Homework 1

The due day is Thursday, April 20th.

1 Part I (50%)

This part is required to be submitted in class.

(1) Exercise 5.1.1.a

(2) Exercise 5.1.3:b,d

(3) Exercise 5.1.6

(4) Exercise 5.1.7

2 Part II (50%)

Population growth is described by an ODE of the form $y'(t) = ry(t)$, where r is the growth rate. In a typical population, the growth rate is not a constant, but is density dependent. For example, as the population grows, there might be less food available, and as a result the growth rate decreases. We consider the following *Logistic Equation*:

$$\begin{cases} y'(t) = r(1 - \frac{y}{K})y, \quad 0 \leq t \leq 50; \\ y(0) = y_0 \end{cases} \tag{1}$$

where $0 < y_0 < K$. Then the exact solution is given by

$$y(t) = \frac{y_0 K}{y_0 + (K - y_0)e^{-rt}}.$$

Solve IVP (1) with $y_0 = 1000$, $r = 0.2$, $K = 4000$ numerically using Euler's method. Choose the step sizes $h = 10, 1, 0.1$, respectively.

a) Compare the solutions to the exact solution in plots of population vs. time. Compare the actual maximal error $\max_i |y(t_i) - w_i|$ with the error bound predicted in Theorem 5.9 (p.271).

b) Discuss the behavior of the solutions as a function of h. What happens for very large step size h?

Requirements Print your pdf file and submit it in the discussion section. Submit to CCLE the code, for example: a MATLAB (or C++, C, etc) function `euler.m` that implements ALGORITHM 5.1 (p.267), and a MATLAB script `main.m` that solves the IVP (1) and plots the approximated solutions versus the exact one.