Math 120

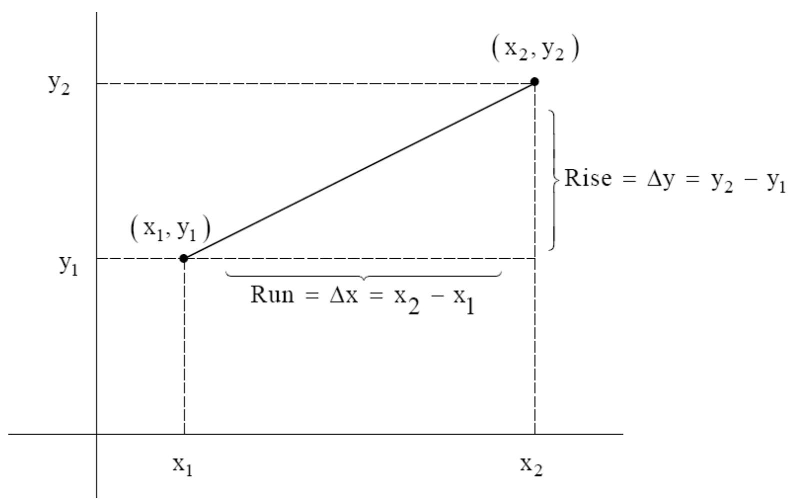
1.4 Linear Functions and Slope

Objectives:

1. Calculate a line's slope.
2. Write the point-slope form of the equation of a line.
3. Write and graph the slope-intercept form of the equation of a line.
4. Graph horizontal and vertical lines.
5. Recognize the general form of a line and use x and y intercepts to graph the equation of the line.
6. Model data with linear functions and make predictions.

Topic #1: The Slope of a Line

A line is determined by 2 points in the plane. The steepness of the line is calculated by finding the ____Slope____ between the 2 points. The slope is the __average rate of change__ between any 2 points on the line; it can also be thought of as "rise over run".

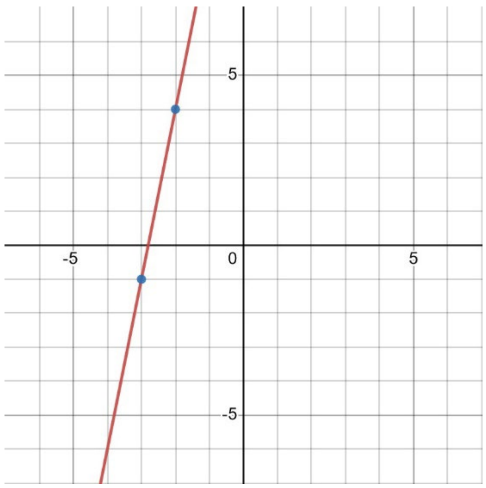


Definition of Slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta \updownarrow}{\Delta \leftrightarrow} = \frac{y_2 - y_1}{x_2 - x_1}$$

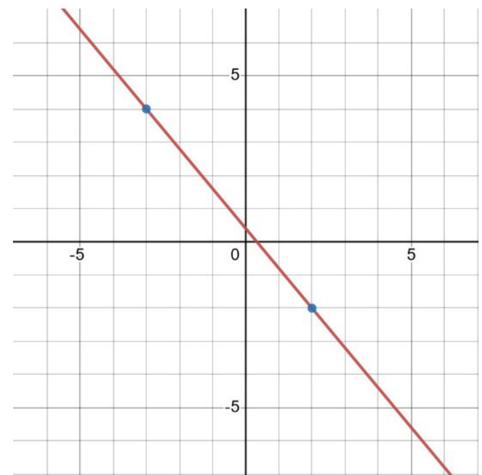
__Example #1__ – Find the Slope of the Line that Passes Through the Given Points

a) $(-3,-1)$ and $(-2,4)$



$m=\frac{\Delta y}{\Delta x}=\frac{(-1)-(4)}{(-3)-(-2)}=\frac{-5}{-1}=\frac{5}{1}$

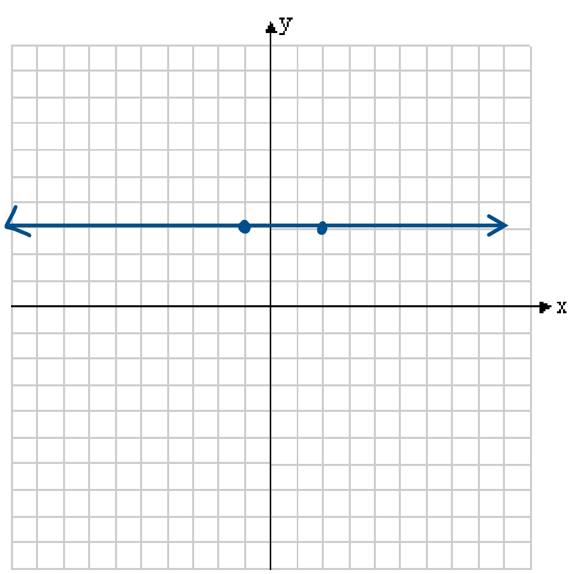
b) $(-3,4)$ and $(2,-2)$



$M=\frac{\Delta y}{\Delta x}=\frac{(4)-(-2)}{(-3)-(2)}=\frac{6}{-5}=-\frac{6}{5}$

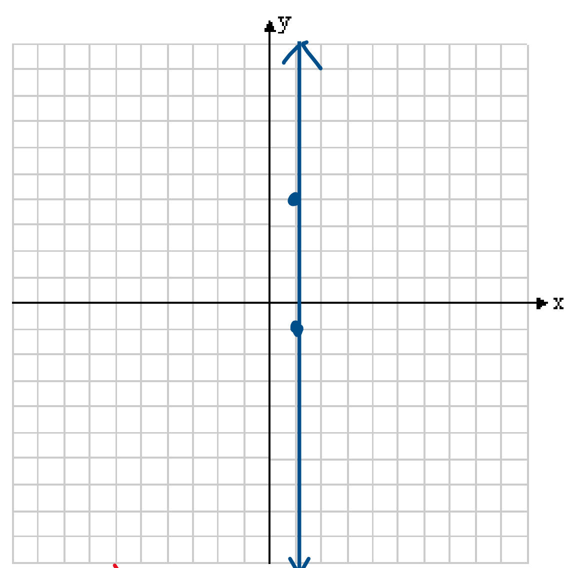
$\frac{0}{K}=0$ $\frac{N}{0}=$ Undefined

c) $(-1,3)$ and $(2,3)$



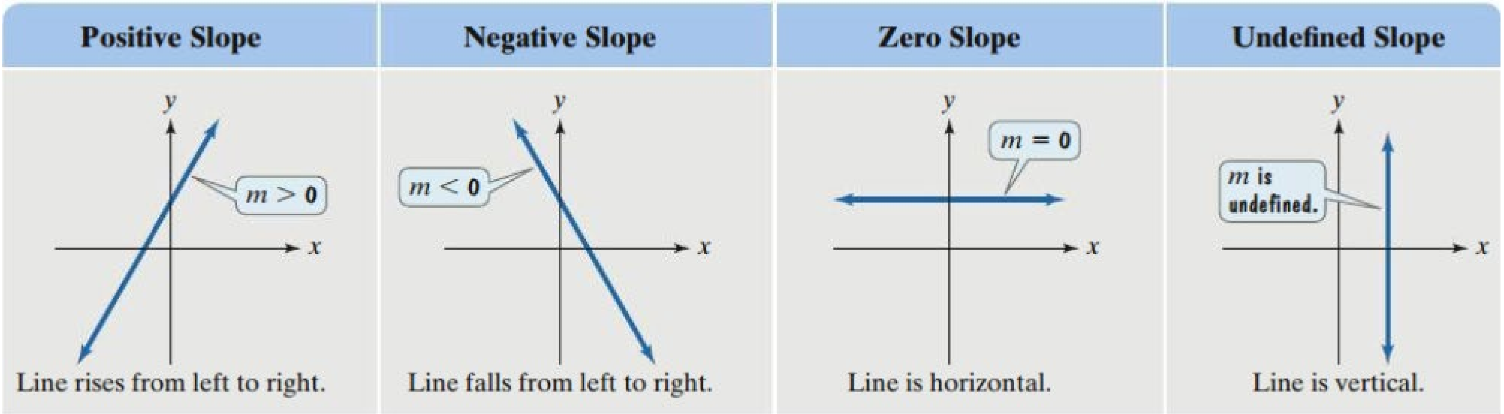
$M=\frac{(3-3)}{(-1-2)}=\frac{0}{-3}=0$

d) $(1,-1)$ and $(1,4)$



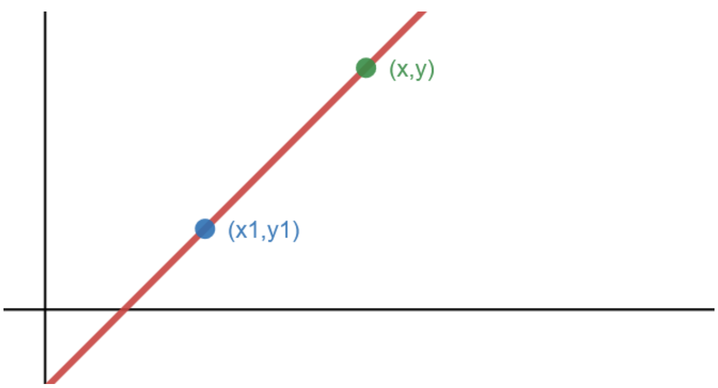
$M=\frac{(-1-4)}{(1-1)}=\frac{-5}{0}=$ undefined

With the exception of the vertical line, the other lines represent Linear Functions where y is a function of x.



<u>Topic #2: The Point-Slope Form of a Line</u>

As stated earlier, it takes 2 points to determine a line; if we have 2 points then we can find the equation of the line that contains the points (and all other points on it). Suppose 1 point is **fixed** at the point (x_1, y_1) and the other point varies at (x, y).



The slope of the line is:

$$m = \frac{y - y_1}{x - x_1}$$

Solving for y gives us the **point-slope form of a line:**

$$y - y_1 = m\,(x - x_1)$$

m = slope

(x_1, y_1) = point

Example #1 – Find the Equation of the Line with the Given Conditions, Write y in Terms of x

a) The slope is 5 and passes through the point $(-3, -1)$.

$x_1 \quad y_1$

$m = 5$

$$y - y_1 = m\,(x - x_1)$$

$$y - (-1) = 5(x - (-3))$$

$$y + 1 = 5(x + 3)$$

$$y + 1 = 5x + 15$$

$$-1 \qquad\qquad -1$$

$$\boxed{y = 5x + 14}$$

b) The slope is $-3/4$ and passes through the origin.

m

$(0,0)$

$$y - 0 = -\frac{3}{4}(x - 0)$$

$$\boxed{y = -\frac{3}{4}x}$$

c) The line passes through the points $(-3,0)$ and $(0,3)$.

$$m = \frac{0-3}{-3-0} = \frac{-3}{-3} = 1$$

$$y - 0 = 1(x-(-3))$$

$$\boxed{y = x + 3}$$

YOU TRY #1

a) Find an equation of a line going through the point (-4, 1) and with slope $\frac{3}{2}$.

$x_1 \quad y_1 \qquad m$

$$y-(1) = \frac{3}{2}(x-(-4))$$

$$y - 1 = \frac{3}{2}x + 6$$

$$+1 \qquad\qquad +1$$

$$\boxed{y = \frac{3}{2}x + 7}$$

b) Find an equation of the line going through the points (2, 5) and (-3, 4).

$$m = \frac{(4)-(5)}{(-3)-(2)} = \frac{-1}{-5} = \frac{1}{5}$$

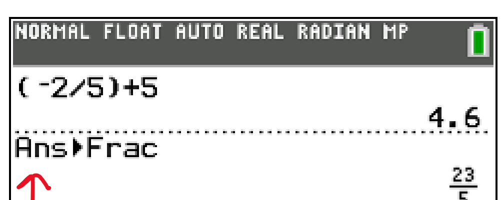
$$y-(5) = \frac{1}{5}(x-(2))$$

$$y - 5 = \frac{1}{5}x - \frac{2}{5}$$

$$+5 \qquad\qquad +5$$

$$y = \frac{1}{5}x + \frac{23}{5}$$

Calculator



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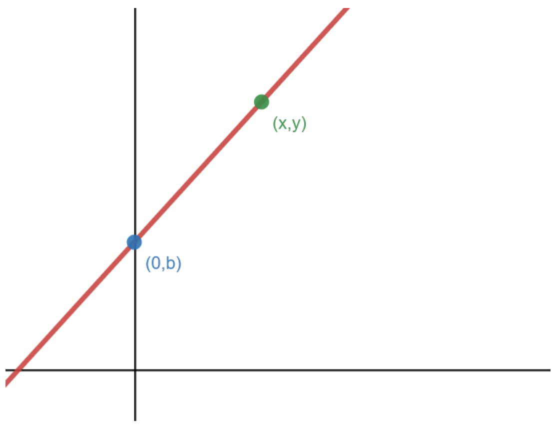
Topic #3: The Slope-Intercept Form of a Line

When a line is expressed in the form $\boldsymbol{y = mx + b}$**,** two main features of the line are revealed: its **slope** and its $\boldsymbol{y}$**-intercept**. As discussed earlier, the slope describes the rate of change (or steepness) of the line. The y-intercept is where the line crosses the y-axis, which is where $x = 0$. The associated ordered pair for any y-intercept is denoted $(0, b)$.



The slope of the line is: $m = \frac{y-b}{x-0}$

Solving for y gives us the **slope-intercept form of a line:**

$y = mx + b$

m = slope

$(0, b)$ = y-int

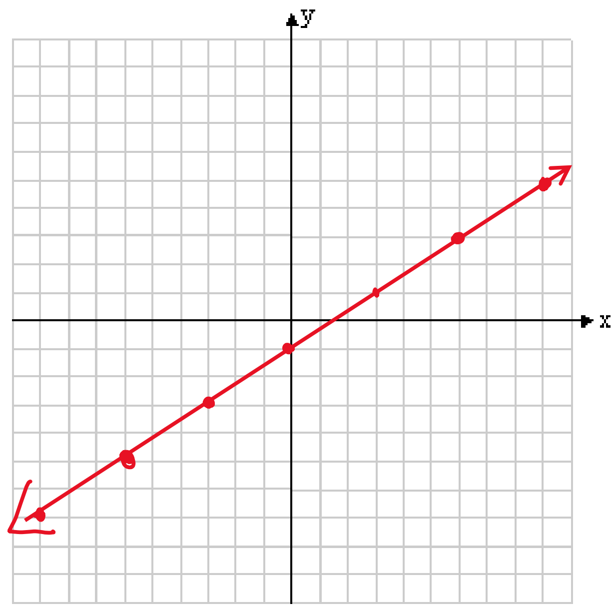
Notice the above examples using point-slope were all simplified to this form where m is the slope and b is where the line crosses the y-axis. For the graphing calculator we need to enter the equation as “Y=”…

Example #1 – State the Slope and y-intercept and graph the line.

a) $y = \frac{2}{3}x - 1$

$m = \frac{2}{3}$

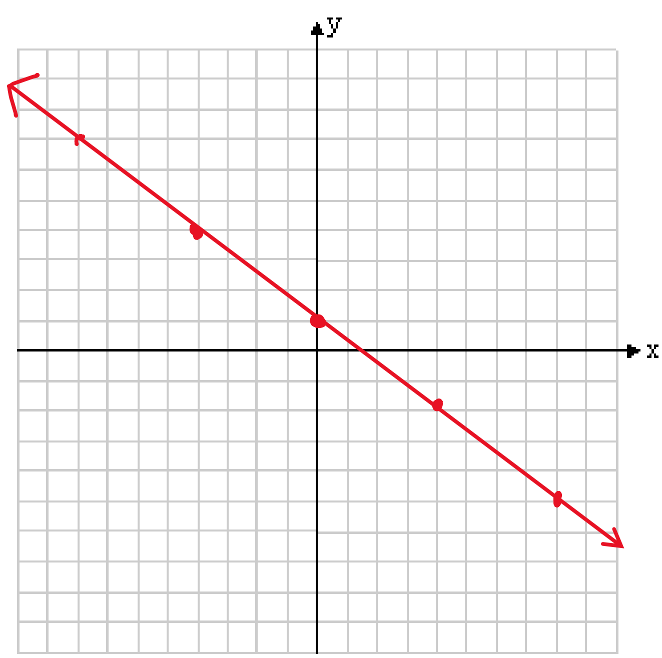
$y\text{-int} = (0, -1)$



b) $y = -\frac{3}{4}x + 1$

$m = -\frac{3}{4} = \frac{-3}{4} = \frac{3}{-4}$

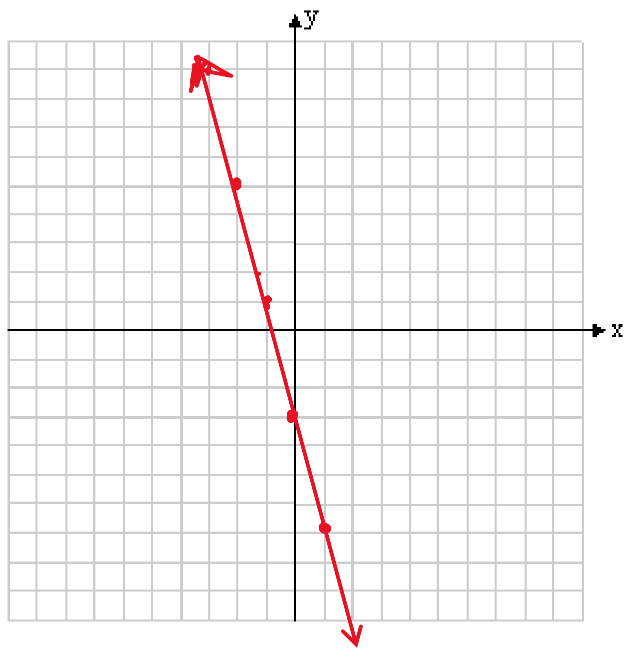
$y\text{-int} = (0, 1)$



c) $4x + y + 3 = 0$

Note this is not in slope-intercept form (it is in Standard form, also called the General Form form for the equation of a line). Let's solve for y by subtracting $4x$ and 3 from both sides of the equation to put in slope intercept like the examples above.

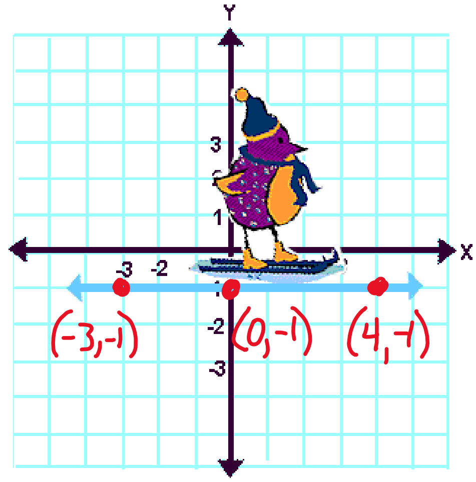
$$\begin{array}{rcl} 4x + y + 3 &=& 0 \\ -4x \quad\quad -3 && -3-4x \\ \hline y &=& -4x - 3 \end{array}$$



Topic #4: Horizontal and Vertical Lines

A horizontal line has a slope of ZERO and passes through the y-axis at the point $(0, b)$. The point $(0, b)$ is also called the y-intercept

- A horizontal line has a slope equal to 0
- (Ski Bird is not working hard to get up a hill, nor is he trying to slow down. His energy level (and his enjoyment level) is at zero.)



$\frac{0}{K} = 0$

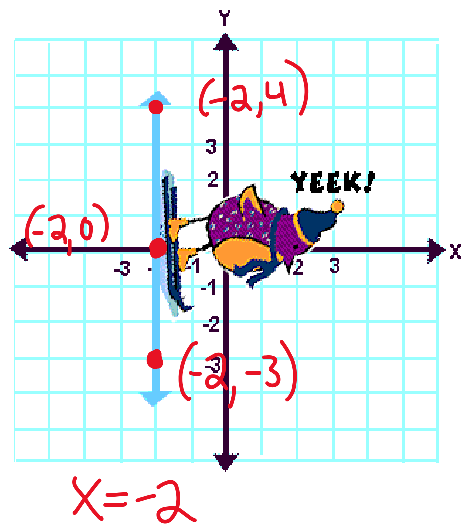
$Y = 0x - 1$

$Y = -1$

$m = \frac{0}{run} = 0$

The resulting equation for a horizontal line is $\boldsymbol{y} = \boldsymbol{b}$ b OR in function notation $\boldsymbol{f(x)} = \boldsymbol{b}$. Since the slope of a horizontal line is ALWAYS zero, we could rewrite the equation in slope-intercept form: $y = 0x + b$ which simplifies back to the equation y=b

A vertical line has an UNDEFINED slope and passes through the x-axis at the point $(a, 0)$. The point $(a, 0)$ is also called the x-int



- A vertical line has an undefined slope
- (Ski Bird cannot ski vertically. **Sheer doom** awaits Ski Bird at the bottom of a vertical hill.)

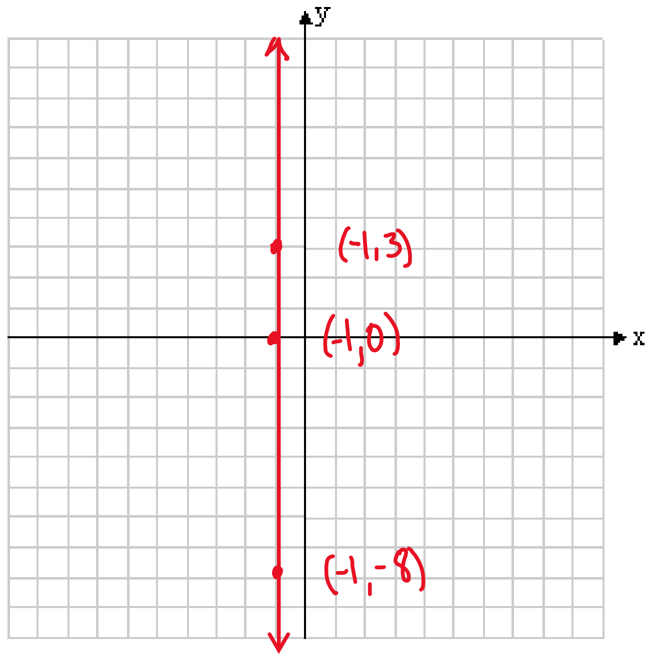
$\frac{N}{0}$ = undefined

$m = \frac{\text{rise}}{0}$ = undefined

The resulting equation for vertical line is $\boldsymbol{x = a}$. ***We cannot use function notation for a vertical line since the x-values repeat so vertical lines are*** NOT functions

__Example #1__ – Graph the Line

a) $x = -1$

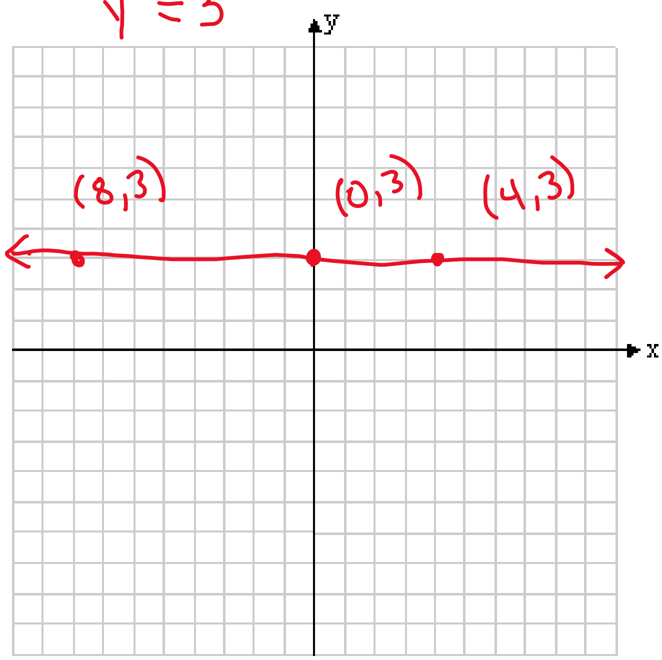


All x values are -1

$X = -1$

b) $f(x) = 3$

$y = 3$



All y values are 3

$y = 3$

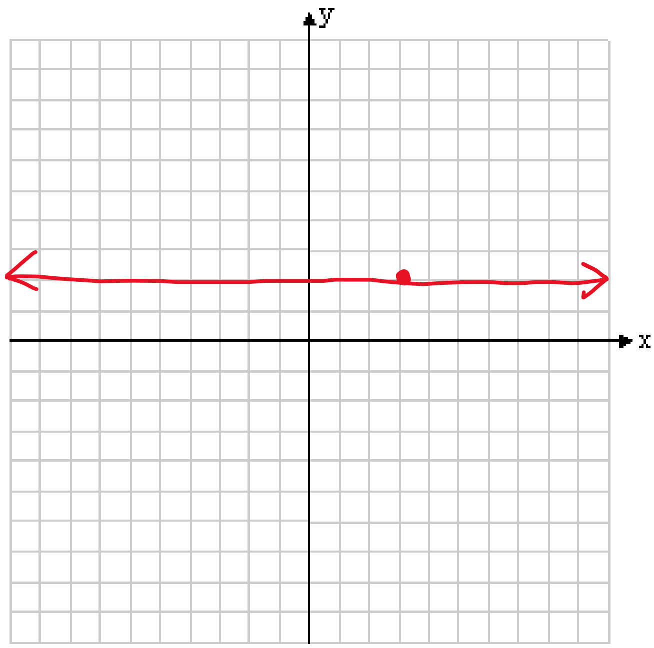
__YOU TRY#2__ –

c) Find the equation of a line going through the point (3,2) and the slope does not exist. Graph the line.

undefined   horizontal

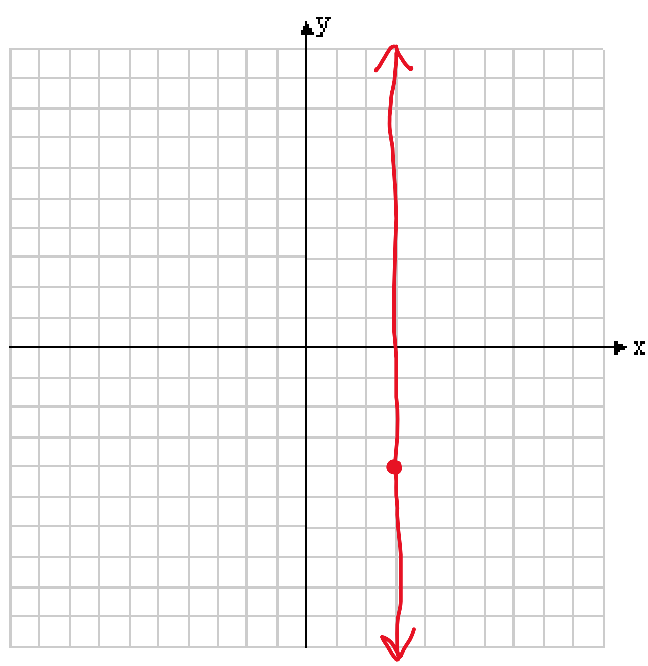
$y = \#$

$f(x) = 2$



vertical

d) Find an equation of the line having slope of 0 and going through the point (3, -4). Graph the line.



$X = \#$

$X = 3$

Topic #5: Modeling Data with a Linear Function

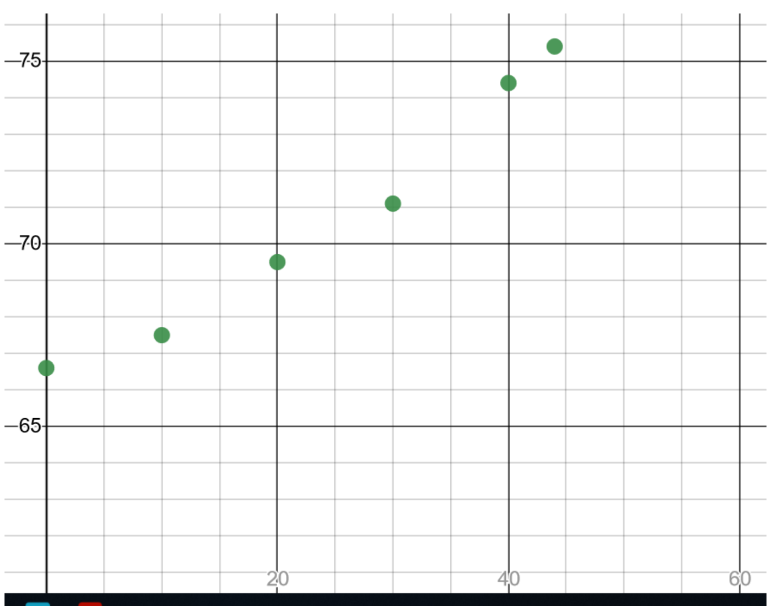
Linear functions can be used to fit data and make predictions. Consider the data set for life expectancy for men born in six selected years from a certain community. Let x represent the years born after 1960 and y represent the life expectancy (in years). This tells us a male born in 1960 corresponds to $x = 0$, a male born in 1970 is $x = 10$, and so on.

Let x be: Years born after 1960

Let y be: life expetancy in years

x	0	10	20	30	40	44
y	66.6	67.5	69.5	71.1	74.4	75.4

We can plot the data points with technology or we can plot the points by hand, just be sure to use an appropriate scale on the axes.



The points do not line up perfectly, but a linear function models the data *pretty well*. We can pick **ANY** 2 points from the data set and come up with a linear model.

Suppose we pick the points associated with a person born in 1980 and 2000; this corresponds to the ordered pairs: $(20,69.5)$ and $(40,74.4)$.

1980 Life expectancy of 69.5 years

To find the equation of a line, we first need the slope:

$$m = \frac{(74.4-69.5)}{40-20} = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{ Life expectancy}}{\Delta \text{ years}} = \frac{4.9}{20} = \frac{0.245}{1} \frac{\text{Life expectancy}}{\text{years}}$$

Every 1 year after 1960 you were born, your life expectancy increases by 0.245 years.

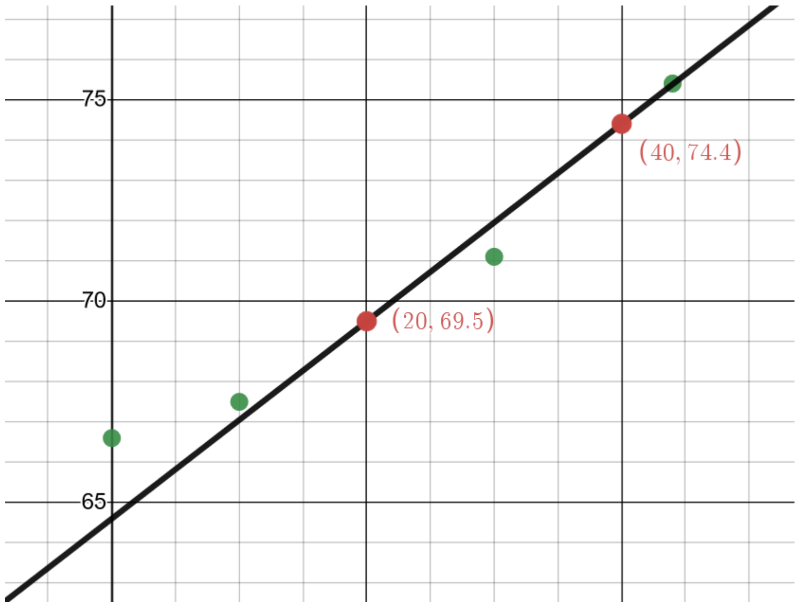
Now we have $m =$ 0.245 using the point in the point-slope equation gives:

$$y-69.5=0.245(x-20)$$

$$y=0.245x+64.6$$

$$f(x)=0.245x+64.6$$

A graph of the line fitting the data follows:



The line does a good job fitting the data and we can use it to make predictions:

Suppose we want to predict the life expectancy of a male born in 2020. This is 60 years after 1960 and corresponds to $x = 60$. Using our linear model from above: $f(x) = 0.245x + 64.6$

$f(60) = 79.3$

The linear model tells us the predicted life expectancy for a male born in 2020 is 79.3 years

<u>YOU TRY #3</u> – Build a Linear Model to Make a Prediction:

x	0	10	20	30	40	44
y	66.6	67.5	69.5	71.1	74.4	75.4

Use the data set above and the points associated with a male born in 1970 and 1990 to build a linear model. Use the model to predict the life expectancy of a male born in 2020.

Let x be: Years born after 1960

Let y be: Life expectancy in years

Looking at the data set, we use the ordered pairs:

$(10, 67.5)$ $(30, 71.1)$

To find the equation of a line, we first need the slope:

$$m = \frac{(71.1-67.5)}{(30-10)} = \frac{3.6}{20} = \frac{0.18}{1} \quad \frac{\text{Life expectancy}}{\text{year}}$$

Using slope and either point we can write the point slope equation and solve it for y:

$$y - 67.5 = 0.18(x-10)$$

$$y = 0.18x + 69.3$$

$$f(x) = 0.18x + 69.3$$

A male born in 2020 corresponds to $x = 60$, use the model to find $f(60)$. Interpret your answer IN A COMPLETE SENTENCE:

$$f(x) = 0.18x + 69.3$$

$$f(60) = 0.18(60) + 69.3 = 80.1$$

The predicted life expectancy of someone born in 2020 is 80.1 years.

Notice our prediction for the life expectancy of a male born in 2020 varied depending on what 2 points we used to construct the model.