EECS3101 notes (Divide and Conquer, recurrence, master theorem, etc.)

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Structure of D and C

- Divide - Divide the problem into smaller sub problems
- Conquer - If the sub problem is small enough, return the solution directly. Else, use recursion.
- Combine - Put it all together into the overall solution.

Example :: Segment sum (code on slide 13)

We can split the array into 2 smaller sub arrays for a start. The problem will be considered small enough once each sub array has one element, i.e. $low == high$, then we return each one individually. If the value is negative, default to 0 since the empty set is also a subset of every set (segment sum of an empty array is always 0).

Recursive analysis of example

Let $T(n)$ be the worst case runtime of the algorithm. The left and right segments are both $T(\frac{n}{2})$. So it is $T(n)$ overall. When analyzing recursive functions, we always assume the base case is true.

The master theorem (formal definition on slide 19)

The function of the following form has a runtime determinable by the master theorem.

$$T(n) = aT(\frac{n}{b}) + f(n)$$

- a is the number of recursive calls.

- b is the rate at which subproblem size decreases.
- $f(n)$ is the runtime of the non recursive portion of the algorithm.

The master theorem has the following cases::

Case 1:: $f(n) = O(n^{\log_b(a)-\epsilon}), \epsilon > 0 : T(n) = \Theta(n^{\log_b(a)})$

Case 2:: $f(n) = \Theta(n^{\log_b(a)}) : T(n) = \Theta(n^{\log_b(a)} \log(n))$

Case 3:: $f(n) = \Omega(n^{\log_b(a)+\epsilon}), \epsilon > 0 : T(n) = \Theta(f(n))$

According to MT, the segment sum discussed earlier is $\Theta(nlog(n))$!

"I don't care about log. I'm not a math prof."-Larry YL Zhang 2023

"Master theorem is P O W E R"-Larry YL Zhang 2023

"log is very important"-Larry YL hang 2023

Recursion trees (slide 32)

When MT isn't appplicable, we can use the general form::

$$T(n) = \sum_{j=0}^{\log_b(n)-1} a^j f(\frac{n}{b^j}) + \Theta(n^{\log_b(a)})$$

"Try doing this proof yourself. It's good practice."-Larry YL Zhang 2023