CST 370 Design and Analysis of Algorithms
Spring 2020
Midterm – II

Name: ______________________________________

Four-digits ID: ________________________________

"On my honor, I have neither given nor received unauthorized aid in doing this assignment."

Signature (Write Your Name)____________________________

- Do not start until told to do so.
- Look over all the questions and observe their point values before you start.
- Use your time wisely—make sure to answer the questions you know first.
- **Read the questions carefully.**

1. (2 points) Consider the following master theorem:

$T(n) = aT(n/b) + f(n)$ where $f(n) \in \Theta(n^d)$, $d \geq 0$

Master Theorem: If $a < b^d$, $T(n) \in \Theta(n^d)$
If $a = b^d$, $T(n) \in \Theta(n^d \log n)$
If $a > b^d$, $T(n) \in \Theta(n^{\log_b a})$

Based on the theorem, select the correct time efficiency for each T(n). You have to **select your answer among 1, 2, 3, 4, and 5 clearly**.

(a) T(n) = 2 * T(n/4) + 4n + 7

1. $\Theta(n^2)$
2. $\Theta(n*log\ n)$
3. $\Theta(n)$
4. $\Theta(n^{log_4 2})$
5. None of the above.

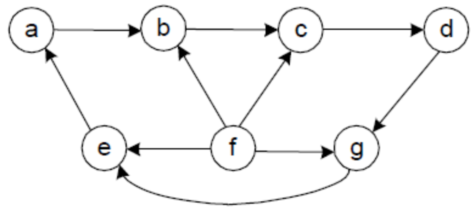
Your answer: ___________

(b) T(n) = 4 * T(n/2) + 3n^2 + 5n

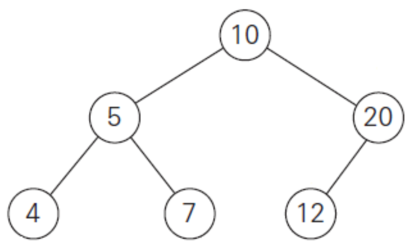
1. $\Theta(n^2)$
2. $\Theta(n*log\ n)$
3. $\Theta(n)$
4. $\Theta(n^{log_4 2})$
5. None of the above.

Your answer: ___________

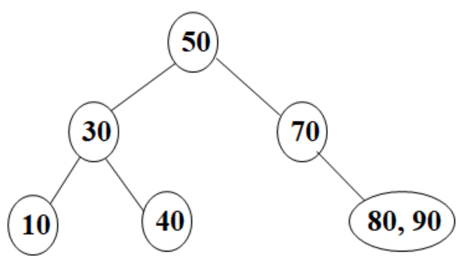
2. (3 points) Is the following graph a DAG (= directed acyclic graph)? (Yes/No)



(b) Is this an AVL tree? (Yes/ No)



(c) Is this a 2-3 tree? (Yes/ No)



3. (1 point) The following algorithm is designed to calculate the number of leaves in a binary search tree. Is this algorithm correct? (Yes / No)

```
Algorithm LeafCounter(T)
//Input: A binary search tree T
//Output: The number of leaves in T
if (T == NULL)
    return 0
else
    return LeafCounter(T_LEFT)+ LeafCounter(T_RIGHT)
```

4. (5 points) Consider the following pseudocode.

```
Algorithm DoSomething ( )
1.  n = 5;
2.  array[5] = {10, 5, 8, 1, 7};
3.  secret = array[n-1];
4.  i = -1;
5.
6.  for j ← 0 to n-1 do
7.  {
8.     if (array[j] < secret)
9.     {
10.       i ← i + 1
11.       swap array[i] and array[j]
12.    }
13. }
14. swap array[i+1] and array[n-1]
15.
16. return;
```

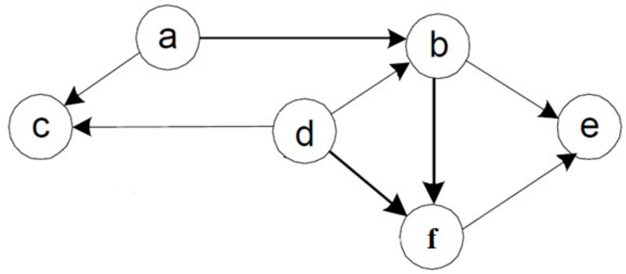
Present the values of "**array**" and "**secrete**" at the end of execution result at the line number 16.

array[0]	
array[1]	
array[2]	
array[3]	
array[4]	
secrete	

5. (2 points) Consider the quicksort algorithm covered in the class. Present the **result of first partitioning operation** for the list “**50 40 30 20 10 70**”. In other words, you have to conduct the operation until the indexes i and j meet and cross over. After that, the pivot value should be swapped. For the problem, you should use the first number, **50, as a pivot** for the partitioning. For the problem, **do not present the intermediate steps**. **Just write the sequence of numbers** after the fist partitioning operation.

Your answer: ____

6. (3 points) For the following graph, you are going to conduct the **topological sorting**.

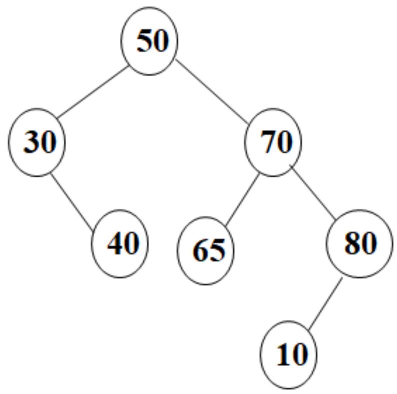


(a) Present the **starting node(s)** for the topological sorting.

(b) Conduct the topological sorting using the **DFS algorithm**. Then, present the **topological order** as we discussed in the class. For the problem, you have to follow our convention of alphabetical order.

Topological Order: ____

[Note] Before solving the problem 7, 8, 9, and 10, read the following description carefully.
In the problem 7, 8, 9 and 10, you have to present the result trees in the **level-by-level order**. This is an example of level-by-level order for a sample tree below. Note that the root value 50 is the level 0. Then, its children (= 30 and 70) should be the level 1. Also, because there's no value in the level 4 and 5, we use "NONE" to indicate them.



A Sample Tree

Level 0	**50**
Level 1	**30, 70**
Level 2	**40, 65, 80**
Level 3	**10**
Level 4	**NONE**
Level 5	**NONE**

Level-By-Level Order

7. (5 points) (a) Consider a binary tree with three nodes with values 10, 20 and 30 in such a way that the inorder and preorder traversals of the tree yield the following lists:

10, 30, 20 (inorder)
10, 20, 30 (postorder).

Note that the problem is asking to **consider only one binary tree**. For the problem, **do not draw the result in the word file**. Instead, **write the values of the result tree level-by-level order**.
If you think that it's not possible to have a binary tree with the given information, explain why.

Level 0	
Level 1	

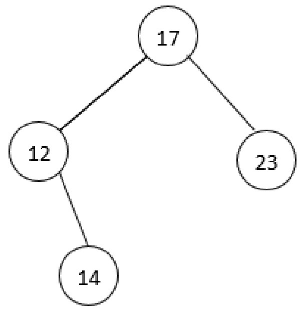
(b) Consider a binary tree with six nodes with the values 10, 20, …, 60 in such a way that the inorder and postorder traversals of the tree yield the following lists:

20, 10, 40, 60, 50, 30 (inorder)
40, 20, 10, 50, 60, 30 (preorder)

Note that the problem is asking to **consider only one binary tree**. If you think that it's not possible to have a binary tree with the given information, explain why.

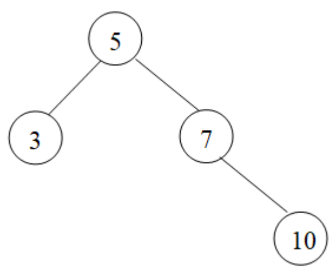
Level 0	
Level 1	
Level 2	
Level 3	

8. (4 points) (a) Assume that you have an AVL tree like below. Add a node with the value **15**. After that, present the result AVL tree using the level-by-level order. For the problem, **do not draw the result in the word file**. Instead, **write the values in the result tree level-by-level order**.



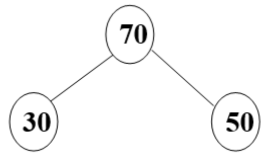
Level 0	
Level 1	
Level 2	
Level 3	

(b) Assume that you have an AVL tree like below. Add a node with the value **8**. After that, **write the values in the result tree level-by-level order**.



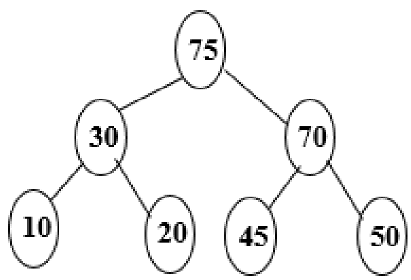
Level 0	
Level 1	
Level 2	
Level 3	

9. (2 points) (a) Add 55 to the following **max heap**. After that, **write the result max-heap using the level-by-level order**.



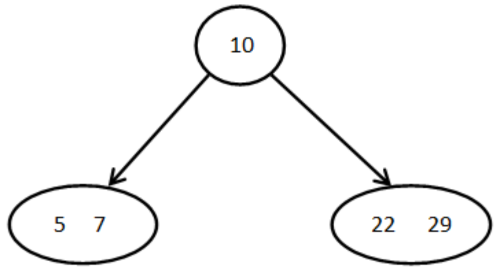
Level 0	
Level 1	
Level 2	

(b) Delete the max value from the following max heap. After that, **write the result max-heap using the level-by-level order**.



Level 0	
Level 1	
Level 2	
Level 3	

10. (3 points) Assume that you have a 2-3 tree like below. Add the two numbers **9** and **27** to the tree one by one. After that, **present the result 2-3 tree using the level-by-level order**



Level 0	
Level 1	
Level 2	
Level 3	