Math 151B Homework 2

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Part 2

Here we have the initial value problem (IVP).

$$\begin{cases} y'(t) = 1 + \frac{y}{t}, 1 \le t \le 2 \\ y(1) = 2 \end{cases}$$

(a) The Taylor's method has the following form

$$\begin{cases} w_{i+1} & = w_i + h(f(t_i, w_i) + \frac{h}{2}(\frac{\partial f(t_i,w_i)}{\partial t} + \frac{\partial f(t_i,w_i)}{\partial y} f(t_i, w_i)) \\ & = w_i + h(1 + \frac{w_i}{t_i} + \frac{h}{2}(-\frac{w_i}{t_i^2} + \frac{1}{t_i}(1 + \frac{w_i}{t_i}))) \\ w_0 & = 2 \end{cases}$$

(b) The midpoint method has the following form $\begin{cases} w_{i+1} = w_i + hf(t_i + \frac{h}{2}, w_i + \frac{h}{2} f(t_i, w_i)) \\ w_0 = 2 \end{cases}$

Results are shown here

```
Taylor's method:
step size: h = 0.200000
Error at t = 2: 0.004975, smaller than 10^-4?: 0
step size: h = 0.100000
Error at t = 2: 0.001248, smaller than 10^-4?: 0
step size: h = 0.050000
Error at t = 2: 0.000312, smaller than 10^-4?: 1
Midpoint method:
step size: h = 0.200000
Error at t = 2: 0.002479, smaller than 10^-4?: 0
step size: h = 0.100000
Error at t = 2: 0.000624, smaller than 10^-4?: 1
step size: h = 0.050000
Error at t = 2: 0.000156, smaller than 10^-4?: 1
```

As we can see from both errors, the approximation gets better as h gets smaller. The plots are shown in the Appendix B as Figure 2. $h = 0.05$ makes the Taylor's method achieve 10^{-4} accuracy while midpoint method only needs $h = 0.1$.
We use linear regression to estimate the order of the method, shown in Figure 1. We set vector $\vec{e}$ to be the error at $t = 2, h = 0.2, 0.1, 0.05$, and we let H_2 to be corresponding square of step sizes. The linear regression $H_2\vec{b} = \vec{e}$ gives us an almost perfect linear plot. Thus we can say we estimate the order of convergence for Taylor's method and Midpoint method. Here, both are 2.

(c) Here we only test the case when there are enough calculation workload, which is $h = 0.05$. The time for Taylor's method is 0.004293s and for midpoint method is 0.002141s. Midpoint method is almost twice as fast. Combining convergent speed and running speed we can confirm that midpoint method is more efficient.

A Code for Exercise 5.10.4

```
% main.m
a = 0; b = 1;
```

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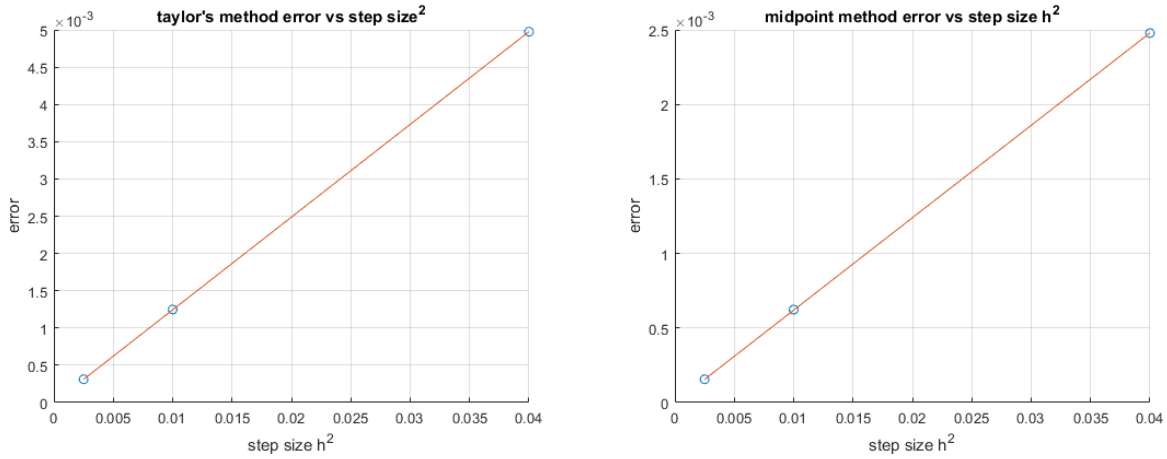

Figure 1: Linear regression of errors and step sizes square h^2

```
f = @(t, y) 1 - y;
y = @(t) 1 - exp(-t);

%%% Step size 0.1
h = 0.1;
mdiff(f, a, b, h, y);

%%% Step size 0.01
h = 0.01;
mdiff(f, a, b, h, y);

% mdiff.m
function [T, W] = mdiff(f, a, b, h, y)
    N = (b - a) / h;
    T = a + h * (0 : N)';
    W = zeros((N + 1), 1);
    W(1) = y(T(1));
    W(2) = y(T(2));

    for i = 2 : N
        W(i + 1) = 4 * W(i) - 3 * W(i - 1) - ...
                   2 * h * f(T(i - 1), W(i - 1));
    end
    fprintf('step size h = %f, result:\n', h);
    disp('T = 0.2');
    disp(W(find(T == 0.2)));
    disp('T = 0.5');
    disp(W(find(T == 0.5)));
    disp('T = 1');
    disp(W(find(T == 1)));
end
```

B Plots for Part 2

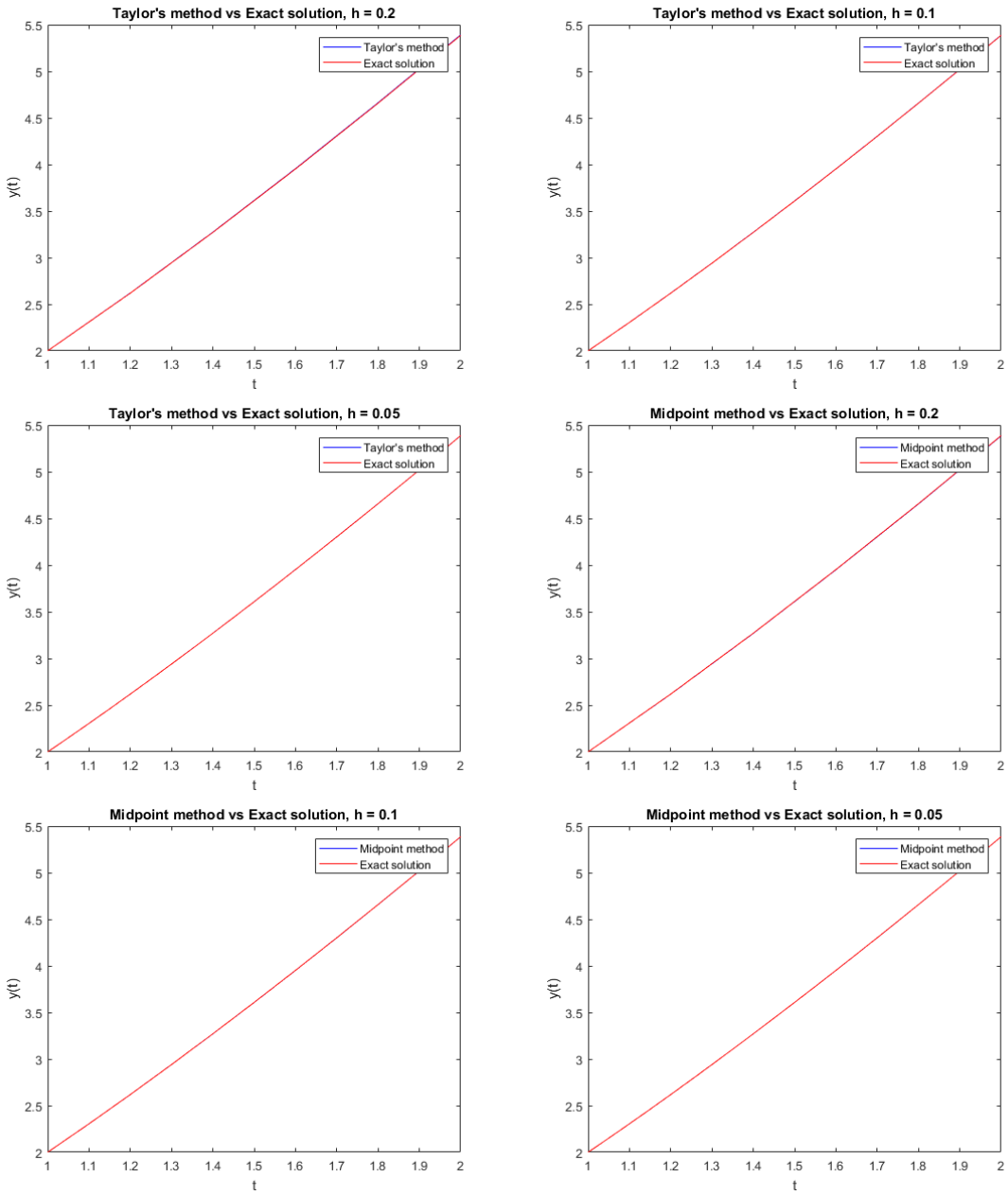

Figure 2: Plot of computed and exact solutions w.r.t. different step size h