Estimating Planetary Habitability via Particle Swarm Optimization of CES Production Functions.

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January 18, 2018

1 Introduction

The search for extra-terrestrial life and potentially habitable extrasolar planets has been an international venture demanding large investments in cost and effort, since Frank Drake's attempt with Project Ozma in the mid-20th century. The first exoplanet was officially confirmed in 1992 which marked the start of a trend that has lasted 25 years and yielded over 3,700 confirmed exoplanets. There have been attempts to assess the habitability of these planets and to assign a score based on their similarity to Earth. Two such habitability scores are the Cobb-Douglas Habitability Score (CDHS) and the Constant Elasticity Earth Similarity Approach (CEESA) score. Estimating these scores involves maximizing a production function while observing a set of constraints on the input variables.

Under most paradigms, maximizing a continuous function requires calculating a gradient. This may not always be feasible for non-polynomial functions in high-dimensional search spaces. Further, subjecting the input variables to constraints, as needed by CDHS and CEESA, are not always straightforward to represent within the model. This paper presents an approach to Constrained Optimization (CO) using the swarm intelligence metaheuristic. Particle Swarm Optimization (PSO) is a method for optimizing a continuous function that does away with the need for a gradient. It employs a large number of particles that traverse the search space converging toward a global best solution encountered by at least one of the particles.

Particle Swarm Optimization is a distributed method that requires simple mathematical operators and short segments of code, making it a lucrative solution where computational resources are at a premium. Its implementation is highly parallelizable. It scales with the dimensionality of the search space. The standard PSO algorithm does not deal with constraints but through variations in initializing and updating particles constraints are straightforward to represent and adhere to, as seen in Section 3.2.

PSO has been adapted to a wide range of design optimization problems including network and VLSI design. It has found applications in machine learning under clustering, feature detection and classification. As a modeling paradigm, it has been used for constructing customer satisfaction models, modeling MIDI music and chaotic time series modeling.

This paper demonstrates the applicability of Particle Swarm Optimization in estimating CDHS and CEESA scores of an exoplanet by maximizing their respective production functions, discussed in Sections 2.1 and 2.2. CDHS considers the planet's Radius, Mass, Escape Velocity and Surface Temperature, while CEESA includes a fifth parameter, the Orbital Eccentricity of the planet. The Exoplanets Catalog hosted by the Planetary Habitability Laboratory, UPR Arecibo records these parameters for each exoplanet in Earth Units. Section 5 reports the performance of PSO and discusses the distribution of the habitability scores of the exoplanets.

2 Habitability Scores

2.1 Cobb-Douglas Habitability Score

Estimating the Cobb-Douglas Habitability Score (CDHS) requires estimating an interior CDHS ($\mathrm{CDHS_i}$) and a surface CDHS ($\mathrm{CDHS_s}$) by maximizing the following production functions,

$$Y_i \;=\; CDHS_i \;=R^{\alpha}.\,D^{\beta} \tag{1a}$$
$$Y_s \;=\; CDHS_s \;=V_e{}^{\gamma}.T_s{}^{\delta} \tag{1b}$$

where, R, D, V_e and T_s are density, radius, escape velocity and surface temperature respectively. α, β, γ and δ are the elasticity coefficients subject to,

$$0 < \alpha, \beta, \gamma, \delta < 1 \tag{2}$$

Equations 1a and 1b are convex under either Constant Returns to Scale (CRS) or Decreasing Returns to Scale (DRS) marked by two constraints on the elasticity coefficients,

$$\text{CRS:} \quad \alpha + \beta = 1, \quad \gamma + \delta = 1, \tag{3a}$$
$$\text{DRS:} \quad \alpha + \beta < 1, \quad \gamma + \delta < 1. \tag{3b}$$

The final CDHS is the convex combination of the interior and surface CDHS values as given by,

$$Y \;=\; w_i.Y_i + w_s.Y_s \tag{4}$$

2.2 Constant Elasticity Earth Similarity Approach

The Constant Elasticity Earth Similarity Approach (CEESA) uses the following production function to estimate the habitability score of an exoplanet,

$$Y = (r.R^{\rho} + d.D^{\rho} + t.T_s{}^{\rho} + v.V_e{}^{\rho} + e.E^{\rho})^{\frac{\eta}{\rho}} \tag{5}$$

where, E is the fifth parameter denoting Orbital Eccentricity. The value of ρ lies within $0 < \rho \leq 1$. The coefficients (r, d, t, v and e) are constrained by,

$$0 < r, d, t, v, e < 1 \tag{6a}$$
$$r + d + t + v + e = 1 \tag{6b}$$

The value of η is constrained by the scale of production used,

$$\text{CRS:} \quad 0 < \eta \leq 1, \tag{7a}$$
$$\text{DRS:} \quad \eta = 1. \tag{7b}$$

3 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a biologically inspired metaheuristic for finding the global minima of a function. Traditionally designed for unconstrained inputs, it works by iteratively converging a population of randomly initialized solutions, called particles, toward a globally optimal solution. Each particle in the population keeps track of its current position and the best solution it has encountered so far, called *pbest*. Each particle also has an associated randomized velocity used to traverse the search space. The swarm keeps track of the overall best solution, called *gbest*. Each iteration of the swarm updates the velocity of the particle towards its *pbest* and the *gbest* values.

Procedure 1 Algorithm for PSO.

Input: $f(x)$, the function to minimize.
Output: global minimum of $f(x)$.

1: **for** each particle $i \leftarrow 1, n$ **do**
2: $\quad p_i \sim U(l,u)^d$
3: $\quad v_i \sim U(-|u-l|, |u-l|)^d$
4: $\quad pbest_i \leftarrow p_i$
5: **end for**
6: $gbest \leftarrow \underset{pbest_i,\, i=1\ldots n}{\arg\min} f(pbest_i)$
7: **repeat**
8: $\quad oldbest \leftarrow gbest$
9: $\quad$ **for** each particle $i \leftarrow 1 \ldots n$ **do**
10: $\quad\quad u_p, u_g \sim U(0,1)$
11: $\quad\quad v_i \leftarrow \mu.v_i + \lambda_g.u_g.(gbest - p_i) + \lambda_p.u_p.(pbest_i - p_i)$
12: $\quad\quad p_i \leftarrow p_i + v_i$
13: $\quad\quad$ **if** $f(p_i) < f(pbest_i)$ **then**
14: $\quad\quad\quad pbest_i \leftarrow p_i$
15: $\quad\quad$ **end if**
16: $\quad$ **end for**
17: $\quad gbest \leftarrow \underset{pbest_i,\, i=1\ldots n}{\arg\min} f(pbest_i)$
18: **until** $|\, oldbest - gbest \,| < threshold$
19: **return** $f(gbest)$

3.1 PSO for Unconstrained Optimization

Let $f(x)$ be the function to be minimized, where x is a d-dimensional vector. $f(x)$ is also called the fitness function. Algorithm 1 outlines the approach to minimizing $f(x)$ using PSO. A set of particles are randomly initialized with a position and a velocity, where l and u are the lower and upper boundaries of the search space. The position of the particle corresponds to its associated solution. The algorithm initializes each particle's *pbest* to its initial position. The *pbest* position that corresponds to the minimum fitness is selected to be the *gbest* position of the swarm.

On each iteration, the algorithm updates the velocity and position of each particle. For each particle, it picks two random numbers u_g, u_p from a uniform distribution, $U(0,1)$ and updates the particle velocity as indicated in line 11. Here, μ is the friction coefficient and λ_g, λ_p are the global and particle learning rates. If the new position of the particle corresponds to a better fit than its *pbest*, the algorithm updates *pbest* to the new position. Once the algorithm has updated all particles, it updates *gbest* to the new overall best position. A suitable termination criteria for the swarm, under convex optimization, is when the *gbest* position has not changed by the end of the iteration.

3.2 PSO with Leaders for Constrained Optimization

Although PSO has eliminated the need to estimate the gradient of a function, as seen in Section 3.1, it still is not suitable for constrained optimization. The standard PSO algorithm does not ensure that the initial solutions are feasible, and neither does it guarantee that the individual solutions will converge to a feasible global solution. Solving the initialization problem is straightforward, resample each random solution from the uniform distribution until every initial solution is feasible. To solve the convergence problem, each particle uses another particle's *pbest* value, called *lbest*, instead of its own to update its velocity. Algorithm 2 describes this process.

On each iteration, for each particle, the algorithm first picks two random numbers u_g, u_p as before. It then selects a *pbest* value from all particles in the swarm that is closest to the position of the

Procedure 2 Algorithm for CO by PSO.

Input: $f(x)$, the function to minimize.
Output: global minimum of $f(x)$.

for each particle $i \leftarrow 1, n$ **do**
  repeat
    $p_i \sim U(l, u)^d$
  until p_i satisfies all constraints
  $v_i \sim U(-|u-l|, |u-l|)^d$
  $pbest_i \leftarrow p_i$
end for
$gbest \leftarrow \underset{pbest_i,\, i=1\ldots n}{\arg\min} f(pbest_i)$
repeat
  $oldbest \leftarrow gbest$
  for each particle $i \leftarrow 1 \ldots n$ **do**
    $u_p, u_g \sim U(0, 1)$
    $lbest \leftarrow \underset{pbest_j,\, j=1\ldots n}{\arg\min} \|pbest_j - p_i\|^2$
    $v_i \leftarrow \mu.v_i + \lambda_g.u_g.(gbest - p_i) + \lambda_p.u_p.(lbest - p_i)$
    $p_i \leftarrow p_i + v_i$
    if $f(p_i) < f(pbest_i)$ **and** p_i satisfies all constraints **then**
      $pbest_i \leftarrow p_i$
    end if
  end for
  $gbest \leftarrow \underset{pbest_i,\, i=1\ldots n}{\arg\min} f(pbest_i)$
until $|\, oldbest - gbest \,| < threshold$
return $f(gbest)$

particle being updated as its *lbest*. The *lbest* value substitutes $pbest_i$ in the velocity update equation. While updating *pbest* for the particle, the algorithm checks if the current fit is better than *pbest*, and performs the update if the current position satisfies all constraints. The algorithm updates *gbest* as before.

4 Representing the Problem

A Constrained Optimization problem can represented as,

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g_k(x) \leq 0, \qquad k = 1 \ldots q, \\ & h_l(x) = 0, \qquad l = 1 \ldots r. \end{aligned}$$

Introducing an error threshold ϵ can convert strict inequalities of the form ${g_k}'(x) < 0$ to non-strict inequalities of the form $g_k(x) = {g_k}'(x) + \epsilon \leq 0$. Adding a tolerance τ transforms equality constraints to a pair of inequalities,

$$\begin{aligned} g_{(q+l)}(x) &= h_l(x) - \tau \quad && \leq 0, \quad l = 1 \ldots r, \\ g_{(q+r+l)}(x) &= -h_l(x) - \tau \quad && \leq 0, \quad l = 1 \ldots r. \end{aligned}$$

Thus, r equality constraints become $2r$ inequality constraints, raising the total number of constraints, denoted by s, to $s = q + 2r$. For each solution p_i, c_i denotes the constraint vector where, $c_{ik} =$

Parameter	Description	Unit
P. Radius	Estimated radius	Earth Units (EU)
P. Density	Density	Earth Units (EU)
P. Esc Vel	Escape velocity	Earth Units (EU)
P. Ts Mean	Mean Surface temperature	Kelvin (K)
P. Eccentricity	Orbital eccentricity	

Table 1: Parameters from the PHL-EC used for the experiment.

$\max\{g_k(p_i), 0\},\ k = 1 \ldots s$. When $c_{ik} = 0,\ \forall k = 1 \ldots s$, the solution p_i lies within the feasible region. When $c_{ik} > 0$, the solution p_i violates the k^{th} constraint.

Following these guidelines to represent a CO problem, CDHS estimation under CRS becomes,

$$\begin{aligned}
&\underset{\alpha,\beta,\gamma,\delta}{\text{minimize:}} && Y_i = -R^{\alpha}.D^{\beta},\ Y_s = -{V_e}^{\gamma}.{T_s}^{\delta} \\
&\text{subject to:} && -\phi + \epsilon \leq 0, \quad \phi - 1 + \epsilon \leq 0, \qquad \forall \phi \in \{\alpha, \beta, \gamma, \delta\} \\
& && (\alpha + \beta - 1) - \tau \leq 0, \quad (\gamma + \delta - 1) - \tau \leq 0, \\
& && (1 - \alpha - \beta) - \tau \leq 0, \quad (1 - \gamma - \delta) - \tau \leq 0,
\end{aligned} \tag{8}$$

but with DRS the last two constraints for Y_i and Y_s are replaced with,

$$\begin{aligned}
\alpha + \beta + \epsilon - 1 \leq 0, \\
\gamma + \delta + \epsilon - 1 \leq 0.
\end{aligned} \tag{9}$$

The CEESA score estimation for DRS is represented as,

$$\begin{aligned}
&\underset{r,d,t,v,e,\rho,\eta}{\text{minimize}} && Y = -(r.R^{\rho} + d.D^{\rho} + t.{T_s}^{\rho} + v.{V_e}^{\rho} + e.E^{\rho})^{\frac{\eta}{\rho}} \\
&\text{subject to} && -\phi + \epsilon \leq 0, \quad \phi - 1 + \epsilon \leq 0, \qquad \forall \phi \in \{r, d, t, v, e, \eta\} \\
& && \rho - 1 \leq 0, \quad \rho - 1 + \epsilon \leq 0, \\
& && (r + d + t + v + e - 1) - \tau \leq 0, \\
& && (1 - r - d - t - v - e) - \tau \leq 0.
\end{aligned} \tag{10}$$

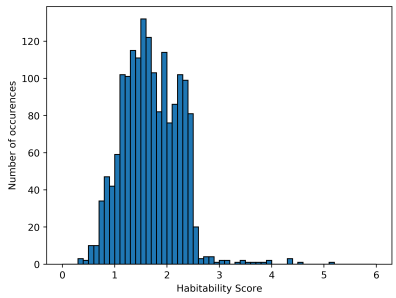
Under CRS there is no need for the parameter η. The objective function for the problem becomes,

$$\underset{r,d,t,v,e,\rho}{\text{minimize}} \quad Y = -(r.R^{\rho} + d.D^{\rho} + t.{T_s}^{\rho} + v.{V_e}^{\rho} + e.E^{\rho})^{\frac{1}{\rho}}. \tag{11}$$

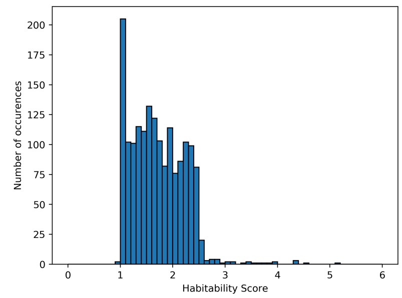
5 Experiment and Results

The data set used for estimating the Habitability Scores of exoplanets, described in Section 2, was the Confirmed Exoplanets Catalog maintained by the Planetary Habitability Laboratory (PHL). The catalog records observed and modeled parameters for exoplanets confirmed by the Extrasolar Planets Encyclopedia. Table 1 describes the parameters from the PHL Exoplanets Catalog (PHL-EC) used for the experiment. Since surface temperature and eccentricity are not recorded in Earth Units, we normalized these values by dividing them with Earth's surface temperature (288 K) and eccentricity (0.017). PHL-EC assumes an Eccentricity of 0 when unavailable. The PHL-EC records empty values for planets whose surface temperature is not known. We chose to drop these records from the experiment.

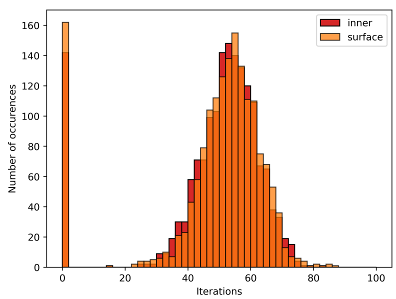
Our implemenation used $n = 25$ particles to traverse the search space, with learning rates $\lambda_g = 0.8$ and $\lambda_p = 0.2$. It regulated velocity through a friction coefficient of $\mu = 0.6$ and upper and lower bounds



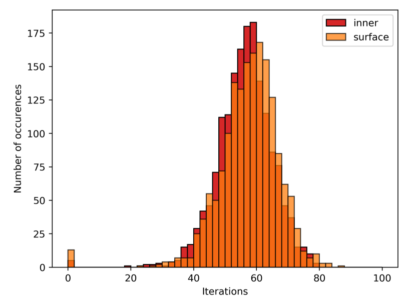
(a) CRS Score Distribution



(b) DRS Score Distribution



(c) CRS Iterations Distribution



(d) DRS Iterations Distribution

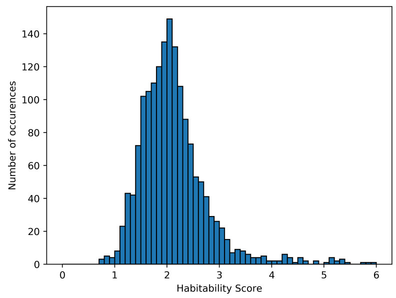
Figure 1: Plots for the Cobb-Douglas Habitability Score.

± 1.0. It used an error threshold of $\epsilon = 1 \times 10^{-6}$ to convert strict inequalities to non-strict inequalities, and a tolerance of $\tau = 1 \times 10^{-7}$ to transform an equality constraint to a pair of inequalities. Further implementation details are discussed in Appendix A.

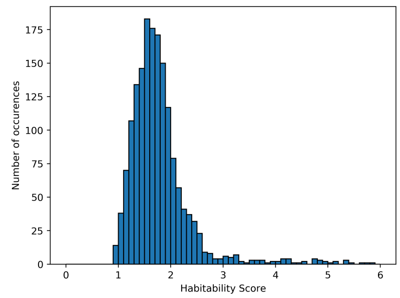
The plots in Figures 1a and 1b describe the distribution of the CDHS across exoplanets tested from the PHL-EC. Figures 1c and 1d show the distribution of iterations required to converge to a global maxima. The spike at 0 is caused by particles converging to a *gbest* that does not shift from the original position (for a more detailed explanation see Appendix **??**). The plots in Figures 2b and 2b describe the distribution of the CEESA score across the exoplanets, while Figures 2d and 2c show the distribution of iterations to convergence. These graphs aggregate the results of optimizing the Habitability Production Functions (Equations 8, 9, 10 and 11) for each exoplanet in the PHL-EC by the method described in Algorithm 2.

Tables 2a and 2b record the CDHS values for a sample of exoplanets under CRS and DRS respectively at $w_i = 0.99$ and $w_s = 0.01$. Tables 3b and 3a record the same for the CEESA scores. All tables also record the number of iterations taken to converge to a stable *gbest* value. As seen in the tables, although CEESA has 7 parameters and 16 constraints under DRS, PSO takes just over twice the number of iterations to converge as in each step of CDHS, which has 2 parameters and 5 constraints. This is a promising result as it indicates that the iterations required for converging increases sub-linearly with the number of parameters in the model.

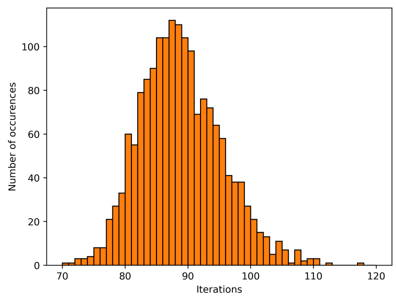
As for real time taken to converge, PSO took 666.85 s ($\approx$ 11 min 7 s) to estimate the CDHS under CRS for 1683 exoplanets, at an average of 198.11 ms for each planet for each part of the CDHS. For CDHS under DRS, it took 638.69 s ($\approx$ 10 min 39 s) at an average of 189.75 ms for each part of the CDHS. The CEESA estimates, requiring one calculation, took a little over half the total time to execute. Under DRS it took a total of 370.86 s ($\approx$ 6 min 11 s) at 220.36 ms per planet, while under CRS it took 356.92 s ($\approx$ 5 min 57 s) at 212.07 ms per planet. This is considerably fast considering the



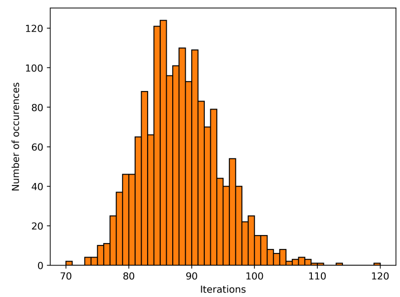
(a) CRS Score Distribution



(b) DRS Score Distribution



(c) CRS Iterations Distribution



(d) DRS Iterations Distribution

Figure 2: Plots for the Constant Elasticity Earth Similarity Approach.

number of parameters and constraints for the problem.

6 Conclusions

A Improving Performance

Matrices P and V represent current position and velocity, where the i^{th} row of each correspond to the position and velocity of particle i. Each row of the matrix L is the leader for the particle in the corresponding row of P. The constraint matrix C is constructed by stacking the constraint vectors c_i described in Section 4. Let r', r'' be two random vectors of length n drawn from the uniform distribution $U(0,1)^n$. Let X_i denote the i^{th} row of matrix X. The implementation of each iteration while updating particles in Algorithm 2 reduces to,

Name	Class	α	β	Y_i	i_i	γ	δ	Y_s	i_s	*CDHS*
GJ 176 b	non	0.460	0.540	1.90	50	0.107	0.893	2.11	61	1.90
GJ 667 C b	non	0.423	0.577	1.71	58	0.692	0.308	1.81	54	1.71
GJ 667 C e	psy	0.129	0.871	1.40	50	0.258	0.742	1.39	55	1.40
GJ 667 C f	psy	0.534	0.466	1.40	48	0.865	0.135	1.39	47	1.40
GJ 3634 b	non	0.409	0.591	1.89	58	0.724	0.276	2.09	48	1.89
HD 20794 c	non	0.260	0.740	1.35	50	0.096	0.904	1.34	58	1.35
HD 40307 e	non	0.168	0.832	1.50	49	0.636	0.364	1.53	63	1.50
HD 40307 f	non	0.702	0.298	1.52	68	0.303	0.697	1.55	45	1.52
HD 40307 g	psy	0.964	0.036	1.82	51	0.083	0.917	1.98	55	1.82
Kepler-186 f	hyp	0.338	0.662	1.17	50	0.979	0.021	1.12	40	1.17
Proxima Cen b	psy	0.515	0.484	1.12	37	0.755	0.245	1.07	0	1.12
TRAPPIST-1 b	non	0.319	0.681	1.09	0	0.801	0.199	0.89	0	1.09
TRAPPIST-1 c	non	0.465	0.535	1.06	0	0.935	0.065	1.14	26	1.06
TRAPPIST-1 d	mes	0.635	0.365	0.77	34	0.475	0.525	0.73	47	0.77
TRAPPIST-1 e	psy	0.145	0.855	0.92	0	0.897	0.103	0.83	55	0.92
TRAPPIST-1 g	hyp	0.226	0.774	1.13	43	0.876	0.124	1.09	0	1.13

(a) Estimated habitability scores under CRS.

Name	Class	α	β	Y_i	i_i	γ	δ	Y_s	i_s	*CDHS*
GJ 176 b	non	0.395	0.604	1.90	59	0.372	0.627	2.11	56	1.90
GJ 667 C b	non	0.781	0.218	1.71	58	0.902	0.097	1.81	57	1.71
GJ 667 C e	psy	0.179	0.820	1.40	49	0.234	0.765	1.39	60	1.40
GJ 667 C f	psy	0.704	0.295	1.40	64	0.398	0.601	1.39	61	1.40
GJ 3634 b	non	0.602	0.397	1.89	59	0.429	0.570	2.09	77	1.89
HD 20794 c	non	0.014	0.985	1.35	50	0.116	0.883	1.34	45	1.35
HD 40307 e	non	0.752	0.247	1.50	60	0.677	0.322	1.53	50	1.50
HD 40307 f	non	0.887	0.112	1.52	51	0.261	0.738	1.55	60	1.52
HD 40307 g	psy	0.300	0.699	1.82	62	0.785	0.214	1.98	56	1.82
Kepler-186 f	hyp	0.073	0.926	1.17	46	0.740	0.259	1.12	51	1.17
Proxima Cen b	psy	0.045	0.954	1.12	57	0.216	0.783	1.07	53	1.12
TRAPPIST-1 b	non	0.102	0.897	1.09	41	0.000	0.000	1.00	65	1.09
TRAPPIST-1 c	non	0.471	0.528	1.06	44	0.227	0.772	1.14	57	1.06
TRAPPIST-1 d	mes	0.000	0.000	1.00	67	0.000	0.000	1.00	59	1.00
TRAPPIST-1 e	psy	0.000	0.000	1.00	55	0.000	0.000	1.00	57	1.00
TRAPPIST-1 g	hyp	0.888	0.111	1.13	47	0.949	0.050	1.09	46	1.13

(b) Estimated habitability scores under DRS.

Table 2: Cobb-Douglas Habitability Scores as estimated by Particle Swarm Optimization. α, β, γ and δ record the parameters of Equation 8 in Table 2a and the parameters of Equation 9 in Table 2b. Y_i and Y_s record the maxima for the objective functions. i_i and i_s specify the number of iterations taken to converge to a stable *gbest* value. Under the Class column there are four categories for the planets — Psychroplanets (psy), Mesoplanets (mes), Non-Habitable planets (non) and Hypopsychroplanets (hyp). *CDHS* is the final score with $w_i = 0.99$ and $w_s = 0.01$.

Name	Class	r	d	t	v	e	ρ	η	*CDHS*	i
GJ 176 b	non	0.304	0.001	0.375	0.271	0.050	0.467	0.808	1.52	85
GJ 667 C b	non	0.297	0.010	0.318	0.052	0.322	0.682	0.730	2.36	90
GJ 667 C e	psy	0.230	0.286	0.137	0.199	0.148	0.551	0.906	1.14	85
GJ 667 C f	psy	0.397	0.035	0.152	0.402	0.014	0.793	0.999	1.31	100
GJ 3634 b	non	0.178	0.175	0.005	0.194	0.447	0.894	0.657	2.07	94
HD 20794 c	non	0.073	0.142	0.452	0.190	0.144	0.953	0.635	1.20	78
HD 40307 e	non	0.156	0.307	0.185	0.033	0.319	0.428	0.939	2.69	88
HD 40307 f	non	0.272	0.231	0.064	0.305	0.127	0.676	0.802	1.28	77
HD 40307 g	psy	0.113	0.219	0.066	0.454	0.148	0.711	0.991	3.26	92
Kepler-186 f	hyp	0.039	0.159	0.116	0.329	0.357	0.253	0.919	1.35	70
Proxima Cen b	psy	0.272	0.173	0.284	0.193	0.079	0.615	0.114	0.99	75
TRAPPIST-1 b	non	0.488	0.151	0.039	0.193	0.129	0.151	0.014	0.99	87
TRAPPIST-1 c	non	0.172	0.236	0.275	0.242	0.075	0.969	0.962	1.06	80
TRAPPIST-1 d	mes	0.106	0.308	0.075	0.218	0.293	0.844	0.017	0.99	93
TRAPPIST-1 e	psy	0.189	0.266	0.192	0.094	0.260	0.371	0.006	0.99	84
TRAPPIST-1 g	hyp	0.326	0.186	0.143	0.278	0.067	0.315	0.021	1.00	76

(a) Estimated habitability scores under DRS.

Name	Class	r	d	t	v	e	ρ	η	*CDHS*	i
GJ 176 b	non	0.194	0.020	0.315	0.465	0.006	0.398	1.000	1.88	86
GJ 667 C b	non	0.162	0.289	0.090	0.087	0.372	0.836	1.000	3.54	107
GJ 667 C e	psy	0.373	0.032	0.134	0.304	0.157	0.217	1.000	1.25	71
GJ 667 C f	psy	0.394	0.006	0.043	0.360	0.196	0.490	1.000	1.44	81
GJ 3634 b	non	0.351	0.122	0.006	0.069	0.453	0.439	1.000	2.89	96
HD 20794 c	non	0.101	0.077	0.691	0.071	0.059	0.756	1.000	1.58	94
HD 40307 e	non	0.069	0.091	0.097	0.173	0.569	0.768	1.000	5.29	94
HD 40307 f	non	0.285	0.161	0.053	0.443	0.058	0.342	1.000	1.42	73
HD 40307 g	psy	0.156	0.010	0.081	0.302	0.451	0.612	1.000	7.15	94
Kepler-186 f	hyp	0.036	0.017	0.082	0.383	0.483	0.929	1.000	1.68	85
Proxima Cen b	psy	0.352	0.383	0.103	0.059	0.103	0.936	1.000	0.89	83
TRAPPIST-1 b	non	0.148	0.147	0.344	0.269	0.093	0.767	1.000	0.94	81
TRAPPIST-1 c	non	0.038	0.060	0.575	0.321	0.005	0.602	1.000	1.17	86
TRAPPIST-1 d	mes	0.023	0.065	0.475	0.391	0.045	0.830	1.000	0.84	79
TRAPPIST-1 e	psy	0.176	0.464	0.253	0.103	0.004	0.920	1.000	0.86	81
TRAPPIST-1 g	hyp	0.060	0.086	0.310	0.540	0.004	0.848	1.000	0.97	86

(b) Estimated habitability scores under CRS.

Table 3: Constant Elasticity Earth Similarity Approach scores as estimated by Particle Swarm Optimization.

r, d, t, v, e, ρ and *eta* record the parameters of Equation 10 in Table 3a and the parameters of Equation 11 in Table 3b. However, since under the CRS constraint, $\eta = 1$, there is no need for the parameter η in the problem. The column *CEESA* records the maxima of the objective function. i specifies the number of iterations taken to converge to the maxima.

$$
\begin{aligned}
&g' = f([gbest]) \\
&L = [\ \underset{pbest_j,\ j=1\ldots n}{\arg\min} \|pbest_j - P_i\|^2 \ \mid\ \forall i = 1\ldots n\ \ ]^T \\
&V = \mu.V + \lambda_g \begin{bmatrix} r_1'(gbest - P_1) \\ r_2'(gbest - P_2) \\ \vdots \\ r_n'(gbest - P_n) \end{bmatrix} + \lambda_p \begin{bmatrix} r_1''(L_1 - P_1) \\ r_2''(L_2 - P_2) \\ \vdots \\ r_n''(L_n - P_n) \end{bmatrix} \\
&V = [\ \ \mathrm{sgn}(v_{ij}) * \max\{|v_{max}|, |v_{ij}|\}\ \mid\ \forall i = 1\ldots n, \forall j = 1\ldots d\ \ ] \\
&P = P + V
\end{aligned}
$$