Two-Dimensional Random Sparse Sampling for High Resolution SAR Imaging Based on Compressed Sensing

Jing Li, Shunsheng Zhang, Junfei Chang
Research Institute of Electronic Science and Technology
University of Electronic Science and Technology of China
Chengdu, China
Email: lijing8609@hotmail.com;zhangss@uestc.edu.cn

Abstract— **High speed analog-to-digital (A/D) sampling and a large amount of echo storage are two basic challenges of high resolution synthetic aperture radar (SAR) imaging. To address these problems, a novel SAR imaging algorithm is proposed based on compressed sensing (CS) in this paper. In particular, this new algorithm provides the approach of receiving echo data via two-dimensional (2-D) random sparse sampling with a significant reduction in the number of sampled data beyond the Nyquist theorem and with an implication in simplification of radar architecture. Then CS technique is used to reconstruct the targets in range and azimuth directions, respectively. The simulation results and error analysis show that the proposed CS-based imaging method presents many important applications and advantages which include less sampled data, lower peak side-lobe ratio (PSLR) and integrated side-lobe ratio (ISLR) and higher resolution than the traditional SAR imaging algorithm.**

I. Introduction

Synthetic aperture radar (SAR) is a radar imaging technology that is capable of producing high resolution images of the stationary surface targets. The main advantages of SAR are that it can reduce the effects of clouds and fog and allow them to be independent of external sources for imaging, having day and night and all-weather imaging capability. Traditional compressions of SAR data utilize the redundancy inherent in sampled data under the Nyquist theorem to achieve compressed representation and profitable transmission. This theory claims that one must sample at least two times faster than the signal bandwidth while capturing it without losing information. Thereby there are large amounts of onboard data that have to be stored and it inevitably results in complex computation and expensive hardware.

Candès, Tao and Romberg[1]and Donoho[2] have proposed an approach, known as compressed sensing(CS), in which a random linear projection is used to acquire efficient representations of compressible signals directly. The theory of CS states that it is possible to recover sparse images from a small number of random measurements, provided that the undersampling results in noise like artifacts in the transform domain and an appropriate nonlinear recovery scheme is used[3]. Because of its compressed sampling ability, compressed sensing has found many applications in radar and remote sensing, and other fields. R.Baraniuk *et al.* proposed the radar imaging system based on CS for the first time[4]. The papers[5,6] use CS in along-track interferometric SAR imaging and moving-target velocity estimation. Some major open questions related with the application of CS to SAR and ISAR are listed in[7].

In the above researches, it has been shown that a successful recovery of a compressible signal depends on the presence of sparse dictionary. These works have made a great contribution to the future research of radar signal processing based on CS, whereas many of these methods take traditional focusing with matched filtering in range direction and perform azimuth focusing via CS[8,9].These methods can not provide practical approaches to simplify radar system and to reduce the sampling rate of the receiver A/D converter.

In this paper, we introduce a novel two-dimensional (2-D) SAR imaging algorithm based on compressed sensing theory. This new algorithm provides the approach of receiving the echo data via 2-D random sparse sampling beyond the Nyquist theorem. This radar system randomly transmits fewer pulses in azimuth direction and samples fewer data than traditional systems at random intervals in range direction. Our imaging approach is in contrast to other compressive radar related algorithms that have only considered using CS as part of one-dimensional analog-to-information conversion. The key idea in our approach is to use CS to reconstruct 2-D targets in the range and azimuth dimension, respectively. Meanwhile, it provides the potential to achieve higher resolution between targets and to reduce on-board storage constraints. More importantly, our method does not use a matched filter and enhances some of these suggestions and provides a proper framework along with general reconstruction techniques.

II. Compressed Sensing

The Shannon-Nyquist sampling theorem requires a signal to be sampled at a frequency of twice its bandwidth to be able to reconstruct it exactly. In CS framework, it uses a low-dimensional,nonadaptive, linear projection to acquire an efficient representation of a sparse signal with just a few measurements, so as to greatly reduce the sampling rate and enhance the data rate. According to the compressed sensing

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theory[1-3], there are three important ingredients:sparse signal representation, measurement operator, and sparse reconstruction algorithms. Consider a discrete signal expressed as a vector $\boldsymbol{x} \in \mathbb{C}^N$ of length N. Suppose $\boldsymbol{x}$ is *K-sparse* if at most $K \ll N$ of its coefficients are nonzero in a basis or more generally a frame $\boldsymbol{\Psi}$, so that $\boldsymbol{x} = \boldsymbol{\Psi s}$, where $\boldsymbol{\Psi} \in \mathbb{C}^{N \times N}$ is a matrix and $\boldsymbol{s} \in \mathbb{C}^N$ is a vector. The signal is acquired through linear projections:

$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi \Psi s} = \boldsymbol{\Theta s} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^M$ is the measurements vector and $\boldsymbol{\Phi} \in \mathbb{C}^{M \times N}$ is the measurement matrix with $M < N$. Since $M < N$, the recovery of signal $\boldsymbol{x}$ from the measurements vector is ill-posed in general. But when the matrix $\boldsymbol{\Theta}$ has the Restricted Isometry Property (RIP)[10], it is possible to reconstruct $\boldsymbol{x}$ from a set of $M = O(K \log(N/K))$ linear measurements. So the signal $\boldsymbol{x}$ can be perfectly recovered via its coefficients $\boldsymbol{s}$ with high probability, by solving the following l_0 minimization problem [11]:

$$\hat{\boldsymbol{s}} = \arg\min \|\boldsymbol{s}\|_{\mathbf{0}} \quad \textit{subject to}\ \boldsymbol{y} = \boldsymbol{\Phi \Psi s} = \boldsymbol{\Theta s} \tag{2}$$

Unfortunately, solving (2) is an NP problem and minimum l_0 norm is too sensitive to noise. Consequently, the researchers [10] present that the recovery of sparse coefficients $\boldsymbol{s}$ can be achieved using optimization by searching for the signal with a l_1 minimization problem:

$$\hat{\boldsymbol{s}} = \arg\min \|\boldsymbol{s}\|_{\mathbf{1}} \quad \textit{subject to}\ \boldsymbol{y} = \boldsymbol{\Phi \Psi s} = \boldsymbol{\Theta s} \tag{3}$$

The optimization problem (3) is often known as Basis Pursuit(BP) and Orthogonal Matching Pursuit (OMP) which can be solved by linear programming methods.

III. Signal Model and Algorithm

A. Signal Model

Fig. 1 shows a simple diagram of stripmap mode SAR and how received echo data via 2-D random sparse sampling is placed in a three-dimensional (3-D) signal model. The range is the direction of signal propagation and the azimuth is the direction parallel to the flight path. As the radar moves along its path in azimuth direction, it transmits pulses at microwave frequencies at a random pulse repetition interval (PRI) which is defined as 1/PRF, where PRF is the pulse repetition frequency. Instead of sampling in range direction with a regular interval, we propose to sample fewer data than traditional systems at random intervals. As shown in Fig.1, the range from the transmitter and receiver to the target can be written as

$$R = 2R(\eta; \boldsymbol{\Gamma}) = 2\sqrt{R_b^2 + (v\eta - x)^2} \tag{4}$$

where η is the slow time, R_b is the range to the sensor position at $\eta = 0$, and a scatter within that scene is located at $\boldsymbol{\Gamma} = (x, y, 0)$. Suppose the transmitted signal is linear frequency modulated (LFM) signal which can be described as

$$s_T(t, \eta) = \mathrm{rect}\left(\frac{t}{T_p}\right) \exp\left\{j2\pi f_c t + j\pi k_r t^2\right\} \tag{5}$$

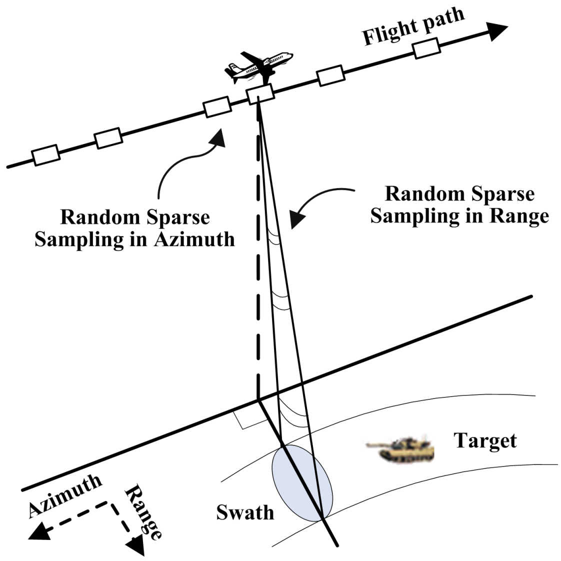

Fig. 1. Geometry of 2-D random sparse sampling SAR

where T_p is the pulse duration, t is the fast time, f_c is the carrier frequency, k_r is the chirp rate and $\mathrm{rect}(\cdot)$ is the stand for the unit rectangular function. After mixing down and quadrature demodulation, the received radar signal is given by

$$s_R(t, \eta) = \mathrm{rect}\left(\frac{t - 2R(\eta; \boldsymbol{\Gamma})/c}{T_p}\right) \times \exp\left\{j\pi k_r \left(t - \frac{2R(\eta; \boldsymbol{\Gamma})}{c}\right)^2 - j\frac{4\pi R(\eta; \boldsymbol{\Gamma})}{\lambda}\right\} \tag{6}$$

where c is the speed of light, and λ is the wavelength of the transmitted signal.

B. CS-based Imaging Algorithm

Traditional SAR imaging algorithms are performed based on matched filter and Nyquist theory. Due to the low computational resources of the acquisition platforms and the steadily increasing resolution of SAR systems, the data cannot generally be processed on board and must be stored or transmitted to the ground where the image formation process is performed. Baraniuk and Steeghs[4] proposed a new application that named random filtering based on CS for SAR. The differences of the two approaches are shown in Fig.2. We can find out that the CS-based SAR imaging method not only reduces the amount of echo data but also does not need a matched filter at the receiver.

Suppose $s_T(t)$ is the transmitted signal and the target is described by $u(t)$, then the received signal $s_R(t)$ can be written as

$$s_R(t) = G \int s_T(t - \xi) u(\xi) d\xi \tag{7}$$

where G represents attenuation due to propagation and reflection. Consider a target reflectivity generated from N Nyquist-rate samples $x(n)$ via $x(n) = u(\Delta t), n = 1, \cdots, N$.We

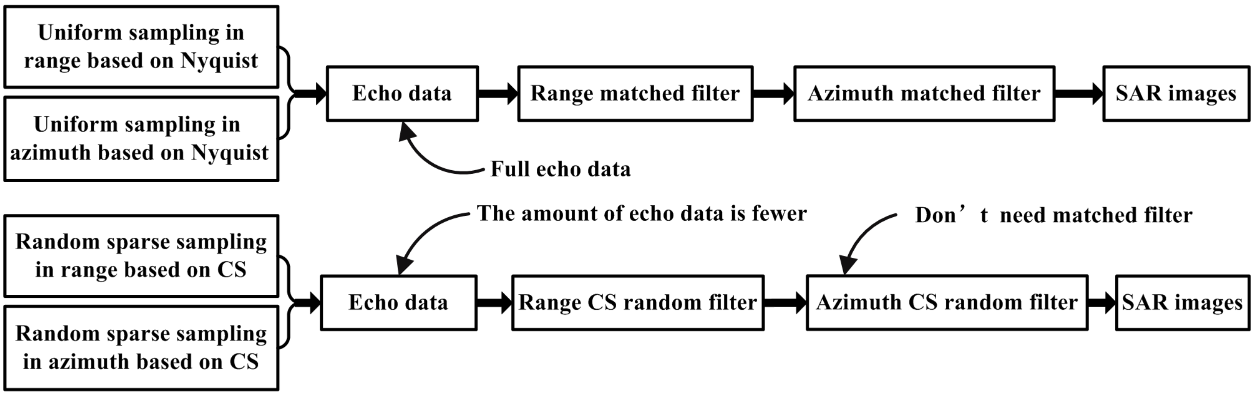

Fig. 2. Comparison of traditional SAR imaging and CS-based SAR imaging

sample the received radar signal $s_R(t)$ not every Δt seconds but rather random $\omega(m)\Delta t$ seconds, where $M = [N/D]$ and $M < N$, $\omega(m)$ is the random sequence of size $1 \times M$, to obtain the M samples,$m = 1, \ldots, M$

$$\begin{aligned} y(m) &= s_R(t)|_{t=\omega(m)\cdot\Delta t} \\ &= G\int_0^{N\Delta t} s_T(\omega(m)\cdot\Delta t - \xi)u(\xi)\,d\xi \\ &= G\sum_{n=1}^{N} s_T(\omega(m)\cdot\Delta t - n)\int_{(n-1)\Delta t}^{n\Delta t} u(\xi)d\xi \\ &= G\sum_{n=1}^{N} s_T(\omega(m) - n)x(n) \end{aligned} \tag{8}$$

where $s_T(n)$is the discrete transmitted signal. The low-rate samples y contain sufficient information to reconstruct the signal x corresponding to the Nyquist-rate samples of the reflectivity $u(t)$ via linear programming or a greedy algorithm[4].

Using the concepts of random filtering, we can apply this approach to 2-D SAR imaging system. Firstly, the measurement matrix of the range dimension is constructed in terms of (8). Suppose D_r represents the down-sampling times in range direction, the range measurement matrix can be expressed as

$$\begin{aligned} \mathbf{\Phi_r}(m,n) &= s_T(\omega(m) - n) \\ &= \mathrm{rect}\left(\frac{\omega(m) - n}{T_p}\right)\exp\left\{j\pi k_r(\omega(m) - n)^2\right\} \end{aligned} \tag{9}$$

where $\mathbf{\Phi_r} \in \mathbb{C}^{M\times N}$,$M = N/D_r$,$m = 1,\ldots M$,$n = 1,\ldots N$. After the targets being reconstructed in range dimension via CS, the signal can be approximated as

$$s_{cs}(t,\eta) \approx \sin c\left(t - \frac{2\cdot R(\eta;\mathbf{\Gamma})}{c}\right)\exp\left\{-j\frac{4\pi R(\eta;\mathbf{\Gamma})}{\lambda}\right\} \tag{10}$$

where $\sin c(\cdot)$ is the Sinc function. The second factor of (10) is the Doppler phase factor. Suppose D_a represents the down-sampling times in azimuth direction, the azimuth measurement matrix can be given by

$$\begin{aligned} \mathbf{\Phi_a}(q,p) &= \exp\left\{-j4\pi\frac{R(q,p;\mathbf{\Gamma})}{\lambda}\right\} \\ &= \exp\left\{-j4\pi\frac{\sqrt{{R_b}^2 + [v(w(q) - p) - x]^2}}{\lambda}\right\} \end{aligned} \tag{11}$$

where $\mathbf{\Phi_a} \in \mathbb{C}^{Q\times P}$,$p = 1,\ldots P$ is the Nyquist sampling sequence in azimuth, $Q = P/D_a$, $q = 1,\ldots Q$ is the down-sampling sequence. $\omega(q)$ is the random sequence of size $1\times Q$.

After constructing $\mathbf{\Phi_r}$ and $\mathbf{\Phi_a}$, equation (3) can be solved by OMP, BP, etc. Finally, we can get the CS-based SAR images. Obviously, the amount of Nyquist-rate data is $N \times P$ and the amount of CS-based data is $M \times Q$. Thus the amount of data has been decreased by $D_r \times D_a$ times as against the traditional imaging algorithm.

IV. Simulations Results

Simulated data have been used to verify the validity of the proposed image formation method. The target space can be regarded as sparse in some special applications in which only a small number of strong scatters distribute in the illuminated scene. In following experiments, we use the solution method

TABLE I
Simulated Radar Parameters

Parameters	Value
Carrier frequency	$10GHz$
Transmitted signal bandwidth	$150MHz$
Platform height	$3000m$
Platform velocity	$150m/s$
Pulse duration	$10\mu s$
PRF	$350Hz$
Range undersampling	2 times
Azimuth undersampling	2 times

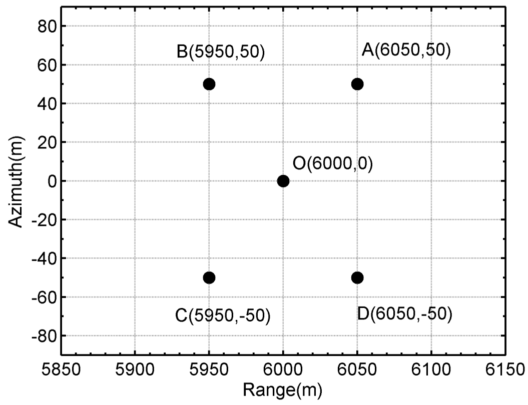

Fig. 3. Simulated scene with 5 point targets

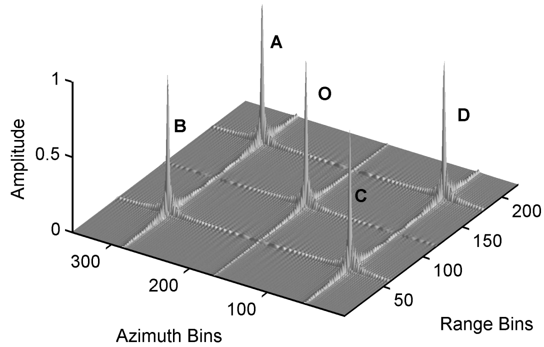

Fig. 4. 3-D reflectivity with traditional reconstruction

of optimization directly as OMP and set the simulation radar parameters as listed in Table I.

A. Point Scatterers Simulation

The simulated scene consists of five point targets, which are shown in Fig.3. Point O is located in the center of the scene, and the other four targets are located on the vertices of a $600m \times 600m$ square. The coordinates are listed as follows (m, m) : $O(6000, 0)$, $A(6050, 50)$, $B(5950, 50)$, $C(5950, -50)$, $D(6050, -50)$.

Using the parameters of table 1, the 5 point targets are reconstructed via traditional SAR imaging algorithm and CS-based algorithm, respectively. The results are shown in Fig. 4 and Fig.5. Fig.4 shows the SAR imaging results with traditional imaging algorithm based on Nyquist theory. The result of the proposed CS-based algorithm is shown in Fig.5.

From Fig.4 and Fig. 5, we can see that the reflectivity function of the targets can be perfectly reconstructed by only using 25% of echo data instead of using all numbers of measurements. The results obviously illustrate that CS-based method has better imaging performance in lower ISLR and PSLR than traditional reconstruction. Thereby, this experiment shows that the focused performance using the CS-based

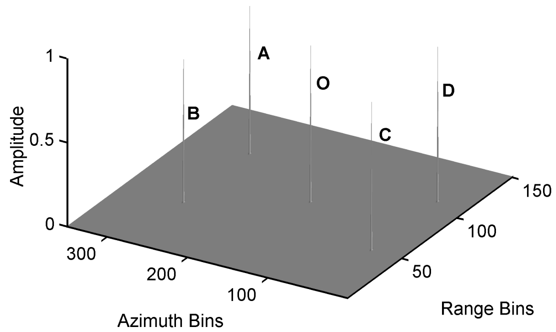

Fig. 5. 3-D reflectivity with the CS-based imaging algorithm

method is much better than the one using the traditional method.

B. Resolution Analysis

We show an important capability of this proposed method that is high resolution, which means that it can reconstruct image details under bandwidth limitations. To demonstrate this property, we apply our method on a synthetic scene composed of two point scatterers which are in different azimuth location as shown in Fig.6. The coordinates of targets are listed as follows (m, m): $M(6000, 1), N(6000, 0)$. This experiment uses the parameters of table I and sets the traditional radar azimuth resolution of $1.5m$. In this case, the traditional imaging algorithm can not distinguish the 2 point targets which their azimuth distance is 1 meter as Fig. 7 shows. On the contrary, the proposed CS-based algorithm can clearly distinguish the targets as Fig.8 shows(after 8-times interpolation). This experiment illustrates CS theory would allow the implementation of wide-swath modes without reducing the azimuth resolution and have an enormous potential application in improving radar resolution.

C. Reconstruction Error Analysis

As mentioned above, D_r represents the range down-sampling times and D_a represents the azimuth down-sampling

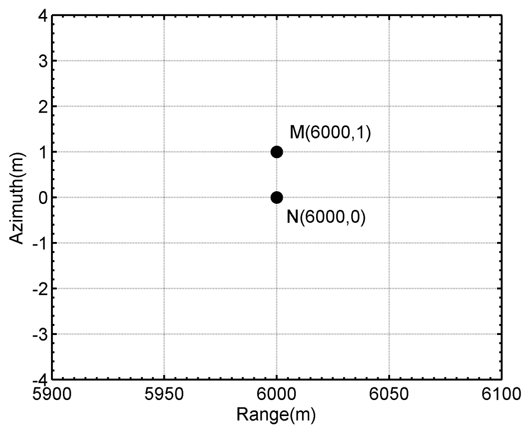

Fig. 6. Two point targets with azimuth location of 1 meter

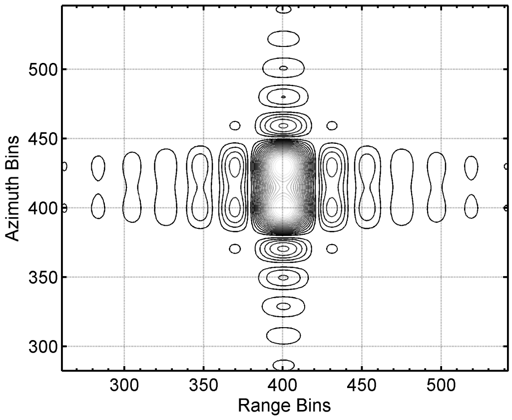

Fig. 7. Contour plots of targets M and N using traditional algorithm

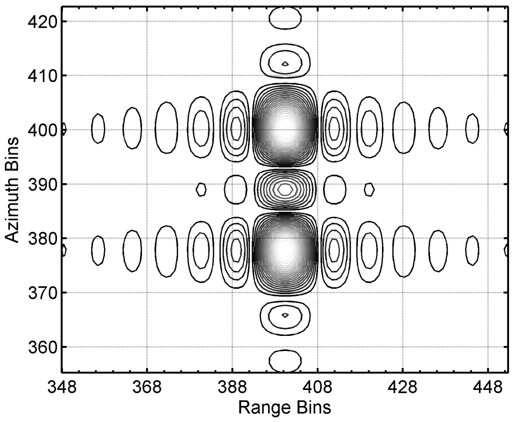

Fig. 8. Contour plots of targets M and N using CS-based algorithm

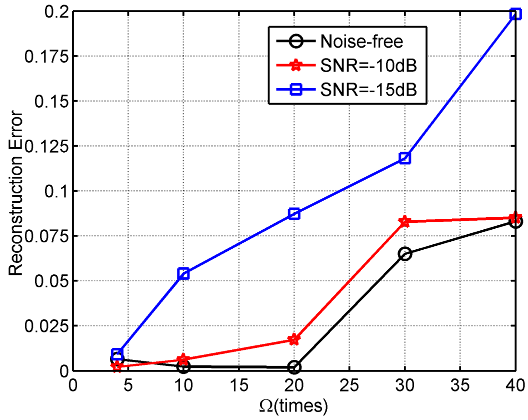

Fig. 9. Reconstruction error for different Ω and SNR

times. And the total down-sampling times Ω is defined as $D_r \cdot D_a$. The reconstruction error is calculated as $\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|_2^2 \,/\, \|\boldsymbol{x}\|_2^2$, where $\hat{\boldsymbol{x}}$ and $\boldsymbol{x}$ are the estimated and true coefficient vectors, respectively. The experiment was repeated for different values of Ω and signal to noise ratio (SNR). The relationships between Ω, SNR and reconstruction error are depicted in Fig.9.

As shown in Fig.9, it is clear that the CS-based algorithm can reconstruct targets in the case of serious under-sampling and low SNR. Note that the error is small for $\Omega < 20$ and $SNR \geq -10dB$, and then increases rapidly for larger values of Ω and smaller values of SNR. This method has high robustness in the presence of serious noise. The experimental results show that the reconstruction error is negligible and the proposed algorithm is valid under certain conditions.

V. Conclusion

In this paper, a novel 2-D SAR imaging algorithm is proposed based on constructing measurement matrixes in range and azimuth dimensions via compressed sensing techniques, respectively. This radar system randomly transmits fewer pulses in azimuth and samples fewer data than traditional systems at random intervals in range. Thereby, this method provides the approach of getting echo data via 2-D random sparse sampling with a significant reduction in the number of sampled data beyond the Nyquist theorem. This will directly impact A/D conversion, and has the potential to reduce the overall data rate and to simplify hardware design. The simulation experiments and reconstruction error analysis verify the validity of the proposed CS-based imaging algorithm which is lower integrated side-lobe ratio ISLR and PSLR, less sampled data, higher resolution than the traditional SAR imaging algorithm.

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