FM2 Assignment #31 Name: Jerry Jiang $9\frac{1}{2}/10$ Very good

1. Find the values that the geometric series : $x + \frac{x^2}{1+x} + \frac{x^3}{(1+x)^2} \dots$ converges.

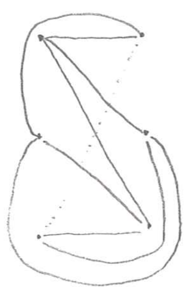
$U_n = \frac{x^n}{(1+x)^{n-1}}$

Ratio Test : $\left|\frac{U_{n+1}}{U_n}\right| = \left|\frac{\frac{x^{n+1}}{(1+x)^n}}{\frac{x^n}{(1+x)^{n-1}}}\right| = \left|\frac{x}{1+x}\right|$, $\left|\frac{x}{1+x}\right| < 1$, then $x > -\frac{1}{2}$.

when $x = -\frac{1}{2}$, the series becomes $\sum_{n=1}^{\infty} \frac{1}{2}(-1)^n$ which is divergent.

Hence, the interval of convergence is $]-\frac{1}{2}, +\infty[$.

2. Draw a simple connected non-planar graph with three or more vertices for which $e \leq 3v - 6$.



$K_{3,3}$, $e = 9$, $v = 6$. $9 \leq 18 - 6 = 12$,

but it's not planar.

3. Prove that a non-Abelian group cannot be isomorphic to an Abelian group.

Suppose a non-Abelian group G is isomorphic to an Abelian group H.

Let $a, b \in G$, and the isomorphism be f.

$f(a \circ b) = f(a) * f(b) = f(b) * f(a) = f(b \circ a)$. $1\frac{1}{2}$

But G isn't abelian so $f(a \circ b) \neq f(b \circ a)$. we want $ab \neq ba$

The contradiction proves what we want.

Since G is non-Abelian there exist $a, b \in G$ s.t. $ab \neq ba$.

$5\frac{1}{2}$

4. The discrete random variable X has the Poisson distribution with mean μ.

(a) Denoting $P(X = k)$ by p_k, show that $p_{k+1} = \frac{\mu}{k+1} \times p_k$.

$$p_k = \frac{e^{-\mu} \cdot \mu^k}{k!} \qquad p_{k+1} = \frac{e^{-\mu} \cdot \mu^{k+1}}{(k+1)!} = \frac{e^{-\mu} \cdot \mu^k \cdot \mu}{k! \cdot (k+1)} = \frac{\mu}{k+1} \cdot p_k.$$

(b) If $\mu = 7.8$, determine the modal value of X.

$$p_1 = \frac{7.8}{0+1} \cdot p_0, \quad p_2 = \frac{7.8}{1+1} \cdot p_1, \quad \cdots \quad p_7 = \frac{7.8}{6+1} p_6,$$

so $p_7 > p_6 > p_5 \cdots > p_1 > p_0$.

$$p_8 = \frac{7.8}{7+1} p_7, \quad p_9 = \frac{7.8}{8+1} p_8, \quad \cdots, \quad \text{so} \quad p_7 > p_8 > p_9 > \cdots.$$

Hence the modal value is 7.

5. Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ given $a_0 = 2$, $a_1 = 7$. Hence find the least n for which $a_n > 1\,000\,000$.

AQE: $r^2 - r - 2 = 0$, $r_1 = 2$, $r_2 = -1$.

$$a_n = \alpha(2)^n + \beta(-1)^n.$$

$$\begin{cases} \alpha + \beta = 2 \\ 2\alpha - \beta = 7 \end{cases} \Rightarrow \begin{cases} \alpha = 3 \\ \beta = -1 \end{cases}$$

$$a_n = 3 \cdot 2^n - (-1)^n > 1000000.$$

$\lceil n \rceil = 19$.

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