FM2 Assignment #36 Name: Jerry Jiang

1. In a corral there are cowboys and an odd number of horses. There are 20 legs in all: how many belong to horses?

From the question, we can infer that there are more than one cowboys and more than one horses. Let's consider the first case where there are 3 horses and 12 legs, which leaves 20−12=8 legs for the 4 cowboys. Another case is that there are 5 horses, but that leaves no legs for cowboys. Hence, the first case is the only possible one.

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1. One horse is possible. [1.5]
2. Should mention the conclusion is by mathematical induction.

Very Good.
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2. Prove that consecutive Fibonacci numbers are relatively prime.

Let's first prove the corollary: if gcd(a,b)=1, then gcd(a,a+b)=1:
Suppose gcd(a,a+b)≠1, then a and a+b can be written as n*x, n*y (x and y are integers), where the positive integer n=gcd(a,a+b) (n>1). So n*y=a+b, y=(a+b)/n=a/n+b/n=x+b/n. For y to be integer, b/n must be an integer, so n is a factor of b. Now we have gcd(a,b)=n>1. This contradiction proves what we want.

Now:
The first two terms of the Fibonacci series are 1,1, gcd(1,1)=1.
Next, suppose the nth term and (n−1)th term of the Fibonacci series are relatively prime, which can be written as gcd(F(n), F(n−1))=1. If we add them up, F(n+1)=F(n−1)+F(n). From the corollary, we can have gcd(F(n), F(n+1))=1.
Therefore, we can conclude that the statement is true for all cases.

3. Find three consecutive integers such that the first is divisible by a square, the second by a cube and the third by a fourth power.

We can simply take 1, 2, and 3, as 1^2=1^3=1^4=1.
If we don't consider 1, suppose we take the square to be 25, the cube to be 27, and the fourth power to be 16.
x≡0 mod 25, x+1≡0 mod 27, x+2≡0 mod 16.
Then x≡0 mod 25, x≡26 mod 27, x≡14 mod 16.
The Chinese Remainder Theorem gives us the answer of x=350, so the three integers are 350, 351, and 352.

4. Let a and b be elements of a group G. We say a is a *conjugate* of b if $a = xbx^{-1}$ for some $x \in G$. Define the relation $\sim$ on G by $a \sim b$ if a is a conjugate of b. Prove that $\sim$ is an equivalence relation on G. What are the equivalence classes when G is Abelian?

It's easy to see that the relation is reflexive by taking x=e.
For the symmetric property, suppose a=pbp^(−1), then p^(−1)ap=b. Since p^(−1) is the inverse of p which must be in G, the equation must work.
Finally, suppose a=xbx^(−1), b=ycy^(−1), for some x and y in G. We can merge the two equations and get a=xycy^(−1)x^(−1). Since (xy)^(−1)=y^(−1)x^(−1), and xy must be in G, we have a~c.
Therefore, we prove that ~ is an equivalence relation.

When G is abelian, we have a=xbx^(−1)=xx^(−1)b=b, so each equivalence class contains one element of the group.

5. Use induction on n to show that the Fibonacci numbers satsify $f_{m+n} = f_{m-1} \cdot f_n + f_m \cdot f_{n+1}, \quad m \geq 1, n \geq 0.$

For n=1, f(m−1)*f1+fm*f2=f(m−1)+fm=f(m+1).
For n=2, f(m−1)*f2+fm*f3=f(m−1)+2fm=f(m+1)+fm=f(m+2).
Now suppose we have f(m+n−1)=f(m−1)*f(n−1)+fm*fn, f(m+n−2)=f(m−1)*f(n−2)+fm*f(n_1). We know that f(m+n)=f(m+n−1)+f(m+n−2), and that is equal to f(m−1)*f(n−1)+fm*fn+f(m−1)*f(n−2)+fm*f(n−1) =f(m−1)*[f(n−1)+f(n−2)]+fm*[fn+f(n−1)]=f(m−1)*fn+fm*f(n+1).
Hence, we can conclude that the equation works for all m≥1 and n≥0.