FM2 Assignment #33 Name: Jerry Jiang $9\frac{1}{2}/10$ Very good

1. The heights in centimetres of a random sample of five Pearson students were 158, 184, 177, 166, 170. Calculate unbiased estimates of the mean and variance of the population of heights of all Pearson students.

$\bar{x} = 171$, $S_{n-1} = 10$ according to the GDC.

Hence, $\hat{\mu} = 171$, $\hat{\sigma}^2 = 10^2 = 100$.

2. Consider the Mersenne number $M_n = 2^n - 1$. Prove that if M_n is prime then n is prime. Is the converse also true?

Suppose M_n is prime while n isn't.

Then $n = ab$, $a > 1$, $b > 1$, $a, b \in \mathbb{N}$.

$M_n = (2^a)^b - 1 = (2^a - 1)[(2^a)^{b-1} + (2^a)^{b-2} + \dots + 1]$

Since $2^a - 1 > 1$, $(2^a)^{b-1} + \dots + 1 > 1$,

M_n is composite.

Hence n must be prime by the contrast.

The converse isn't true as

$2^{11} - 1 = 2047 = 23 \times 89$.

3. Currently, the largest known prime number is the Mersenne prime $2^{82\,589\,933} - 1$.

(a) How many digits does this number have in base 16?

$\lceil \log_{16} 2^{82589933} \rceil = \lceil 20647483.25 \rceil = 20647484$.

The power isn't odd, so the last digit can't be 0. "-1" won't have effects on digit number. So the value have 20647484 digits.

(b) How many digits does this number have in base 10?

$\lceil \log 2^{82589933} \rceil = 24862048$.

The power of 2 never ends in 0, so "-1" still has no effect on digit number. Hence, the value has 24862048 digits.

10

4. Pippin chocolates are packed in boxes of 25. The weight in grams of a Pippen chocolate is distributed $N(10, 2^2)$.

(a) What is the probability that the contents of a box of Pippin chocolates weighs more than 245 grams?

$S \sim N(250, 25 \cdot 2^2)$

$P(S > 245) =$ normal cdf $(245, \infty, 250, \sqrt{25 \cdot 4}) = 0.691$ (3 s.f.).

(b) What is the probability that the mean weight of the chocolates in a box is between 9.9 and 10.1 grams?

$M \sim N(10, \frac{2^2}{25})$.

$P(9.9 \leq M \leq 10.1) =$ normalcdf $(9.9, 10.1, 10, \sqrt{\frac{4}{25}}) = 0.197$ (3s.f.).

5. Consider the permutations (1 2) and (1 2 3) in the symmetric group S_3.

(a) Let $H = \langle(1\ 2)\rangle$.

i. Determine the left cosets of H in S_3 giving your answers in cycle notation.

$\{(13), (123)\}$. $\{(23), (132)\}$.) also $\{e, (12)\}$

ii. Determine the right cosets of H in S_3 giving your answers in cycle notation.

$\{(13), (132)\}$. $\{(23), (123)\}$.) also $\{e, (12)\}$

$1\frac{1}{2}$

(b) Describe the group $\langle(1\ 2), (1\ 2\ 3)\rangle$.

$(12)(12) = e$.

$(12)(123) = (23)$

$(123)(123) = (132)$

$(123)(132) = e$.

$(12)(132) = (13)$

so $\langle(12), (123)\rangle = S_3$.

3!