

Homework 4

Due: Thursday, May 22.

1 Part I (50%)

This part is required to be submitted in class.

- (1) Let $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $g(\vec{x}) = \|F(\vec{x})\|_2^2$. Show that

$$\nabla g(\vec{x}) = 2J(\vec{x})^T F(\vec{x}),$$

where $J(\vec{x})$ is the Jacobian matrix of $F(\vec{x})$. In particular, if $F(\vec{x}) = A\vec{x} - \vec{b}$, then

$$\nabla g(\vec{x}) = 2A^T(A\vec{x} - \vec{b}).$$

- (2) Exercise 10.4.1. (a, b).

- (3) Exercise 10.5.3. (a, b).

- (4) Exercise 10.2.1. (a).

2 Part II (50%)

Consider the following nonlinear system

$$\begin{cases} 15x_1 + x_2^2 - 4x_3 = 13 \\ x_1^2 + 10x_2 - x_3 = 11 \\ x_2^3 - 25x_3 = -22. \end{cases}$$

Solve the system by applying the following methods with $\vec{x}^{(0)} = \vec{0}$, and compare the performance of each method in terms of the number of iterations, the running time, and the error, i.e., $\|\vec{x}^{(k)} - \vec{x}^*\|_\infty$ where \vec{x}^* is the actual solution.

- (a) Use Newton's method with the stopping criteria $\|\vec{x}^{(k)} - \vec{x}^{(k-1)}\|_\infty < 10^{-6}$.
- (b) Use the method of Steepest Descent to approximate the solution.
- (c) Use the homotopy method with the midpoint method and the Runge-Kutta method of order four respectively, where $N = 10; 20; 50$. Which way performs better?

Requirements Submit to CCLE a file `lastname_firstname_hw7.zip` containing the following files:

- MATLAB functions: `Newton.m` that implements Newton's method, `steep.m` that implements the method of steepest descent, `homotopy.m` that implements the homotopy method with various IVP solvers¹, e.g., midpoint method and RK4; and a MATLAB script `main.m` that solves the given nonlinear system.
- A PDF report that displays the solutions and answers the above questions.

REMARK: Please try to make your codes as efficient as possible so that the comparison is fair, especially to reduce the number of function evaluations. For instance, you may create some intermediate variables to avoid duplicate computations. The running time varies with programs and computer configurations.

¹One input argument should be set as a string to specify which IVP solver is to be used.