Math 120

8.1 Sequences and Summation Notation

Objectives:

1. Find particular terms of a sequence given the equation for the general term.

Topic #1: Sequences

Introduction to Sequences

A sequence is a string of numbers that follow a pattern or a rule to get from one term to the next. Consider the sequence:

$2, 4, 6, 8, \ldots$

The first term is 2, the second 4, the third 6, the fourth 8, and so on. To get the next term, the pattern suggests to "add 2". The sequence goes on forever and is an

infinite sequence

Suppose that we were only interested in the first ten terms:

$2, 4, 6, 8, 10, 12, 14, 16, 18, 20$

This is now a finite sequence since there are a fixed number of terms (specifically, $n = 10$ terms).

Definition of a Sequence

The terms of a sequence are a function of their position along the sequence. The first term is denoted as a_1 and occurs when $n = 1$, the second term is a_2 and occurs when $n = 2$, and so on. This gives the formal definition of an infinite sequence:

$$a_1, a_2, a_3, a_4, \ldots a_n \ldots$$

where a_n is the nth term and n is a positive integer.

Each term is a function of its position along the sequence.

The domain is the set of positive integers n and the range are the associated terms a_n.

The terms of the sequence:

$$2, 4, 6, 8, \dots$$

can be defined as $a_n = 2n$, giving any term along the sequence as a function of its position:

when $n = 1,\ a_1 = 2(1) = 2$
when $n = 2,\ a_2 = 2(2) = 4$
when $n = 3,\ a_3 = 2(3) = 6$
when $n = 4,\ a_4 = 2(4) = 8$

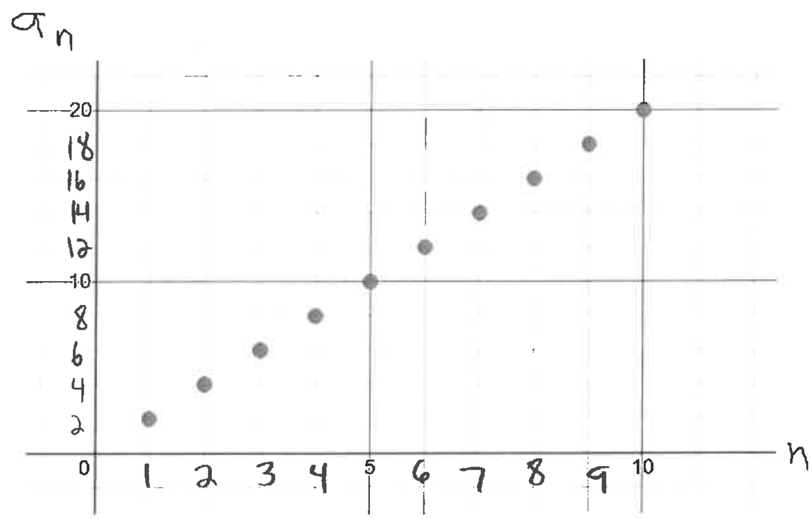
Using the rule, the 99th term in the sequence is:

$$n = 99,\ a_{99} = 2(99) = 198$$

Note that we cannot evaluate the term of the sequence unless n is a positive integer (also called a counting/natural number).

For example, values such $n = 1/2$ or $n = -2$ is out of the domain since they are not counting numbers.

Here is a graph of the first ten terms of the sequence from the previous page. Notice that the domain is only $n = 1,2,3,4,5,6,7,8,9,10$ and there are gaps between each term.



Example #1 – Write the First 4 Terms of the Sequence

a) $a_n = 5n + 1$

Plug in each position into the definition to get its term:

when $n = 1$, $a_1 = 5(1)+1 = 6$

when $n = 2$, $a_2 = 5(2)+1 = 11$

when $n = 3$, $a_3 = 5(3)+1 = 16$

when $n = 4$, $a_4 = 5(4)+1 = 21$

Notice the next term is 5 more than the previous term.

b) $a_n = 3^n$

Plug in each position into the definition to get its term:

when $n = 1$, $a_1 = 3^1 = 3$

when $n = 2$, $a_2 = 3^2 = 9$

when $n = 3$, $a_3 = 3^3 = 27$

when $n = 4$, $a_4 = 3^4 = 81$

Notice the next term is 3 times the previous term.

c) $a_n = (-4)^n$

when $n = 1$, $a_1 = (-4)^1 = -4$

when $n = 2$, $a_2 = (-4)^2 = 16$

when $n = 3$, $a_3 = (-4)^3 = -64$

when $n = 4$, $a_4 = (-4)^4 = 256$

Notice the terms alternate between negative/positive and next term is -4 times the previous term.

d) $a_n = (-1)^{n+1}(n-2)$

when $n = 1$, $a_1 = (-1)^{1+1}(1-2) = (-1)^2(-1) = -1$

when $n = 2$, $a_2 = (-1)^{2+1}(2-2) = (-1)^3(0) = 0$

when $n = 3$, $a_3 = (-1)^{3+1}(3-2) = (-1)^4(1) = 1$

when $n = 4$, $a_4 = (-1)^{4+1}(4-2) = (-1)^5(2) = -2$

This is another alternating sequence.

e) $a_n = \dfrac{2n}{n+6}$

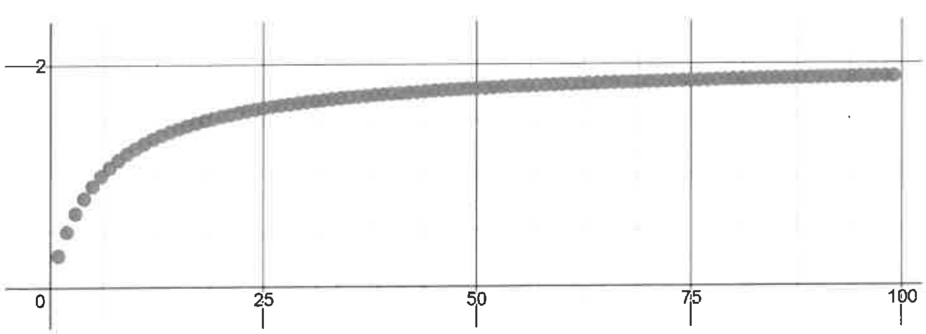
Plug in each position into the definition to get its term:

$a_1 = \dfrac{2(1)}{1+6} = \dfrac{2}{7}$

$a_2 = \dfrac{2(2)}{2+6} = \dfrac{4}{8} = \dfrac{1}{2}$

$a_3 = \dfrac{2(3)}{3+6} = \dfrac{6}{9} = \dfrac{2}{3}$

$a_4 = \dfrac{2(4)}{4+6} = \dfrac{8}{10} = \dfrac{4}{5}$



A graph of the first 100 terms shows they approach the value 2 as n gets bigger.