

# On the Robustness of Second-Price Auctions in Prior-Independent Mechanism Design

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## Motivation

- mech design: how to optimally sell things
- classical theory too detail-dependent, so we **relax the common prior assumption** (Wilson doctrine)

## Problem Formulation

- Optimize over direct mechanisms  $(x, p)$  selling one indivisible item to  $n$  buyers.
- mechanism is **prior-independent**
  - no need to know  $F$  (“detail-free” or “robust”)
  - performance guarantee over all  $F \in \mathcal{F}$
- We consider many dist classes on  $[0, 1]^n$ .
- **dominant strategy** IC+IR
  - each buyer need not know other buyers’ dists
- Objective = “regret” on revenue
- Benchmark = maximum possible revenue when valuation is known =  $\max(\mathbf{v})$ .

## Research question:

What is an optimal detail-free mechanism and how well can we perform?

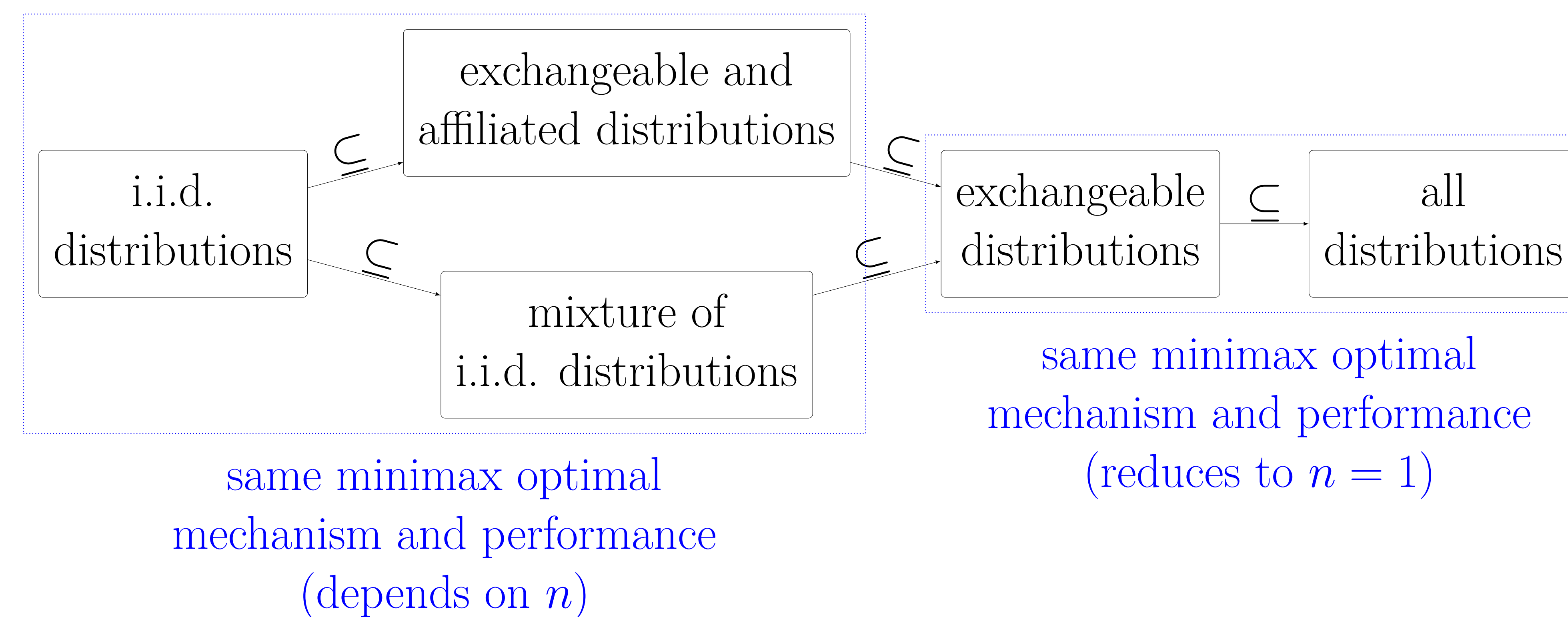
## Challenges

- The space of all mechanisms is large.
- The space of all bounded dists is large.
- The problem is **nonconvex** due to class restriction in  $\mathcal{F}$  e.g. i.i.d.

## Minimax Problem For Each Distribution Class $\mathcal{F}$

$$\min_{\text{mech } (x,p)} \max_{\mathbf{F} \in \mathcal{F}} \mathbb{E}_{\mathbf{v} \sim \mathbf{F}} \left[ \underbrace{\max(\mathbf{v})}_{\text{benchmark}} - \underbrace{\sum_{i=1}^n p_i(\mathbf{v})}_{\text{revenue}} \right]$$

## Main Result



**Second Price Auction with Random Reserve is minimax optimal across many distribution classes!**

## Theorem

Under the distribution class of {i.i.d., mixture of i.i.d., exchangeable and affiliated}, the minimax regret admits as an optimal mechanism a second-price auction with random reserve price with cumulative distribution  $\Phi_n^*$  on  $[r_n^*, 1]$  given by

$$\Phi_n^*(v) = \left( \frac{v}{v - r_n^*} \right)^{n-1} \log \left( \frac{v}{r_n^*} \right) - \sum_{k=1}^{n-1} \frac{1}{k} \left( \frac{v}{v - r_n^*} \right)^{n-1-k},$$

where  $r_n^* \in (0, 1/n)$  is the unique solution to

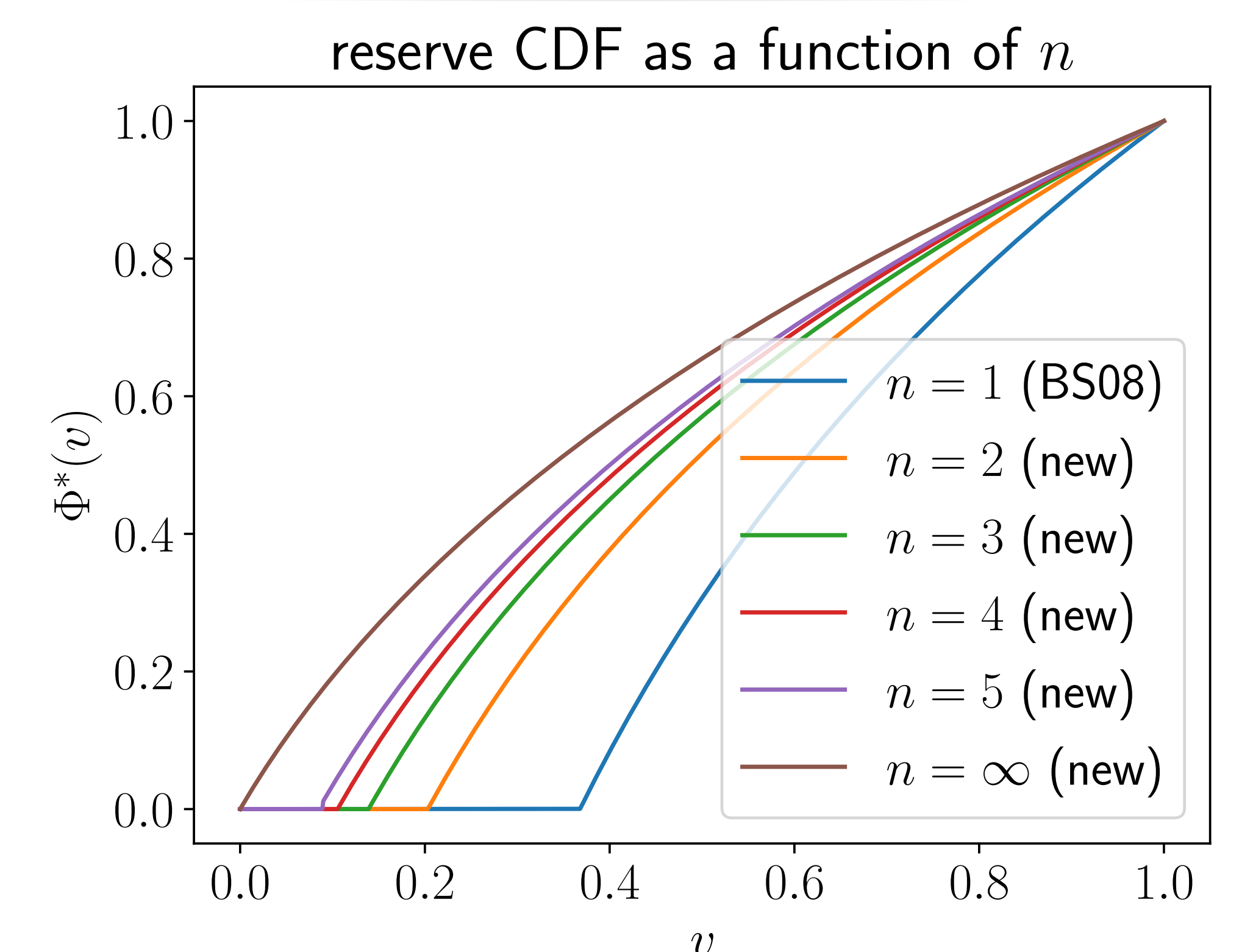
$$(1 - r^*)^{n-1} + \log(r^*) + \sum_{k=1}^{n-1} \frac{(1 - r^*)^k}{k} = 0.$$

## Our Approach

- **saddle point approach**: find  $m^*, F^*$ 

$$R(m^*, \mathbf{F}) \leq R(m^*, \mathbf{F}^*) \leq R(m, \mathbf{F}^*) \quad \forall m, \mathbf{F}$$
- Conjecture that  $m^* = \text{SPA}(\Phi^*)$ .
- $\Phi^*$  minimizes  $R(\Phi, F^*)$ , which is linear in  $\Phi \Rightarrow$  pins down  $F^*$
- $F^*$  maximizes  $R(\Phi^*, F)$  which is a function of  $F(\cdot) \Rightarrow$  pins down  $\Phi^*$

## Insights



$n$	OPT	SPA(0)	SPA( $r^*$ )
1	0.3679	1.0000	0.5000
2	0.3238	0.5000	0.4444
3	0.3093	0.4444	0.4219
4	0.3021	0.4219	0.4096
5	0.2979	0.4096	0.4019
10	0.2896	0.3874	0.3855
$\infty$	0.2815	0.3679	0.3679

- value of competition positive as  $n \rightarrow \infty$
- significantly outperforms benchmarks (no & optimal deterministic reserves)