Optimizing the Cobb Doughlas Habitability Production function using Particle Swarm Optimization

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1 Introduction

Under most paradigms, optimization of a continuous function requires the calculation of a gradient. This might not always be feasible for non-polynomial functions in high-dimensional search spaces. This is further complicated when the optimization must be performed under constraints. Particle Swarm Optimization is a metaheuristic for continuous function optimization that does away with the need for a gradient. It employs a large number of particles that traverse the search space converging toward a global best solution encountered by at least one of the particles.

Particle Swarm Optimization is a distributed method that requires simple mathematical operators and few lines of code, making it a lucrative solution where computation resources are at a premium. Its implementation is highly parallelizable. It scales with the dimensionality of the search space. Constrained optimization can be incorporated within the standard algorithm through the use of a penalty function.

Particle Swarm Optimization has been successfully adapted to a wide range of design optimization problems, e.g., networks and VLSI design. Particle Swarm Optimization has also found applications in machine learning under clustering, feature detection and classification. As a modelling paradigm, it has been used for constructing customer satisfaction models, MIDI music models, friction models, etc.

In this paper, Particle Swarm Optimization has been applied to estimate the Cobb Douglas Habitability Score (CDHS) of an exoplanet. The score is estimated by maximizing the Cobb-Douglas Habitability Production Function (CD-HPF). The function considers four parameters, radius, mass, escape velocity and surface temperature, taken from the Exoplanets Catalog hosted by the Planetary Habitability Laboratory, UPR Arecibo. Since the Cobb-Douglas function is concave under certain constraints, the problem can be reduced to a convex optimization over the negative of the production function.

2 Cobb-Douglas Habitability Score

2.1 Cobb-Douglas Production Function

The general form of the Cobb-Douglas production function (CD-PF) is given by,

$$Y = k \cdot (x_1)^{\alpha} \cdot (x_2)^{\beta}, \tag{1}$$

where k is a constant that may be set according to requirement, x_1 and x_2 are the input parameters to the function, Y is the total output production value, and α and β are called the elasticity coefficients.

The CDHS is estimated by maximizing the CD-PF when the returns to scale are either decreasing or constant. This is marked by two constraints on the elasticity coefficients,

- When $\alpha + \beta < 1$, the CD-PF observes decreasing returns to scale (DRS).
- When $\alpha + \beta = 1$, the CD-PF observes constant returns to scale (CRS).

Further, if $0 < \alpha, \beta < 1$, the CD-PF is concave, and therefore has a maxima. The CD-PF can be adapted as a Habitability Production Function by setting the input parameters to the measurable parameters of an exoplanet and using these to estimate the elasticity coefficients.

2.2 Cobb-Douglas Habitability Score Estimation

Two types of Habitability Scores are calculated for each planet, the interior CDHS ($\mathrm{CDHS_i}$) and the surface CDHS ($\mathrm{CDHS_s}$), given by the following equations,

$$Y_1 = \ CDHS_i \ = D^{\alpha} \ \cdot \ R^{\beta} \ \ , \tag{2a}$$

$$Y_2 = \ CDHS_s \ = {V_e}^{\gamma} \ \cdot \ {T_s}^{\delta} \ \ , \tag{2b}$$

where, D, R, V_e and T_s are density, radius, escape velocity and surface temperature respectively. The final CDHS score is convex combination of the interior and surface CDHS values as given by,

$$Y = w' \cdot Y_1 + w'' \cdot Y_2, \tag{3}$$

3 Particle Swarm Optimization

Should include,

- Equations. Modification for max.
- Constrained optimization modifications.
- Algorithm for PSO.

Procedure 1 Estimating CDHS values.

Input: A dataset of exoplanets with density, radius, escape velocity and surface temperature in Earth Units.

Output: The $CDHS$ value for each exoplanet in the input dataset.

$CDHS \leftarrow \{\}$

for all exoplanets e **do**

$D \leftarrow e.density$

$R \leftarrow e.radius$

$V_e \leftarrow e.escape\_velocity$

$T_s \leftarrow e.surface\_temperature$

$Y_1 \leftarrow \max\ D^{\alpha} \cdot R^{\beta}, \quad \text{subject to:} \quad \alpha + \beta < 1; \quad 0 < \alpha, \beta < 1$

$Y_2 \leftarrow \max\ {V_e}^{\gamma} \cdot {T_s}^{\delta}, \quad \text{subject to:} \quad \gamma + \delta < 1; \quad 0 < \gamma, \delta < 1$

$Y.DRS \leftarrow w' \cdot Y_1 \ + \ w'' \cdot Y_2$

$Y_1 \leftarrow \max\ D^{\alpha} \cdot R^{\beta}, \quad \text{subject to:} \quad \alpha + \beta = 1; \quad 0 < \alpha, \beta < 1$

$Y_2 \leftarrow \max\ {V_e}^{\gamma} \cdot {T_s}^{\delta}, \quad \text{subject to:} \quad \gamma + \delta = 1; \quad 0 < \gamma, \delta < 1$

$Y.CRS \leftarrow w' \cdot Y_1 \ + \ w'' \cdot Y_2$

append Y to $CDHS$

end for

4 Experiment

Should include,

- Discussing the data set.
- Parameters for the constrained optimization.
- Implementation method.
- Ensuring convergence.

5 Results

Should include,

- The values and proximity to earth's habitability score.
- Values that do not converge. Or were hard to converge.
- Speed of convergence graphs.
- Graph1: Iterations to convergence vs. Number of particles.
- Graph2: Distribution of number of iterations to convergence.
- Graph3: Iterations to convergence vs. Constraint parameters.

6 Conclusions

Should include,

- Why is the speed so important?
- Parallelizable.