FM2 Assignment #28 Name: Jerry Jiang $\frac{9}{10}$ V. good

1. Lebron James's probability of making a free throw is 75%. After practice one day, he decides he must make 50 free throws before he can go home. How many free throw attempts should he expect to take?

$X \sim NB(50, 0.75)$

$E(X) = \frac{r}{p} = \frac{50}{0.75} = 66.7$ (3s.f.)

2. Prove that the complement of a complete bipartite graph does not possess a spanning tree.

The complement of a complete bipartite graph will have all points connected to each other within both of the two parts, but not one connection between the two parts. As the two parts are not connected, there's no possibility to have a spanning tree, which contains every vertex of the graph.

3. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \\ 5 & 5 & 5 & 0 \end{pmatrix}$. Find all row vectors $\vec{y}$ such that $\vec{y}A^T = (2 \quad 2 \quad 3)$.

$A\vec{y}^T = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$ is better.

$(a \; b \; c \; d) \begin{pmatrix} 1 & 4 & 5 \\ 2 & 3 & 5 \\ 3 & 2 & 5 \\ 4 & 1 & 0 \end{pmatrix} = (2 \; 2 \; 3)$

$$\begin{cases} a + 2b + 3c + 4d = 2 \\ 4a + 3b + 2c + d = 2 \\ 5a + 5b + 5c + 0d = 3 \end{cases}$$

$5a + 5b + 5c + 5d = 4$

$\therefore 5d = 1, \quad d = \frac{1}{5}$

$a + b + c = \frac{3}{5}$

$$\begin{cases} b + 2c = \frac{3}{5} \\ 2a + b = \frac{3}{5} \end{cases}$$

therefore, $c = \frac{3}{10} - \frac{1}{2}b, \quad a = \frac{3}{10} - \frac{1}{2}b.$

$(a \; b \; c \; d) = (\frac{3}{10} - \frac{1}{2}b \quad b \quad \frac{3}{10} - \frac{1}{2}b \quad \frac{1}{5}).$

$= (\frac{3}{10} \quad 0 \quad \frac{3}{10} \quad \frac{1}{5}) + b(-\frac{1}{2} \quad 1 \quad -\frac{1}{2} \quad 0),$

for all $b \in R$.

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4. Solve the differential equation $\cos x \dfrac{dy}{dx} + y\cos^2 x \csc x = \sin 2x$ where $x \in]-\pi/2, \pi/2[$.

Multiply both sides by $\frac{\sin x}{\cos x}$.

$\sin 2x = 2\sin x \cos x$

$\sin x \frac{dy}{dx} + y\cos x = 2\sin^2 x$

$\int (\sin x \cdot y)' = 2\int \sin^2 x$

$y \cdot \sin x = 2 \cdot \frac{1}{3}\sin^3 x + C$

$y = \dfrac{\frac{2}{3}\sin^3 x + C}{\sin x}$

$\int \sin^2 x \, dx$

$= \int \frac{1-\cos 2x}{2} dx$

$= \frac{1}{2}\left[\int 1 \, dx - \int \cos 2x \, dx\right]$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]$

$y \cdot \sin x = 2 \cdot \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$

$y = \frac{x}{\sin x} - \frac{1}{2} \cdot 2 \sin x \cos x + C = \frac{x}{\sin x} - \cos x + C/\sin x$

5. Let X be the score on the throw of a fair die.

(a) Show that the pgf for X is $G(t) = \frac{1}{6}t(1-t^6)(1-t)^{-1}$.

$G(t) = \frac{1}{6}t + \frac{1}{6}t^2 + \frac{1}{6}t^3 + \frac{1}{6}t^4 + \frac{1}{6}t^5 + \frac{1}{6}t^6$

$= \frac{1}{6}t \dfrac{(1-t^6)}{(1-t)}$

$= \frac{1}{6}t(1-t^6)(1-t)^{-1}$.

(b) Hence determine the probability of a sum of 14 when four fair dice are thrown.

The pgf in this case is:

$G(t) = \left[\frac{1}{6}t(1-t^6)(1-t)^{-1}\right]^4$

$= \frac{1}{1296} t^4 (1-t^6)^4 (1-t)^{-4}$

$= \frac{1}{1296} t^4 (t^{24} - 4t^{18} + 6t^{12} - 4t^6 + 1)\left(1 + \binom{-4}{1}x + \binom{-4}{2}x^2 + \binom{-4}{3}x^3 + \dots\right)$

coefficient for $t^{14} = \frac{1}{1296}\left[(-4)\binom{-4}{4} + 1 \cdot \binom{-4}{10}\right]$.

$= \frac{1}{1296}\left[(-4) \cdot 35 + 286\right]$

$= \frac{146}{1296}$

$= \frac{73}{648}$

Therefore, the probability for sum of 14 is $\frac{73}{648}$.

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