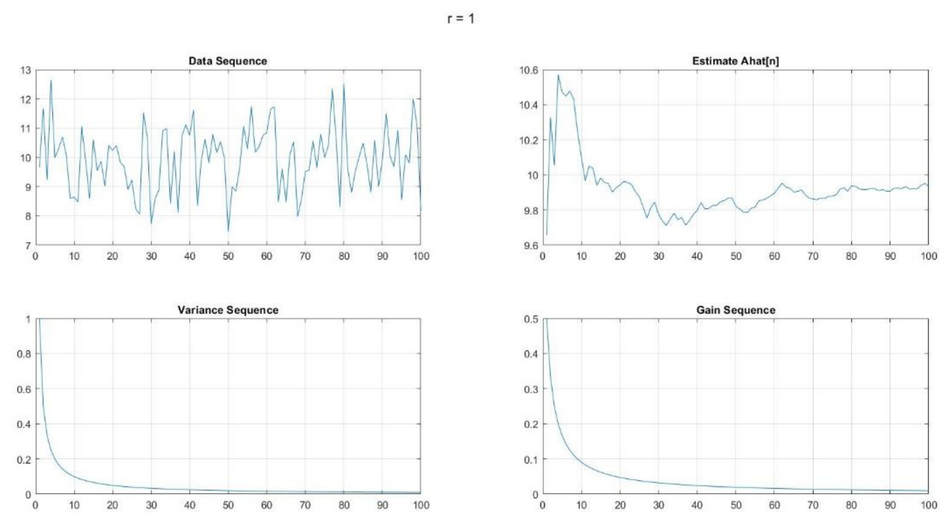
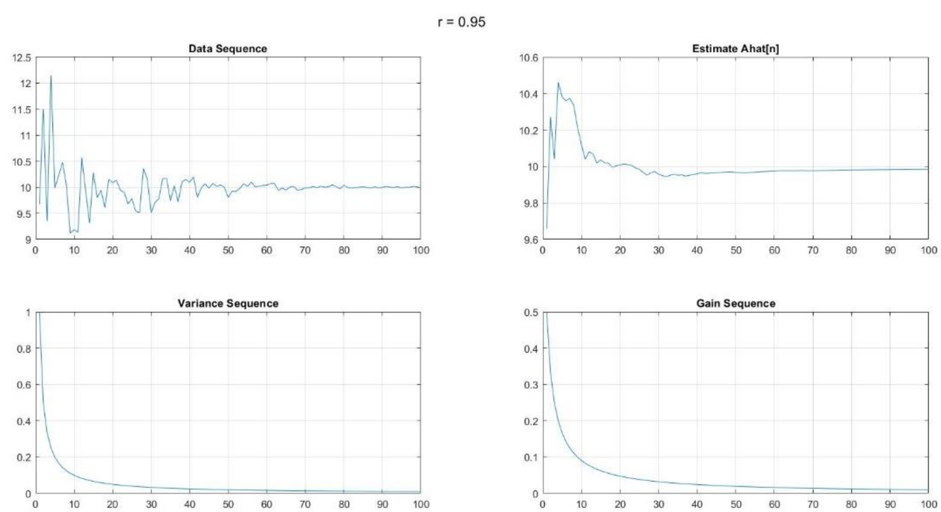
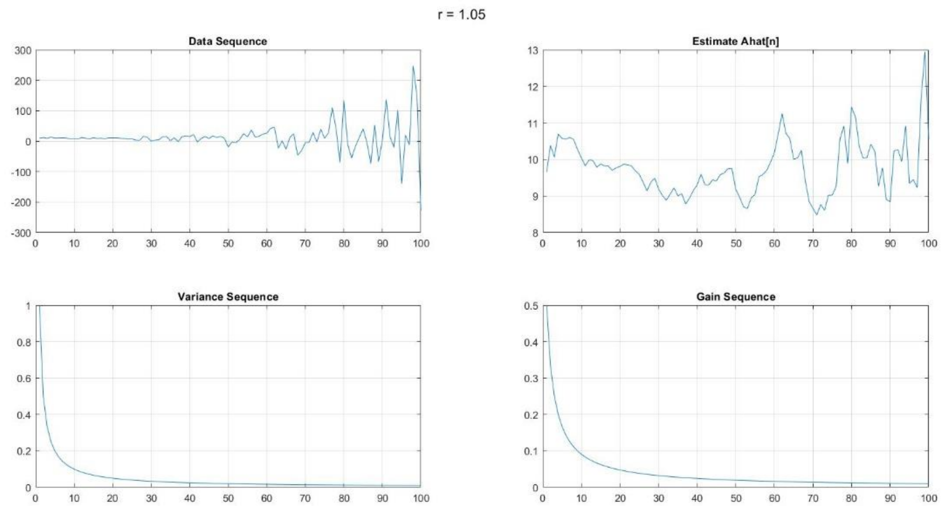
8.22

Before this problem, well, the author made a mistake of WGN (standard deviation, not variances) function in his figures' solution. r^n should be sqrt(r^n) in next formula. So, I separately use two different parameters (variances and standard deviation) to show the results.

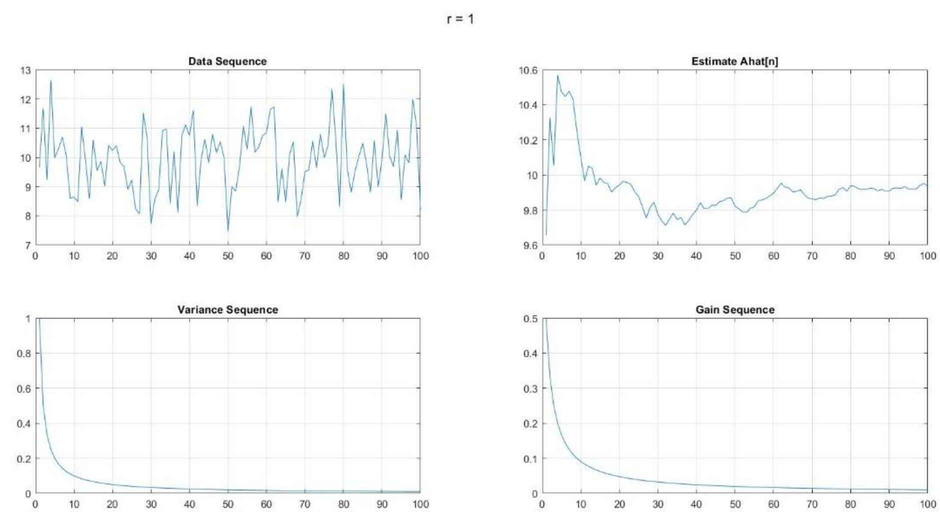
The first three pictures are the standard deviation' like the author,



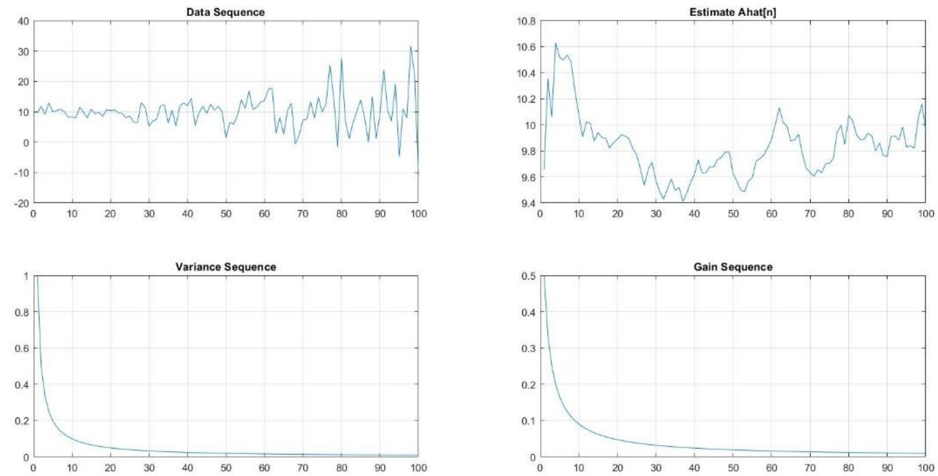




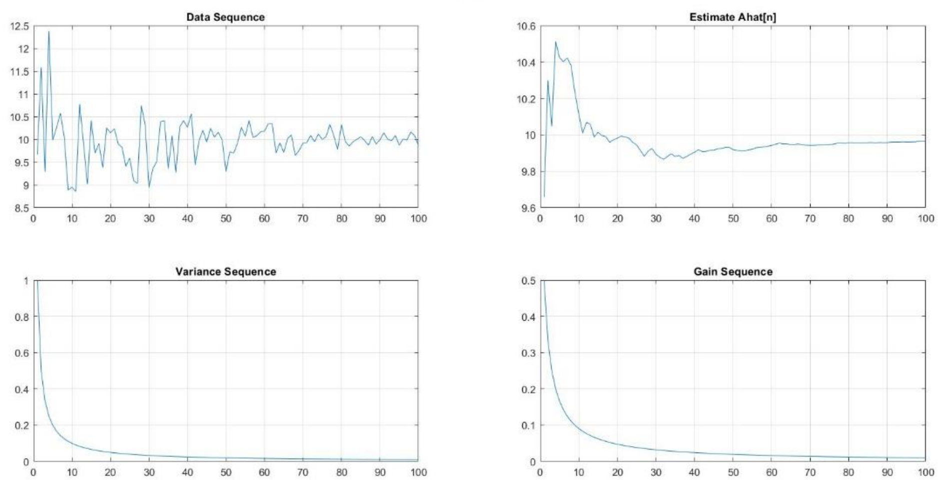
another three pictures are the variances' which are the true questions.



r = 1.05


Data Sequence
Estimate Ahat[n]
Variance Sequence
Gain Sequence

r = 0.95


Data Sequence
Estimate Ahat[n]
Variance Sequence
Gain Sequence

MATLAB CODE

```
function homework9ch8_22
% 6865
% Zeyu Liu
% 10/28/2019
% By implementing a Monte Carlo computer simulation
plot A[N] hat as given
% by (8.40). The data x[n]=A+w[n], w[n]is zero mean WGN
delta(n)^2 = r^n
% Use A =10, and r=1, 0.95, 1.05. Initialize the
estimator by using
% A[0] hat = x[0] and var(A[0] hat)= var(x[0])=
delta(0)^2=1.
% Plot the gain and variance sequences.


% Well, the author make a mistake of variance function
in his figures'
% solution. r^n should be sqrt(r^n) in next formula.
But I use two here.

A = 10;

% because matlab figure is begin at(1,1)
% 1. r = 1

% Data Sequence
s = rng(5);% set a seed to control the result
w = randn(1,100);
for n=1:100,
    x(n) = A + 1^n.*w(n);
    %x(n) = A + sqrt(1^n).*w(n);
end;
plot(x);

% Estimate Ahat[n]

Ahat(1) = A + w(1);
for N = 1:99
    x(N+1) = A + 1^(N+1).*w(N+1);
    % x(N+1) = A + sqrt(1^(N+1)).*w(N+1);
    Ahat(N+1) = Ahat(N) + 1/(N+2)*(x(N+1)-Ahat(N));
```

```
end;
plot(Ahat);

% Variance Sequence
for N = 1:100,
    VAhat(N) = 1/N;
end;
plot(VAhat);

% Gain Sequence
for N = 1:100,
    K(N) = 1/(N+1);
end;
plot(K);


subplot(221),plot(x),title('Data Sequence');
grid on;
subplot(222),plot(Ahat),title('Estimate Ahat[n]');
grid on;
subplot(223),plot(VAhat),title('Variance Sequence');
grid on;
subplot(224),plot(K),title('Gain Sequence');
grid on;
suptitle('r = 1');

pause;

% 2. r = 0.95

% Data Sequence
s = rng(5);% set a seed to control the result
w = randn(1,100);
for n=1:100
    x(n) = A + 0.95^n.*w(n);
    %x(n) = A + sqrt(0.95^n).*w(n);
end;
plot(x);

% Estimate Ahat[n]

Ahat(1) = A + w(1);
for N = 1:99
    x(N+1) = A + 0.95^(N+1).*w(N+1);
```

```
    %x(N+1) = A + sqrt(0.95^(N+1)).*w(N+1);
    Ahat(N+1) = Ahat(N) + 1/(N+2)*(x(N+1)-Ahat(N));
end;
plot(Ahat);

% Variance Sequence
for N = 1:100,
    VAhat(N) = 1/N;
end;
plot(VAhat);

% Gain Sequence
for N = 1:100,
    K(N) = 1/(N+1);
end;
plot(K);


subplot(221),plot(x),title('Data Sequence');
grid on;
subplot(222),plot(Ahat),title('Estimate Ahat[n]');
grid on;
subplot(223),plot(VAhat),title('Variance Sequence');
grid on;
subplot(224),plot(K),title('Gain Sequence');
grid on;
suptitle('r = 0.95');
pause;
% 3. r = 1.05

% Data Sequence
s = rng(5);% set a seed to control the result
w = randn(1,100);
for n=1:100
    x(n) = A + 1.05^n.*w(n);
    % x(n) = A + sqrt(1.05^n).*w(n);
end;
plot(x);

% Estimate Ahat[n]

Ahat(1) = A + w(1);
for N = 1:99
    x(N+1) = A + 1.05^(N+1).*w(N+1);
```

```
    %x(N+1) = A + sqrt(1.05^(N+1)).*w(N+1);
    Ahat(N+1) = Ahat(N) + 1/(N+2)*(x(N+1)-Ahat(N));
end;
plot(Ahat);

% Variance Sequence
for N = 1:100,
    VAhat(N) = 1/N;
end;
plot(VAhat);

% Gain Sequence
for N = 1:100,
    K(N) = 1/(N+1);
end;
plot(K);



subplot(221),plot(x),title('Data Sequence');
grid on;
subplot(222),plot(Ahat),title('Estimate Ahat[n]');
grid on;
subplot(223),plot(VAhat),title('Variance Sequence');
grid on;
subplot(224),plot(K),title('Gain Sequence');
grid on;
suptitle('r = 1.05');

pause;
```

Solution:
Compare with the original plot, the smaller variance can plot a better estimator.