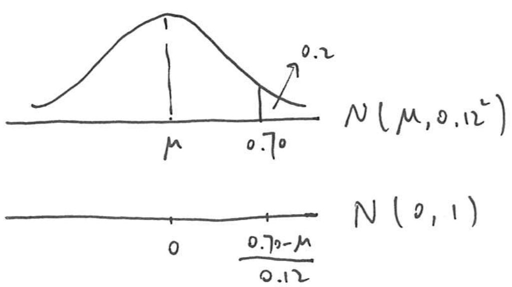
FM2 Test #5 Name: Jerry Jiang

1. Find unbiased estimates of the population parameters μ and σ^2 if a random sample of size 10 from this population gives $\sum x_i^2 = 180$ and $\sum x_i = 30$.

$$\hat{\mu} = \bar{x} = \frac{30}{10} = 3.$$

$$\hat{\sigma}^2 = E(S_{n-1}^2) = \frac{n}{n-1} E(S_n^2) = \frac{10}{9}\left(\frac{180}{10} + \left(\frac{30}{10}\right)^2\right) = 30.$$

2. The lengths of king fish are normally distributed with mean μ metres and standard deviation 0.12 metres. If 20% of king fish are longer than 0.70 metres, find the value of μ.



$$\frac{0.70-\mu}{0.12} = \text{invNorm}(0.8, 0, 1) = 0.842 \text{ (3s.f.)}$$

$$\mu = 0.599 \text{ (3s.f.)}$$

3. Email messages arrive at an office at an average rate of four per hour. Find the probability that more than two messages are received between 11.00 am and 11.45 am on Monday morning.

$$\mu = 4 \times \frac{45}{60} = 3.$$

$$P(X > 2) = 1 - \text{poissoncdf}(3, 2) = 0.577 \text{ (3s.f.)}$$

反馈

1, The final calculation is a subtract not an add. [4]

4. Remember 3 s.f. in the IB for final answers which are not exact.

6. (b) \bar{X}/4 as E(aX)=aE(X) [3]

Intuitively, on average the random variable \var{X}/4 has the value 1/p, where p is a fixed parameter of the parent distribution.

9. The parent distribution has variance 1/12. [3]

10. (b) The second normal curve has mean \mu and the first 60. You appear to have mixed them up. [2]

Very Good

分数

88 / 100

4. A random sample of 400 pet dogs shows a mean life expectancy of 10.3 years. Assuming that $\sigma = 3.2$ years, find a 90% confidence interval for the average life-span of all pet dogs.

inv Norm $(0.95, 0, 1) = 1.64$ (3s.f.)

$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow 10.3 \pm 1.64 \cdot \frac{3.2}{\sqrt{400}} = 10.3 \pm 0.263$, or $[10.037, 10.563]$.

5. The maximum load an elevator can carry is 550 kg. The weights of men are normally distributed with mean 70 kg and standard deviation 10 kg. The weights of women are normally distributed with mean 60 kg and standard deviation 5 kg. Find the probability that the elevator will be overloaded by five men and three women, if their weights are independent.

$M \sim N(70, 10^2)$, $W \sim N(60, 5^2)$.

$M_1 + M_2 + M_3 + M_4 + M_5 + W_1 + W_2 + W_3 \sim N(530, 575)$

P = normalcdf$(550, \infty, 530, \sqrt{575}) = 0.202$ (3s.f.).

6. The sample mean of the random variable $X \sim \text{NB}(4, p)$ is $\bar{X}$.

(a) Find $\text{E}(\bar{X})$.

$E(\bar{x}) = \mu = \frac{4}{p}$.

(b) Find an unbiased estimator for $1/p$ in terms of $\bar{X}$.

$\frac{4}{p} = E(\bar{x})$, $\frac{1}{p} = \frac{E(\bar{x})}{4}$.

is this what you want, or something like $\frac{3}{\bar{x}-1}$, what is the question asking and what is an intuitive way of doing it?

7. A large number of leaves of species A were measured and their lengths were found to have a mean of 64 mm and a standard deviation of 8 mm. A particular bush is discovered with leaves that are visually similar to species A. A random sample of 100 leaves from this bush were collected and they were shown to have a mean length of 61 mm. Do you think the bush is species A?

1. $H_0: \mu = 64$ mm.

   $H_1: \mu \neq 64$ mm.

2. $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$

3. p-value $= 1.77 \times 10^{-4}$

4. There's sufficient evidence at 5% level of significance to reject H_0 in favor of H_1, so the bush isn't species A.

8. The normal range of phosphorus in the blood is 26 to 48 mg/l. Alice is a patient who is thought to have kidney disease. Her blood is tested on six different occasions giving phosphorous readings in mg/l of

56, 53, 46, 48, 57, 61.

If her phosphorus level varies normally, is there evidence at the 5% level of significance that Alice has a mean phosphorous level that exceeds 48 mg/l?

1. $H_0: \mu = 48$ mg/l.

   $H_1: \mu > 48$ mg/L.

2. $t = \frac{\bar{x} - \mu}{S_{n-1}/\sqrt{n}}$

3. $p = 0.0320$ (3 s.f.)

4. There's sufficient evidence at 5% level of significance to reject H_0 in favor of H_1.

9. A student proposes a new method to produce random numbers from the interval $[0, 1]$. The method is implemented on a computer. His command to generate 100 random numbers gives a mean value $\bar{x} = 0.5784$. Is this result significant at the 1% level($\alpha = 0.01$)? What does this result tell you about his new method?

What is the variance of the original population?

Suppose it has mean μ and variance σ^2. then for the sample mean, the distribution is $N(0.5, \frac{\sigma^2}{100})$ according to the CLT.

Put $\text{invNorm}(0.995, 0.5, \frac{x^2}{100})$ into the function, $x = 1.74$ when the value gets to 0.5784 and 1.74 is a really big standard deviation.

Hence, the result is really likely to be significant and the new method isn't that reliable.

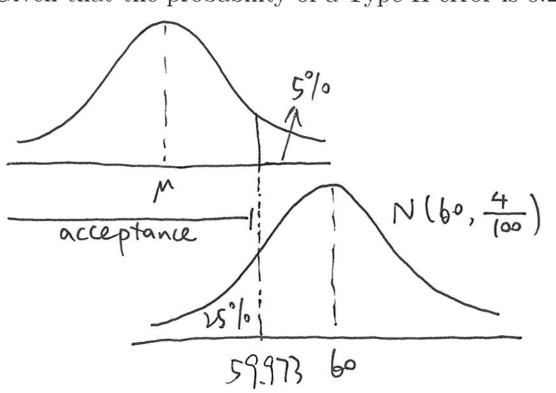
Have I misunderstood the question?

10. A random sample of size 100 is taken from a population with mean μ and variance 4 to test the null hypothesis that $\mu = 60$ against the alternative hypothesis that $\mu > 60$ at the 5% level of significance.

(a) Write down the probability of a Type I error.

$\alpha = 0.05$.

(b) Given that the probability of a Type II error is 0.25, find the actual value of μ.



$$\frac{59.973 - \mu}{\sqrt{4}} = \text{invNorm}(0.95, 0, 1) = 1.64485$$

$\mu = 56.7$ (3 s.f.)