# Facial Recognition Using PCA and Eigenfaces

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#### 1 Introduction

Facial recognition can be applied in a variety of problems such as image processing, personal identification and criminology.

In this report, we use the technique called eigenface, a model implemented for a particular face (of a person), and classify them using Principal Component Analysis, a method that rotates the dataset in a way such that the rotated features are statistically uncorrelated and selects the features that are relevant enough in explaining the data.

Our goal is to implement the model for a particular face and distinguish it from a large number of stored faces in dataset.

#### 2 Data set

We will use the images from the Labeled Faces in the Wild dataset. This dataset contains face images of celebrities downloaded from the Internet, and it includes faces of politicians, singers, actors, and athletes from the early 2000s. We will use grayscale, scaled down versions of the images for faster processing. The data set contains 2,530 images, each  $87 \times 65$  pixels large, belonging to 45 different people.

# 3 Terminologies

#### 3.1 Principal Component Analysis

PCA uses a vector space transform to reduce the dimensionality of large data sets, producing a set of important variables called Principal Components.

In facial recognition algorithm we are building, we reduce the facial structure to orthogonal and uncorrelated set of variables (eigenfaces). A facial image can be expressed as a weighted sum of the eigenfaces.

When the new facial image is given, our algorithm calculates the weights for each eigenvalue for the face and compares that with those of the existing training set of facial images. The algorithm then classifies the image using the K-nearest Neighbors or Support Vector Machine algorithm.

PCA uses an orthogonal linear transformation that transforms the data to a new coordinate system such that the greatest variance by some projection of the data comes to first coordinate (or first principal component).

The principal components of a data matrix X, with sample mean shifted to zero and each of the n rows represent different repetition of an experiemnt, is given as T = XW, where W is a p-by-p matrix whose columns are the eigenvectors of  $X^TX$ .  $W^T$  is also called whitening.

The *k*th component can be found by subtracting the first k-1 principal components from **X**:

$$\hat{\mathbf{X}}_k = \mathbf{X} - \sum_{s=1}^{k-1} \mathbf{X} \mathbf{w}_{(s)} \mathbf{w}_{(s)}^T$$

The weight vectors has to satisfy:

$$\mathbf{w}_{(1)} = \arg\max\{\frac{\mathbf{w}^T \mathbf{X}^T \mathbf{X} \mathbf{w}}{\mathbf{w}^T \mathbf{w}}\}$$

$$\mathbf{w}_{(k)} = \arg\max_{\|\mathbf{w}\|=1} \{\|\hat{\mathbf{X}}_k \mathbf{w}^2\|\} = \arg\max_{\mathbf{w}} \{\frac{\mathbf{w}^T \hat{\mathbf{X}}_k^T \hat{\mathbf{X}}_k \mathbf{w}}{\mathbf{w}^T \mathbf{w}}\}$$

, which yields the eigenvectors of  $\mathbf{X}^T\mathbf{X}$ 

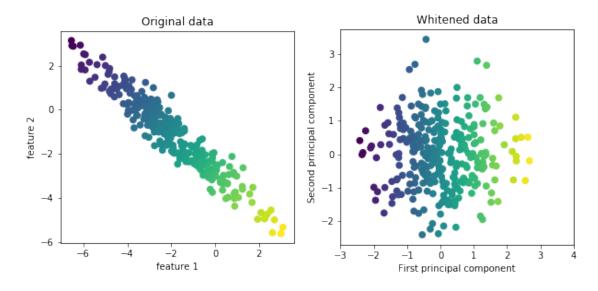
### 3.2 Whitening

Whitening is a generalization of feature normalization, which makes the data independent (i.e. decorrelates) by transforming it against a transformed input covariance matrix.

It corresponds to not only rotating the data, but also rescaling it so that the center panel is a circle instead of an ellipse. The whitening process can be summarized as below:

- 1. Project the dataset onto the eigenvectors. This rotates the dataset so that there is no correlation between the components.
- 2. Normalize the the dataset to have a variance of 1 for all components. This is done by simply dividing each component by the square root of its eigenvalue.

The transformation of data with PCA using whitening is done as follows:



### 3.3 Eigenfaces

Eigenfaces are the set of features in the form of vector (i.e. eigenvectors) that denotes the variation between faces. They are the weighted sum of the some component of faces. A set of eigenfaces can be generated by performing PCA on a large set of images depicting different human faces. Any human faces can be considered as linear combination of the components.

Eigenfaces are generated using PCA as below:

- 1. The training set is taken and are normalized (whitened).
- 2. The eigenvectors of the covariance matrix of the face image are extracted.
- 3. Eigenvectors are formed by converting the image matrix into vector form and the vector should satisfy the eigenvalue equation.

### 3.4 k-nearest Neighbours Algorithm

*k*-nearest Neighbours Algorithm is a supervised learning algorithm that classifies the given data by comparing with the *k* nearest neighbors in the existing data set.

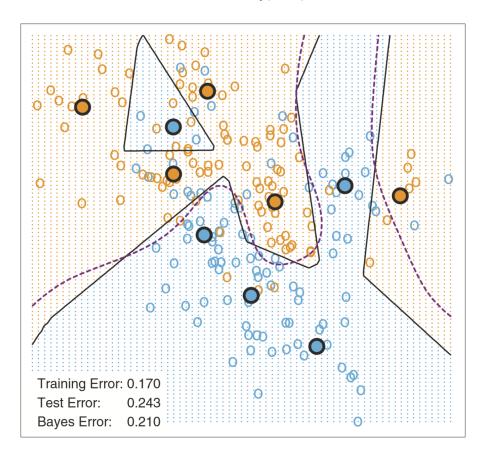
The algorithm can be summarized as below:

- 1. Choose the number of neighbors *k* and a distance metric
  - We will use the Euclidean distance, which is a Minkowski distance with p = 2 (i.e. root of the sum of squares of each difference between two points, based on the Pythagorean formula).

• 
$$d(x^{(i)}, x^{(j)}) = \sqrt[p]{\sum_k |x_k^{(i)} - x_k^{(j)}|^p}\Big|_{p=2}$$

- 2. Find the *k* nearest neighbors from the existing data set.
- 3. Assign the class determined by the majority.
  - The majority will be weighted by inverse of the distance.

K-means - 5 Prototypes per Class



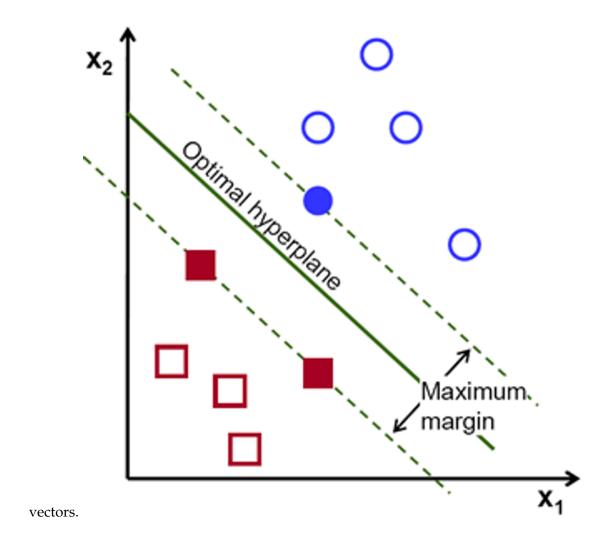
## 3.5 Support Vector Machine

Suport Vector machine is another supervised learning algorithm that can be used for classifying data. The algorithm classifies the given data by creating planes that separates each class. The planes are created such that they maximize the margin between different classes (optimal hyperplanes).

We maximize the distance to the nearest training data points of any class because doing so lowers the classification error. The test points or query points are then mapped into that same space and predicted to belong to a category based on which side of the gap they fall on.

In Python, the Gamma parameter defines how far the influence of a single training example reaches, with low values meaning 'far' and high values meaning 'close'.

The C parameter trades off misclassification of training examples against simplicity of the decision surface. A low C makes the decision surface smooth, while a high C aims at classifying all training examples correctly by giving the model freedom to select more samples as support



# 4 Experiment

```
In [2]: %matplotlib inline
```

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import itertools

from mpl_toolkits.mplot3d import Axes3D
from sklearn.model_selection import train_test_split
from sklearn.datasets import fetch_lfw_people
from sklearn.neighbors import KNeighborsClassifier
from sklearn.decomposition import PCA
from sklearn.svm import SVC
from sklearn.metrics import classification_report
from sklearn.metrics import confusion_matrix
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
```

```
plt.rcParams['image.cmap'] = "gray"
```

First, we fetch faces from the Labeled Faces in the Wild dataset.











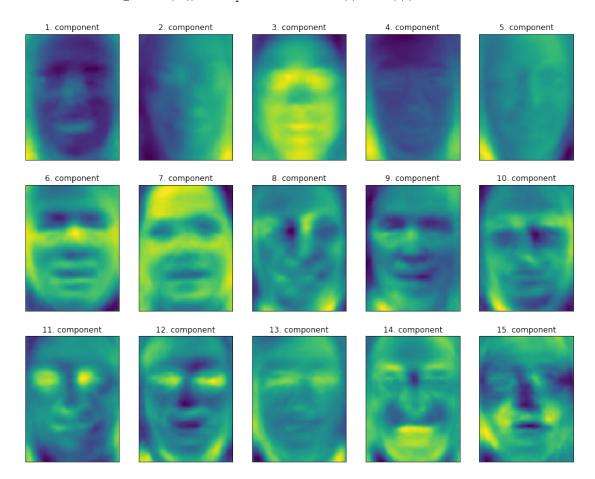
There are 2,530 images, with each  $87 \times 65$  pixels large, belonging to 45 different people.

Alejandro Toledo	39	Alvaro Uribe	35	Amelie Mauresmo	21
Andre Agassi	36	Ariel Sharon	77	Atal Bihari Vajpayee	24
Bill Clinton	29	Colin Powell	236	David Beckham	31
Donald Rumsfeld	121	George Robertson	22	George W Bush	530
Gerhard Schroeder	109	Gloria Macapagal Arroyo	44	Gray Davis	26
Hamid Karzai	22	Hans Blix	39	Hugo Chavez	71
Igor Ivanov	20	Jack Straw	28	Jacques Chirac	52
Jean Chretien	55	Jennifer Aniston	21	Jennifer Capriati	42
Jennifer Lopez	21	John Ashcroft	53	Juan Carlos Ferrero	28
Junichiro Koizumi	60	Kofi Annan	32	Laura Bush	41
Lindsay Davenport	22	Lleyton Hewitt	41	Megawati Sukarnoputri	33
Michael Bloomberg	20	Pete Sampras	22	Ricardo Lagos	20
Rudolph Giuliani	26	Saddam Hussein	23	Serena Williams	52
Tiger Woods	23	Tom Daschle	25	Tom Ridge	33
Tony Blair	144	Vicente Fox	32	Vladimir Putin	49

As data seems to be too skewed towards George W Bush and Colin Powell, we will only take up to 50 per person.

```
In [6]: # data too skewed, take 50 per person
       mask = np.zeros(people.target.shape, dtype=np.bool)
        for target in np.unique(people.target):
            mask[np.where(people.target == target)[0][:50]] = 1
        X_people = people.data[mask]
        y_people = people.target[mask]
        # rescale the grey-scale values to be between 0 and 1
        X_people = X_people / 255
In [7]: # split the data in training and test set
        X_train, X_test, y_train, y_test = train_test_split(
            X_people, y_people, stratify=y_people, random_state=0)
        # build a KNeighborsClassifier with using one neighbor:
        knn = KNeighborsClassifier(n_neighbors=1)
        knn.fit(X_train, y_train)
        print("Test set score of 1-nn: {:.2f}".format(knn.score(X_test, y_test)))
Test set score of 1-nn: 0.24
```

*K*-nearest Neighbours Algorithm itself can yield 24% of accuracy. Let's see what it can do with PCA. The principal components can be seen below:



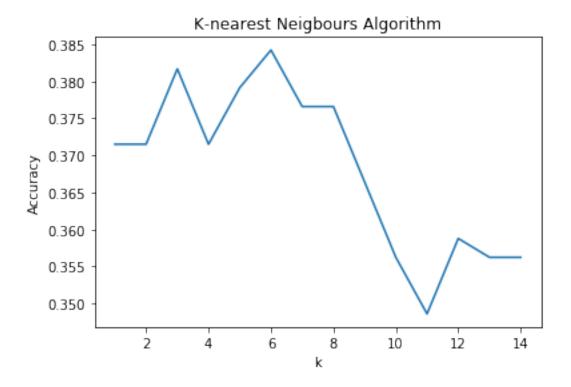
While they don't resemble how humans perceive other people's faces, they do seem to figure out some distinguishing features, such as shape of eyes, noses, chins, and cheekbones.

### 4.1 Comparing KNN and SVM

For *k*-nearest Neighbours Algorithm, we will use *k* from 1 to 14.

```
score[i] = knn.score(X_test_pca, y_test)
            print("{:2d}
                           {:.4f}".format(k[i], knn.score(X_test_pca, y_test)), end='
        fig = plt.figure()
        plt.plot(k, score, '-')
        plt.title('K-nearest Neigbours Algorithm')
        plt.xlabel('k')
        plt.ylabel('Accuracy')
k
     acc.
     0.3715
 1
 2
     0.3715
     0.3817
 3
     0.3715
 5
     0.3791
     0.3842
 7
     0.3766
     0.3766
 9
     0.3664
10
     0.3562
     0.3486
11
12
     0.3588
     0.3562
13
     0.3562
14
```

Out[9]: Text(0,0.5,'Accuracy')



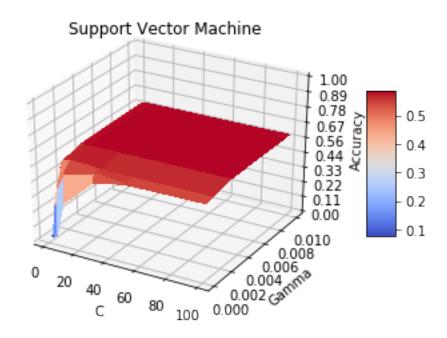
Changing k doesn't seem to affect the accuracy significantly. The maximum accuracy is 38.42% with k = 6.

For SVM, we will use varied values of C from 1 to 100 and Gamma from 0.0001 to 0.01, as seem below:

```
In [10]: c = [1,3,5,10,30,50,100]
        gamma = [0.0001, 0.0003, 0.001, 0.003, 0.01]
         score = np.zeros((7,5))
                      .0001
        print(" c
                                .0003
                                         .001
                                                 .003
                                                           .01\n")
        for i in range(len(c)):
            print("{:3d}".format(c[i]), end='
             for j in range(len(gamma)):
                 svc = SVC(C=c[i], gamma=gamma[j])
                 svc.fit(X_train_pca, y_train)
                 score[i][j] = svc.score(X_test_pca, y_test)
                 print("{:.4f}".format(score[i][j]), end=' ')
             print("\n")
         fig = plt.figure()
         ax = fig.gca(projection='3d')
         ax.set_title('Support Vector Machine')
         ax.set_xlabel('C')
         ax.set_ylabel('Gamma')
         ax.set_zlabel('Accuracy')
         c, gamma = np.meshgrid(c, gamma)
         # Plot the surface.
         surf = ax.plot_surface(c , gamma, np.transpose(score), cmap=cm.coolwarm,
                                linewidth=0, antialiased=False)
         # Customize the z axis.
         ax.set_zlim(0,1)
         ax.zaxis.set_major_locator(LinearLocator(10))
         ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))
         # Add a color bar which maps values to colors.
        fig.colorbar(surf, shrink=0.5, aspect=5)
      .0001
                .0003
                         .001
                                  .003
  С
                                           .01
              0.0305 0.2774 0.4504
  1
     0.0305
                                         0.5522
  3
     0.0305
              0.2621
                       0.4962
                                0.5725
                                          0.5852
  5
    0.0509
              0.3893
                       0.5598
                                0.5802
                                          0.5878
 10
    0.2977
              0.5064
                       0.5725
                               0.5852
                                        0.5878
```

```
30
      0.5064
               0.5649
                         0.5547
                                  0.5852
                                            0.5878
 50
      0.5496
               0.5394
                         0.5547
                                   0.5852
                                            0.5878
100
      0.5598
               0.5420
                         0.5522
                                   0.5852
                                            0.5878
```

Out[10]: <matplotlib.colorbar.Colorbar at 0x29ae3032780>



Higher values of C and Gamma improve the algorithm, with maximum accuracy is 58.78% at c = 100 and gamma = 0.01.

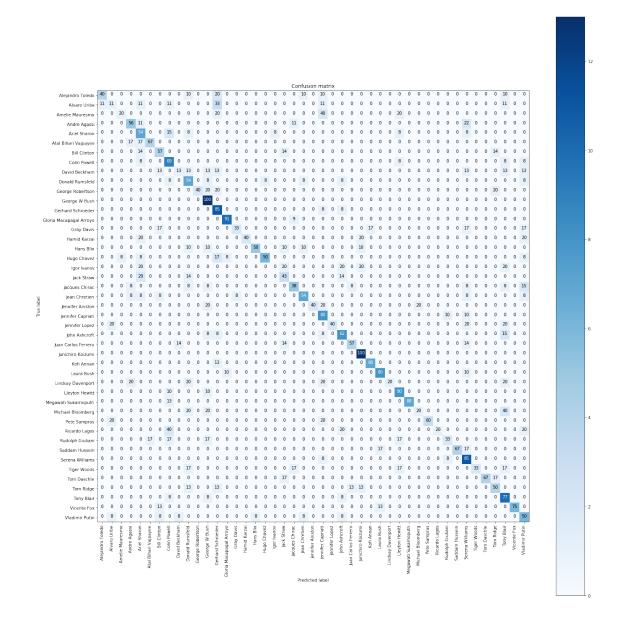
#### 4.2 Confusion Matrix

We can use confusion matrix and classification report to delve a little deeper into the performance on a per class basis:

```
fig.set_figheight(20)
             fig.set_figwidth(20)
             plt.imshow(cm, interpolation='nearest', cmap=cmap)
             plt.title(title)
             plt.colorbar()
             tick_marks = np.arange(len(classes))
             plt.xticks(tick_marks, classes, rotation=90)
             plt.yticks(tick_marks, classes)
             if normalize:
                 cm2 = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]
             else:
                 cm2 = cm
             thresh = cm.max() * 0.50
             for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
                 plt.text(j, i, int(cm2[i, j]*100+0.5),
                          horizontalalignment="center",
                          color="white" if cm[i, j] > thresh else "black")
             plt.tight_layout()
             plt.ylabel('True label')
             plt.xlabel('Predicted label')
In [12]: # suc with c=100 and gamma = 0.01
         y_pred=svc.predict(X_test_pca)
         cm=confusion_matrix(people.target_names[y_test], people.target_names[y_pred], labels=pe
         plot_confusion_matrix(cm,people.target_names,normalize=True)
         print(classification_report(y_test, y_pred, target_names=people.target_names))
                         precision
                                      recall f1-score
                                                          support
       Alejandro Toledo
                              0.80
                                        0.40
                                                  0.53
                                                               10
           Alvaro Uribe
                              0.25
                                        0.11
                                                  0.15
                                                                9
                                        0.20
                                                  0.29
                                                                5
        Amelie Mauresmo
                              0.50
                                        0.56
                                                  0.56
                                                                9
           Andre Agassi
                              0.56
           Ariel Sharon
                              0.39
                                        0.54
                                                  0.45
                                                               13
  Atal Bihari Vajpayee
                              0.80
                                        0.67
                                                  0.73
                                                                6
           Bill Clinton
                              0.44
                                        0.57
                                                  0.50
                                                                7
           Colin Powell
                              0.47
                                        0.69
                                                  0.56
                                                               13
          David Beckham
                              0.33
                                        0.12
                                                  0.18
                                                                8
        Donald Rumsfeld
                              0.41
                                        0.54
                                                  0.47
                                                               13
                                        0.40
                                                  0.57
                                                                5
       George Robertson
                              1.00
          George W Bush
                              0.54
                                        1.00
                                                  0.70
                                                               13
      Gerhard Schroeder
                              0.46
                                        0.85
                                                  0.59
                                                               13
Gloria Macapagal Arroyo
                              0.83
                                        0.91
                                                  0.87
                                                               11
```

fig = plt.figure()

Gray Davis	0.67	0.33	0.44	6
Hamid Karzai	1.00	0.40	0.57	5
Hans Blix	0.83	0.50	0.62	10
Hugo Chavez	0.86	0.50	0.63	12
Igor Ivanov	0.00	0.00	0.00	5
Jack Straw	0.38	0.43	0.40	7
Jacques Chirac	0.62	0.38	0.48	13
Jean Chretien	0.64	0.54	0.58	13
Jennifer Aniston	1.00	0.40	0.57	5
Jennifer Capriati	0.44	0.80	0.57	10
Jennifer Lopez	1.00	0.40	0.57	5
John Ashcroft	0.53	0.62	0.57	13
Juan Carlos Ferrero	0.67	0.57	0.62	7
Junichiro Koizumi	0.76	1.00	0.87	13
Kofi Annan	0.88	0.88	0.88	8
Laura Bush	0.80	0.80	0.80	10
Lindsay Davenport	1.00	0.20	0.33	5
Lleyton Hewitt	0.62	0.80	0.70	10
Megawati Sukarnoputri	1.00	0.88	0.93	8
Michael Bloomberg	0.50	0.20	0.29	5
Pete Sampras	1.00	0.60	0.75	5
Ricardo Lagos	1.00	0.20	0.33	5
Rudolph Giuliani	0.50	0.33	0.40	6
Saddam Hussein	1.00	0.67	0.80	6
Serena Williams	0.48	0.85	0.61	13
Tiger Woods	1.00	0.33	0.50	6
Tom Daschle	1.00	0.67	0.80	6
Tom Ridge	0.57	0.50	0.53	8
Tony Blair	0.43	0.77	0.56	13
Vicente Fox	1.00	0.75	0.86	8
Vladimir Putin	0.38	0.50	0.43	12
avg / total	0.64	0.59	0.57	393



Looks like there are some cases where Ricardo Lagos is misclassified as Colin Powell, Michael Bloomberg as Tony Blair and Amelie Mauressimo as Jennifer Capriati.

```
In [13]: def show_misclassified(row_total, col_total, test, pred=-1, notpred=False):
    fig = plt.figure()
    fig.set_figheight(4*row_total)
    fig.set_figwidth(3*col_total)
    # set pred = -1 to show all faces from a person from test
    # set notpred = True to set filter to NOT equal pred
    if pred == -1:
        print('All faces of ' + people.target_names[test] + ':')
        filter = (y_test==test)
    elif notpred:
```

```
if test == pred:
                     print('Faces of ' + people.target_names[test] + ' misclassified:')
                 else:
                     print('Faces of ' + people.target_names[test] + ' NOT classifed as ' + peop
                 filter = (y_test==test) & (y_pred != pred)
             else:
                 if test == pred:
                     print('Faces of ' + people.target_names[test] + ' classified correctly:')
                     print('Faces of ' + people.target_names[test] + ' misclassified as ' + peop
                 filter = (y_test==test) & (y_pred == pred)
             misclassified = X_test[filter]
             total = sum(filter)
             for row, col in itertools.product(range(row_total), range(col_total)):
                 ind = row*col_total + col
                 if ind < total:
                     ax = plt.subplot2grid((row_total, col_total), (row, col))
                     ax.axis('off')
                     ind = row*col_total + col
                     ax.set_title(people.target_names[test])
                     ax.imshow(misclassified[ind].reshape(87,65),cmap='gray')
             plt.tight_layout()
In [14]: show_misclassified(4, 7, 35, 7)
         show_misclassified(4, 7, 35, 7, True)
         show_misclassified(4, 7, 7)
Faces of Ricardo Lagos misclassified as Colin Powell:
Faces of Ricardo Lagos NOT classifed as Colin Powell:
All faces of Colin Powell:
```

Ricardo Lagos



Ricardo Lagos



Ricardo Lagos



Ricardo Lagos



Ricardo Lagos





While Lagos and Powell do look similar, there aren't significant differences between those who were clasified as Powell and those who weren't. As Lagos performed quite badly (20%), let's see those faces that were classified correctly.

In [15]: show\_misclassified(4, 7, 35, 35)

Faces of Ricardo Lagos classified correctly:

# Ricardo Lagos



The one photo that were classified correctly has glasses that distinguished the photo from the others. Also, the other photos tend to be aligned poorly.

Let's look at Bloomberg and Blair.

Faces of Michael Bloomberg misclassified as Tony Blair: Faces of Michael Bloomberg NOT classifed as Tony Blair: All faces of Tony Blair:

Michael Bloomberg



### Michael Bloomberg











Again, they do seem to look alike a lot, especially in such a low resolution and monochrome photos.

Bloomberg also has an accuracy of 20%

In [17]: show\_misclassified(4, 5, 33, 33)

Faces of Michael Bloomberg classified correctly:

Michael Bloomberg



Looks like the shades helped him classified correctly.

Faces of Amelie Mauresmo misclassified as Jennifer Capriati: Faces of Amelie Mauresmo NOT classifed as Jennifer Capriati: All faces of Jennifer Capriati:

Amelie Mauresmo



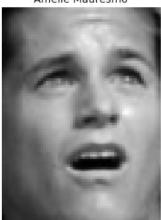
Amelie Mauresmo



Amelie Mauresmo



Amelie Mauresmo



Amelie Mauresmo





They not only do look alike, but also contain frowning photos. As they are both Tennis players, lots of photos of them seem to be taken during a match.

Let's see what photos of Mauressimo classfied correctly.

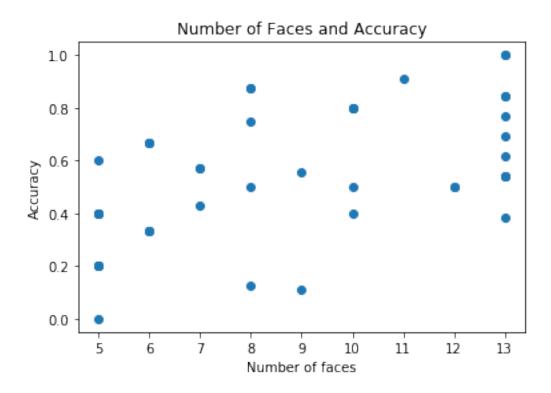
In [19]: show\_misclassified(4, 5, 2, 2)

Faces of Amelie Mauresmo classified correctly:

### Amelie Mauresmo



It could be the shades this time too, but Capriati also have shady photos.



People with more number of faces tend to have higher accuracy, as expected.

### 5 Conclusion

Both KNN and SVM are robust algorithms, despite their simplicity. However, it may perform differently in real life dataset. For example, coloured images with higher dataset or images with similar faces such as twins may result in lower accuracy or slower computation.

The confusion matrix indicates that the algorithm seems to struggle with images with similar facial expression and poor alignment. It also seems to be affected by the shades within the images. This indicates that a more advanced pre-processing algorithm may improve the performance significantly. Also, focusing on domain knowledge such as shape of eye, nose, chin etc. may increase

the accuracy as the algorithm doesn't use the same metrics as humans use. As people with higher number of faces performed better, gathering more data may also help.

With cutting edge, propreietary ML algorithms, facial recognition algorithm that is as accurate as human brain may be achievable. Facebook's facial recognition project, DeepFace, has achieved the accuracy of 97.25%. Humans can perform same task with 97.53% accuracy.

## 6 Acknowledgements

I would like to thank Prof. David A. Stephens for this opportunity to research and his assitance through the research.

### 7 References

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