Math 120
5.1 System of Linear Equations

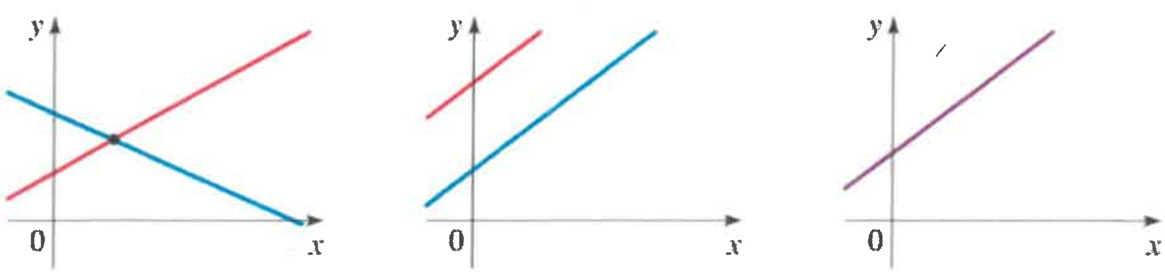
Objectives:

1. Determine whether an ordered pair is a solution of a linear system.
2. Solve linear systems by substitution.
3. Solve linear systems by addition.
4. Identify systems that do not have exactly 1 ordered pair solution.
5. Solve problems using systems of linear equations.

Topic #1 - System of Linear Equations in Two Variables/Two Equations

A system of two linear equations can be thought of as two lines in the plane. The solution set is an ordered pair (x, y) that satisfies both equations/falls on both lines.

From this geometric standpoint, three possibilities emerge:



(a) Lines intersect at a single point. The system has one solution.

(b) Lines are parallel and do not intersect. The system has no solution.

(c) Lines coincide—equations are for the same line. The system has infinitely many solutions.

Example #1 – Determine if the Ordered Pair is a Solution to the System

a. $\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases}$ Ordered Pair: $(2,1)$

This is a system, one line is in the form $y = mx + b$, the other in the form $Ax + By = C$. To determine if the given point works, plug into both systems where $x = 2$ and $y = 1$:

$1 = 4(2) - 7$ $\quad$ $3(2) - 4(1) = 2$

$1 = 8 - 7$ $\quad$ $6 - 4 = 2$

$1 = 1$ $\quad$ $2 = 2$

Both equations are true when plugging the values into the system. The ordered pair (2,1) Solution to the system.

b. $\begin{cases} 4x + 3y = 13 \\ 2x - 4y = 18 \end{cases}$ Ordered Pair: $(4, -1)$

To determine if the given point works, plug into both systems where $x = 4$ and $y = -1$.

$4(4) + 3(-1) = 13$ $\quad$ $2(4) - 4(-1) = 18$

$13 = 13$ $\quad$ $12 = 18$

The first equation is true, but the second one is false. The ordered pair $(4, -1)$ NOT a solution to the system.

Topic #2 – Solving a System of Linear Equations: Substitution Method

Consider the system from the previous example:

$$\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases}$$

Notice the first equation gives y in terms of x, we can rewrite the second equation accordingly and solve for x:

$$3x - 4(4x-7) = 2$$

$$x = 2$$

Since $x =$ 2, we can go back to the first equation and solve for y:

$$y = 4(2) - 7$$

$$y = 1$$

Therefore the solution to the system is (2,1)
We can check by plugging into both equations to confirm!

<u>Example #1</u> – Solve the System Using Substitution

a. $\begin{cases} 2x + 5y = 0 \\ x - 3y = 0 \end{cases}$

Isolate x in the second equation and it will be in terms of y:

$x = 3y$

$2(3y) + 5y = 0$

$11y = 0$

$y = 0$

Rewrite the first equation accordingly and solve for y:

$x = 0$

Since ______________________ we can go back to the second equation and solve for x:

Therefore the solution to the system is $(0,0)$

Plug into the system to confirm.

b. $\begin{cases} x + 3y = 2 \\ 2x + 5y = 2 \end{cases}$ $\longrightarrow$

$$\begin{aligned} x + \cancel{3y} &= 2 \\ -\cancel{3y} &\quad -3y \\ \hline x &= \underline{2-3y} \end{aligned}$$

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$$2(2-3y)+5y=2$$
$$4-6y+5y=2$$
$$4-y=2$$
$$-4 \qquad -4$$
$$-y=-2$$
$$y=2$$

$$x+3(2)=2$$
$$x+6=2$$
$$x=-4$$

$$\boxed{(-4,2)}$$

Topic #3 – Solving a System of Linear Equations: Addition Method

Consider the system:

$$\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases}$$

$-4x \quad -4x$

$\longrightarrow (-4x + y = -7) \cdot 4$

$3x - 4y = 2$

The first equation can be rewritten in $Ax + By = C$ form by subtracting $4x$:

There is a theorem that states an equation can be multiplied by any constant and that any two equations can be added together.

Suppose we multiply/scale all terms in the first equation by 4

$$\begin{array}{rl} & -16x + \cancel{4y} = -28 \\ + & 3x - \cancel{4y} = 2 \\ \hline & -13x = -26 \end{array}$$

$$\begin{cases} -16x + 4y = -28 \\ 3x - 4y = 2 \end{cases}$$

$x = 2$

Notice the y values are OPPOSITES, we can add the two equations together and the y values drop out:

$y = 1$

$$\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases}$$

Therefore the solution to the system is (2, 1)

We can check by plugging into both equations to confirm!

Example #1 – Solve the System Using Addition

a. $\begin{cases} 2x + 5y = 0 \\ -2(x - 3y = 0) \end{cases}$ $\begin{matrix} \rightarrow 2x+5y=0 \\ \rightarrow -2x+6y=0 \end{matrix}$

Both equations are in $Ax + By = C$ form, so there is no need to rewrite either equation accordingly.
Multiply all terms in the second equation by
___-2___ to get OPPOSITE ___x-value___

$2x+5(0)=0$

$2x=0$

$x=0$

$\frac{11y}{11}=\frac{0}{11}$

$y=0$

Therefore the solution to the system is ___(0,0)___
Plug into the system to confirm.

b. $\begin{cases} -2(x + 3y = 2) \rightarrow -2\cancel{x} - 6y = -4 \\ 2x + 5y = 2 \qquad\qquad\ \ \cancel{2x} + 5y = 2 \end{cases}$

Both equations are in $Ax + By = C$ form, so there is no need to rewrite either equation accordingly.
Multiply all terms in the ~~second equation~~ by ____________ to get OPPOSITE ____________________

$$-y = -2$$
$$\frac{-y}{-1} = \frac{-2}{-1}$$
$$y = 2$$

$(-4, 2)$

Topic #4 – Solving a System of Linear Equations: Graphing Method

The solution to a system of two linear equations is where the two lines intersect

Consider the system:

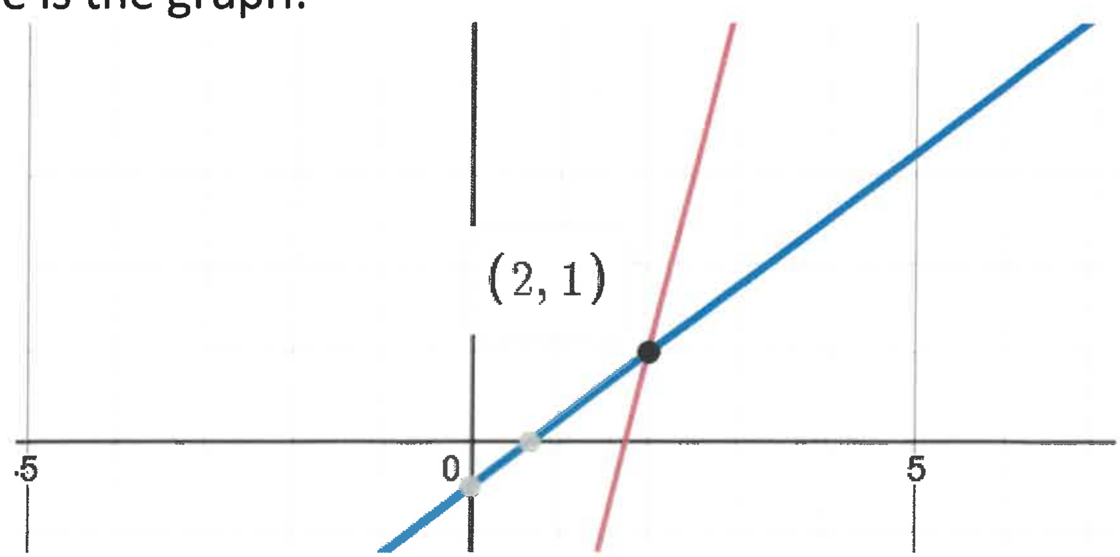
$$\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases} \longrightarrow$$

$$\begin{array}{rcl} 3x - 4y &=& 2 \\ -3x & & -3x \\ \hline \frac{-4y}{-4} &=& \frac{2-3x}{-4} \end{array}$$

If using a graphing calculator, both equations must be solved for y:

$$\begin{cases} y = 4x - 7 \\ 3x - 4y = 2 \end{cases} \rightarrow \begin{cases} y = 4x - 7 \\ y = \dfrac{(2 - 3x)}{-4} \end{cases}$$

Here is the graph:



The solution to the system is where the lines meet, at the point (2,1).

Example #1 – Solve the System Using Graphing

a. $\begin{cases} 2x + 5y = 0 \\ x - 3y = 0 \end{cases}$

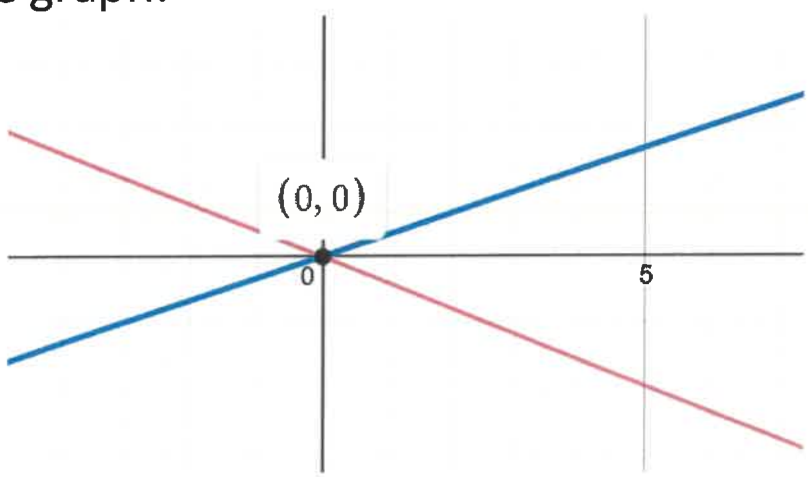
Solve both equations for y if using a graphing calculator:

$$\begin{cases} 2x + 5y = 0 \\ x - 3y = 0 \end{cases} \rightarrow$$

$$\begin{aligned} 2x + 5y &= 0 \\ -2x \quad &\quad -2x \\ \hline 5y &= -2x \\ \frac{5y}{5} &= \frac{-2x}{5} \\ y &= \frac{-2x}{5} \end{aligned}$$

$$\begin{aligned} x - 3y &= 0 \\ -x \quad &\quad -x \\ \hline -3y &= -x \\ \frac{-3y}{-3} &= \frac{-x}{-3} \\ y = \frac{-x}{-3} &= \frac{x}{3} \end{aligned}$$

Here is the graph:



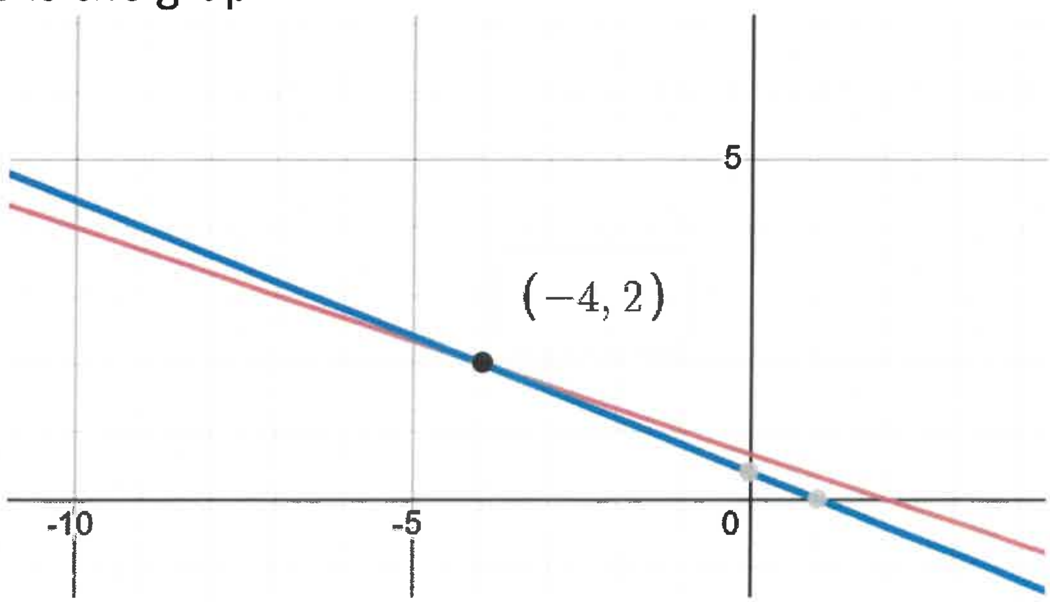
The solution to the system is where the lines meet, at the point ___(0,0)___

b. $\begin{cases} x + 3y = 2 \\ 2x + 5y = 2 \end{cases}$

Solve both equations for y if using a graphing calculator:

$$\begin{cases} x + 3y = 2 \\ 2x + 5y = 2 \end{cases} \rightarrow$$

Here is the graph:



The solution to the system is where the lines meet, at the point ______________________________

Topic #5 – System of Linear Equations with No Solution

A system with **no solution** is two lines in the plane that DO NOT meet parallel lines. A graph will show this and solving with addition or substitution will produce a FALSE statement when both variables drop out.

Consider the system:

$$-3\begin{cases}(2x+3y=6) \\ 6x+9y=12\end{cases} \rightarrow \begin{matrix} -\cancel{6x} - \cancel{9y} = -18 \\ \cancel{6x} + \cancel{9y} = 12\end{matrix}$$

By the addition method, multiply the terms of first equation by -3:

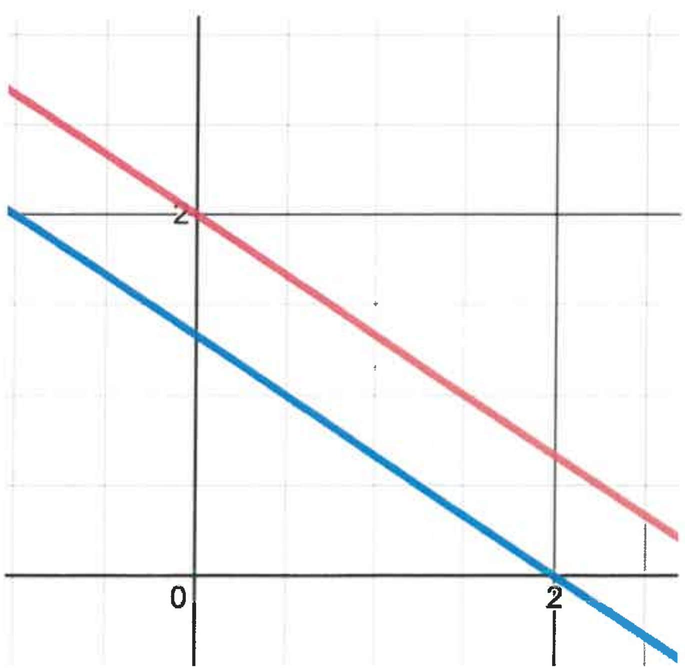
$0 \neq -6$

$\boxed{\text{No solution}}$

Adding the equations results in both variables dropping out and False statement

A graph confirms the lines are ______________________
which also indicates NO SOLUTION:

$$\begin{cases} 2x + 3y = 6 \\ 6x + 9y = 12 \end{cases} \rightarrow$$



Feel free to try out the substitution method; it also will cause both variables to drop out and leave a FALSE statement.

Topic #6 – System of Linear Equations with Infinitely Many Solutions

A system with **infinitely many solutions** are two lines that coincide Meet everywhere, same line

A graph will show this and solving with addition or substitution will produce a True statement when both variables drop out.

Consider the system:

$$\begin{cases} y = 3x - 2 \\ 15x - 5y = 10 \end{cases}$$

By the substitution method, the first equation has y in terms of x. Rewrite the second equation accordingly to solve for x:

$15x - 5(3x-2) = 10$

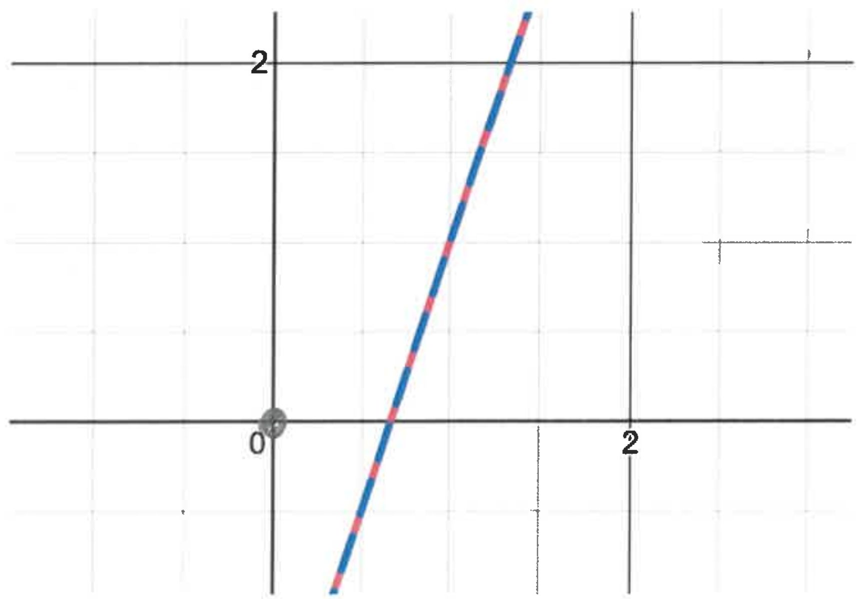
$10 = 10$

The variables drop out and the statement is TRUE, which indicates infinetly many solution

A graph confirms the lines are the SAME, which also indicates INFINITELY MANY SOLUTIONS:

$$\begin{cases} y = 3x - 2 \\ 15x - 5y = 10 \end{cases} \rightarrow$$

$\{(x,y) \mid 15x - 5y = 10\}$



Feel free to try out the addition method; it also will cause both variables to drop out and leave a TRUE statement.

Topic #7 – Application of Systems of Linear Equations: Revenue, Cost, and Profit Functions

iPhones

Suppose a company produces and sells x units of a product. The revenue is the money generated by selling x units and the cost is the amount spent producing x units.

$x =$ iphone

Revenue Function: $R(x) = (\text{price per unit sold})x$

Cost Function: $C(x) = \text{fixed cost} + (\text{cost per unit produced})x$

The profit is what is left over from the revenue after the cost is taken out:

Profit Function: $P(x) = R(x) - C(x)$

The break-even point is where

$P(x) = 0$ OR where $R(x) = C(x)$

Example #1 – Build and Solve a System of Linear Equations

A company manufactures and sells a certain model of Bluetooth speakers. The fixed cost to operate daily production is $18,000 and it costs $20 to produce each speaker. The selling price is $80 per speaker. Assume each unit produced is sold. $X = \#$ of speakers

a. Write the cost function.

$C(x) = 18000 + 20x$

b. Write the revenue function.

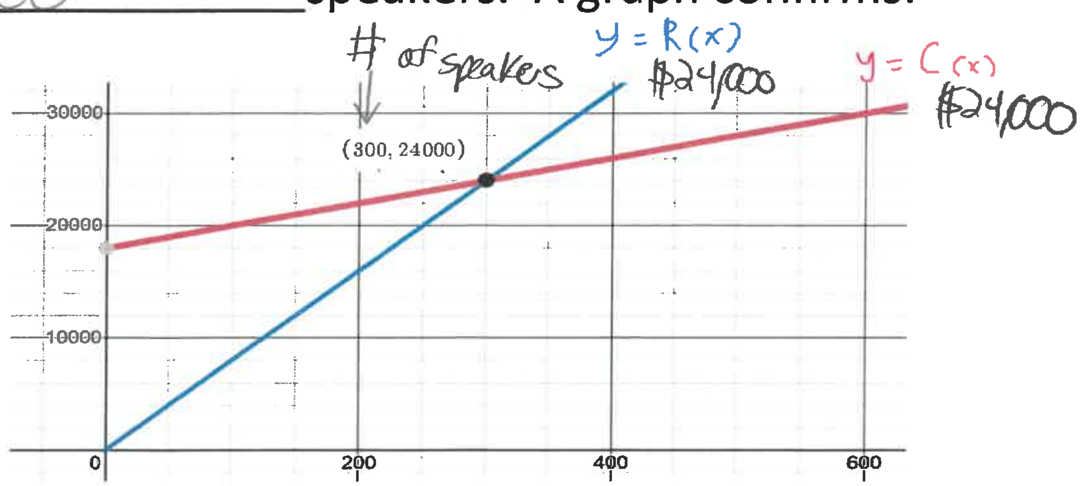
$R(x) = 80x$

c. Determine the break-even point and interpret the meaning in context.

$80x = 20x + 18000$

$X = 300$

This means that the company will break-even (neither make nor lose money) by producing and selling 300 speakers. A graph confirms:



d. Will the company make a profit by producing and selling 250 units? Explain.

Using the definition:

$P(x) = R(x) - C(x)$

$P(x) = 80x - (20x + 18000)$ $\quad\quad$ $P(x) = 60x - 18000$

When $x = 250$ $P(250) = 60(250) - 18000 = -3000$

This means the company will loss money by producing and selling 250 units; lose - $3000