Math 151B Homework 4

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1 Part 2

Here we have $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $F(0) = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$, x^* is solved by MATLAB function `fsolve`.

$$F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} = \begin{bmatrix} 15x_1 + x_2^2 - 4x_3 - 13 \\ x_1^2 + 10x_2 - x_3 - 11 \\ x_2^3 - 25x_3 + 22 \end{bmatrix}$$

$$J(x) = \begin{bmatrix} 15 & 2x_2 & -4 \\ 2x_1 & 10 & -1 \\ 0 & 3x_2^2 & -25 \end{bmatrix}$$

(a) Newton's method only needs 5 iterations and gives us $x^{(5)} = \begin{bmatrix} 1.036400470329211 \\ 1.085706550741678 \\ 0.931191442315390 \end{bmatrix}$.
The elapsed time is 0.002179s and the error $\left\|x^{(5)} - x^*\right\|_\infty \approx 4 \times 10^{-16}$.
Note: actually 4 iterations but I chose the new value in my implementation so it returns 5.

(b) Steepest Grad Descent method needs 32 iterations and gives us $x^{(5)} = \begin{bmatrix} 1.036428746313132 \\ 1.085624560400561 \\ 0.931161508291562 \end{bmatrix}$.
The elapsed time is 0.010363s and the error $\left\|x^{(5)} - x^*\right\|_\infty \approx 8.2 \times 10^{-5}$.

(c) Runge-Kutta method of order 4 performs better accuracy per iteration.
However, it is noticeably slower than midpoint method.

Method	N	$x^{(N)}$	Time	$\left\|x^{(N)} - x^*\right\|_\infty$
Midpoint	10	$\begin{bmatrix} 1.036317612131312 \\ 1.085658625483103 \\ 0.931120256555617 \end{bmatrix}$	0.000344s	$\approx 8.3 \times 10^{-5}$
	20	$\begin{bmatrix} 1.036379815156702 \\ 1.085694815560816 \\ 0.931173584106285 \end{bmatrix}$	0.000892s	$\approx 2.1 \times 10^{-5}$
	50	$\begin{bmatrix} 1.036397171471843 \\ 1.085704696867131 \\ 0.931188580170288 \end{bmatrix}$	0.000674s	$\approx 3.3 \times 10^{-6}$
Runge-Kutta order 4	10	$\begin{bmatrix} 1.036400475424716 \\ 1.085706552365897 \\ 0.931191458219360 \end{bmatrix}$	0.000365s	$\approx 1.6 \times 10^{-8}$
	20	$\begin{bmatrix} 1.036400470643249 \\ 1.085706550841369 \\ 0.931191443298061 \end{bmatrix}$	0.000580s	$\approx 9.8 \times 10^{-10}$
	50	$\begin{bmatrix} 1.036400470337180 \\ 1.085706550744201 \\ 0.931191442340366 \end{bmatrix}$	0.001519s	$\approx 2.5 \times 10^{-11}$