Math 120 Final Exam Review

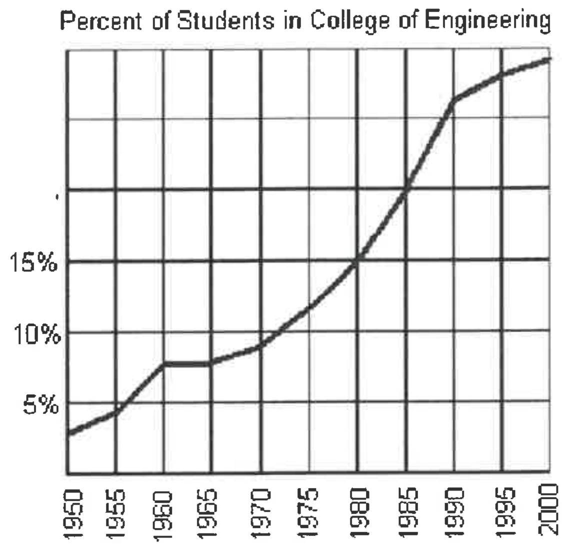
Section 1.2

1. Evaluate the function at the given value of the independent variable and simplify.

$$f(x)=\frac{x^2-6}{x^3+3x};\quad f(-1)$$

$f(-1)=\dfrac{(-1)^2-6}{(-1)^3+3(-1)}=\dfrac{-5}{-4}=\dfrac{5}{4}$

2. The graph below shows the percentage of students enrolled in the College of Engineering at Stale University. Use the graph to answer the question.



a. Does the graph represent a function?

yes

b. Refer to the graph above #2. If f represents the function, find $f(1995)$.

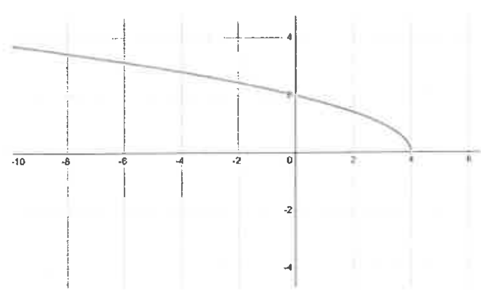
≈ 28%

c. If $f(x) = 12\%$, what year is represented by x?

1975

Section 1.3

3. Use the graph below, f(x), to determine the domain and range. State if the graph represents a function.



D: $(-\infty, 4]$

R: $[0, \infty)$

yes

4. Find and simplify the difference quotient for the given functions.

a. $f(x)=7x-4$

$\dfrac{7(x+h)-4-(7x-4)}{h}$

$\dfrac{7x+7h-4-7x+4}{h}$

$\dfrac{7h}{h}=7$

b. $f(x)=x^2+8x+2$

$\dfrac{(x+h)^2+8(x+h)+2-(x^2+8x+2)}{h}$

$\dfrac{\cancel{x^2}+2xh+h^2+\cancel{8x}+8h+\cancel{2}-\cancel{x^2}-\cancel{8x}-\cancel{2}}{h}$

$\dfrac{2xh+h^2+8h}{h}=2x+h+8$

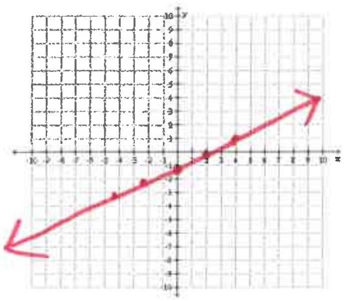
Section 1.4

5. Use the given information to write an equation for the line in point-slope form
   Passing through (3, 5) and (-5, 6).

$m = \frac{6-5}{-5-3} = -\frac{1}{8}$ $\quad y-5 = -\frac{1}{8}(x-3)$ or $y-6 = -\frac{1}{8}(x+5)$

6. Graph the following equation in a rectangular coordinate system: $y = \frac{1}{2}x - 1$

$m = \frac{1}{2} \quad b = -1$



7. The average value of a certain type of automobile was \$14,760 in 1991 and depreciated to \$4440 in 1994. Let y be the average value of the automobile in the year x, where $x=0$ represents 1991. Write a linear equation that models the value of the automobile in terms of the year x.

(0, 14760) (3, 4440)

$m = \frac{14760 - 4440}{0 - 3}$

$m = -3440$

$\boxed{y = -3440x + 14760}$

Section 1.5

8. Use the given conditions to write an equation for the line in slope-intercept form
   Passing through (4, 3) and perpendicular to the line whose equation is $y = \frac{1}{8}x + 8$.

$m = \frac{1}{8}$

$m_\perp = -8$

$-8(x-4) = y-3$

$-8x + 32 = y - 3$

$\boxed{y = -8x + 35}$

9. Find the average rate of change of the function $f(x) = \sqrt{2x}$ from $x_1 = 2$ to $x_2 = 8$.

$f(2) = \sqrt{2(2)}$ $\quad f(8) = \sqrt{2(8)}$ $\quad (2,2)\ (8,4)$ $\quad m = \frac{4-2}{8-2} = \frac{2}{6} = \boxed{\frac{1}{3}}$

$f(2) = 2$ $\quad f(8) = 4$

10. Along with incomes, people's charitable contributions have steadily increased over the past few years. The table below shows the average deduction for charitable contributions reported on individual income tax returns for the period 1993 to 1998. Find the average rate of annual increase between 1995 and 1997.

Year	Charitable Contributions
1993	\$1650
1994	\$2400
1995	\$2480
1996	\$2830
1997	\$3050
1998	\$3180

$\frac{1997-1995}{3050-2480}$

$\frac{3050-2480}{1997-1995}$

$\boxed{\$285\text{ / year}}$

Section 1.7

11. For $f(x)=x^2+2x-2$, and $g(x)=x^2-2x-4$ find the composition $(f\circ g)(-2)$.

$g(-2)=(-2)^2-2(-2)-4$ $\quad f(4)=4^2+2(4)-2$ $\quad \boxed{22}$

$g(-2)=4+4-4=4$ $\quad f(4)=16+8-2$

12. Express the given function h as a composition of two functions f and g so that $h(x)=(f\circ g)(x)$

$h(x)=\dfrac{1}{x^2-4}$. $\quad g(x)=x^2-4$ $\quad f(x)=\frac{1}{x}$

Section 1.8

13. The function $f(x)=(x+7)^3$ is one-to-one. Find an equation for $f^{-1}(x)$, the inverse function.

$y=(x+7)^3$ $\quad x=(y+7)^3$ $\quad \boxed{f^{-1}(x)=\sqrt[3]{x}-7}$

$\sqrt[3]{x}=y+7$

$\sqrt[3]{x}-7=y$

Section 2.1

14. Find all values of x satisfying the given conditions: $f(x)=8x+4(3+x), g(x)=3(x-6)+10x$, and $f(x)=g(x)$.

$8x+4(3+x)=3(x-6)$ $\quad \boxed{x=30}$

$8x+12+4x=3x-18+10x$

$-x=-30$

15. Find the zero of the function $f(x)=2[2x-(3x-7)]-6(x-7)$.

$0=2(-x+7)-6x+42$ $\quad \frac{-56}{-8}=\frac{-8x}{-8}$ $\quad \boxed{x=7}$

$0=-2x+14-6x+42$

16. Solve the equation $36\cdot\frac{8x}{9}-36\cdot x=36\cdot\frac{x}{36}-\frac{5}{4}\cdot 36$

LCD 36

$4(8x)-36x=x-5(9)$ $\quad 32x-36x=x-45$

$-5x=-45$

$\boxed{x=9}$

17. The following rational equation has denominators that contain variables.

a. Write the value or values of the variable that make a denominator zero. 3, -3

b. Keeping the restrictions in mind, solve the equation.

$(x-3)(x+3)\dfrac{9}{x+3}-\dfrac{7}{x-3}(x+3)=\dfrac{2}{x^2-9}(x+3)(x-3)$

$x+3=0$ $\quad$ LCD

$x=-3$ $\quad (x+3)(x-3)$ $\quad 9(x-3)-7(x+3)=2$

$x-3=0$ $\quad 9x-27-7x-21=2$ $\quad \boxed{x=25}$

$x=3$ $\quad \frac{-2x}{-2}=\frac{-50}{-2}$

18. The function $f(x)=\dfrac{24{,}000+280x}{x}$ models the average cost per unit, $f(x)$, for Electrostuff to manufacture x units of Electrogadget IV. How many units must the company produce to have an average cost per unit of $440? (Round to nearest whole number if necessary)

$x \cdot 440=\dfrac{24000+280x}{x}\cdot x$ $\quad 440x=24000+280x$ $\quad \boxed{150 \text{ units}}$

Section 2.2

19. When 80% of a number is added to the number, the result is 144. What is the number(s)? $\boxed{80}$

$.8x+x=144$

20. You inherit $10,000 with the stipulation that for the first year the money must be invested in two stocks paying 6% and 11% annual interest, respectively. How much should be invested at each rate if the total interest earned for the year is to be $900? (Round to the nearest cent, if necessary.)

amt 6% x

amt 11% $10000-x$

$.06x+.11(10000-x)=900$

$4000 @ 6%

$6000 @ 11%

21. After a 9% price reduction, a boat sold for $28,210. What was the boat's price before the reduction? (Round to the nearest cent, if necessary.)

$x-.09x=28210$ $\quad$ $31,000

22. The length of a rectangular room is 7 feet longer than twice the width. If the room's perimeter is 194 feet, what are the room's dimensions? (Round to nearest whole number if necessary)

w

7+2w

$P=2L+2w$

$194=2(7+2w)+2w$

w = 30 ft

L = 67 ft

Section 2.3

23. Perform the operation and write the expression in the standard form a + bi.

a. $(3-7i)-(-2+4i)$ $\quad \boxed{5-11i}$

b. $4i(5-7i)$ $\quad 20i-28i^2=\boxed{28+20i}$

c. $(2+2i)(4-9i)$ $\quad \boxed{26-10i}$

$8-10i-18i^2$

d. $5\sqrt{-36}+10\sqrt{-25}$

$5(6i)+10(5i)$

$30i+50i=\boxed{80i}$

e. $\dfrac{10i}{3+6i}\cdot\dfrac{3-6i}{3-6i}=\dfrac{30i-60i^2}{9-36i^2}=\dfrac{60+30i}{45}$ $\quad \boxed{\frac{4}{3}+\frac{2}{3}i}$

f. $(5+2i)^2$

$(5+2i)(5+2i)=25+20i+4i^2=\boxed{21+20i}$

g. $\dfrac{10-\sqrt{-45}}{15}=\dfrac{10-3i\sqrt{5}}{15}=\boxed{\frac{2}{3}-\frac{\sqrt{5}}{5}i}$

Section 2.4

24. Solve the equation:

a. $x^2=x+20$ $\quad \boxed{x=5,-4}$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x-5=0 \quad x+4=0$

b. $7x^2+34x-5=0$ (a b c)

$\dfrac{-34\pm\sqrt{34^2-4(7)(-5)}}{2(7)}$

$\dfrac{-34\pm\sqrt{1296}}{14}$

$\dfrac{-34\pm 36}{14}$

$\dfrac{2}{14}=\frac{1}{7}$

$\dfrac{-70}{14}=-5$

25. Solve the equation: $\overset{a}{6x^2} + \overset{b}{10x} + \overset{c}{3} = 0$. Be sure to simplify answer. Give exact answer, using radicals as needed. Express complex numbers in terms of i.

$x = \dfrac{-10 \pm \sqrt{10^2 - 4(6)(3)}}{2(6)}$ $\quad x = \dfrac{-10 \pm \sqrt{28}}{12} = \dfrac{-10 \pm 2\sqrt{7}}{12} = \boxed{\dfrac{-5}{6} \pm \dfrac{\sqrt{7}}{6}}$

26. Solve the equation: $\overset{a}{x^2} \overset{b}{-2x} + \overset{c}{5} = 0$. Give exact answer, using radicals as needed. Express complex numbers in terms of i.

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \dfrac{2 \pm \sqrt{-16}}{2} = \dfrac{2 \pm 4i}{2}$ $\quad \boxed{1+2i,\ 1-2i}$

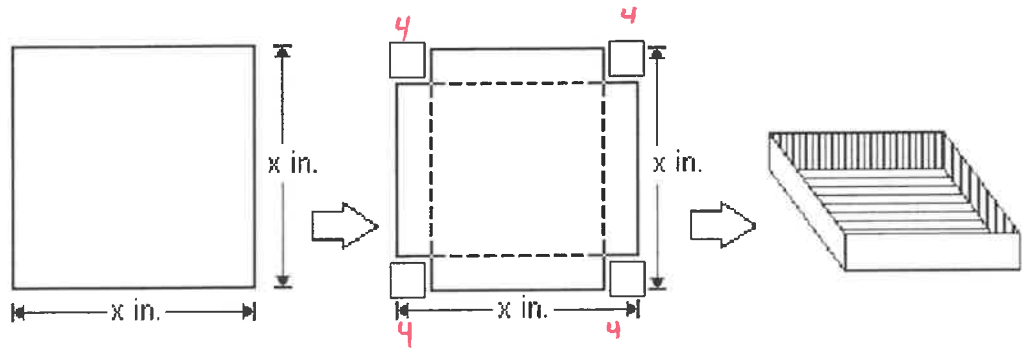
27. A ladder that is 10 feet long is 6 feet from the base of a wall. How far up the wall does the ladder reach? Make sure to include units in your answer.

$x^2 + 6^2 = 10^2$

$x^2 = 64$

$\boxed{8 \text{ ft}}$

28. Suppose that an open box is to be made from a square sheet of cardboard by cutting out 4-inch squares from each corner as shown and then folding along the dotted lines. If the box is to have a volume of 400 cubic inches, find the original dimensions of the sheet of cardboard.



$4(x-8)(x-8) = 400$

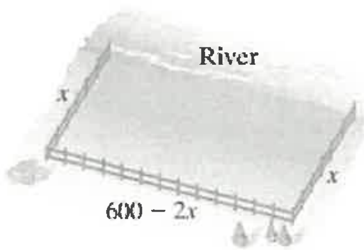
$(x-8)^2 = 100$

$x - 8 = 10 \qquad x - 8 = -10$

$x = 18 \qquad x = \cancel{-2}$

$\boxed{18 \text{ in by } 18 \text{ in}}$

29. You have 600 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?



$A(x) = x(600 - 2x)$

$A(x) = 600x - 2x^2$

$600 - 2(150) = \boxed{300 \text{ ft}}$

$x = -\dfrac{600}{2(-2)}$

$\boxed{x = 150 \text{ ft}}$

Area $= 150(300)$

$\boxed{45000 \text{ ft}^2}$

Section 2.6

30. Solve the linear inequality $-24x + 20 \le -4(5x - 6)$. Write your answer in interval notation. If the answer is empty set, use $\varnothing$.

$-24x + 20 \le -20x + 24$

$-4x \ge 4$

FLIP $x \ge -1$

$\boxed{[-1, \infty)}$

31. Use interval notation to represent all values of x satisfying the given conditions.

$f(x) = 4x - 5,\ g(x) = 3x - 9$, and $f(x) > g(x)$.

$4x - 5 > 3x - 9$

$x > -4$

$\boxed{(-4, \infty)}$

32. Solve the compound inequality $14 \le \frac{5}{3}x + 9 < 29$. Write your answer in interval notation. If the answer is empty set, use $\varnothing$. Graph the solution set on a number line.

$3 \cdot \quad .3 \quad .3 \quad .3$

$42 \le 5x + 27 < 87$

$15 \le 5x < 60$

$3 \le x < 12$

$\boxed{[3, 12)}$

33. Greg is opening a car wash. He estimates his cost function as $C(x) = 9000 + 0.09x$ and his revenue function as $R(x) = 1.95x$, where x is the number of cars washed in a six-month period. Find the number of cars that must be washed in a six-month period for Greg to make a profit. Round your answer to the nearest whole number and write your answer in interval notation. Rev > COST

$1.95x > 9000 + .09x$

$1.86x > 9000$

$\frac{1.86x}{1.86} > \frac{9000}{1.86}$

$x > 4838.709677$

$\boxed{[4839, \infty)}$

Section 5.1

34. Solve the systems of equations given below.

a. $\begin{cases} y = -\frac{1}{2}x + 4 \\ y = 3x - 3 \end{cases}$

$3x - 3 = -\frac{1}{2}x + 4$

$6x - 6 = -x + 8$

$7x = 14$

$x = 2$

$y = 3(2) - 3$

$y = 3$

$\boxed{(2, 3)}$

b. $\begin{cases} 3(-x + 4y = -12) \\ 3x - 6y = 6 \end{cases}$

$-3x + 12x = -36$

$3x - 6y = 6$

$6y = -30$

$y = -5$

$-x + 4(-5) = -12$

$-x = 8$

$x = -8$

$\boxed{(-8, -5)}$

c. $\begin{cases} 6\left(\frac{x}{3} - \frac{y}{2} = 1\right) \\ 4\left(\frac{x}{4} + y = -2\right) \end{cases}$

$2x - 3y = 6$

$x + 4y = -8 \Rightarrow$

$x = -8 - 4y$

$2(-8 - 4y) - 3y = 6$

$-16 - 8y - 3y = 6$

$-11y = 22$

$y = -2$

$x = -8 - 4(-2)$

$x = -8 + 8$

$x = 0$

$\boxed{(0, -2)}$

Section 1.9

35. For coordinates given below, find:
   a. The distance between the points. Express your answer in simplified radical form and also as a decimal approximation rounded to the hundredth place
   b. The midpoint of the line segment that connects them. Give your answer an ordered pair

(1, -4) and (5, 2)

a.

$d = \sqrt{(5-1)^2 + (2--4)^2}$

$d = \sqrt{4^2 + 6^2}$

$d = \sqrt{52}$

$\boxed{d = 2\sqrt{13} \approx 7.21}$

b. $m = \left(\frac{1+5}{2}, \frac{-4+2}{2}\right)$

$m = \boxed{(3, -1)}$

36. For the equation given, complete the square to write it in standard form. Then, give the center and radius of the circle.

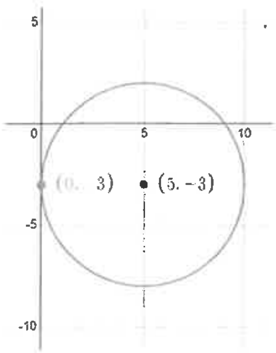
$x^2 - 6x + y^2 + 2y = -1$

$x^2 - 6x + \underline{9} + y^2 + 2y + \underline{1} = -1 + 9 + 1$

$\boxed{(x-3)^2 + (y+1)^2 = 9}$

$\boxed{C(3, -1) \quad r = 3}$

37. Using the graph of this circle write the equation of the circle in standard form.



$r = \sqrt{(5-0)^2 + (-3 - -3)^2}$

$r = 5$

$\boxed{(x-5)^2 + (y+3)^2 = 25}$

Section 3.1

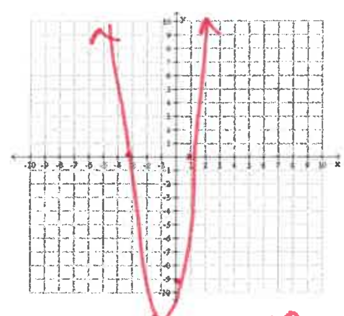
38. Use the vertex and intercepts to sketch the graph of the quadratic functions. Give the equation for the parabola's axis of symmetry. Use the parabola to identify the function's domain and range.

$$f(x) = 3(x+1)^2 - 12$$ up

a. Write the vertex as an order pair. $(-1, -12)$
b. Is the vertex a maximum or minimum? minimum
c. Write the axis symmetry equation. $x = -1$
d. Identify the zeros of the function. Write your answers as ordered pairs. $(1, 0)$ $(-3, 0)$
e. Find the y-intercept. Write your answer as an ordered pair. $(0, -9)$
f. Graph the function.

d. $0 = 3(x+1)^2 - 12$
$4 = (x+1)^2$
$x+1 = 2$ $x+1 = -2$
$x = 1$ $x = -3$
$(1, 0)$ $(-3, 0)$

e. $f(0) = 3(0+1)^2 - 12$
$f(0) = 3 - 12$
$(0, -9)$



39. Given the function $f(x) = 6x^2 + 10x + 3$

a. Find the vertex.
b. Is the vertex a max or min? min
c. Find the zeros of the function.

a. $x = \frac{-10}{2(6)} = -\frac{5}{6}$

$f(-\frac{5}{6}) = 6(-\frac{5}{6})^2 + 10(-\frac{5}{6}) + 3$

$\boxed{(-\frac{5}{6}, -\frac{7}{6})}$

40. Determine whether the quadratic function $f(x) = -x^2 - 2x - 9$ has a minimum value or maximum value. Then find the coordinates of the minimum or maximum point.

down $\boxed{\text{max value } (-1, -8)}$

$x = \frac{-(-2)}{2(-1)}$

$x = -1$

$f(-1) = -(-1)^2 - 2(-1) - 9$

$f(-1) = -8$

41. A rain gutter is made from sheets of aluminum that are 18 inches wide by turning up the edges to form right angles. Determine the depth of the gutter that will maximize its cross-sectional area and allow the greatest amount of water to flow.

$A(x) = x(18 - 2x)$

$A(x) = 18x - 2x^2$

$x = \frac{-18}{2(-2)}$

$\boxed{x = 4.5 \text{ in}}$

Section 3.2

42. Find the y-intercept and the zeros of the polynomial function $f(x) = (x+1)(x-2)(x-1)^2$ Then state the multiplicities of each zero and whether it would touch and turn around or cross the x-axis at each zero. Determine the maximum number of turning points.

$f(0) = (0+1)(0-2)(0-1)^2$ $(0, -2)$
$f(0) = (1)(-2)(1)$ max turning = 3

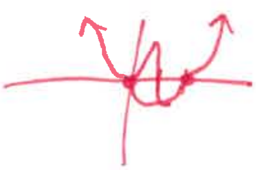
-1	K=1	Cross
2	K=1	Cross
1	K=2	turns

43. Use the Leading Coefficient Test to determine the end behavior of the polynomial function $f(x) = 4x^2 + 5x^3 - x^5$

Odd

Neg. lead. coeff.

44. Write the equation of a polynomial function P(x) with the given characteristics. Use a leading coefficient of 1 or -1 and make the degree of the function as small as possible.
Touches the x-axis at 0 and crosses the x-axis at 2; lies below the x-axis between 0 and 2



$P(x) = x^2(x-2)$

Section 3.5

45. Find the domain of the rational function $f(x) = \dfrac{x+2}{x^3 - 16x}$.

$x^3 - 16x \neq 0$
$x(x+4)(x-4) \neq 0$
$x \neq 0, -4, 4$

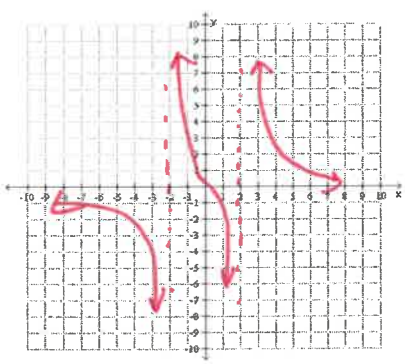
$(-\infty, -4) \cup (-4, 0) \cup (0, 4) \cup (4, \infty)$

46. For the following problems, find the vertical and horizontal asymptotes, if any, of the graph of the given rational function as well as the x and y-intercepts (if they exist).

	Vert	Horiz	X-int	y-int
a) $f(x) = \dfrac{x-2}{x(x+1)}$	$x \neq 0$ $x = -1$	$y = 0$	$(2, 0)$	None
b) $f(x) = \dfrac{10x}{2x^2+1}$	NON	$y = 0$	$(0, 0)$	$(0, 0)$
c) $f(x) = \dfrac{-4x+3}{5x-3}$	$5x - 3 = 0$ $x = \frac{3}{5}$	$y = -\frac{4}{5}$	$(\frac{3}{4}, 0)$	$(0, -1)$

47. Graph the rational function $f(x) = \dfrac{2x}{x^2 - 4}$, including vertical asymptotes (if any) sketched as dashed lines.

$x^2 = 4$
$x = 2, -2$



48. A drug is injected into a patient and the concentration of the drug is monitored. The drug's concentration, C(t), in milligrams after t hours is modeled by $C(t) = \frac{5t}{2t^2+2}$.
What is the horizontal asymptote for this function? Describe what this means in practical terms.

$y=0$

Section 3.7

49. y varies directly as z. $y = 160$ when $z = 16$. Find y when $z = 15$

$y = Kz \quad 160 = K(16) \quad 10 = K$

$y = 10z$
$y = 10(15)$

$\boxed{y = 150}$

50. The intensity I of light varies inversely as the square of the distance D from the source. If the intensity of illumination on a screen 28 ft from a light is 2.5 foot-candles, write an equation that expresses the relationship and find the intensity on a screen 40 ft from the light.

$I = \frac{K}{D^2} \quad 2.5 = \frac{K}{28^2} \quad I = \frac{1960}{D^2} \quad I = \frac{1960}{40^2}$

$\boxed{1.225 \text{ foot candles}}$

51. Body-mass index, or BMI, takes both weight and height into account when assessing whether an individual is underweight or overweight. BMI varies directly as one's weight, in pounds, and inversely as the square of one's height, in inches. In adults, normal values for the BMI are between 20 and 25. A person who weighs 170 pounds and is 71 inches tall has a BMI of 23.71. What is the BMI, to the nearest tenth, for a person who weighs 122 pounds and who is 66 inches tall?

$BMI = \frac{Kw}{h^2} \quad 23.71 = \frac{K(170)}{71^2} \quad 703.0112353 = K$

$BMI = \frac{(703.07)(122)}{66^2}$

$\boxed{BMI = 19.7}$

Section 4.1

52. A city is growing at the rate of 0.8% annually. If there were 3,980,000 residents in the city in 1995, find how many (to the nearest ten-thousand) are living in that city in 2000. Use $y = 3{,}980{,}000(2.7)^{0.008t}$ where t is the time since 1995 and y is the population at time, t. Round your answer to the nearest person.

$y = 3{,}980{,}000(2.7)^{.008(5)}$

$\boxed{4{,}141{,}309 \text{ residents}}$

53. The size of the raccoon population at a national park increases at the rate of 4.6% per year. If the size of the current population is 127, find how many raccoons there should be in 5 years. Use the function $f(x) = 127e^{0.046t}$ and round to the nearest whole number, where f(x) represents the racoon population and t is the time in years.

$f(5) = 127e^{.046(5)}$

$\boxed{160 \text{ racoons}}$

54. Suppose that you have \$4000 to invest. Which investment yields the greater return over 7 years: Choice A: 8.75% compounded continuously or Choice B: 8.9% compounded semiannually? How much do they each earn? Which is the better investment? Round to nearest cent.

A

$A = 4000e^{.0875(7)}$

\$7380.15

better

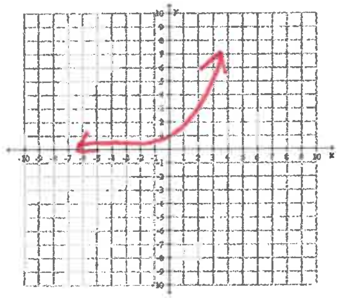
B

$A = 4000\left(1 + \frac{.089}{2}\right)^{2(7)}$

\$7358.31

Section 4.2

55. Graph the function $f(x)=\left(\dfrac{5}{2}\right)^x$



56. Use the formula $R=\log\left(\dfrac{a}{T}\right)+B$ to find the intensity R on the Richter scale, given that amplitude a is 443 micrometers, time T between waves is 2.8 seconds, and B is 2.1. Round answer to one decimal place.

$R=\log\left(\frac{443}{2.8}\right)+2.1$ $\boxed{R=4.3}$

57. Students in a psychology class took a final examination. As part of an experiment to see how much of the course content they remembered over time, they took equivalent forms of the exam in monthly intervals thereafter. The average score, $f(t)$, for the group after t months is modeled by the function

$76-18(\log 6+1)$

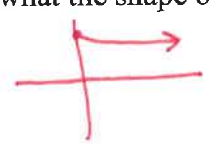
$76-18(\log 2+1)$

$$f(t)=76-18\log(t+1),\quad t \text{ where } 0\le t\le 12$$

$76-18(\log 8+1)$

$76-18(\log 4+1)$

a) What was the average score when the exam was first given? 76
b) What was the average score after 2 months? 4 months? 6 months? 8 months? one year? Round to the nearest whole number. 67, 63, 61, 59
c) Use the results from parts a) and b) to graph f. Describe what the shape of the graph indicates in terms of the material retained by the students.



58. The formula

$$t=\frac{1}{c}\ln\left(\frac{A}{A-N}\right)$$

describes the time, t, in weeks, that is takes to achieve mastery of a portion of a task. In the formula, A represents maximum learning possible, N is the portion of the learning that is to be achieved, and c is a constant used to measure an individual's learning style. A 50-year-old man decides to start running as a way to maintain good health. He feels that the maximum rate he could ever hope to achieve is 12 miles per hour. How many weeks will it take before the man can run 5 miles per hour if $c=0.06$ for this person? Round to the nearest week.

$t=\frac{1}{.06}\ln\left(\frac{12}{12-5}\right)$ $\boxed{\text{9 weeks}}$

$=8.9832$

Section 4.3

59. Use properties of logarithms to expand the logarithmic expression $\log_5\left(\frac{\sqrt{7}x^3}{y^8}\right)$ as much as possible.

$\frac{1}{2}\log_5 7 + 3\log_5 x - 8\log_5 y$

60. Use properties of logarithms to condense the following logarithmic expression $4\log_b y + \frac{1}{3}\log_b z - \log_b x$. Write the expression as a single logarithm whose coefficient is 1.

$\log_b \frac{y^4\sqrt[3]{z}}{x}$

61. Evaluate the expression $e + \ln e^2 - 4\ln\sqrt{e}$. Give the exact answer.

$e + 2 - 4(\frac{1}{2})$ $\boxed{e}$

Section 4.4

62. Solve the equation $2^{(5+7x)} = 8$. Give the exact answer.

$2^{5+7x} = 2^3$

$5+7x=3$

$7x=-2$

$\boxed{x = -\frac{2}{7}}$

63. Solve the exponential equation $3e^{2x-7} = 15$. Give the exact answer.

$\ln e^{2x-7} \ln = 5$

$2x-7 = \ln 5$

$2x = \ln 5 + 7$

$\boxed{x = \frac{\ln 5 + 7}{2}}$

64. Solve the logarithmic equation $\log_4(x-2) = -3$. Give the exact answer.

$4^{-3} = x-2$

$x = 2 + 4^{-3}$

$\boxed{x = \frac{129}{64}}$

65. The function $f(x) = 1 + 1.3\ln(x+1)$ models the average number of free-throws a basketball player can make consecutively during practice as a function of time, where x is the number of consecutive days the basketball player has practiced for two hours. After how many days of practice can the basketball player make an average of 7 consecutive free throws? Round to the nearest day.

$7 = 1 + 1.3\ln(x+1)$

$\frac{6}{1.3} = \ln(x+1)$

$e^{\frac{6}{1.3}} - 1$

$\boxed{100 \text{ days}}$

Section 4.5

66. The formula $A = 226e^{0.037t}$ models the population of a particular city, in thousands, t years after 1998. When will the population of the city reach 327 thousand (round to the nearest year)?

$\frac{327}{226} = \frac{226e^{.037t}}{226}$

$\ln \frac{327}{226} \ln = e^{.037t}$

$\frac{\ln\left(\frac{327}{226}\right)}{.037} = t$

$t = 10$

$\boxed{2008}$

67. A fossilized leaf contains 28% of its normal amount of carbon 14. How old is the fossil (to the nearest year)?
Use $A = A_0 e^{-0.000121t}$

$\ln .28 = \ln e^{-.000121t}$ $\frac{\ln .28}{-.000121}$ $\boxed{10{,}520 \text{ years}}$

68. The logistic growth function $f(t) = \dfrac{36{,}000}{1+1199e^{-1.6t}}$ models the number of people who have become ill with a particular infection t weeks after its initial outbreak in a particular community.

a. How many people were ill after 3 weeks (round to the nearest number of people)?
b. What is the limiting value?

36000

$f(3) = \dfrac{36000}{1+1199e^{-1.6(3)}}$

$\boxed{3313 \text{ people}}$

Section 8.1

69. Write the first four terms of the sequence(s):

a. $a_n = (-1)^{n+1}(n+4)$

$(-1)^{1+1}(5)$ $(-1)^{2+1}(6)$ $(-1)^{3+1}(7)$ $(-1)^{4+1}(8)$

5 −6 7 −8

b. $a_n = \left(\frac{1}{3}\right)^{n+1}$

$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$ $\left(\frac{1}{3}\right)^5 = \frac{1}{243}$

Section 8.2

70. Find a_{200} when $a_1 = -40,\ d = 5$.

$a_{200} = -40 + (200-1)(5)$

$\boxed{955}$

71. Write a formula for the general (nth) term of the arithmetic sequence given. Find a_{20}
2, 7, 12, 17, …

$a_1 = 2$
$d = 5$

$a_{20} = 2 + (20-1)(5)$

$\boxed{97}$

72. Find $1+2+3+4+\ldots+100$, the sum of the first 100 natural numbers.

$S_N = \frac{100}{2}(1+100)$

$\boxed{S_N = 5050}$

Section 8.3

73. Find a_{30} when $a_1 = 8000,\ r = -\frac{1}{2}$.

$a_{30} = 8000\left(-\frac{1}{2}\right)^{29} = \boxed{-.0000149011612}$

74. Write a formula for the general (nth) term of the geometric sequence given. Then find a_7.

$5, -1, \frac{1}{5}, -\frac{1}{25}, \ldots$

$a_1 = 5$
$r = -\frac{1}{5}$

$a_N = 5\left(-\frac{1}{5}\right)^{N-1}$ $a_7 = 5\left(-\frac{1}{5}\right)^6$

$\boxed{\frac{1}{3125} \text{ or } .00032}$

75. Find the sum of the first 11 terms of the geometric sequence: 4, -12, 36, -108, …

S_{11}

$\boxed{177{,}148}$

76. Find the sum of the infinite geometric series: $3+\frac{3}{4}+\frac{3}{4^2}+\frac{3}{4^3}+\ldots$

$r = \frac{1}{4}$

$S_N = \dfrac{3}{1-\frac{1}{4}}$

$\boxed{S_N = 4}$