FM2 Assignment #25 Name: Jerry Jiang $\frac{10}{10}$ Excellent!

1. The linear transformation L maps $(2, 1)$ to $(4, 1)$ and $(2, 4)$ to $(0, 6)$. Find $L(3, 3)$.

$M\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix}, \quad M\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}0\\6\end{pmatrix}.$

$M\begin{pmatrix}3\\3\end{pmatrix} = M\left[\begin{pmatrix}2\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}2\\4\end{pmatrix}\right] = M\begin{pmatrix}2\\1\end{pmatrix} + \frac{1}{2}M\begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}.$

✓

2. Find the shortest distance between a point on the circle $x^2+y^2 = 9$ and a point on the circle $x^2+y^2-12x+6y+41 = 0$.

$x^2+y^2 = 9$: $(0, 0)$. $r = 3$.

$x^2+y^2-12x+6y+41 = 0$: $(6, -3)$. $r = 2$.

$D_{min} = \sqrt{6^2+3^2} - (3+2) = 3\sqrt{5} - 5.$ ✓

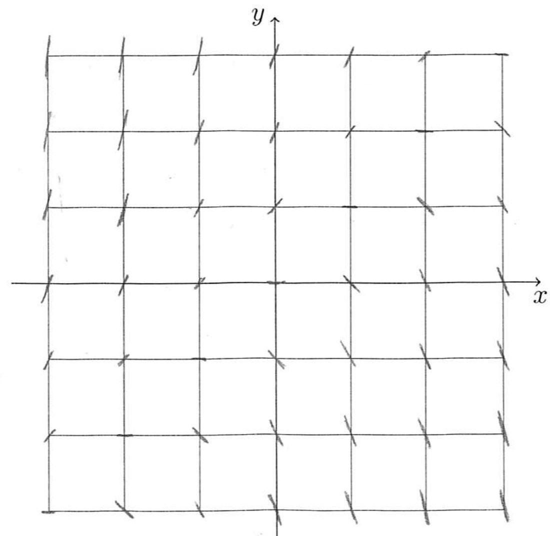
3. Sketch the slope field of the differential equation $\dfrac{dy}{dx} = y - x$ using a window of $[-3, 3] \times [-3, 3]$ and a rectangular grid of lattice points. Identify the isoclines and write down a particular solution to the differential equation.

isoclines: $y - x = k$. ✓

solution: $y = x + 1 - 0 \cdot e^x$. ✓

(general solution: $y = x + 1 + k \cdot e^x$.)

more usual here.



✓

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4. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = y - x$ with $f(0) = 2$. Give the recurrence relation found by applying Euler's method with a step size of 0.1. Hence approximate $f(1)$ aided by the GDC.

$$\begin{cases} x_n = x_{n-1} + 0.1 \\ y_n = y_{n-1} + 0.1\,(y_{n-1} - x_{n-1}). \end{cases}$$

$f(1) \approx 4.59$ (3 s.f.).

5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \frac{y}{8}(6 - y)$ with $f(0) = 8$.

(a) Use Euler's method in tabular form with a step size of 0.5 to approximate $f(1)$.

n	x_n	y_n	h	$h \cdot f(x_n, y_n)$
0	0	8	0.5	-1
1	0.5	7	0.5	-0.4375
2	1.0	6.5625	0.5	/

$f(1) \approx 6.5625$.

(b) Find the second degree Maclaurin polynomial for f and use it to approximate $f(1)$.

$P_2(x) = 8 + (-2)x + \frac{5}{4}x^2$

$f(1) \approx P_2(1) = 8 + (-2) + \frac{5}{4}$

$f(1) \approx 7.25$.

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