1. Females enter Superstore at an average rate of two a minute and males enter it at an average rate of one a minute. Find the probability that three people enter Superstore in a given minute. Could you have got your answer in another way?

①
$$X \sim P_0(z)$$
, $Y \sim P_0(1)$. $X + Y \sim P_0(1+2)$.
 $P(X+Y=3) = \frac{e^{-3} \cdot 3^3}{3!} = 0.224(35.f.)$

$$P(X=0) = 0.1353353... P(Y=0) = 0.3678794... P(X+Y=3) = \sum_{i=0}^{3} P(X=i) \cdot P(Y=3-i)$$

$$P(X=i) = 0.2706706... P(Y=i) = 0.3678794... = 0.224(35.4.)$$

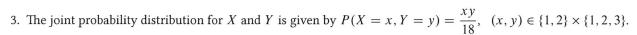
$$P(X=2) = 0.2706706... P(Y=2) = 0.1839397...$$

$$P(X=2) = 0.2706706... P(Y=2) = 0.183731/...$$

2. Consider the matrix $\begin{pmatrix} k & 1 & 1 \\ k & 2 & k-1 \\ k & 0 & k-2 \end{pmatrix}$. For which values of k is the matrix invertible?

Determinant =
$$k(2k-4) - (k^2-2k-k^2+k) + (-2k)$$

= $2k^2-4k+k-2k$
= $2k^2-5k \neq 0$
 $k(2k-5) \neq 0$, $k \neq 0$ or $\frac{5}{2}$.



(a) Tabulate the joint probability distribution for X and Y.

XX	1	2	3	
1	1	2	3	$\frac{b}{18}$
٤	18	18	18	12
	18	6 18	9 18	

(b) Tabulate the probability distribution for X + Y.



4. As we saw in class, the Poisson distribution can be derived as the limiting case of a binomial distribution with $p = \mu/n$ and $q = 1 - \mu/n$. Write down the pgf for the binomial distribution and show that the pgf for the Poisson distribution can be obtained from this by letting n go to infinity.

$$\begin{array}{lll} \chi \sim P_{0}(M), & \chi \sim B(N, \frac{M}{n}) \text{ as } n \rightarrow \infty. \\ \\ \text{for } B(N, \frac{M}{n}), \\ \\ G(t) = \sum_{n=0}^{N} {n \choose n} {n \choose n}^{n} {1 - M \choose n}^{n-n} t^{n} \\ \\ = \sum_{n=0}^{N} \frac{n(n-1) - (n-n+1) \cdot M^{n}}{n! \cdot n^{n}} \left(1 - \frac{M}{n}\right)^{n-n} t^{n} \\ \\ = \sum_{n=0}^{N} \frac{M^{n}}{n!} \cdot \left[\frac{n}{n} \cdot \frac{n-1}{n} - \frac{n-n+1}{n}\right] \left(1 + \frac{m}{n}\right)^{n-n} t^{n} \\ \\ \text{when } n \rightarrow \infty, \\ \\ G(t) = \sum_{n=0}^{N} \frac{M^{n}}{n!} \cdot \left[\frac{n}{n} \cdot \frac{n}{n} - \frac{n}{n}\right] \left(1 + \frac{m}{n}\right)^{n} t^{n} \\ \\ = \sum_{n=0}^{N} \frac{M^{n}}{n!} \cdot \left[\frac{n}{n} \cdot \frac{n}{n} - \frac{n}{n}\right] \left(1 + \frac{m}{n}\right)^{n} t^{n} \\ \\ = \sum_{n=0}^{N} \frac{M^{n}}{n!} \cdot e^{-M} t^{n} \quad \text{as expected.} \end{array}$$

5. Use the pigeon hole principle to show that some pair of any five points in a unit square will be at most $\frac{1}{\sqrt{2}}$ units apart. Divide the square into four smaller squares by connecting mid-points of opposite sides.

As there're 5 points and 4 regions, there must be a region with 2 points. Since the diagonal of that square region is $\frac{1}{42}$ units long, that's as far as the two points in the region can possibly get.

