FM2 Assignment #34 Name: Jerry Jiang $9\frac{1}{2}$ / 12 Very good.

1. The weights of Pearson students are distributed normally. The weights in kilograms of a random sample of five Pearson students are 53, 72, 65, 58 and 61. Find a 95% confidence interval for the mean weight of all Pearson students.

$\bar{x} = 61.8$ C.I.: $61.8 \pm t^* \cdot \frac{7.19}{\sqrt{5}}$. $df = 5-1 = 4$.

$S = 7.19$ $t^* = invT(0.975, 4) = 2.78$.

C.I.: $61.8 \pm 2.78 \cdot \frac{7.19}{\sqrt{5}} = 61.8 \pm 8.94$.

$[52.9, 70.7]$. ✓

2. Prove that every simple planar graph contains a vertex whose degree is at most five.

Suppose a simple planar graph have all vertices' degrees more than 5.

Let $v = n$. We have $e = \frac{\sum deg(v)}{2} \geq \frac{6n}{2} = 3n$. $1\frac{1}{2}$

For simple planar graphs, $e \leq 3v - 6$ for $v \geq 3$. However $e \geq 3n > 3n-6 = 3v-6$.

The contradiction proves the result.

Also connected and $n \geq 3$. This should be considered in your proof.

3. Solve the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ given $y(1) = 2$.

$\frac{dy}{dx} = \frac{x}{2y} + \frac{y}{2x}$. Let $y = ux$. $\frac{dy}{dx} = \frac{du}{dx} \cdot x + u$.

$\frac{y}{x} = u$. $\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{u} + \frac{1}{2} \cdot u = \frac{du}{dx} \cdot x + u$.

$\frac{1-u^2}{2u} = \frac{du}{dx} \cdot x$, $\frac{2u}{1-u^2} du = \frac{1}{x} \cdot dx$.

$\int \frac{2u}{1-u^2} du = \int \frac{1}{x} \cdot dx \to \int \frac{2u}{1-u^2} du = -\int \frac{2u}{u^2-1} du = -\ln(u^2-1)$

$-\ln(u^2-1) = \ln x + C. \to C = \ln x(u^2-1) = \ln x \cdot \frac{y^2-x^2}{x^2} = \ln \frac{y^2-x^2}{x}$

$e^C = \frac{y^2-x^2}{x} \to c \cdot x + x^2 = y^2. \to y = \sqrt{x^2+cx} \to y(1) = 2, C = 3$.

Hence, $y = \sqrt{x^2+3x}$. ✓

$5\frac{1}{2}$

4. Let G be a simple graph. Prove that G and its complement $\bar{G}$ cannot both be disconnected.

- Suppose G is disconnected. It has n separate parts where vertices within each part are connected. $n \geq 2$. Suppose $n=3$.

A B C

For $\bar{G}$, there will be a path from every vertex in Part A to all vertices in B and C, so A, B, and C are connected between each other. Also, there will be a path from every vertex in Part B to all vertices in A and C. There's always a path for two vertices within A by first going to B and then return to A. Hence, A, B, C are connected within itself and between each other. So $\bar{G}$ must be connected.
- For cases when $\bar{G}$ is disconnected, we can show G must be connected using the same logic. Therefore, G and $\bar{G}$ can't be both disconnected.

5. Suppose $\phi: \mathbb{Z}_{50} \to \mathbb{Z}_{15}$ is a group homomorphism with $\phi(7) = 6$.

(a) Write down $\phi(21)$.

$\phi(21) = \phi(7+7+7) = \phi(7) + \phi(7) + \phi(7) = 18$, but $\phi(21)$ should be less than 15.
so $\phi(21) = 18 - 15 = 3$.

(b) Find $\phi(1)$.

$\phi(7) = \phi(1+1+1+1+1+1+1) = 7\phi(1)$, $\phi(1)$ have to be integer, so
$6+15=21$. $\phi(1) = 21 \div 7 = 3$.

(c) Find ran(ϕ).

			4th	5th
$\phi(1) = 3$,	$\phi(2) = 6$,	$\phi(3) = 9$,	$\phi(4) = 12$.	$\phi(5) = 0$.
$\phi(6) = 3$,	$\phi(7) = 6$,	$\phi(8) = 9$,	$\phi(9) = 12$,	$\phi(10) = 0$.
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Hence, $\text{ran}(\phi) = \{0, 3, 6, 9, 12\}$.

(d) Find ker(ϕ).

$\ker(\phi) = \{0, 5, 10, 15, 20, 25, 30, 35, 40, 45\}$ by inspection from the 5th column in (c).

(e) Solve the equation $\phi(x) = 12$, $x \in \mathbb{Z}_{50}$.

By inspection of the 4th column in (c), $x = 4, 9, 14, 19, 24, 29, 34, 39, 44, 49$.

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