Day 1 - 1.2 Basics of Functions

Thursday, August 14, 2025 8:12 PM

Intro

- Hello, Welcome, introduce self
- Learning college algebra - will meet here some time MW
- Usually will do mix of lecture, group work
- Its early, dont want to start w/ syllabus (boring), do group warmup instead

Warmup

- Greet, introduce selves. Learn each others' names
- Try first problem. Its okay if yall get stuck.
- Discuss answers together

① Average salary of registered nurse increased from $71,011 in 2015 to $80,010 in 2020. Determine average change in salary per year.

have points (2015, 71011) and (2020, 80010)

want slope of line — slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in salary}}{\text{change in years}} = \$/\text{year}$

$\frac{80010-71011}{2020-2015} = \frac{\$8999}{5 \text{ yrs}} = \$1799.80/\text{year}$

a) ≈ $ 9000 per year

b) ≈ $ 1800 per year

c) ≈ $ 4000 per year

② A typical 2-year old child knows about 200 words. After that, a typical child's vocabulary increases by about 280% every year. How many words would a typical 6-year old know?

a) about 19,800 words

b) about 2000 words

c) about 10,000 words

$f(x) \approx 42 \times 2.79^x$

b) about 2000 words
c) about 10,000 words

(3) Finn and Paige are both dental hygienists.
Finn can usually clean a patients teeth in 10 minutes
Paige can usually do it in 15 minutes
If they managed to somehow work together and clean the same patient's teeth at the same time, how long would they take?

a) 6 minutes
b) 12.5 minutes
c) 5 minutes

Some of these are hard problems. We don't know how to approach them. This course will teach us how, but first we need to fill our toolbox.

Hand out notes

1.2 Basics of Functions and Their Graphs MODIFIED

Math 120
1.2 Basics of Functions and Their Graphs

Objectives:
1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Determine whether an equation represents a function.
4. Evaluate a function.
5. Graph functions by plotting points.
6. Use the vertical line test to identify functions.
7. Obtain information about a function from its graph.
8. Identify the domain and the range of a function from its graph.
9. Identify the intercepts from a function's graph.

We begin this class with learning some fundamentals about functions and their graphs. **This whole class is about different types of functions and how we can use functions to model real world situations!** Let's start by looking at the application problem on the last page of your Study Guide – our lesson today will prepare you to answer these types of questions about functions.

Topic #1: Introduction to Relations and Functions

Relations: Two quantities that are related can be grouped in an ordered pair (x, y)

Usually just numbers, can be other things (days of the week, names, cities etc)

x: Time (PM)	y: Temp (°F)
1 PM	70° → (1, 70°)
2 PM	72° (2, 72°)
3 PM	75° (3, 75)

Example #1 - Consider the set of 5 ordered pairs for the following relation:
$\{(0,9.1), (10, 6.7), (20, 10.7), (30, 13.2), (42, 21.7)\}$

This is a relation since each ordered pair relates an x value with a corresponding y value.
The Domain is the set of the x-values:
The Range is the set of the y-values:

Functions: A function is a special relation where each number in the Domain (the x-values) corresponds to only one number in the Range (the y-values). In other words, a relation is a **function** if x-values don't repeat.

Example #1 continued -
Is this a function?
$\{(0,9.1), (10, 6.7), (20, 10.7), (30, 13.2), (42, 21.7)\}$

yes!

Example #2 - Is this a function?

Celebrity	Earnings (millions of dollars)
Stern	95
Cowell	95
Beck	90
Winfrey	82
McGraw	82

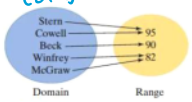

FIGURE 1.15(a) Celebrities correspond to earnings.

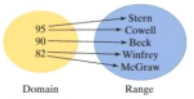

FIGURE 1.15(b) Earnings correspond to celebrities.

YES How much does each celeb make?

NO what celeb makes $?

YOU TRY #1:

Determine if the Relation is a Function, explain why or why not and state the Domain and Range in either case.

a) {(1,6), (2,6), (3,8), (4,9)}

yes.

No x-values repeat

D: {1, 2, 3, 4}

R: {6, 8, 9}

b) {(6,1), (6,2), (8,3), (9,4)}

NO

6 repeats

D: {6, 8, 9}

R: {1, 2, 3, 4}

c) {(0, 2), (-1, 1), (2, 0), (-1, 2)}

NO

-1 repeats

D: {-1, 0, 2}

R: {0, 1, 2}

Topic #2: Functions as Equations

Functions are normally expressed as an equation where...

x is the Independent Variable or input

and y is the ~~dependent variable~~ or output

An equation is a **function** If x does not repeat.

To determine if an equation represents a function, Solve for y

- If x repeats, then y is not a function of x.
- If x doesn't repeat, then y is a function of x (more on this statement soon).

Example 1 - Determine if the Equation is a Function

Plug in values of x to check

a) $x^2 + y = 4$

$-x^2 \quad -x^2$

$y = 4 - x^2$

OR $y = -x^2 + 4$

$y = 4 - (1)^2 = 4 - 1 = 3$ (1, 3)

$y = 4 - (-1)^2 = 4 - 1 = 3$ (-1, 3)

$y = 4 - (2)^2 = 4 - 4 = 0$ (2, 0)

Function

b) $x^2 + y^2 = 1$

$-x^2 \quad -x^2$

$y^2 = 1 - x^2$

$\sqrt{y^2} = \sqrt{1 - x^2}$

$y = \pm\sqrt{1 - x^2}$

$y = \pm\sqrt{1 - (3)^2} = \pm\sqrt{8}$

$(3, \sqrt{8}) \quad (3, -\sqrt{8})$

x repeats

NOT a Function

YOU TRY #2: $3x + y = 10$

$-3x \quad -3x$

$y = -3x + 10$

All linear equations are

$y = -3(1) + 10 = 7$ (1, 7)

$y = -3(5) + 10 = -5$ (5, -5)

$y = -3x + 10$

All linear equations are Functions

$(1, 7)$

$y = -3(5) + 10 = -5$

$(5, -5)$

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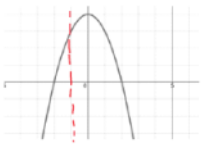
Topic #3: The Vertical Line Test (abbreviated as VLT)

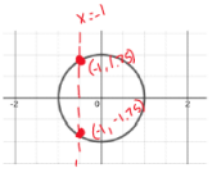
A function exists when the elements of the domain (x-values) DO NOT Repeat Not all equations are functions, but they all have a graph that shows the relationship geometrically.

Consider the graph of the equations: $x^2 + y = 4$ and $x^2 + y^2 = 1$



$x^2 + y = 4$

Yes!



$x^2 + y^2 = 1$

No

From a graphical standpoint, the graph on the left **is a function** since X don't repeat Drawing a vertical line confirms this; notice that no vertical lines intersect the graph more than one time.

Topic #4: Function Notation

If an equation is a function, write $y = f(x)$

output input

"y is a function of x"

the function is named "f"

For example, suppose $y = 4 - x^2$

$f(x) = 4 - x^2$

$f(\square) = 4 - \square^2$

For example,

$f(3) = 4 - (3)^2 = 4 - 9 = -5$

$f(x)$ means $x = 3$

So $f(3) = -5$ or $(3, -5)$ (x, y)

$f(a) = 4 - (a)^2 = 4 - a^2$

$f(a) = 4 - a^2$

$f(a+1) = 4 - (a+1)^2$

$= 4 - (a^2 + 2a + 1)$

$= 4 - a^2 - 2a - 1 = -a^2 - 2a + 3$

Must FOIL $(a+1)^2 = (a+1)(a+1)$

$= a^2 + a + a + 1$

$= a^2 + 2a + 1$

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YOU TRY #4 – Evaluate the Function

Consider the function $f(x) = x^2 + 3x + 5$ use to evaluate:

a) $f(2) = (2)^2 + 3(2) + 5$
$= 4 + 6 + 5$
$= 15$

Replace x with 2

$f(2) = 15$

$(2, 15)$

b) $f(x+1) = (x+1)^2 + 3(x+1) + 5$
$= (x^2 + 2x + 1) + 3x + 3 + 5$
$= x^2 + 5x + 9$

FOIL
$(x+1)(x+1)$
$x^2 + x + x + 1$
$x^2 + 2x + 1$

c) $f(-x) = (-x)^2 + 3(-x) + 5$
$= x^2 - 3x + 5$

Topic #5: Graphs of Functions

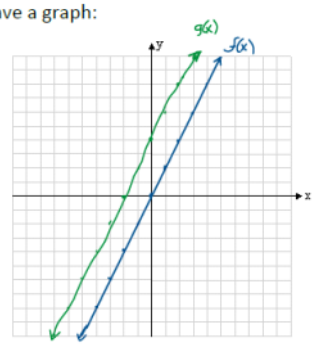
The graph of a function is the set of its ordered pairs (that satisfy the equation) plotted on the coordinate plane system.

Consider the functions: $f(x) = 2x$ and $g(x) = 2x + 4$

USE A GRAPHING CALCULATOR TO PLOT. Both have ordered pairs that satisfy the equations and both functions have a graph:

Y1 = 2x
Y2 = 2x + 4

Look at Table



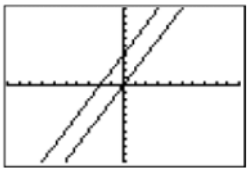
Notice that the graph for function g is the graph of function f shifted up 4 units.

In addition to a graph of the functions, we could also look at a table of values:

X	Y1	Y2
-3	-6	-2
-2	-4	0
-1	-2	2
0	0	4
1	2	6
2	4	8
3	6	10

X=-3

Although we could generate graphs and tables by hand, it is more efficient to use technology.



The graph and table tell us the same information and we can evaluate a function with both. For example:

$f(-3) = -6$ $g(-3) = -2$ $f(0) = 0$ $g(0) = 4$

$(-3,-6)$ $(-3,-2)$ $(0,0)$ $(0,4)$

Topic #6: Analyzing the Graph of a Function

( ) NOT included

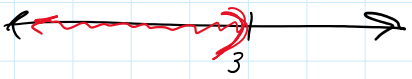
[ ] included

REMINDER:

Set builder notation and Interval Notation

"x such that..." lowest, highest

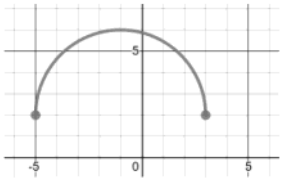
	Set Builder Notation	Interval Notation
$x < 3$	$\{x \mid x < 3\}$ "such that"	$(-\infty, 3)$
$x \geq 3$	$\{x \mid x \geq 3\}$	$[3, \infty)$
$-2 < x \leq 6$	$\{x \mid -2 < x \leq 6\}$	$(-2, 6]$



(1) Domain and Range

Recall the domain represent all x-values for the function and the range represents all y-values.

Example #1 - Find the domain and range using the graph.

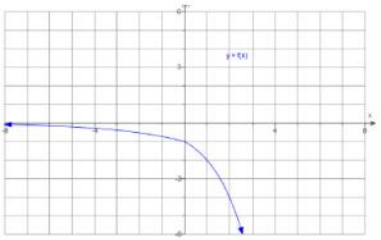


Looking from left to right (along the **x-axis**) the **domain** is all numbers between -5 and 3. To express the values, we can write the domain as an interval OR as a set

closed circle means included $[-5,3]$ or $\{x \mid -5 \leq x \leq 3\}$

Looking from bottom to top (along the **y-axis**) what is the **range** of the function?

$[2,6]$



Find the Domain: Lowest to highest
L to R
$(-\infty, \infty)$

Find the Range:

Lowest to highest
Down to up
$(-\infty, 0)$

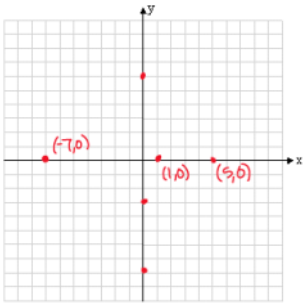
(2) Intercepts

Intercepts occur where the graph crosses the x-axis (x-intercept) and where the graph crosses the y-axis (y-intercept).

What is always true for every point on the x-axis? $y=0$ OR $f(x)=0$

(-7,0)
(1,0)
(5,0)

What is always true for every point on the y-axis? $x=0$



(0,6)
(0,3)
(0,-8)

Example #3 – Find the x and y intercepts of the graph.

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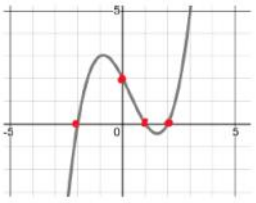

The graph crosses the x-axis three times. This graph shows that the x-intercepts are at $\boldsymbol{x = -2, 1, 2}$.

(-2,0) (1,0) (2,0)

The graph crosses the y-axis one time. **What is the y-intercept?**

(0,2)

Example #4 –

$y=0$ or $f(x)=0$

If the x-intercepts of a function are 9 and -8, then

f(9)= 0 f(-8)equals 0

(9,0) (-8,0)

The x-intercepts, 9 and -8,

are called the zeroes of the function.

Example #5 –

Find the x-intercept and y-intercept for the following function:

$f(x) = 3x + 10$

x-int: $y=0$ or $f(x)=0$

$f(x)=3x+10$

$0=3x+10$

$-10 \quad -10$

$\frac{-10}{3}=\frac{3x}{3}$

$-\frac{10}{3}=x$

$\left(-\frac{10}{3}, 0\right)$

y-int: $x=0$

$f(0)=3(0)+10$

$=10$

$(0, 10)$

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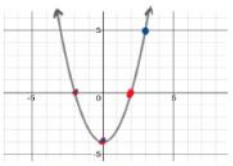
YOU TRY #5 – Analyze the Function

Use the graph of the function $y = f(x)$ to answer the questions:



a) Find the domain and range in interval notation and set notation. D: $(-\infty, \infty)$ or $\{x \mid x \in \mathbb{R}\}$

↑ x is any real #

R: $[-4, \infty)$ or $\{x \mid x \geq -4\}$

b) Find the intercepts.

x-int: $f(x)=0$ or $y=0$ (-2,0) (2,0)

y-int: $x=0$ $f(0)$ (0,-4)

c) Evaluate $f(3), f(0), f(-2)$

(3,5) (0,-4) (-2,0)

d) For what x value(s) is $f(x) = -3$? $y=-3$ (-1,-3) (1,-3)

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Example #1 – Interpret the Function in Context

Use the bar graph and function model described above to answer the questions below:

a) Use the function to evaluate $g(80)$ and interpret the meaning in a complete sentence.

x in this problem is age

$g(80) = -2.9(80) + 229 = 55$

(80: Age; 55: survival)

There is a 55% chance of surviving to age 80 years.

b) Compare the value from the model to the actual value. How far off are the values?

Graph shows survival of 58%.

YOU TRY #6 - Use the function to evaluate $g(86)$ and interpret its meaning.

$g(86) = -2.9(86) + 287 = 37.6$

There is a 37.6% chance of surviving to 86 years.