Information Theory | **Prof. I-Hsiang Wang**

Lecture Notes 1

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Information is the resolution of uncertainty.

1 Source channel separation

Shannon came with the brilliant idea to **separate** the process of communication into two parts: *source* and *channel*. Moreover, Shannon proved that the scheme is **without loss of optimality**! Also, Shannon use **bits** as an universal currency in the channel.

- Source: Compress the source code and remove redundancy.
- Channel: Combat with the noise.

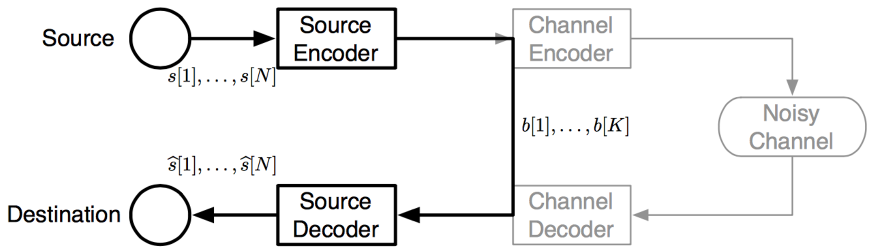
Also, Shannon use **random process** to simulate the process in source terminal. The scheme is an **average-case** paradigm.On the other hand, the channel terminal consider an uniform source code, which has no redundancy. (more details later) The concern here is **noise**, which is also simulated by random process.

The issues we concerned in this model is the **fundamental limits** for a given random process. (source and noise) The reason why Shannon's idea is so brilliant is that it starts with simplify the framework and separate the tasks without loss of optimality.

2 Source coding (Data compression)

Here, we remove the channel part in our framework. Namely, we do not consider the noise in the communication and focus on the removal of redundancy. There are two main issues in source coding: *redundancy* and *uncertainty*. Redundancy refers to the different likeliness of appearance for different symbols and uncertainty means that what we deal with is a random process.

To remove redundancy and face we the uncertainty, we have to construct a general scheme in the source terminal. And this consists of two parts: source encoding and source decoding.



Basically, what happens here is to encode the length N source code into a length K bit stream and then recover it back to a estimated source code with length N. Intuitively, we want to compress the original length N data into a K long big stream in a lossless way. (we may consider the lossy case in the near future) A question immediately appears:

$$What's\ the\ minimum\ K\ we\ can\ achieve?$$

With some observation, we can find out that $K = \Theta(N)$. As a result, it's useless to only minimize K since it will scale up with N. Thus, we focus on minimizing the term $R := \frac{K}{N}$, which is also called **compression rate**.

A quick answer for out concern is source cosing theorem, which gives us the fundamental limit of code rate in a lossless paradigm.

Theorem 1 *Source coding theorem*
As long as $R \geq H(S)$, which is the entropy rate of source, we can recover the source code in a lossless way.

3 Channel coding (Data transmission)

Here, we remove the source terminal and only focus on the transmission part of the communication. With this separation, we only need to consider 2 things: *uniform message* and *noise*. The reason why we model with uniform message is that we assume the redundancy has been removed in the source terminal. That way, we can focus on dealing with noise. As to the noise, we model it with a random process: $P(y|x)$. Details we be discussed later.

As a result, in channel coding, our goal is to **add redundancy to combat with noise**. Thus, we encode the input bit stream with length K into a compressed channel code with length N and then decode it back to recovered string with length K. A question immediately comes up:

$$What's\ the\ minimum\ N\ we\ can\ achieve?$$

Similar to source coding, with some non-trivial analysis, we will find out that $N = \Theta(K)$, which tells us that we should focus on maximizing the term: $R := \frac{K}{N}$, which is also called **code rate**.

Also, there's a quick answer for the maximization of R in lossless channel:

Theorem 2 *Channel coding theorem*
The code rate $R < C$, which is the capacity of the channel.

4 Conclusion

Shannon provides a simplification scheme for us to analyze a communication process. He divides it into two parts: source coding and channel coding. Such source-channel separation enables us optimize each part without loss of optimality in the whole process. As a result,

$$Source\ is\ responsible\ for\ compression;$$

$$Channel\ is\ responsible\ for\ combatting\ with\ noise.$$