Homework 2

Due: May 4th.

1 Part I (50%)

(1) Show that the Modified Euler method is of order two.

(2) Use Theorem 5.20 to show that the Runge-Kutta method of order four is consistent.

(3) Exercise 5.10.4 a,b,c,d

(4) Exercise 5.4.30.

(5) Exercise 5.4.32.

2 Part II (50%)

Consider the following well-posed IVP:

$$\begin{cases} y'(t) = 1 + \dfrac{y}{t}, \quad 1 \le t \le 2; \\ y(1) = 2, \end{cases} \tag{1}$$

with the exact solution $y(t) = t \ln t + 2t$. Choose the step sizes $h = 0.2, 0.1, 0.05$, respectively.

(a) Use Taylor's method of order two to approximate the solution. Discuss the behavior of the approximated solution as a function of h, and compare it with the exact solution in plots of t versus y. Estimate the order of the method from the error. Which value of h do you need to choose (approximately) to achieve an accuracy of 10^{-4} for $y(2)$?

(b) Use Midpoint method (p.286) to redo Part (a).

(c) Compare the results and running times[1] of Part (a) and (b). What does the comparison of error and running time tell us about the efficiency of the two methods?

Requirements

- Submit the code file to CCLE : A MATLAB (or other software) function `taylor2.m` that implements Taylor's method of order two, a MATLAB function (or other software) `midpt.m` that implements Midpoint method, and a MATLAB (or other software) script `main.m` that solves the IVP (1) and plots the approximated solutions versus the exact one.
- Print a PDF report to your TA.

[1] `tic` and `toc` can be used to record the running time. See `http://www.mathworks.com/help/matlab/ref/tic.html` and `http://www.mathworks.com/help/matlab/ref/tic.html` for more details.