Math 120
2.2 Models and Applications

Objectives:

1. Use linear equations to solve problems

Topic #1: Modeling and Applications with Linear Equations

When solving linear equations in context (word problems), we are often responsible for coming up with an equation for a given situation, and need to translate the words of the problem into algebra.

Here is the basic strategy:

1. Read the problem carefully (2-3x) and identify all quantities. Assign a variable to the unknown quantities.
2. Write expressions for any unknown quantities in terms of your assigned variable.
3. Write an equation in your variable that models the problem.
4. Solve the equation and answer the questions
5. Check the solution to see that answer makes sense not just in the equation.

Example #1 **– Construct and Solve the Linear Equation**

According to market research, the average cost of a new car in 2014 was $37,600. Every year after 2014, the average cost of a new car is expected to increase by about $1250 per year. Assume that this trend will continue for many years.

Let x be: years since 2014

Let C(x) be: Cost of a car ($)

a. Write a function that models the average cost C of a new car x years after 2014.

This is a linear model.

The slope of the line is the rate of change $\frac{\Delta y}{\Delta x} = \frac{\$}{\text{years}} = \frac{\$1250}{1 \text{ year}}$

y-intercept is the initial value $37,600

Using slope-intercept form: $C(x) = 1250x + 37600$

b. In how many years after 2014 will the average price of a new car cost $48,850?

This is asking when $C(x) = 48,850$

$48,850 = 1250x + 37600$

$-37600 \quad -37600$

$\frac{11250}{1250} = \frac{1250x}{1250}$

$x = 9$

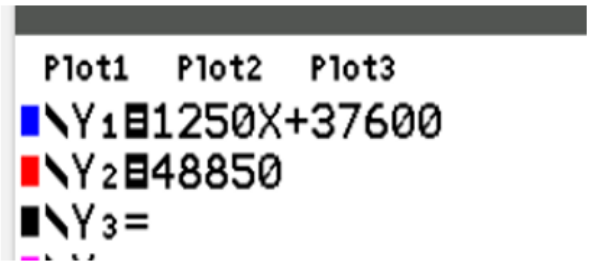
9 years after 2014

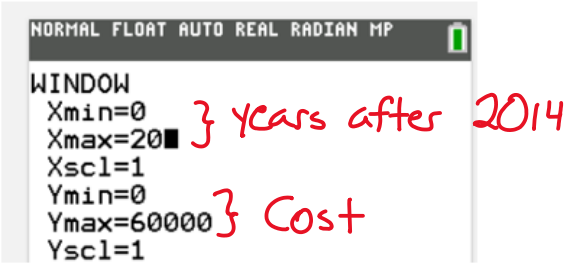
Interpret your answer in a sentence. **Does your answer make sense?** In 2023, the cost of the car is $48,850.

c. Graph the solution to part b.

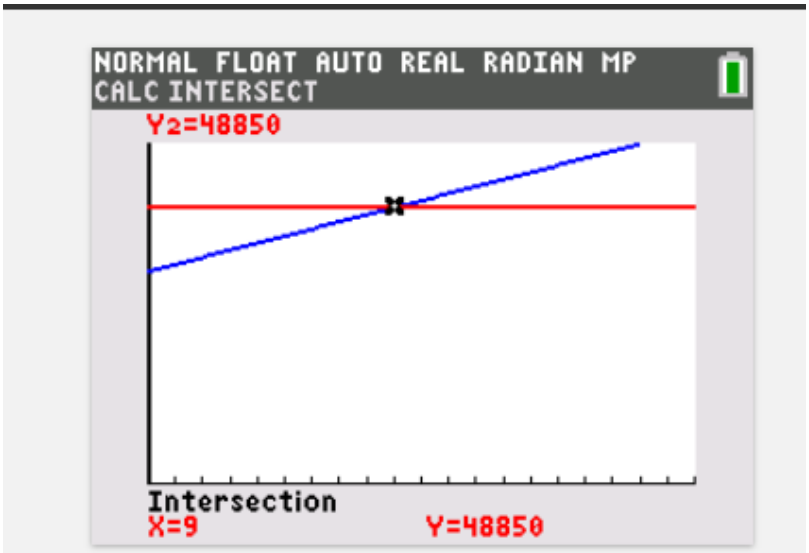
Let the left side of the equation be y_1 and the right side be y_2.

To view the graph, the window will need adjusting. Here, we know x and y must both be positive. Moreover, the y maximum needs to be at least 48850 to see the second line.





The intersection of the lines is the solution:



Example #2 **– Construct and Solve the Linear Equation**

In 2020, there were 12,200 students at college A, with a projected enrollment increase of 1000 students per year. In the same year, there were 24,200 students at college B, with a projected enrollment decrease of 500 students per year.

Let x be: years after 2020

a. Write a function that models enrollment x years after 2020 for college A.

$y = mx + b$

$m = \frac{1000 \text{ students}}{\text{year}}$ b = initial amount = 12,200

$f(x) = 1000x + 12{,}200$

b. Write a function that models enrollment x years after 2020 for college B.

$m = \frac{-500 \text{ students}}{\text{year}}$ $b = 24{,}200$

$g(x) = -500x + 24200$

c. Based on the projections, when will the colleges have the same enrollment? What will the enrollment be at that time?

$f(x) = g(x)$

$1000x + 12200 = -500x + 24200$

Y_1 $x = 8$ Y_2

In 8 years (2028) enrollment for both colleges is 20,200.

d. Use the table to confirm the solution.

What equations are entered for y_1 $1000x+12200$ and y_2 $-500x+24200$

X	Y1	Y2
1	13200	23700
2	14200	23200
3	15200	22700
4	16200	22200
5	17200	21700
6	18200	21200
7	19200	20700
8	20200	20200
9	21200	19700
10	22200	19200
11	23200	18700

The table confirms both y values are equivalent when $x =$ 8

Moreover, the table shows those y values are both 20200

Does your answer make sense?

Yes! College A increased and college B decreased.

Example #3 **– Construct and Solve the Linear Equation**

After a 35% reduction, a person purchases a guitar for $780. What was the original price of the guitar?

Let $x =$ Original price.

There is a discount of 35%, which means the person is only paying 65% of original

The equation says "65% of the original price is $780"

0.65 · x = 780

$$\frac{.65x}{.65} = \frac{780}{.65}$$

$x = \$1200$

The original price of the car was $1200.

Does your answer make sense?

Yes! The original price must be higher than the discounted price. $1200 is also a reasonable price for guitar.

Example #4 **– Construct and Solve the Linear Equation**

Including a 7% sales tax, a hotel charges $235.40 per night for a room. Find the cost of the room *before* taxes.

Let $x =$ original price

There is a tax of 7%, which means the hotel is adding 7% to 100% of the original cost.

In words, the hotel is charging 107% of the original price for the room. This is equal to the total cost of $235.40

The equation says
"107% of the original price is $235.40"?

$1.07 \cdot x = 235.40$

$$\frac{1.07x}{1.07} = \frac{235.40}{1.07}$$

$x = \$220$

Does your answer make sense?

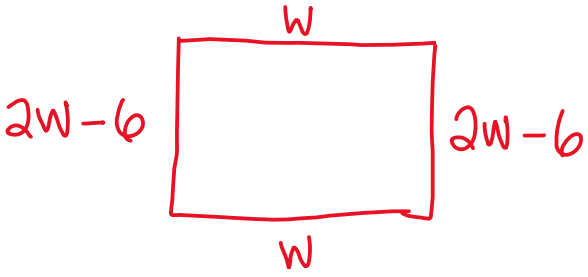
Yes! Our answer is lower than the taxed price and $220 is a reasonable cost of a hotel room.

Example #5 **– Construct and Solve the Linear Equation**

The length of a rectangular pool is 6 meters less than twice the width. If the perimeter of the pool is 126 meters, what are its dimensions? outside Add all sides

There are two unknown quantities, the length and width of the pool.

Since the relationship between the quantities is given, we have an equation:



Let length = $2w-6$

Let width = w

The known quantity is the perimeter, $P = 126$. Using the formula for perimeter, we have another equation:

P = Add all sides

$P = w + (2w-6) + w + (2w-6)$

$126 = 6w - 12$

$+12 \quad +12$

$138 = 6w$

$w = 23$

This tells us the width is $23m$

The length is 6 less than double the width $2(23)-6 = 40m$

Does your answer make sense? $P = 23m + 40m + 23m + 40m$

Yes! $P = 126m$

Example #6 **– Construct and Solve the Linear Equation**

Three friends discuss their hourly wage. The first friend makes double the second friend and the third friend makes $5 more per hour than the second friend. If the combined hourly wage of the friends is $49 per hour, find how much each friend makes per hour.

There are three unknown quantities, the wages of the 1st friend, 2nd friend, and 3rd friend.

1st = 2x

2nd = x

3rd = x+5

The known quantity is the sum of the wages, 49

This gives the equation: $2x + x + x+5 = 49$

$4x+5=49$

$-5 \quad -5$

$\frac{4x}{4} = \frac{44}{4}$ $x = 11$

This tells us the 2nd friend makes $11 per hour

The 1st friend makes 2(11) = $22 per hour

The 3rd friend makes 11+5 = $16 per hour

We can confirm the values are correct by adding up the wages 11 + 22 + 16 = 49

YOU TRY #1 **–** Does it make sense?

a. The projected enrollment, y, of a college where there were 25,000 students decreasing at a rate of 500 students per year, x, can be modeled by y=500x+25,000.

No! Decreasing so -500

b. According to market research, the average cost of a new car in 2014 was $37,600. Every year after 2014, the average cost of a new car is expected to increase by about $1250 per year. For the year 2025 the new cost of the car will be $39,000.

NO! 11 years have gone by but less than $2000 increase.

c. After a 35% reduction, a computer's price is $780, so I determined the original price, *x*, by solving $x - 0.35 = 780$

NO! This means 65% OF original

$0.65 \cdot x = 780$

d. I should check my answer with the original wording of the problem and not the equation I constructed in case I made a mistake and my equation doesn't model the problem.

Yes!

YOU TRY #2 – Construct and solve the linear equations

a. In 2014, the average price of a new car was $37,600. New car prices increased by approximately $1250 per year. Write an equation modeling this situation. If this trend continues, how many years after 2014 will the price of the car average $46,350? In which year will this occur.

Let x be years since 2014

Let $f(x)$ be price of car

$m = \frac{\$1250}{\text{year}}$ $b = 37{,}600$

$f(x) = 1250x + 37600$

$46350 = 1250x + 37600$

$-37600 \qquad -37600$

$\frac{8750}{1250} = \frac{1250x}{1250}$

$7 = x$

↑ years since 2014

In 2021, the car will cost $46,350.

b.

Let B = banana w/peel

Peel is $\frac{1}{8}$ of Total weight.

So $\frac{7}{8}B$ is weight of unpeeled banana.

unpeeled balances peeled plus $\frac{7}{8}$ oz

$$B = \frac{7}{8}B + \frac{7}{8}$$

LCD = 8

$$\frac{8}{1}\cdot B = \frac{8}{1}\cdot\frac{7}{8}B + \frac{8}{1}\cdot\frac{7}{8}$$

$$8B = 7B + 7$$

$$-7B \quad -7B$$

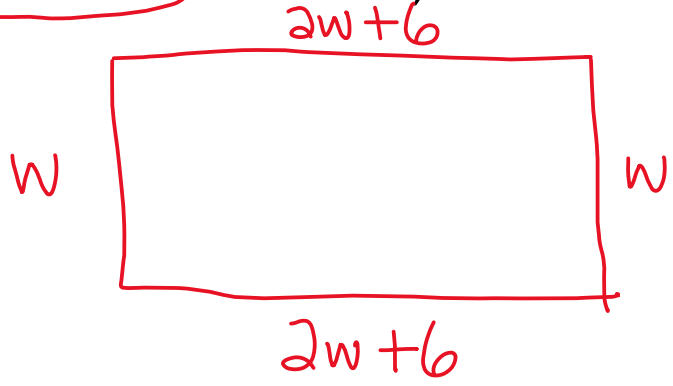
$$B = 7$$

Unpeeled banana is 7 ounces.

c. The length of the rectangular tennis court at Wimbledon is 6 feet longer than twice the width. If the court's perimeter is 228 feet, what are the court's dimensions?

Let length = $2w+6$

Let width = w



Perimeter = Add all sides

$$228 = (2w+6) + w + (2w+6) + w$$

$$228 = 6w + 12$$

$$-12 \qquad -12$$

$$\frac{216}{6} = \frac{6w}{6}$$

$$36 = w$$

$L = 2(36)+6 = 78$

The Wimbledon courts are 36 Ft by 78 Ft.