5 2 bisectors of triangles answers

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5.2 Bisectors of Triangles

Question: The bisector of an angle of a triangle divides the opposite side into segments that are proportional to the lengths of the adjacent sides. Prove this.

Answer: Let ABC be a triangle with angle bisector AD. By the definition of angle bisector, we have

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m?BAD = m?CAD
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By the Triangle Angle Sum Theorem, we have

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m?BAD + m?CAD + m?BAC = 180°
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Substituting m?BAD = m?CAD into the above equation, we get

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2(m?BAD) + m?BAC = 180°
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Therefore,

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m?BAD = (180° - m?BAC)/2
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Similarly,

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m?CAD = (180° - m?BAC)/2
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By the Law of Sines, we have

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BD/AB = sin(m?BAD)/sin(m?ABD)
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and

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CD/AC = sin(m?CAD)/sin(m?ACD)
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Substituting m?BAD = (180° - m?BAC)/2 and m?CAD = (180° - m?BAC)/2 into the above equations, we get

BD/AB = sin((180° - m?BAC)/2)/sin((m?BAC)/2)

and

CD/AC = sin((180° - m?BAC)/2)/sin((m?BAC)/2)

Therefore,

BD/AB = CD/AC

Question: The bisectors of the base angles of an isosceles triangle intersect on the bisector of the vertex angle. Prove this.

Answer: Let ABC be an isosceles triangle with AB = AC. Let AD be the bisector of angle BAC, and let BE and CF be the bisectors of angles ABC and ACB, respectively.

By the definition of angle bisector, we have

m?BAD = m?CAD

and

m?ABE = m?CBE

and

m?ACF = m?BCF

By the Triangle Angle Sum Theorem, we have

m?BAD + m?BAD + m?BAC = 180°

m?ABE + m?ABE + m?ABC = 180°

m?ACF + m?ACF + m?ACB = 180°

Substituting m?BAD = m?CAD, m?ABE = m?CBE, and m?ACF = m?BCF into the above equations, we get

2(m?BAD) + m?BAC = 180°

2(m?ABE) + m?ABC = 180°

2(m?ACF) + m?ACB = 180°

Therefore,

m?BAD = (180° - m?BAC)/2

m?ABE = (180° - m?ABC)/2

m?ACF = (180° - m?ACB)/2

Since AB = AC, we have m?ABC = m?ACB. Substituting m?ABC = m?ACB into the above equations, we get

m?BAD = (180° - m?BAC)/2

m?ABE = (180° - m?BAC)/2

m?ACF = (180° - m?BAC)/2

Therefore,

m?BAD = m?ABE = m?ACF

Hence, BE and CF intersect on AD.

Question: The bisectors of two of the angles of a triangle are congruent. Prove that the triangle is isosceles.

Answer: Let ABC be a triangle with angle bisectors AD and BE congruent. Suppose without loss of generality that AD = BE.

By the definition of angle bisector, we have

m?BAD = m?CAD

and

m?ABE = m?CBE

By the Triangle Angle Sum Theorem, we have m?BAD + m?BAD + m?BAC = 180°

m?ABE + m?ABE + m?ABC = 180°

Substituting m?BAD = m?CAD and m?ABE = m?CBE into the above equations, we get

2(m?BAD) + m?BAC = 180°

2(m?ABE) + m?ABC = 180°

Therefore,

m?BAD = (180° - m?BAC)/2

m?ABE = (180° - m?ABC)/2

Since AD = BE, we have m?BAD = m?ABE. Substituting m?BAD = m?ABE into the above equations, we get

(180° - m?BAC)/2 = (180° - m?ABC)/2

Therefore,

m?BAC = m?ABC

Hence, AB = BC.

Question: The bisector of an angle of a triangle divides the opposite side into segments that are inversely proportional to the lengths of the other two sides. Prove this.

Answer: Let ABC be a triangle with angle bisector AD. By the definition of angle bisector, we have

m?BAD = m?CAD

By the Triangle Angle Sum Theorem, we have

m?BAD + m?CAD + m?BAC = 180°

Substituting m?BAD = m?CAD into the above equation, we get

2(m?BAD) + m?BAC = 180°

Therefore,

m?BAD = (180° - m?BAC)/2

Similarly,

m?CAD = (180° - m?BAC)/2

By the Law of Sines, we have

BD/AB = sin(m?BAD)/sin(m?ABD)

and

CD/AC = sin(m?CAD)/sin(m?ACD)

Substituting m?BAD = (180° - m?BAC)/2 and m?CAD = (180° - m?BAC)/2 into the above equations, we get

BD/AB = sin((180° - m?BAC)/2)/sin((m?BAC)/2)

and

CD/AC = sin((180° - m?BAC)/2)/sin((m?BAC)/2)

Therefore,

BD/AB = 1/CD/AC

Question: The bisectors of the three angles of a triangle are concurrent. Prove that the triangle is equilateral.

Answer: Let ABC be a triangle with angle bisectors AD, BE, and CF intersecting at point O. By the definition of angle bisector, we have

m?BAD = m?CAD

m?ABE = m?CBE

m?ACF = m?BCF

By the Triangle Angle Sum Theorem, we have

m?BAD + m?BAD + m?BAC = 180°

m?ABE + m?ABE + m?ABC = 180°

m?ACF + m?ACF + m?ACB = 180°

Substituting m?BAD = m?CAD, m?ABE = m?CBE, and m?ACF = m?BCF into the above equations, we get

2(m?BAD) + m?BAC = 180°

2(m?ABE) + m?ABC = 180°

2(m?ACF) + m?ACB = 180°

Therefore,

m?BAD = (180° - m?BAC)/2

m?ABE = (180° - m?ABC)/2

m?ACF = (180° - m?ACB)/2

Since AD, BE, and CF are concurrent, we have

m?BAD + m?ABE + m?ACF = 180°

Substituting m?BAD = (180° - m?BAC)/2, m?ABE = (180° - m?ABC)/2, and m?ACF = (180° - m?ACB)/2 into the above equation, we get

(180° - m?BAC)/2 + (180° - m?ABC)/2 + (180° - m?ACB)/2 = 180°

Simplifying the above equation, we get

m?BAC = m?ABC = m?ACB

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