**MINIMUM VALUE CALCULATION**

Over all real numbers, find the minimum value of a positive real number, y such that

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y = sqrt((x+6)^2 + 25) + sqrt((x-6)^2 + 121)
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**Solution:**

$$y = \sqrt{(x + 6)^2 + 25} + \sqrt{(x - 6)^2 + 121}$$

differentiating , using chain rule

$$\frac{d_y}{d_x} = \frac{2(x + 6)}{2\sqrt{(x + 6)^2 + 25}} + \frac{2(x - 6)}{2\sqrt{(x - 6)^2 + 121}}$$

$$\frac{d_y}{d_x} = \frac{(x + 6)}{\sqrt{(x + 6)^2 + 25}} + \frac{(x - 6)}{\sqrt{(x - 6)^2 + 121}}$$

To calculate minimum value we find the stationary points where, dy/dx = 0

$$\frac{x + 6}{\sqrt{(x + 6)^2 + 25}} + \frac{x - 6}{\sqrt{(x - 6)^2 + 121}} = 0$$

$$\frac{x + 6}{\sqrt{(x + 6)^2 + 25}} = \frac{6 - x}{\sqrt{(x - 6)^2 + 121}}$$

Squaring both sides,

$$\frac{(x + 6)^2}{(x + 6)^2 + 25} = \frac{(6 - x)^2}{(x - 6)^2 + 121}$$

$$\frac{x^2 + 12x + 36}{x^2 + 12x + 61} = \frac{x^2 - 12x + 36}{x^2 - 12x + 157}$$

Cross multiplying

$$\left(x^2 + 12x + 36\right)\left(x^2 - 12x + 157\right) = \left(x^2 - 12x + 36\right)\left(x^2 + 12x + 61\right)$$

Simplifying,

$$x^4 + 49x^2 + 1452x + 5652 = x^4 - 47x^2 - 300x + 2196$$

$$49x^2 + 47x^2 + 1452x + 300x + 5652 - 2196 = 0$$

$$96x^2 + 1752x - 3456 = 0$$

Solving the above equation quadratically, we get

$$x_1 = 1.796$$

$$x_2 = -20.046$$

Substituting x1 and x2 respectively, to get the minimum value of y

$$y = \sqrt{(x + 6)^2 + 25} + \sqrt{(x - 6)^2 + 121}$$

At x = 1.796,

$$y(1.796) = \sqrt{(1.796 + 6)^2 + 25} + \sqrt{(1.796 - 6)^2 + 121}$$

y = 21.04

At x = -20.046

$$y(-20.046) = \sqrt{(-20.0466 + 6)^2 + 25} + \sqrt{(-20.046 - 6)^2 + 121}$$

y = 43.18

Therefore, **x = 1.796** is a minimum and the minimum value is **y = 21.04**