Department of Statistics

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Dear Editors of *Annals of Statistics*,

I would like to submit the manuscript, "Subspace Estimation with Dimension and Variable Selection in Sufficient Dimension Reduction" by Zeng, myself and Zhang, for possible publication in *Annals of Statistics*.

Sufficient dimension reduction (SDR) methods target at finding lower-dimensional representations of a multivariate predictor $\mathbf{X} \in \mathbb{R}^p$ such that all the information about the conditional distribution of $Y \mid \mathbf{X}$ is preserved. The reduction is commonly achieved by projecting the predictor onto a low-dimensional subspace $\mathcal{S} \subseteq \mathbb{R}^p$. The smallest such subspace is known as the Central Subspace (CS), and is the key parameter of interest for most SDR methods. In this article, we propose a unified and flexible framework for estimating the CS in high dimensions. Our approach generalizes a wide range of model-based and model-free SDR methods to high-dimensional settings, where the CS is assumed to involve only a subset of the predictors for interpretability. We formulate the problem as a quadratic convex optimization so that the global solution is feasible. The proposed estimation procedure simultaneously achieves the structural dimension selection and coordinate-independent variable selection of the CS. Theoretically, our method achieves both dimension selection and subspace estimation consistency under mild conditions. We also demonstrate the effectiveness and efficiency of our method with extensive simulation studies and real data examples.

We confirm that this manuscript has not been published elsewhere and is not under consideration by another journal. Many thanks in advance for your consideration!

Sincerely,

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Enclosures