Homework 3

Due: Thursday, May 18th.

Part I (50%)

(1) Exercise 5.10.4.d

(2) Exercise 5.10.7

(3) Exercise 5.11.10

(4) Exercise 5.11.11

Part II (50%)

Consider the following IVP

$$\begin{cases} y'(t) = -20y + 20t^2 + 2t, \quad 0 \le t \le 1; \\ y(0) = 1/3 \end{cases}$$

with the exact solution $y(t) = t^2 + 1/3e^{-20t}$. Use the time step sizes $h = 0.2, 0.125, 0.1, 0.02$ for all methods. Solve the IVP using the following methods

(a) Euler's method

(b) Runge-Kutta method of order four

(c) Adams fourth-order predictor-corrector method (see ALGORITHM 5.4 p.311)

(d) Milne-Simpson predictor-corrector method which combines the explicit Milne's method

$$w_{i+1} = w_{i-3} + \frac{4h}{3}[2f(t_i, w_i) - f(t_{i-1}, w_{i-1}) + 2f(t_{i-2}, w_{i-2})],$$

and the implicit Simpson's method

$$w_{i+1} = w_{i-1} + \frac{h}{3}[f(t_{i+1}, w_{i+1}) + 4f(t_i, w_i) + f(t_{i-1}, w_{i-1})].$$

Compare the results to the actual solution in plots, compute $|w_i - y_i|$, and specify which methods become unstable. Based on the values of h that were chosen, can you make a statement about the region of absolute stability for Euler's method and Runge-Kutta method of order four?

Requirements Submit to CCLE a file `lastname_firstname_hw3.zip` containing the following files:

- A MATLAB function `abm4.m` that implements Adams fourth-order predictor-corrector method, a MATLAB function `ms.m` that implements Milne-Simpson predictor-corrector method, and a MATLAB script `main.m` that solves the given IVP and plots the approximated solutions versus the exact one. (Please include `euler.m` and `rk4.m` for completeness.)
- A PDF report that shows the plots and answers the above questions.