Math 151B Homework 3

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1 Part 2

Here we have the initial value problem (IVP).

$$\begin{cases} y'(t) = -20y + 20t^2 + 2t, 0 \leq t \leq 1 \\ y(0) = 1/3 \end{cases}$$

1.1 Errors

Plots are shown in Figure 1 and Figure 2. Maximum errors $\max(|w_i - y_i|)$ are shown below

1. Euler's method

```
step size: h = 0.200000
Maximum error at t = 1.000000: 83.440000
step size: h = 0.125000
Maximum error at t = 1.000000: 8.696899
step size: h = 0.100000
Maximum error at t = 0.100000: 0.388445
step size: h = 0.020000
Maximum error at t = 0.040000: 0.030416
```

2. Runge-Kutta method of order 4

```
step size: h = 0.200000
Maximum error at t = 1.000000: 1083.320000
step size: h = 0.125000
Maximum error at t = 0.125000: 0.193870
step size: h = 0.100000
Maximum error at t = 0.100000: 0.067666
step size: h = 0.020000
Maximum error at t = 0.060000: 0.000037
```

3. Adams 4th order predictor-corrector method

```
step size: h = 0.200000
Maximum error at t = 1.000000: 2812.043704
step size: h = 0.125000
Maximum error at t = 1.000000: 0.234790
step size: h = 0.100000
Maximum error at t = 1.000000: 0.272144
step size: h = 0.020000
Maximum error at t = 0.100000: 0.000138
```

4. Milne-Simpson predictor-corrector method

```
step size: h = 0.200000
Maximum error at t = 1.000000: 2861.973498
step size: h = 0.125000
Maximum error at t = 1.000000: 15.606907
step size: h = 0.100000
Maximum error at t = 0.800000: 0.714792
step size: h = 0.020000
Maximum error at t = 0.120000: 0.000081
```

Step size $h = 0.02$ is a pretty big step size and here almost every method converges to the true solution with $h = 0.02$, and non of them blows up when $h = 0.1$. I would like to call every method stable and convergent. However, if we hold our standard high, say $h = 0.125$, Euler's method and Milne-Simpson predictor-corrector method become unstable since they blow up.

1.2 Stability

1. Euler's method with step size h: apply $y' = f = \lambda y, \lambda < 0$ with $y(0) = w_0 = \alpha$). Then

$$w_{i+1} = w_i + h\lambda w_i = (h\lambda + 1)w_i \Rightarrow w_i = (h\lambda + 1)^i \alpha$$

$i \to \infty \Rightarrow |w_i| \to 0$ implies $|1 + h\lambda| < 1$. Let $z = h\lambda$, the region of absolute stability is

$$R = \{z \in \mathbb{C} \ : \ |1 + z| < 1\}$$

From the graph, ideally we want $h \in (0, 0.02]$, but we can also make the argument that $h \in (0, 0.1]$ since it didn't blow up at $h = 0.1$.

2. Runge-Kutta method of order 4 with step size h:
apply $y' = f = \lambda y, \lambda < 0$ with $y(0) = w_0 = \alpha$). Then according to calculation for Exercise 5.11.10

$$\begin{aligned} w_{i+1} &= (1 + (h\lambda) + \frac{1}{2}(h\lambda)^2 + \frac{1}{6}(h\lambda)^3 + \frac{1}{24}(h\lambda)^4)w_i \\ \Rightarrow w_i &= (1 + (h\lambda) + \frac{1}{2}(h\lambda)^2 + \frac{1}{6}(h\lambda)^3 + \frac{1}{24}(h\lambda)^4)^i \alpha \end{aligned}$$

$i \to \infty \Rightarrow |w_i| \to 0$ implies $\left|1 + (h\lambda) + \frac{1}{2}(h\lambda)^2 + \frac{1}{6}(h\lambda)^3 + \frac{1}{24}(h\lambda)^4\right| < 1$.
Let $z = h\lambda$, the region of absolute stability is

$$R = \left\{z \in \mathbb{C} \ : \ \left|1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}\right| < 1\right\}$$

From the graph, we should have $h \in (0, 0.125]$.

1.3 Plots

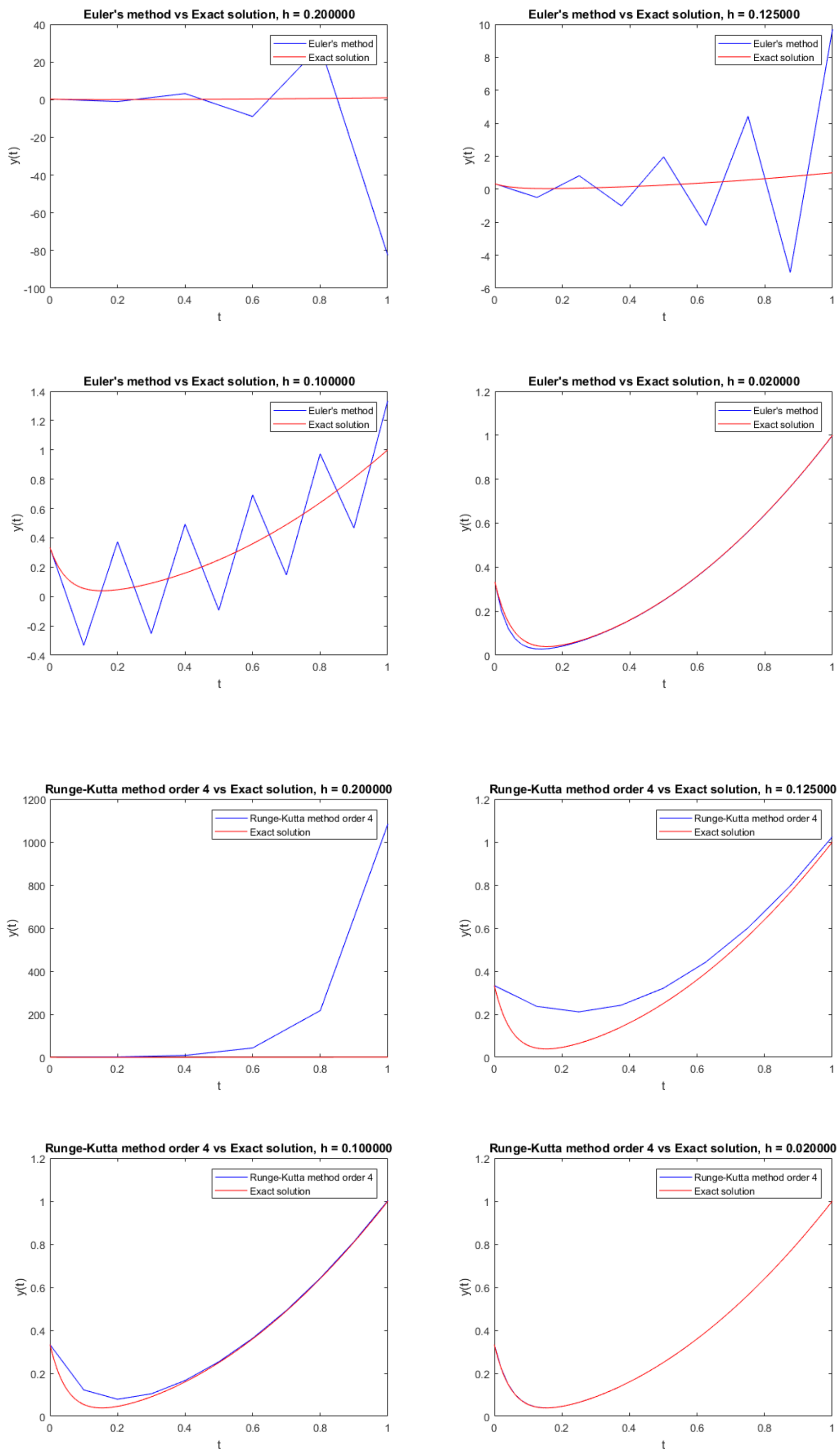

Figure 1: Plots of two one-step methods and step size h

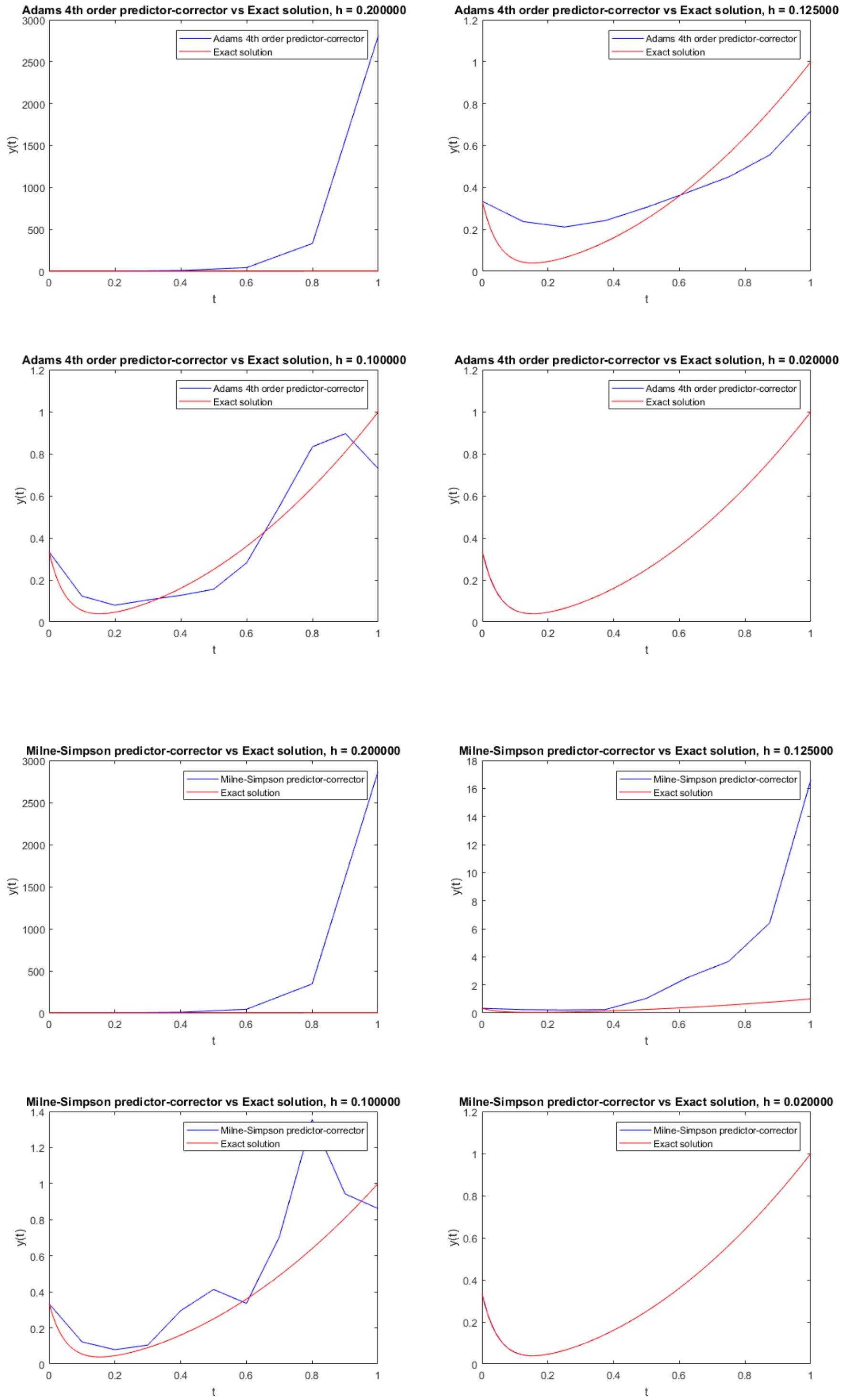

Figure 2: Plots of two predictor-corrector methods and step size h