FURTHER MATHEMATICS
HIGHER LEVEL
PAPER 1

Tuesday 18 February 2020

Name in block letters

2 hour 30 minutes

J	E	R	R	Y		J	I	A	N	G	

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- A graphic display calculator is required for this paper.
- A clean copy of the formula booklet is required for this paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

$\frac{129}{150} = 86\%$

Very good

$\boxed{7}$

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. Use L'Hôpital's rule to find $\lim_{x \to 0} \frac{\tan x - x}{1 - \cos x}$.

$\lim_{x\to 0} \tan x - x = \lim_{x \to 0} 1 - \cos x = 0.$

Apply the L'Hôpital's rule,

$L = \lim_{x\to 0} \frac{\sec^2 x - 1}{1+\sin x} = 0.$ $L = \lim_{x\to 0} \frac{\sec^2 x - 1}{\sin x}$

x

Apply again.

$L = \lim_{x \to 0} \frac{\sec x \tan x \cdot 2\sec x}{\cos x} = 0.$

/5

2. Let $n \in \mathbb{N}$ and define $n\mathbb{Z} = \{nx \mid x \in \mathbb{Z}\}$.

   (a) Simplify

      i. $(3\mathbb{Z} \cap 6\mathbb{Z}) \cup 18\mathbb{Z}$

      ii. $6\mathbb{Z} \cap 15\mathbb{Z}$.

   (b) Let $n_1, n_2 \in \mathbb{N}$. Giving reasons, state whether the following assertions are true or false.

      i. $n_1\mathbb{Z} \cap n_2\mathbb{Z} = m\mathbb{Z}$ for some $m \in \mathbb{N}$.

      ii. $n_1\mathbb{Z} \cup n_2\mathbb{Z} = m\mathbb{Z}$ for some $m \in \mathbb{N}$.

(a) i. $(3\mathbb{Z} \cap 6\mathbb{Z}) \cup 18\mathbb{Z} = 6\mathbb{Z} \cup 18\mathbb{Z} = 6\mathbb{Z}$. ✓

ii. $\operatorname{lcm}(6, 15) = 30$ $\quad 6\mathbb{Z} \cap 15\mathbb{Z} = 30\mathbb{Z}$. ✓

(b) i. $n_1\mathbb{Z} \cap n_2\mathbb{Z} = \operatorname{lcm}(n_1, n_2)\,\mathbb{Z}$ ✓

$m = \operatorname{lcm}(n_1, n_2)$.

There's always a m as $\operatorname{lcm}(n_1, n_2)$ always exists.

True

ii. False. when $\gcd(n_1, n_2) \neq n_1$ or n_2, there isn't an $m \in \mathbb{N}$

That satisfies the equation.

For instance, $2\mathbb{Z} \cup 3\mathbb{Z} \neq m\mathbb{Z}$ for all $m \in \mathbb{N}$.

✓

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3. The graph G with vertices P, Q, R, S, T has the following adjacency table.

	P	Q	R	S	T
P	0	1	0	1	2
Q	1	0	1	0	0
R	0	1	0	1	1
S	1	0	1	0	0
T	2	0	1	0	0

(a) Make a drawing of G illustrating that G is planar.

(b) Giving reasons, state whether or not G is

i. simple;

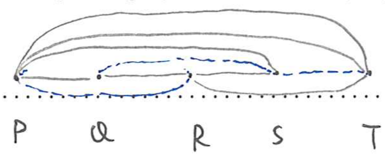
ii. connected;

iii. bipartite.

(c) Explain why G has an Eulerian trail but not an Eulerian circuit.

(d) Find the maximum number of edges that can be added to the graph G, not including loops or further multiple edges, whilst still keeping it planar.

(a)



(b) i not simple. 2 edges between P and T.

ii ~~connected,~~ no vertex has degrees 0. not connected.

iii. bipartite, the two parts are {P, R}, {Q, S, T}.

There's a path between any two vertices.

(c) T and R has odd degrees, so only Eulerian trail is present.

(d) K_5 is not planar, so it's impossible to connect all PR, QS, QT, and ST.

Q: is there another way?

But as in (a), blue edges between PR, QS, ST can be added, so this is as far as we can get.

We can add at most three more edges.

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4. (a) Consider the functions from $\mathbb{N}$ to $\mathbb{N}$ with rules $f(x) = \lfloor x/2 \rfloor$, $g(x) = x$, $h(x) = 1 + x$.

   i. Write down the function which is injective but not surjective.

   ii. Write down the function which is surjective but not injective.

   (b) Write down a function from $\mathbb{N}$ to $\mathbb{N}$ which is neither injective nor surjective.

   (c) If the function $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ with rule $f(x, y) = (2x + y, x + y)$ possesses an inverse find its rule, otherwise explain why no such inverse exists.

(a) i. $h(x) = 1 + x$. ✓

ii. $f(x) = \lfloor \frac{x}{2} \rfloor$. ✓

(b) $h \circ f(x) = 1 + \lfloor \frac{x}{2} \rfloor$. ✓

(c) f is bijective. ✓

$$\begin{cases} x' = 2x + y, \\ y' = x + y. \end{cases} \longrightarrow \begin{cases} x = x' - y' \\ y = 2y' - x' \end{cases}$$

$f^{-1}(x, y) = (x - y, 2y - x)$. ✓

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5. (a) Show that the vectors $\begin{pmatrix}1\\2\\3\end{pmatrix}$, $\begin{pmatrix}2\\3\\1\end{pmatrix}$, $\begin{pmatrix}5\\2\\5\end{pmatrix}$ form a basis for $\mathbb{R}^3$.

(b) Express the vector $\begin{pmatrix}12\\14\\16\end{pmatrix}$ as a linear combination of the above vectors.

(a) $M = \begin{pmatrix}1 & 2 & 5\\2 & 3 & 2\\3 & 1 & 5\end{pmatrix}$. $|M| = -30 \neq 0$.

They're basis for R^3.

(b). $\begin{pmatrix}12\\14\\16\end{pmatrix} = 3\begin{pmatrix}1\\2\\3\end{pmatrix} + 2\begin{pmatrix}2\\3\\1\end{pmatrix} + \begin{pmatrix}5\\2\\5\end{pmatrix}$, by inspection.

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6. Consider the two independent random variables X and Y where $X \sim \mathrm{Po}(3)$ and $Y \sim \mathrm{Po}(4)$.

   (a) Calculate $\mathrm{E}(2X + 7Y)$.

   (b) Calculate $\mathrm{Var}(4X - 3Y)$.

   (c) Calculate $\mathrm{E}(X^2 - Y^2)$.

(a) $E(2X+7Y) = 2E(X) + 7E(Y) = 2\times 3 + 7\times 4 = 34$

(b) $\mathrm{Var}(4X-3Y) = 16\,\mathrm{Var}(X) + 9\,\mathrm{Var}(Y) = 16\times 3 + 9\times 4 = 84.$

(c) $E(X^2-Y^2) = E(X^2) - E(Y^2)$

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$, Hence, $E(X^2) = 12$.

$\mathrm{Var}(Y) = E(Y^2) - [E(Y)]^2$. Hence, $E(Y^2) = 20$.

Therefore, $E(X^2 - Y^2) = 12 - 20 = -8$.

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7. Consider the system of equations $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 2 & -1 \\ 3 & 5 & -4 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \\ k \end{pmatrix}$.

(a) Find the rank of the coefficient matrix.

(b) Find the value of k for which the system has a solution.

(c) For this value of k determine the solution.

(a) $M = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 2 & -1 \\ 3 & 5 & -4 \\ 3 & 1 & 1 \end{pmatrix}$. $\operatorname{rref}(M) = \begin{pmatrix} 1 & 0 & \frac{3}{4} \\ 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. $\operatorname{rank}(M) = 2$. ✓

(b). $\left\{ \begin{array}{l} x - y + 2z = 5 \\ 2x + 2y - z = 3. \\ 3x + 5y - 4z = 1. \\ 3x + y + z = k. \end{array} \right.$ $\rightarrow$ $\left\{ \begin{array}{l} x = \frac{13}{4} - \frac{3}{4}z \\ y = -\frac{7}{4} + \frac{5}{4}z \\ z = z. \end{array} \right.$

$k = 3\left(\frac{13}{4} - \frac{3}{4}z\right) + \left(-\frac{7}{4} + \frac{5}{4}z\right) + z = 8.$ ✓

(c) from (b),

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{13}{4} \\ -\frac{7}{4} \\ 0 \end{pmatrix} + z \begin{pmatrix} -\frac{3}{4} \\ \frac{5}{4} \\ 1 \end{pmatrix}.$ ✓

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8. The function f is defined by $f(x) = e^x \cos x$.

   (a) Show that $f''(x) = -2e^x \sin x$.

   (b) Determine the fourth degree Maclaurin polynomial for $f(x)$.

   (c) By differentiating your polynomial, determine the cubic Maclaurin polynomial for $e^x \sin x$.

(a) $f'(x) = -e^x \sin x + e^x \cos x$

$f''(x) = -(e^x \cdot \cos x + e^x \sin x) + (e^x \cos x - e^x \sin x)$

$= -2e^x \sin x$.

(b) $f'''(x) = -2(e^x \sin x + e^x \cos x)$

$f''''(x) = -2[(e^x \sin x + e^x \cos x) + (e^x \cos x - e^x \sin x)]$

$= -2[2e^x \cos x]$.

$f(0) = 1$, $f'(0) = 1$, $f''(0) = 0$, $f'''(0) = -2$, $f''''(0) = -4$.

$P_4(x) = 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4$.

(c) $f'(x) = -e^x \sin x + f(x)$.

$e^x \sin(x) = f(x) - f'(x)$

$= (1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4) - (1 - x^2 - \frac{2}{3}x^3)$

$= 1 + x - \frac{1}{3}x^3 - \frac{1}{6}x^4 - 1 + x^2 + \frac{2}{3}x^3$

$= x + x^2 + \frac{1}{3}x^3 + R_4$.

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9. (a) The point $T(at^2, 2at)$ lies on the parabola $y^2 = 4ax$. Show that the tangent to the parabola at T has equation $x - ty + at^2 = 0$.

(b) The distinct points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$, where $p, q \neq 0$, also lie on the parabola $y^2 = 4ax$. If the line (PQ) passes through the focus show that the tangents to the parabola at P and Q intersect at the directrix.

(a) ~~$y = \pm\sqrt{2ax}$ $\quad y' = \frac{\sqrt{2a}}{\sqrt{2ax}} = \frac{1}{\sqrt{x}}$~~

~~For $T(at^2, 2at)$, $y' = \frac{1}{\,}$~~

Let $S(as^2, 2as)$. l_{ST}: $2x - (s+t)y + 2ast = 0$.

let S be infinitely close to T to get the tangent: $s = t$,

so $2x - 2ty + 2at^2 = 0$. $\quad x - ty + at^2 = 0$.

(b) L_{PQ}: $2x - (p+q)y + 2apq = 0$. pass through $(a, 0)$

$2a + 2apq = 0$, $\quad pq = -1$.

T_P: $x - py + ap^2 = 0$. $\Rightarrow$ $x = a$, so intersect at directrix

T_Q: $x - \frac{1}{p}y + a\frac{1}{p^2} = 0$.

$x = -a$ is

directrix.

$$\begin{cases} x - py + ap^2 = 0. & \rightarrow y = \dfrac{x + ap^2}{p} \\ x + \frac{1}{p}y + a\frac{1}{p^2} = 0 & \rightarrow y = -p\left(x + a\frac{1}{p^2}\right) \end{cases}$$

$x + ap^2 = -xp^2 - a$

$x(1+p^2) = -a(1+p^2)$

$x = -a.$

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10. (a) Write down the values of p for which the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges.

(b) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{2n^2+1}$.

(c) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

(a) $p > 1$.

(b) compare the limit to $\frac{1}{n^{\frac{3}{2}}}$.

$\frac{3n^2}{2n^2+1} = \frac{3}{2}$ as $n \to \infty$, so the limit and $\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ behaves the same.

since $p = \frac{3}{2} > 1$, it converges.

Therefore $\sum_{n=1}^{\infty} \frac{3\sqrt{n}}{2n^2+1}$ converges as well.

(c) Ratio Test.

$\left| \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} \right| = \left| \frac{n^n}{(n+1)^n} \right|$. $\frac{(n+1)^n}{n^n} = \left(1+\frac{1}{n}\right)^n = e$ as $n \to \infty$

so ratio $= \frac{1}{e} < 1$.

The series converges.

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11. The weights of peaches are normally distributed with mean 98 grams and standard deviation 16 grams.

(a) A shopkeeper places 100 randomly chosen peaches on a weighing machine. Find the probability that their total weight exceeds 10 kilograms.

(b) Find the minimum number of randomly selected peaches needed to ensure their total weight exceeds 10 kilograms with probability greater than 0.95.

(a) $A \sim N(100 \times 98, 100 \times 16^2)$.

$P(A > 10000) = \text{normalcdf}(10000, \infty, 9800, 160)$

$= 0.106$ (3s.f.)

(b) $B \sim N(98x, 256x)$.

$\frac{10000 - 98x}{16\sqrt{x}} = \text{invNorm}(0.05, 0, 1) = -1.645$ (4s.f.)

$x = 105$ (3s.f.)

the minimum number 105.

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12. The points A, B have coordinates $(-3,0)$, $(5,0)$ respectively. Consider the circle $\mathscr{C}$ with centre $(13,0)$ which is the locus of the point P where $PA : PB = k : 1$ for $k \neq 1$.

(a) Find the radius of $\mathscr{C}$.

(b) If M is a point on $\mathscr{C}$ and N is the x-intercept of $\mathscr{C}$ between A and B, prove $\angle AMN = \angle NMB$.

(a) $\frac{PA}{PB} = \frac{k}{1} = \frac{16-r}{r-8} = \frac{16+r}{r+8}$. $r = \pm 8\sqrt{2}$.

Hence, the radius is $8\sqrt{2}$. ✓

(b) Since M and N are both on the circle,

$\frac{MA}{MB} = k$, $\frac{NA}{NB} = k$.

According to the converse of the angle bisector theorem,

$\angle AMN = \angle NMB$. ✓

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13. (a) State Lagrange's theorem.

(b) Prove that every group of prime order is cyclic.

(c) The Abelian group G is generated by the distinct group elements a and b where $|a| = |b| = 3$.

i. What is the order of G?

ii. How many proper subgroups does G have?

finite group.

finite.

(a) The order of all proper subgroups of G divides the order of G.

(b) only 1 and $|G|$ divides group G if $|G|$ is prime.

the order of all elements divides the order of the finite group.

element with order 1 is the identity, while all others with order $|G|$ are generators. More explanation is needed

Hence, group of prime order is cyclic.

(c) i. $|G| = 3 = |a| = |b|$.

ii. Since $G = |a|$, $G = |b|$, then $a^{-1} = b$.

there're only 3 elements in group G: a, a^{-1}, e.

Therefore, G have no proper subgroups as the order is prime.

Not what I had in mind but perhaps the question is ambiguous.

(c). $G = \langle a, b \rangle$.

i. $G = \{a, a^2, b, b^2, e\}$. $|G| = 5$.

ii. 2 proper subgroups: $\{a, a^2, e\} \leq G$,

$\{b, b^2, e\} \leq G$.

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14. Consider the matrix $A = \begin{pmatrix} 11 & \sqrt{3} \\ \sqrt{3} & 9 \end{pmatrix}$.

(a) Find the eigenvalues and eigenvectors of A.

(b) The ellipse $\mathscr{E}$ has equation $X^T A X = 24$ where $X^T = (x \quad y)$.

i. Show that $\mathscr{E}$ can be rotated about the origin onto the ellipse $\mathscr{E}'$ having equation $2x^2 + 3y^2 = 6$.

ii. Find the acute angle through which $\mathscr{E}$ must be rotated to coincide with $\mathscr{E}'$.

(a) $\lambda^2 - 20\lambda + 96 = 0$

$\lambda_1 = 10 + \sqrt{10}$, $\lambda_2 = 10 - \sqrt{10}$

$\vec{v} = \begin{pmatrix} \sqrt{3} \\ 1-\sqrt{10} \end{pmatrix}$ $\quad \vec{v} = \begin{pmatrix} \sqrt{3} \\ 1+\sqrt{10} \end{pmatrix}$

(b) $\begin{pmatrix} -1-\sqrt{10} \\ 3\sqrt{3} \end{pmatrix}$ $\quad \begin{pmatrix} \sqrt{3} \\ 1+\sqrt{10} \end{pmatrix}$

(a) $\lambda^2 - 20\lambda + 96 = 0$.

$\lambda_1 = 8 \quad \vec{v} = \begin{pmatrix} -1/2 \\ \sqrt{3}/2 \end{pmatrix}$ $\qquad \begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$

$\lambda_2 = 12 \quad \vec{v} = \begin{pmatrix} \sqrt{3}/2 \\ +1/2 \end{pmatrix}$ $\qquad \begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{pmatrix}$

(b) $\left[(x \quad y)\begin{pmatrix} +1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & +1/2 \end{pmatrix}\right]\begin{pmatrix} 8 & 0 \\ 0 & 12 \end{pmatrix}\left[\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\right] = 24$.

$(x' \quad y')\begin{pmatrix} 8 & 0 \\ 0 & 12 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} = 24$.

$(x' \quad y')\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} = 6$.

ii. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$.

~~答~~ $\cos\theta = \frac{1}{2}$, $\theta = 60°$, anticlockwise.

15. Let $y = f(x)$ be the solution to the differential equation $y' + y\sec x = x(\sec x - \tan x)$ with $f(0) = 3$.

(a) Use Euler's method in table form with a step length of 0.1 to approximate $f(0.2)$.

(b) i. Determine the second degree Maclaurin polynomial for $f(x)$.

ii. Use this polynomial to approximate $f(0.2)$.

(c) i. Solve this differential equation to find $y = f(x)$.

ii. Hence determine which of the above two approximations for $f(0.2)$ is closer to the true value.

(a)

n	x_n	y_n	h	$h \cdot f'(x_n, y_n)$
0	0	3	0.1	-0.3
1	0.1	2.7	0.1	-0.262
2	0.2	2.438	0.1	—

$f(0.2) \approx 2.44$ (3s.f.) ✓

(b) i. $y'' = x(\sec x\tan x - \sec^2 x) + (\sec x - \tan x) - y'\sec x - y\cdot\sec x\tan x$

$f(0) = 3$, $f'(0) = -3$, $f''(0) = 4$.

$P_2(x) = 3 - 3x + 2x^2$. ✓

ii. $P_2(0.3) = 2.18$? $3 - 0.6 + 0.08 = 2.48$.

(c) i. $(\sec x + \tan x)y' + y(\sec^2 x + \tan x\sec x) = x$

$\int [(\sec x + \tan x)y]' = \int x\,dx$

$(\sec x + \tan x)y = \frac{1}{2}x^2 + c$

$y = \dfrac{\frac{1}{2}x^2 + c}{\sec x + \tan x}$

$c = 3$

$y = \dfrac{\frac{1}{2}x^2 + 3}{\sec x + \tan x}$

ii. $f(0.2) = 2.47$ (3s.f.)

(a) is closer.

✓

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Correction for February Block Week

Jerry Jiang

February 24, 2020

1. Question 1: when differentiating $1-\cos x$, I got wrong with the constant term and wrote $1+\sin x$. The correct steps afterwards should be:
$$L = \lim_{x\to 0} \frac{\sec^2 x - 1}{\sin x} = \lim_{x\to 0} \frac{\sec x \tan x \cdot 2\sec x}{\cos x} = 0.$$
2. Question 3, (b), ii: My explanation for connected graph was "no vertex has degrees 0" and it didn't effectively express my idea. The correct explanation should be: there is a path between any two vertices in the graph.
3. Question 9, (b): I messed up the sign of $\frac{1}{p}y$ and memorized the wrong equation for the directrix for the parabola, it should be $x=-a$ instead of $x=a$. Therefore, the correct solution is:
$$\begin{cases} T_P : x - py + ap^2 = 0, \\ T_Q : x + \frac{1}{p}y + a\frac{1}{p^2} = 0. \end{cases} \quad \Rightarrow x = -a.$$
4. Question 13:
   (a) I thought infinite groups don't have orders but in fact their order is infinity. Hence, the Lagrange's theorem is: the order of all subgroups of the finite group G divides $|G|$.
   (b) I didn't put in enough explanation for this question. The correct solution should be: the order of all elements divides the order of the finite group, so when a group is prime with order p, only p and 1 divide p. Hence, all non-identity elements are the generator of the group with order p (as the only element with order 1 is the identity).
   (c) For this one, I misunderstood the question. It's $G =< a, b >$instead of $G =< a >=< b >$. Hence, $G = \{a, a^2, b, b^2, ab, ab^2, a^2b, a^2b^2, e\}$ and its order is 9. There are four proper subgroups: $\{a, a^2, e\} \le G$, $\{b, b^2, e\} \le G$, $\{ab, a^2b^2, e\} \le G$, and $\{a^2b, ab^2, e\} \le G$.
5. Question 14: I made such a silly mistake: "$\sqrt{3}\times\sqrt{3} = 9$". So to calculate the eigenvalues, we have:
$$\lambda^2 - 20\lambda + 96 = 0. \Rightarrow \begin{cases} \lambda_1 = 8, \quad \vec{v} = \begin{pmatrix} 1/2 \\ -\sqrt{3}/2 \end{pmatrix}, \\ \lambda_2 = 12, \quad \vec{v} = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}. \end{cases}$$
   The new ellipse is:
$$\left[\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix}\right] \begin{pmatrix} 8 & 0 \\ 0 & 12 \end{pmatrix} \left[\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}\right] = 24.$$
   So,
$$\begin{pmatrix} x' & y' \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} = 6, \text{ while } \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$
   Therefore the ellispse can be written as $2x^2 + 3y^2 = 6$ and the linear transformation represented by the matrix is 60° anticlockwise.
6. Question 15, (b), ii: in this part, I made a stupid algebraic mistake. I got $p_2(x) = 3 - 3x + 2x^2$ in part 1 which is correct, but I somehow calculated $p_2(0.3) = 2.18$. The correct answer should be $p_2(0.3) = 3 - 0.6 + 0.08 = 2.48$. Accordingly, the conclusion in (c), ii should now be Maclaurin polynomial provides a better approximation for $f(0.2)$ than the Euler's method.