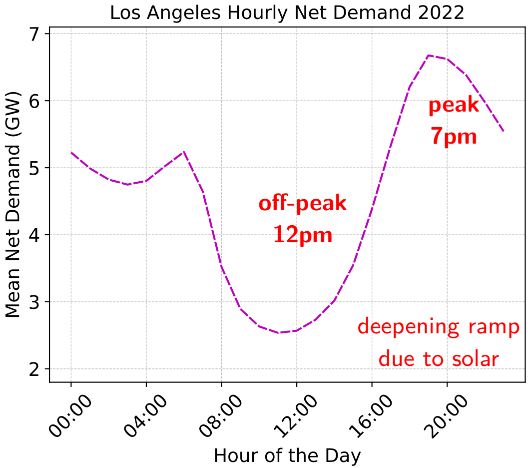

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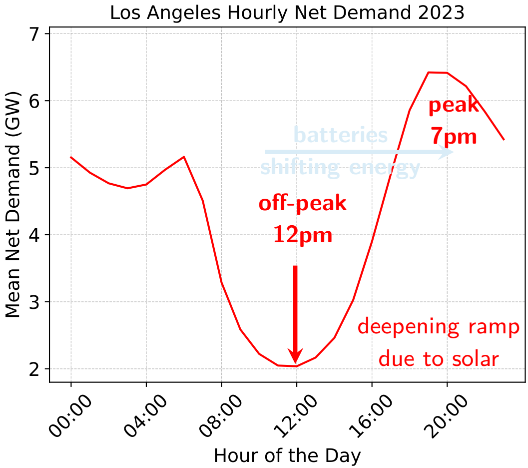

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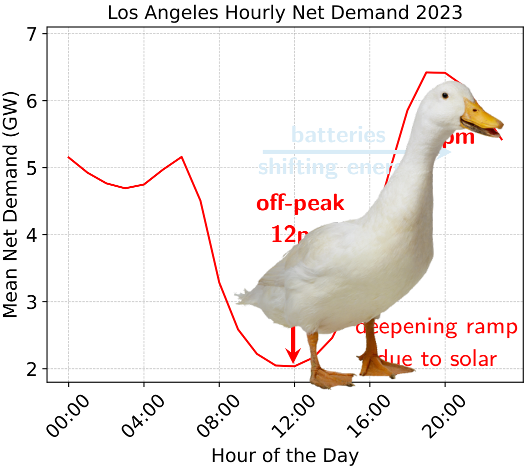

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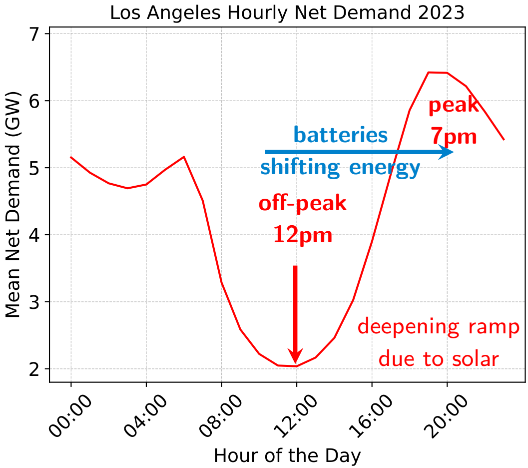

# Research Questions

**How do batteries operate in electricity markets?**

**How does the strategic behavior of decentralized batteries distort decisions compared to centralized batteries?**

**What is the impact of strategic behavior on system performance?**

## Outline of the Talk

- Electricity markets are complex → **tractable analytical model**
- Identify 3 types of distortions
  - quantity withholding
  - shift from day-ahead to real-time
  - reduction in real-time responsiveness
- Quantify the loss resulting from strategic behavior
  - Price of Anarchy is nontrivial but **bounded**
  - Calibration with **real data** from California and Texas
- Analyze competition and market power mitigation measures
- Discuss extensions of the model

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# Main Features of the Model

- two-stage market clearing
- heterogeneity in generator ramp speeds and costs
- duck curve net demand trend (peak and off-peak)
- demand stochasticity and correlation

3 entities: $\underbrace{\text{net demand, conventional generators,}}_{\text{non-strategic}}$ $\underbrace{\text{batteries}}_{\text{strategic}}$

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Spoiler Alert: **Battery market power is bounded!**

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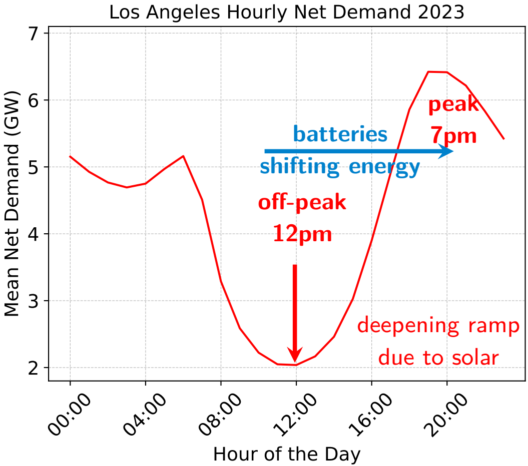
# Two-Period Model Captures Duck Curve Arbitrage

$$(D_{\mathsf{peak}}, D_{\mathsf{off}}) \sim \pi$$



# Electricity Markets Clear in Two Stages

**Day-Ahead Market (DA)**

**based on forecast**

(1) forward market reduces uncertainty

(2) slow generators take time to start and ramp up

**financially but NOT physically binding**

**Real-Time Market (RT)**

**based on realized demand**

demand must equal supply at all times

**financially AND physically binding**

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# Two Types of Conventional Generators

Assume **two types** of conventional generators:

| | supply curve |
|---|---|
| "slow" (DA only, e.g. coal & nuclear) | $G_s(p)$ |
| "fast" (DA + RT, e.g. gas) | $G_f(p)$ |

(mass of generators with cost below $p$)

Let $k_f$ be the share of fast generators. We assume ▸ California's supply stack

$$G_s(p) = (1 - k_f)G(p) \quad \text{and} \quad G_f(p) = k_f G(p).$$

$G(\cdot)$ is the total supply curve: price $\rightarrow$ quantity.

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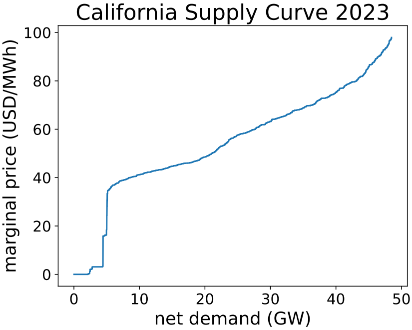
# Shape of the Supply Curve

Assume $G^{-1}(d) = \alpha + \beta d$ where $\alpha, \beta \geq 0$.

Linearity assumption: Sioshansi (2010, 2014), Ito and Reguant (2016).




**Linearity captures first-order features.**

We can also derive results under convex supply curves. ⏩ formulas

⏩ California's supply stack

## Electricity Markets Clear in Two Stages

**Day-Ahead Market (DA)** → **Real-Time Market (RT)**

$T = 2$ periods, peak and off-peak

**demand** $\underbrace{\mathbb{E}[D_{\text{peak}}], \mathbb{E}[D_{\text{off}}]}_{\text{DA demand (forecast)}}$ $\underbrace{D_{\text{peak}}, D_{\text{off}}}_{\text{RT demand (realized)}}$

**decisions** $z^{DA}_{\text{peak}}, z^{DA}_{\text{off}}$ $\underbrace{z^{RT}_{\text{peak}}(D_{\text{peak}}), z^{RT}_{\text{off}}(D_{\text{peak}}, D_{\text{off}})}_{\text{depending on realized demand history}}$

Discharge ($z > 0$) or charge ($z < 0$)

state-of-charge constraints: $z^{DA}_{\text{peak}} + z^{DA}_{\text{off}} = 0$ and $z^{RT}_{\text{peak}}(D_{\text{peak}}) + z^{RT}_{\text{off}}(D_{\text{peak}}, D_{\text{off}}) = 0$. ▸▸ battery net position data

# The Battery Decides Discharges $z$ in DA and RT

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# Market Clearing: Day-Ahead (DA) + Real-Time (RT)

**battery decisions** $z_t^{DA}, z_t^{RT}(\cdot)$ **affect prices** $p_t^{DA}, p_t^{RT}(\cdot)$

For each time period $t \in \{\text{peak}, \text{off}\}$,

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supply from "slow" — supply from "fast" — net demand − battery discharge

SAME — DIFFERENT

Let $k_f$ be the share of fast gens. Write $G_s(p) = (1 - k_f)G(p)$ and $G_f(p) = k_f G(p)$.

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# Battery Decisions in Three Regimes

battery decisions ($z$) $\rightarrow$ prices ($p$) $\rightarrow$ generation cost | battery profit

**No Battery (NB)**

"Status quo" benchmark.

**Centralized Battery (CN)**

Minimizing generation cost. expressions

**Decentralized Battery (DCN)**

Maximizing battery profit. expressions

# Generation Cost From DA+RT Supply Curves

▸▸ Three Regimes

Slow generators clear in DA at price $p_t^{DA}$.

Fast generators clear in RT at price $p_t^{RT}$.

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# Battery Profit - from Energy Arbitrage



profit = price × quantity for each time period, for DA and RT

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## Outline of the Talk

- Electricity markets are complex $\rightarrow$ **tractable analytical model**
- Identify 3 types of distortions
  - quantity withholding
  - shift from day-ahead to real-time
  - reduction in real-time responsiveness
- Quantify the loss resulting from strategic behavior
  - Price of Anarchy is nontrivial but **bounded**
  - Calibration with **real data** from California and Texas
- Analyze competition and market power mitigation measures
- Discuss extensions of the model

# Results: Battery Behavior

Both CN and DCN are **convex infinite-dimensional opt problems**.

## Centralized Battery Discharge (CN)

$$z^{DA}_{\text{peak}} = \frac{1}{2}(\mu_{\text{peak}} - \mu_{\text{off}})$$

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centralized battery
equates net demands
from two periods
**BUT THEN**
two periods have the
same price
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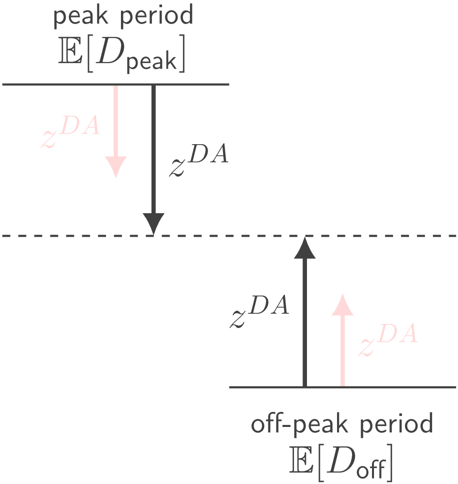


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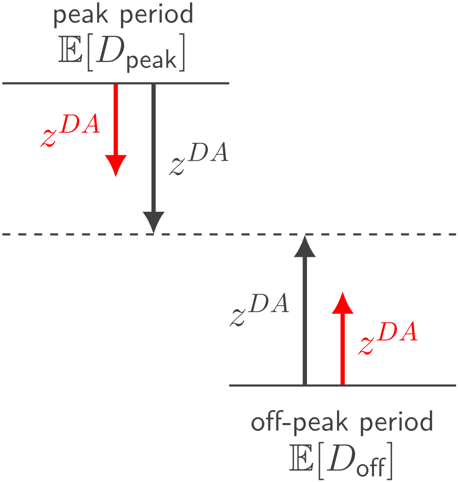


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# Distortion 2: Shift from Day-Ahead to Real-Time

▸▸ QUANTIFY ▸▸ NON-GRAPHICAL INTUITION

**battery hides capacity in DA and makes it available later in RT**

Simplest case: no randomness, identical markets with price function $P(\cdot)$.

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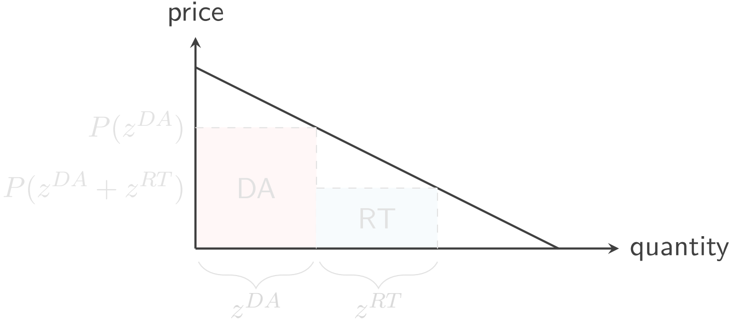

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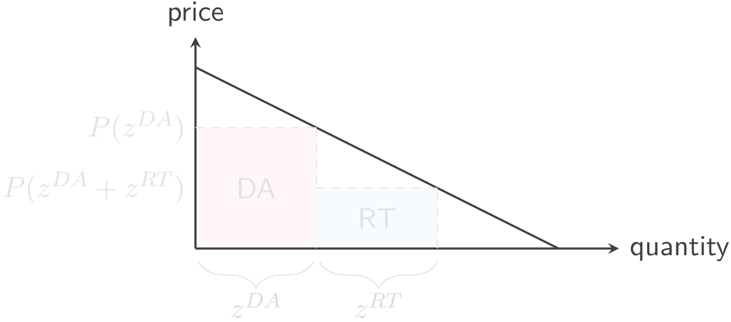

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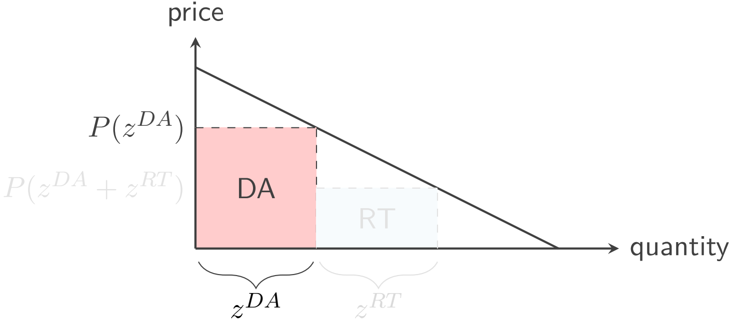

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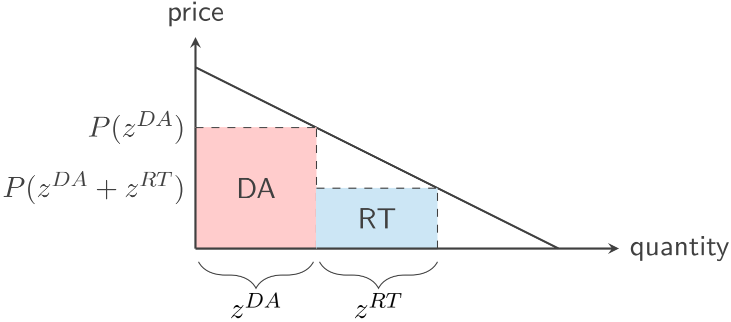

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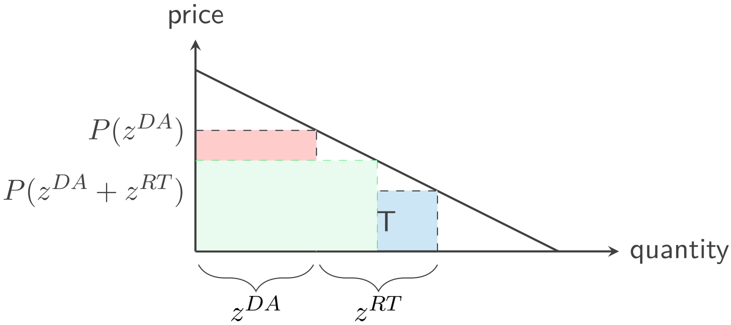

# Distortion 2: Shift from Day-Ahead to Real-Time

## Centralized Battery Discharge (CN)

$$z^{DA}_{\text{peak}} = \frac{1}{2}(\mu_{\text{peak}} - \mu_{\text{off}})$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \mathbf{0}(\boldsymbol{\mu}_{\text{peak}} - \boldsymbol{\mu}_{\text{off}}) + \frac{1}{2}\left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \frac{1}{2}\left(\mu_{\text{off}|D_{\text{peak}}} - \mu_{\text{off}}\right)$$

## Decentralized Battery Discharge (DCN)

$$z^{DA}_{\text{peak}} = \frac{(2-k_f)}{2(4-k_f)}(\mu_{\text{peak}} - \mu_{\text{off}})$$

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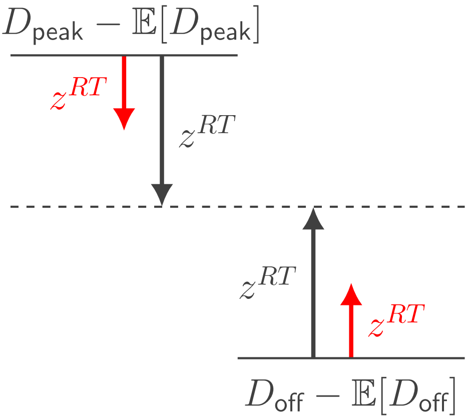
quantity withholding, **shift from DA to RT**, reduction in RT responsiveness

# Distortion 3: Reduction in Real-Time Responsiveness

⏩ QUANTIFY

**battery responds less to the higher-than-forecast realized demand**

$\approx$ RT-withholding (versus Distortion 1 = DA-withholding)



# Distortion 3: Reduction in Real-Time Responsiveness

$$(D_{\text{peak}} - \mu_{\text{peak}}) - z_{CN} = \mathbb{E}[z_{CN} + (D_{\text{off}} - \mu_{\text{off}})|D_{\text{peak}}]$$
$$\Rightarrow z_{CN} = \tfrac{1}{2}\left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \tfrac{1}{2}\left(\mu_{\text{off}|D_{\text{peak}}} - \mu_{\text{off}}\right).$$

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# Results: Generation Costs

Define the Price of Anarchy (PoA) as an incentive misalignment metric:

$$\text{PoA} = \frac{\text{GenCost(NoBattery)} - \text{GenCost(Centralized)}}{\text{GenCost(NoBattery)} - \text{GenCost(}\textbf{De}\text{centralized)}}$$

PoA $\geq 1$. Lower PoA means better alignment.

**Theorem**

Assume the demand is jointly normal, then **for every market**,

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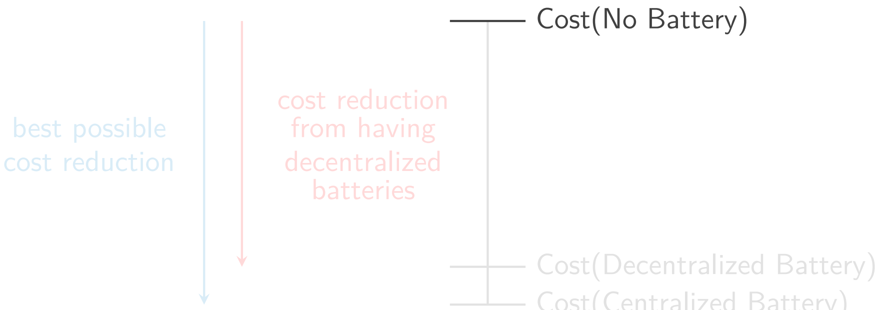

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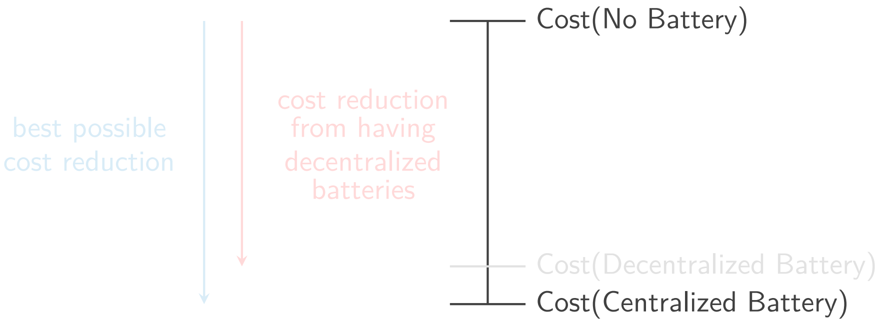

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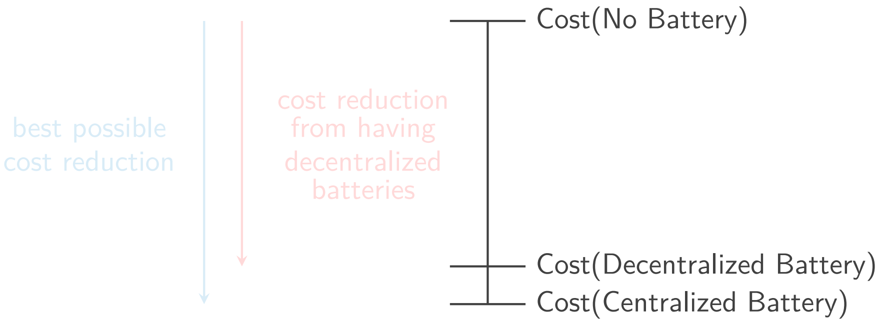

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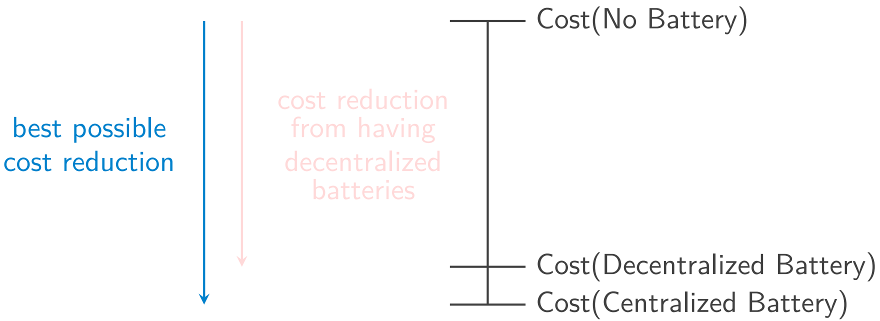

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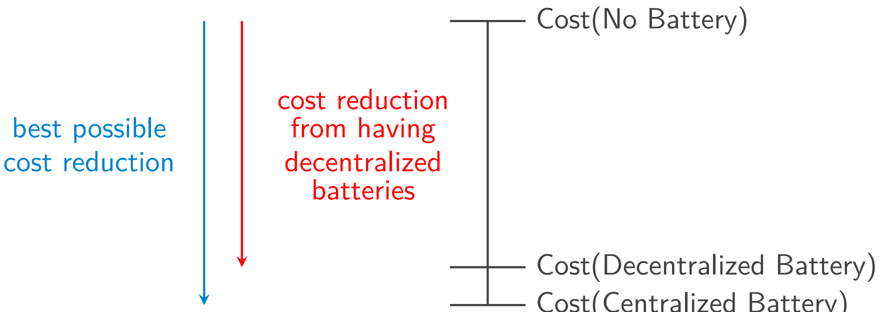

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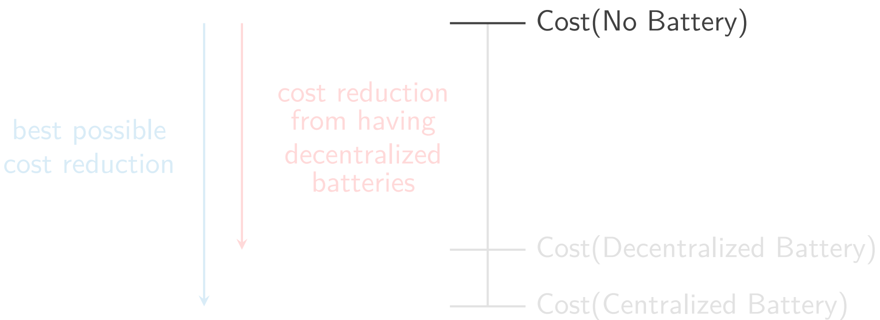

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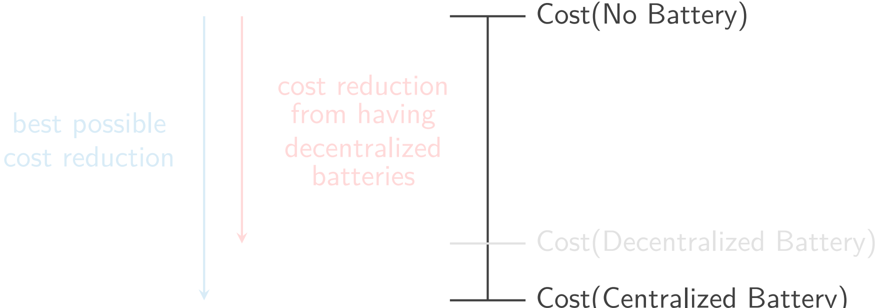

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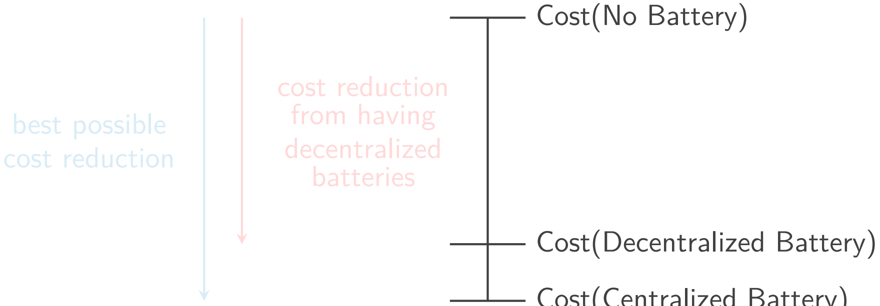

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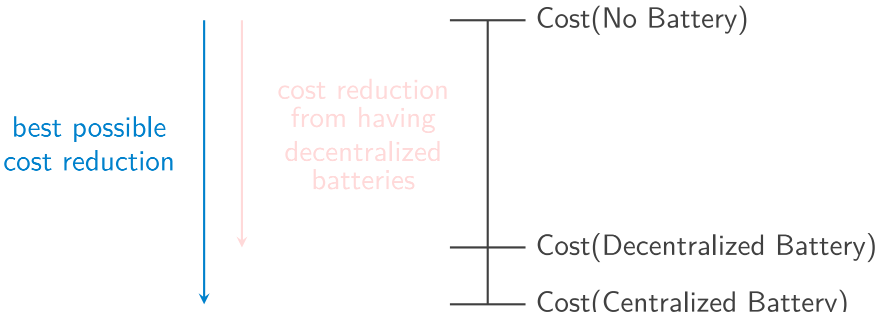

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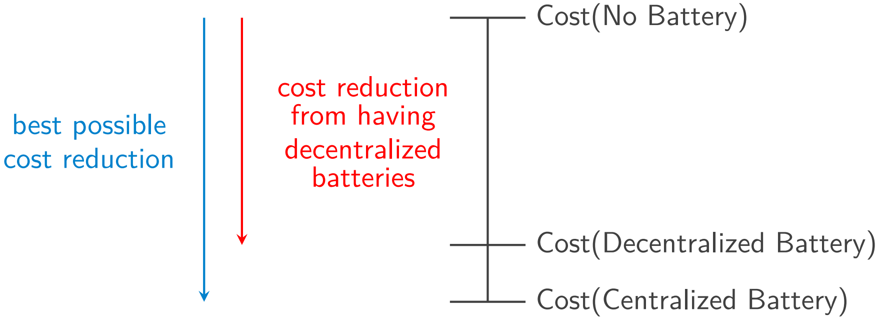

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# Impact of Competition

We now consider $n$ big batteries in Cournot competition.

We can derive battery strategies in closed form. formulas

**Theorem**

PoA is decreasing in $k_f$ and

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and the bounds are tight.

PoA decreases to 1 at a rate of $1/n^2$ worst case, $1/n^4$ best case.

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The shift from DA to RT is easiest to observe for regulators.

California's regulators note that ⏩ graphs

- in DA, battery bids $\gg$ clearing prices
- in RT, battery bids $\approx$ clearing prices

so batteries avoid being scheduled in DA.

The regulator can:

- require that batteries **discharge zero in expectation in RT**
- introduce **virtual bidders** (to arbitrage DA and RT)

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We can **subsidize battery discharge** if it has positive externalities

$\approx$ "production tax credit" for batteries

The regulator pays $sz$ for a discharge of $z$.

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# Extensions

- battery capacity ⏩ details
- battery investments and operations ⏩ details
- convex supply curves ⏩ details
- battery inefficiency ⏩ details
- multiple time periods ⏩ details

⏩ Takeaways ⏩ Main Model Features

## Takeaways

- tractable analytical model
- 3 forms of distortions
  - quantity withholding
  - shift from day-ahead to real-time
  - reduction in real-time responsiveness
- loss from market power is nontrivial but bounded
- competition is effective
- market power mitigation measures backfire
- building block for future work

# Takeaways

- tractable analytical model
- 3 forms of distortions
  - quantity withholding
  - shift from day-ahead to real-time
  - reduction in real-time responsiveness
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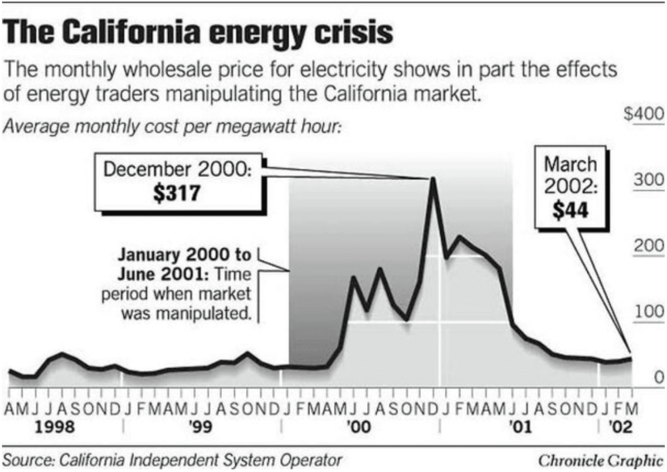
# Appendix

# Regulators concerned about battery market power

*"This short-term strategic rebidding to capitalise on market conditions had the effect of exacerbating high prices. Again, while this behaviour may not be a breach of the rules, the ability of these* ***batteries*** *to increase price through these rebidding strategies highlights the market power that participants may be able to exercise at certain times."*

**— Australian Energy Regulator**

# 2000–01 California Energy Crisis ⏩ MAIN



# Shift from DA to RT in California ⏩ MAIN ⏩ MITIGATION

**Figure 2.3.1 Hourly average day-ahead bids and nodal prices (by quarter)**

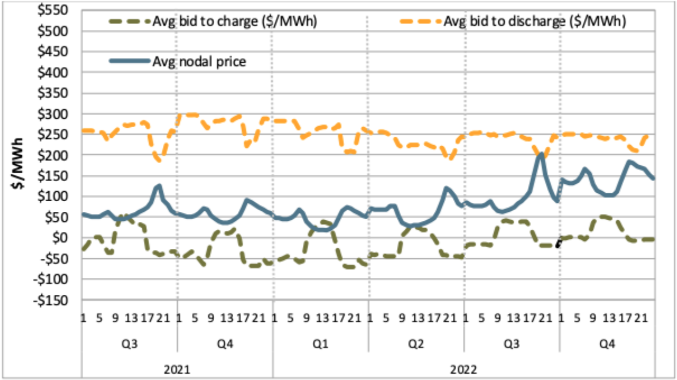



Figure: Day-Ahead discharge bid $\gg$ price (avoid DA scheduling)

# Shift from DA to RT in California

MAIN MITIGATION

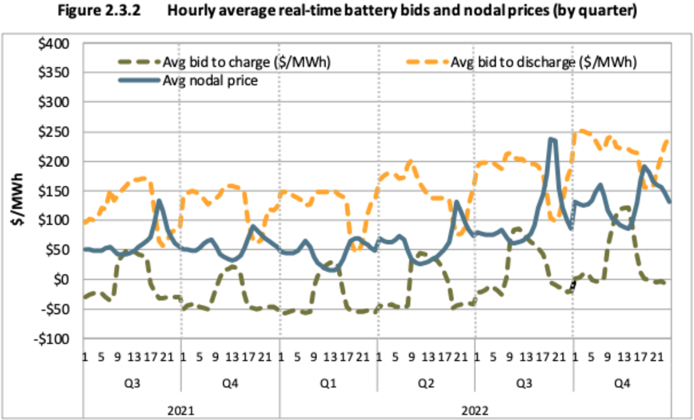



Figure: Real-time discharge bid ≈ price (batteries suddenly show up in RT)

# Suggestive Evidence of Battery Withholding

MAIN

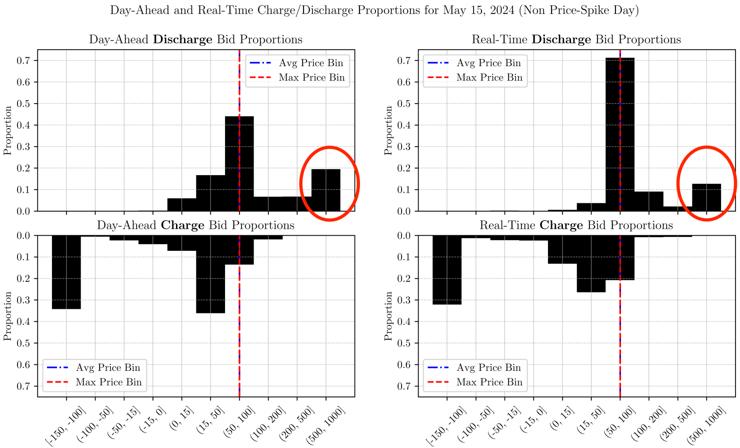



Figure: A lot of very high bids (above $500/MWh) even on a normal day ...

# Suggestive Evidence of Battery Withholding

▸▸ MAIN

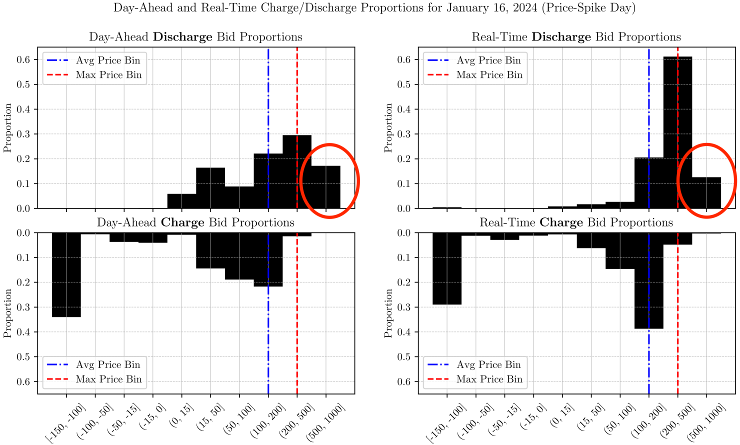



Figure: Fewer high bids on a price-spike day! Clear more, withhold less.

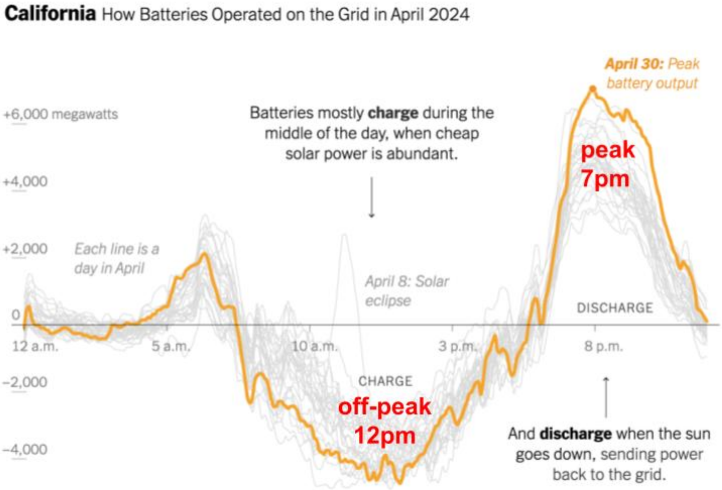
# Batteries Typically Have Negligible Net Daily Discharge ⏩ MAIN

| Year | Battery Net Position (% of Total Capacity) |
|---|---|
| 2021 | 1.0% |
| 2022 | 2.9% |
| 2023 | 1.1% |

Table: battery net position = mean absolute daily discharge over the year

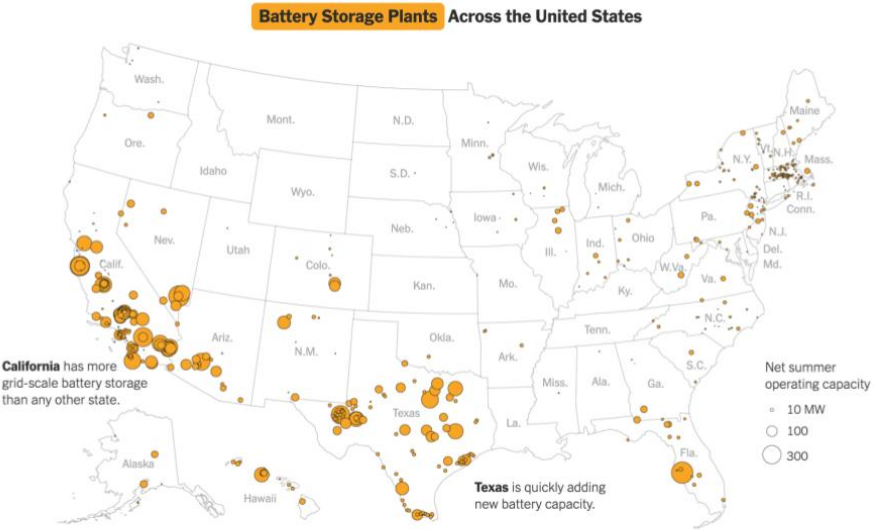
The daily charge cycle of batteries is by design: less than 7% of installed storage have duration exceeding 4 hours (NREL).

# Battery Operations

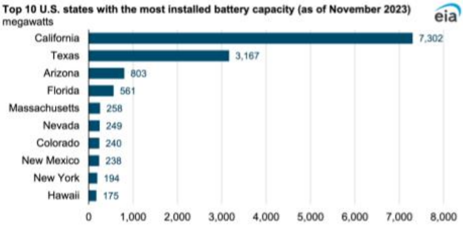



New York Times (2024)

# Battery Locations ⏩ MAIN



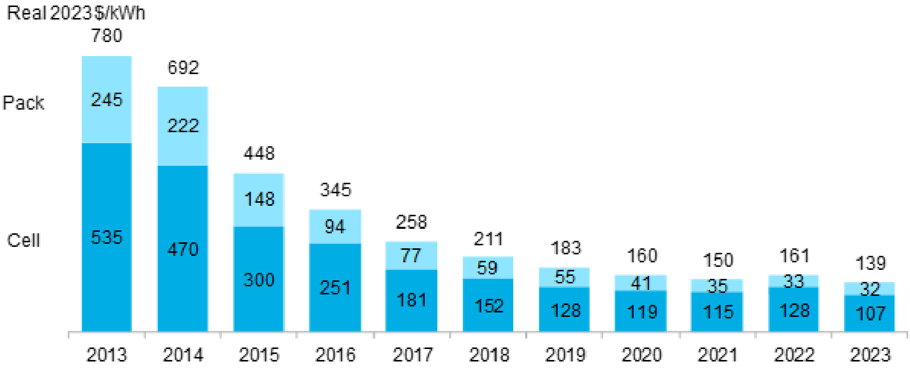
# The Growth of Battery Capacity ⏩ MAIN

# Lithium-Ion Battery Pack $139/kWh

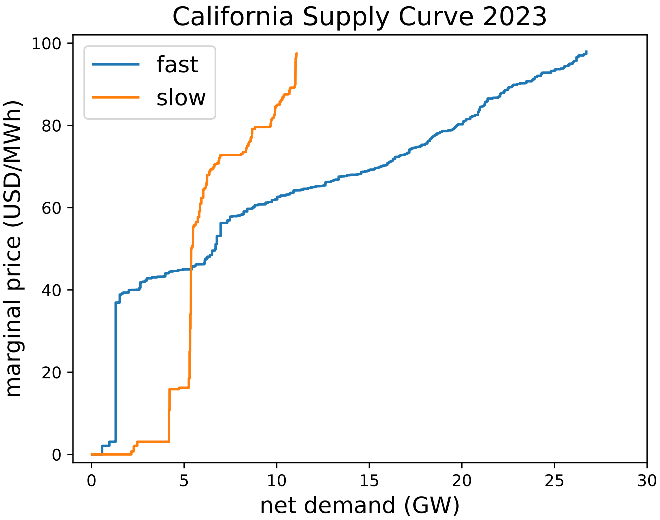
MAIN

**Figure 1: Volume-weighted average lithium-ion battery pack and cell price split, 2013-2023**




*Source: BloombergNEF. Historical prices have been updated to reflect real 2023 dollars. Weighted average survey value includes 303 data points from passenger cars, buses, commercial vehicles, and stationary storage.*

# California's Supply Stack: Fast vs Slow ⏩ MAIN



# Distortion 1: Quantity Withholding

▸▸ INTUITION

$$
\begin{aligned}
\text{quantity withholding} &= 1 - \frac{\text{total DCN discharge}}{\text{total CN discharge}} \\
&= 1 - \frac{\left(z^{DA}_{\text{peak}} + \mathbb{E}[z^{RT}_{\text{peak}}]\right)_{DCN}}{\left(z^{DA}_{\text{peak}} + \mathbb{E}[z^{RT}_{\text{peak}}]\right)_{CN}} \\
&= \frac{2 - k_f}{4 - k_f}
\end{aligned}
$$

(decreasing in $k_f$)

more fast generators $\Rightarrow$ less price impact $\Rightarrow$ less quantity withholding

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INTUITION

$$\text{shift from DA to RT} = \frac{\text{RT DCN discharge}}{\text{DA+RT DCN discharge}}$$

$$= \frac{\left(\mathbb{E}[z^{RT}_{\text{peak}}]\right)_{DCN}}{\left(z^{DA}_{\text{peak}}\right)_{DCN} + \left(\mathbb{E}[z^{RT}_{\text{peak}}]\right)_{DCN}}$$

$$= \frac{k_f}{2}$$

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Suppose the battery has 10 MWh to sell in DA and RT.

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▸▸ INTUITION

reduction in RT responsiveness

$$= 1 - \frac{\text{part of } z^{DCN}_{\text{peak}} \text{ depending on } D_{\text{peak}}}{\text{part of } z^{CN}_{\text{peak}} \text{ depending on } D_{\text{peak}}}$$

$$= 1 - \frac{\frac{1}{4}\left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \frac{1}{4}\left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}{\frac{1}{2}\left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \frac{1}{2}\left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}$$

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# Parameters for Los Angeles ⏩ MAIN

| Market | $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ | $k_f$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|---|---|---|
| LA Q1 | 5.94 | 0.39 | 2.21 | 1.01 | 0.25 | 0.93 | 5.39 | 2.77 |
| LA Q2 | 5.49 | 0.62 | 1.14 | 0.55 | 0.47 | 0.93 | 5.39 | 2.77 |
| LA Q3 | 7.82 | 1.10 | 2.86 | 1.20 | 0.80 | 0.93 | 5.39 | 2.77 |
| LA Q4 | 6.40 | 0.68 | 1.95 | 0.94 | 0.42 | 0.93 | 5.39 | 2.77 |

Table: Parameter values for different markets and quarters in 2023

# Parameters for Houston ⏩ MAIN

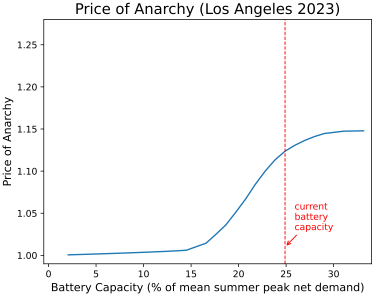
| Market | $\mu_1$ | $\mu_2$ | $\sigma_1$ | $\sigma_2$ | $\rho$ | $k_f$ | $\alpha$ | $\beta$ |
|---|---|---|---|---|---|---|---|---|
| HOU Q1 | 3.17 | 2.73 | 0.91 | 0.92 | 0.67 | 0.66 | 0.00 | 0.73 |
| HOU Q2 | 4.24 | 3.06 | 1.01 | 0.70 | 0.76 | 0.66 | 0.00 | 0.73 |
| HOU Q3 | 6.06 | 3.93 | 0.53 | 0.51 | 0.39 | 0.66 | 0.00 | 0.73 |
| HOU Q4 | 3.74 | 2.92 | 0.73 | 0.59 | 0.41 | 0.66 | 0.00 | 0.73 |

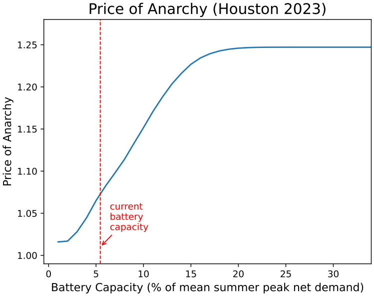
Table: Parameter values for different markets and quarters in 2023

# Battery Capacity ⏩ BACK

Assume that the monopoly battery has a **given capacity** $C$, so $z^{DA}_{\text{peak}} \leq C$ and $z^{DA}_{\text{peak}} + z^{RT}_{\text{peak}}(D_{\text{peak}}) \leq C$ for every $D_{\text{peak}}$.

No closed form, but can be approximated with SAA.



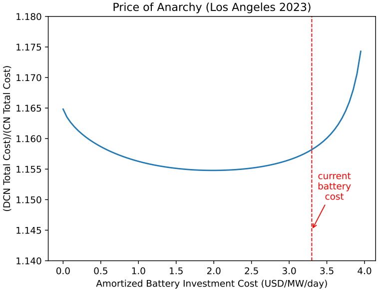



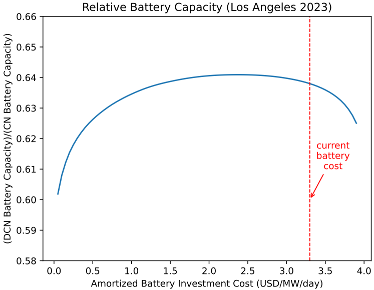
# Battery Investments and Operations ⏩ BACK

Assume a cost $c_{\text{inv}}$ per unit of battery capacity.

Decision variables: investment $C$ and operations $z^{DA}_{\text{peak}}, z^{RT}_{\text{peak}}(D_{\text{peak}})$.

$$\begin{aligned} \text{centralized}: &\quad \min \text{TotalCost} = c_{\text{inv}} C + \text{GenCost} \\ \text{decentralized}: &\quad \max \text{NetProfit} = \text{ArbitrageProfit} - c_{\text{inv}} C \end{aligned}$$



# Convex Supply Curves ⏩ BACK

Consider the supply curve $G^{-1}(d) = \alpha + \beta d + \gamma d^2$.

Technique: write

$$z(\gamma) = \underbrace{z(0)}_{\bar{z}} + \gamma \underbrace{z'(0)}_{\hat{z}} + O(\gamma^2)$$

for DA and RT and solve for $\bar{z}$ and $\hat{z}$ from FOCs via perturbations.

# Convex Supply Curves ⏩ BACK

## Centralized Battery Discharge

$$z^{DA}_{\text{peak}} = \bar{z}^{DA}_{\text{peak}} + O(\gamma^2)$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \bar{z}^{RT}_{\text{peak}}(D_{\text{peak}}) - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{2k_f}\frac{\gamma}{\beta} + O(\gamma^2)$$

## Decentralized Battery Discharge

$$z^{DA}_{\text{peak}} = \bar{z}^{DA}_{\text{peak}} - \frac{\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}}{2k_f(4-k_f)}\frac{\gamma}{\beta} + O(\gamma^2)$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \bar{z}^{RT}_{\text{peak}}(D_{\text{peak}}) + \left(\frac{\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}}{4(4-k_f)} - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{4k_f}\right)\frac{\gamma}{\beta} + O(\gamma^2)$$

If $\sigma^2_{\text{peak}} > \sigma^2_{\text{off}}$, DA goes down, and RT goes up by a lesser amount!

quantity withholding **up**, shift to RT **up**, reduction in responsiveness **same**

# Convex Supply Curves

BACK

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## Other Extensions

BACK

### Battery Inefficiency

Assume the round trip efficiency $\eta \in [0, 1]$. (Li-ion battery has $\eta \approx 0.9$.)

- Both CN and DCN battery strategies are in closed form. formulas
- All 3 types of distortions are **the same**.
- The bounds PoA $\in [9/8, 4/3]$ still hold.

### Multiple Time Periods

We can extend our framework to $T$ periods with $(D_1, D_2, \dots, D_T) \sim \pi$.
Decision variables:

$$z_1^{DA}, \dots, z_T^{DA}, z_1^{RT}(D_1), z_2^{RT}(D_1, D_2), \dots, z_T^{RT}(D_1, \dots, D_T)$$

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- Both CN and DCN battery strategies are in closed form. ⏩ formulas
- All 3 types of distortions are **the same**.
- The bounds PoA $\in [9/8, 4/3]$ still hold.

## Multiple Time Periods

We can extend our framework to $T$ periods with $(D_1, D_2, \ldots, D_T) \sim \pi$.
Decision variables:

$$z_1^{DA}, \ldots, z_T^{DA}, z_1^{RT}(D_1), z_2^{RT}(D_1, D_2), \ldots, z_T^{RT}(D_1, \ldots, D_T)$$

Both CN and DCN battery strategies are in closed form.

# Battery Behavior with Curvature Correction

MAIN

Assume $G^{-1}(d) = \alpha + \beta d + \gamma d^2$.

## Centralized Battery Discharge

$$z_{\text{peak}}^{DA} = \boxed{\frac{1}{2}(\mu_{\text{peak}} - \mu_{\text{off}})} + O(\gamma^2)$$

$$z_{\text{peak}}^{RT}(D_{\text{peak}}) = \frac{1}{2}(D_{\text{peak}} - \mu_{\text{peak}}) - \frac{1}{2}(\mu_{\text{off}|D_{\text{peak}}} - \mu_{\text{off}}) - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{2k_f}\frac{\gamma}{\beta} + O(\gamma^2)$$

## Decentralized Battery Discharge

$$z_{\text{peak}}^{DA} = \boxed{\frac{(2-k_f)}{2(4-k_f)}(\mu_{\text{peak}} - \mu_{\text{off}})} - \frac{\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}}{2k_f(4-k_f)}\frac{\gamma}{\beta} + O(\gamma^2)$$

$$z_{\text{peak}}^{RT}(D_{\text{peak}}) = \boxed{\frac{k_f}{2(4-k_f)}(\mu_{\text{peak}} - \mu_{\text{off}})} + \frac{1}{4}(D_{\text{peak}} - \mu_{\text{peak}}) - \frac{1}{4}(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}})$$
$$+ \left(\frac{\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}}{4(4-k_f)} - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{4k_f}\right)\frac{\gamma}{\beta} + O(\gamma^2)$$

# $n$ Batteries under Cournot Competition ⏩ MAIN

## Centralized Battery Discharge (Total)

$$z_{\text{peak}}^{DA} = \boxed{\frac{1}{2}(\mu_{\text{peak}} - \mu_{\text{off}})}$$

$$z_{\text{peak}}^{RT}(D_{\text{peak}}) = \frac{(D_{\text{peak}} - \mu_{\text{peak}}) - \left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}{2}$$

## Decentralized Battery Discharge (Each Battery)

$$z_{\text{peak}}^{DA} = \boxed{\frac{(n+1-k_f)}{2((n+1)^2 - nk_f)}(\mu_{\text{peak}} - \mu_{\text{off}})}$$

$$z_{\text{peak}}^{RT}(D_{\text{peak}}) = \boxed{\frac{k_f}{2((n+1)^2 - nk_f)}(\mu_{\text{peak}} - \mu_{\text{off}})} + \frac{(D_{\text{peak}} - \mu_{\text{peak}}) - \left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}{2(n+1)}$$

# $n$ Competing Batteries with Curvature Correction

▸▸ MAIN

## Centralized Battery Discharge (Total)

$$z^{DA}_{\text{peak}} = \boxed{\frac{1}{2}(\mu_{\text{peak}} - \mu_{\text{off}})} + O(\gamma^2)$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \frac{(D_{\text{peak}} - \mu_{\text{peak}}) - \left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}{2} - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{2k_f}\frac{\gamma}{\beta} + O(\gamma^2)$$

## Decentralized Battery Discharge (Each Battery)

$$z^{DA}_{\text{peak}} = \boxed{\frac{(n+1-k_f)}{2((n+1)^2 - nk_f)}(\mu_{\text{peak}} - \mu_{\text{off}})} - \frac{\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}}{2k_f((n+1)^2 - nk_f)}\frac{\gamma}{\beta} + O(\gamma^2)$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \boxed{\frac{k_f}{2((n+1)^2 - nk_f)}(\mu_{\text{peak}} - \mu_{\text{off}})} + \frac{(D_{\text{peak}} - \mu_{\text{peak}}) - \left(\mu_{2|D_{\text{peak}}} - \mu_{\text{off}}\right)}{2(n+1)}$$
$$+ \left(\frac{n\left(\sigma^2_{\text{peak}} - \sigma^2_{\text{off}}\right)}{2(n+1)((n+1)^2 - nk_f)} - \frac{\sigma^2_{\text{off}|D_{\text{peak}}}}{2(n+1)k_f}\right)\frac{\gamma}{\beta} + O(\gamma^2)$$

# Battery Inefficiency ⏩ main

Assume battery round trip efficiency of the circle $\eta \in [0,1]$.

## Centralized Battery Discharge

$$z^{DA}_{\text{peak}} = \frac{\eta^2 \mu_{\text{peak}} - \eta \mu_{\text{off}}}{1+\eta^2}$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \frac{\eta^2 \left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \eta \left(\mu_{\text{off}|D_{\text{peak}}} - \mu_{\text{off}}\right)}{1+\eta^2}$$

## Decentralized Battery Discharge

$$z^{DA}_{\text{peak}} = \frac{(2-k_f)}{(4-k_f)} \frac{\eta^2 \mu_{\text{peak}} - \eta \mu_{\text{off}}}{1+\eta^2}$$

$$z^{RT}_{\text{peak}}(D_{\text{peak}}) = \frac{k_f}{(4-k_f)} \frac{\eta^2 \mu_{\text{peak}} - \eta \mu_{\text{off}}}{1+\eta^2} + \frac{\eta^2 \left(D_{\text{peak}} - \mu_{\text{peak}}\right) - \eta \left(\mu_{\text{off}|D_{\text{peak}}} - \mu_{\text{off}}\right)}{2(1+\eta^2)}$$

# $T$ Time Periods ⏩ MAIN

## Centralized Battery Discharge

$$z_t^{DA} = \mu_t - \bar{\mu}$$

$$z_t^{RT} = \frac{(T-t)}{(T-t+1)}(d_t - \mu_t) - \sum_{t'=1}^{t-1} \frac{1}{(T-t'+1)}(d_{t'} - \mu_{t'})$$
$$- \frac{1}{(T-t+1)} \sum_{i=t+1}^{T} (\mu_{i|d_{1:t}} - \mu_i) + \sum_{t'=1}^{t-1} \sum_{i=t'+1}^{T} \frac{1}{(T-t')(T-t'+1)} (\mu_{i|d_{1:t'}} - \mu_i)$$

## Decentralized Battery Discharge

$$z_t^{DA} = \frac{(2-k_f)}{(4-k_f)}(\mu_t - \bar{\mu})$$

$$z_t^{RT} = \frac{k_f}{(4-k_f)}(\mu_t - \bar{\mu}) + \frac{(T-t)}{2(T-t+1)}(d_t - \mu_t) - \sum_{t'=1}^{t-1} \frac{1}{2(T-t'+1)}(d_{t'} - \mu_{t'})$$
$$- \frac{1}{2(T-t+1)} \sum_{i=t+1}^{T} (\mu_{i|d_{1:t}} - \mu_i) + \sum_{t'=1}^{t-1} \sum_{i=t'+1}^{T} \frac{1}{2(T-t')(T-t'+1)} (\mu_{i|d_{1:t'}} - \mu_i)$$