Problem Diary

Jerry Hong $z5479221$

November 28, 2024

Problem Set #7

Very very hard problem.

The intuition for DP was quite clear however the recurrence was not very obvious to me.

I was confused about the state for a very long time, until I realised (And Laeque also hinted) that the condition for the continuation of the game was contingent upon a monotonically increasing selection of numbers.

This means we can reduce the state to just keeping track of two things, the turn number (how many number we have picked), and the last number we pick.

Defining the subproblem: $dp(x, y)$ denoting the probability (kept as an integer) of winning, have picked number x as last number, in the yth turn.

The recurrence is thus as follows:

$$dp(x, y) = \text{P(picking another x)} + \sum_{k=x+1}^{n} freq(k) \times dp(k, y + 1)$$

This represents two disjoint possibilities:

1. Picking another of the same number at the yth turn and winning
2. Picking a higher number, and winning next turn

Calculating the probability of picking another of the same number will require us to keep track of frequency of all numbers. To speed this up, we can keep a cumulative sum of the following expression:

$$sum[x][y] = \sum_{k=x}^{n} freq(k) \times dp(k, y + 1)$$

Which can be turned into a recurrence

$$sum[x][y] = sum[x+1][y] + freq(x) \times dp(x, y+1)$$