Outline

1. Simulation
   - Monte Carlo methods
   - random numbers
2. Repeat Until
   - binary expansion
   - break statement
3. Double For Loops
   - lists of lists to represent matrices
   - searching a list of lists
4. Summary + Assignments

MCS 260 Lecture 12
Introduction to Computer Science
Jan Verschelde, 8 February 2016

Simulation

Monte Carlo methods

- In a mathematical model with uncertainties, events occur with assigned probabilities.
- Simulation consists in the repeated drawing of samples according to a probability distribution.
  We count the number of successful samples.
- The Law of Large Numbers states that the arithmetic average of the observed successes converges to the expected value or mean of the experiment, as the number of experiments increases.
- Monte Carlo methods are listed among the Top Ten Algorithms of the 20th century.

Running Simulations
repeat until: break

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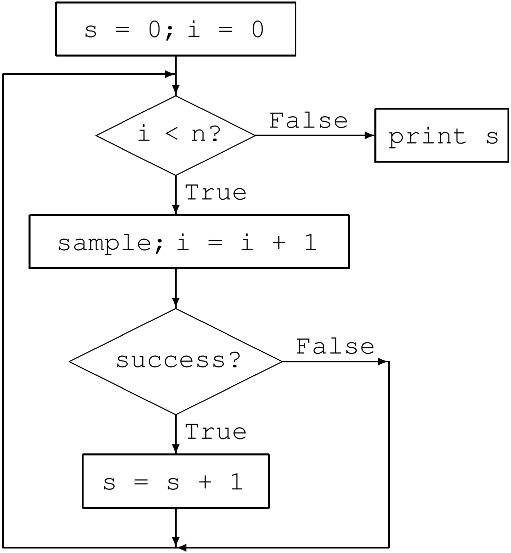
flowchart for simulations



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Random Numbers

as available in Python

Random number generators are in the module `random`.

Three things we need to know:

1. `import random` loads the module into a session.
   Afterwards, `help(random)` shows a description of the definitions and functions offered by the module.

2. `random.seed()`
   Giving a fixed number as argument results in the same sequence of random numbers.

3. `r = random.uniform(a,b)`
   `r` is a randomly generated number, drawn from a uniform distribution over the interval `[a,b)`.

using random numbers

A sample program randuse.py:

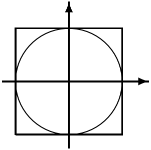
```
"""
Illustration of using random numbers.
"""
import random          # use module random
random.seed(21342342) # get same sequence
print(’uniformly distributed random numbers’)
LOWER = float(input(’give lower bound : ’))
UPPER = float(input(’give upper bound : ’))
RND = random.uniform(LOWER, UPPER) # generate a number
print(’a random number in [%.2f, %.2f] : %.15f’ \
    % (LOWER, UPPER, RND))
```

Estimating Areas and Volumes

high dimensional integrals

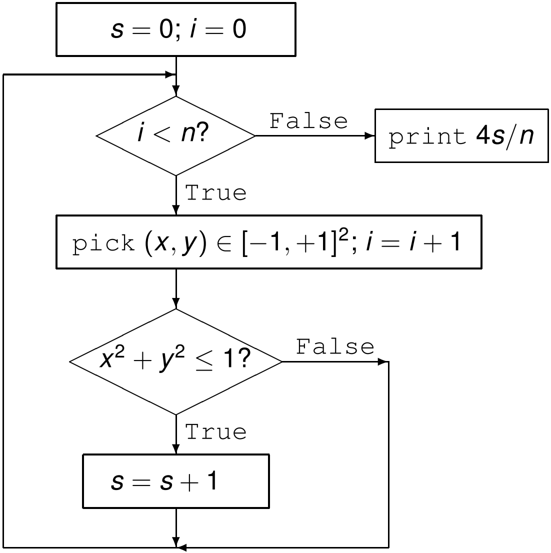
- Expected values are expressed as integrals.
  When many parameters are involved, the integration is high dimensional and only estimation is possible.

- The area of the unit disk is π.



  Generate random uniformly distributed points with coordinates $(x, y) \in [-1, +1] \times [-1, +1]$.
  We count a success when $x^2 + y^2 \leq 1$.

Flowchart for Estimating π



estimating π with the script `mc4pi.py`

```
"""
We count the number of samples (x, y)
that lie in the unit disk.
"""
from random import uniform as u

print('Monte Carlo simulation for Pi')
NBR = int(input('Give number of runs : '))
INDISK = 0
for i in range(NBR):
    (X, Y) = (u(-1, 1), u(-1, 1))
    if X**2 + Y**2 <= 1:
        INDISK = INDISK + 1
print('After %d runs : %f' % (NBR, 4*INDISK/NBR))
```

Why multiply by 4? 4 is the area of $[-1,+1]^2$.

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Converting Numbers

Converting 123, from decimal into binary format:

n	$n/2$	$n \bmod 2$	
123	61	1	$123 = 61 \times 2 + 1$
61	30	1	$61 = 30 \times 2 + 1$
30	15	0	$30 = 15 \times 2 + 0$
15	7	1	$15 = 7 \times 2 + 1$
7	3	1	$7 = 3 \times 2 + 1$
3	1	1	$3 = 1 \times 2 + 1$
1	0	1	$1 = 0 \times 2 + 1$

$$
\begin{aligned}
123 &= 1 + 2 \times 61 = 1 + 2 \times (1 + 2 \times 30) \\
&= 1 + 2 \times (1 + 2 \times (0 + 2 \times 15)) \\
&= 1 + 2 \times (1 + 2 \times (0 + 2 \times (1 + 2 \times 7))) \\
&= \ldots = 1111011 = 7B.
\end{aligned}
$$

The table shows the progression of the values of the variables in the loop, each row is one iteration.

Binary Expansions: repeat until loops

The bits of a number are the remainders of division by 2. divmod() is

an intrinsic operation:

```
>>> divmod(9,2)
(4, 1)
```

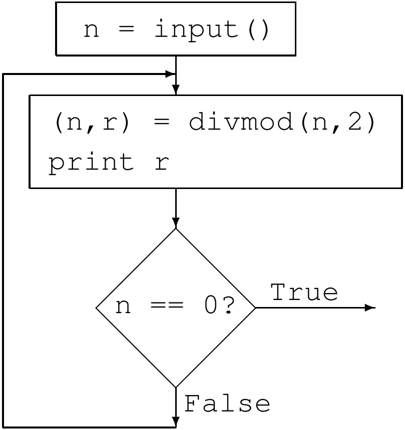
Use as `(n,r) = divmod(n,2)`
to obtain remainder `n%2` in `r` and to replace n by n/2.

Pseudocode to compute the binary expansion:

```
n = input()
repeat
    (n,r) = divmod(n,2)
    print r
until (n == 0).
```

Flowchart of Binary Expansion

picture of repeat until



a first Python solution

```
"""
This first version prints the bits
in the order as they are computed.
We use divmod(), an intrinsic operation
on numeric types.
"""
print(’computing the binary expansion’)
NBR = int(input(’Give a number : ’))
(NBR, REST) = divmod(NBR, 2)
print(REST)
while NBR > 0:
    (NBR, REST) = divmod(NBR, 2)
    print(REST)
```

avoid duplication of code

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The break Statement: repeat until as while true break

To exit a loop inside the body of a loop, the statement `break` occurs usually within an `if` statement.

```
repeat
    < body of loop >
until < condition >
```

is realized in Python as

```
while True:
    < body of loop >
    if < condition > :
        break
```

The `while True` starts an infinite loop,
terminated when `< condition >` becomes True.

binary expansions with break: a better solution

The program below avoids the duplication of code:

```
"""
Use of break for repeat until.
The script also shows how to avoid a line break
when printing the bits in the expansion.
"""
print(’computing the binary expansion’)
NBR = int(input(’Give a number : ’))
while True:
    (NBR, REST) = divmod(NBR, 2)
    print(REST, end=’ ’)  # no line break
    if NBR == 0:
        break
```

Exercise: how to print bits in correct order?

Two Loops, Two Breaks – scope of a break

As long as the number typed in by the user is nonnegative,
the loop continues.

```
"""
A break only effects one loop.
"""
print('computing the binary expansion')
while True:
    NBR = int(input('Give a number (< 0 to exit) : '))
    if NBR < 0:
        break
    while True:
        (NBR, REST) = divmod(NBR, 2)
        print(REST)
        if NBR == 0:
            break
```

A break only effects the one loop it is in.

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accessing lists by entry or by index

```
>>> L = list(range(3, 10))
>>> L
[3, 4, 5, 6, 7, 8, 9]
```

We can go over the elements of `L` by entry:

```
>>> for x in L: print(x, end=’ ’)
...
3 4 5 6 7 8 9 >>>
```

Or we can go over the elements of `L` by index:

```
>>> for k in range(len(L)): print(L[k], end=’ ’)
...
3 4 5 6 7 8 9 >>>
```

matrices as lists of lists

We view a matrix as a list of rows.

Generating a random matrix `A` of `N` rows and `M` columns:

```
from random import randint
MAT = []
for i in range(N):
    ROW = []
    for j in range(M):
        ROW.append(randint(10, 99))
    MAT.append(ROW)
```

A typical double loop:

- `i` runs over all the rows, from 0 to `N-1`, and
- `j` runs over all the columns, from 0 to `M-1`.

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searching a list of lists

Problem statement:

Input: A is a matrix of n rows and m columns,
x is some number.

Output: if $A[i][j]$ equals x, then print $[i][j]$,
else print x does not occur in A.

We develop an interactive program:

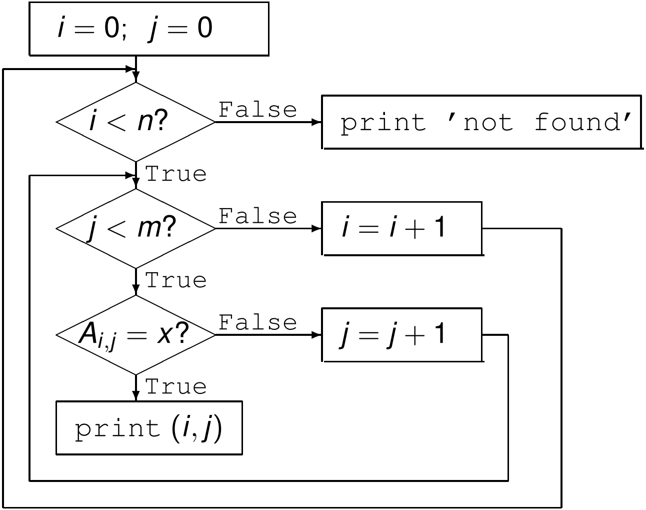
1. The user provides n and m, and
2. the computer generates an n-by-m matrix A of random integer numbers in the interval $[10, +99]$.
3. The program prompts the user for x, and
4. searches A for x and prints search result.

running the code at the command prompt

```
$ python findelem.py
give the number of rows : 3
give the number of columns : 5
random 3-by-5 matrix :
[90, 47, 93, 98, 95]
[55, 70, 51, 71, 50]
[31, 41, 23, 43, 59]
give a number : 41
found 41 at [2][1]
```

If the given number does not occur, then
`'%d does not occur in matrix'` is printed.

Flowchart



$i = 0; \quad j = 0$
$i < n?$
False
print 'not found'
True
$j < m?$
False
$i = i + 1$
True
$A_{i,j} = x?$
False
$j = j + 1$
True
print (i, j)

start of findelem.py

```
"""
Illustration of a double for loop to find
an element in a two dimensional matrix.
"""
# first we make a random matrix
from random import randint
ROWS = int(input(’give the number of rows : ’))
COLS = int(input(’give the number of columns : ’))
MAT = []
for i in range(ROWS):
    MAT.append([randint(10, 99) for _ in range(COLS)])
print(’random %d-by-%d matrix :’ % (ROWS, COLS))
for row in MAT:
    print(row)
```

the double for loop

Search an *n*-by-*m* matrix *A* for *x*:

```
# then we ask for a number and search
NBR = int(input('give a number : '))
FOUND = False
for i in range(0, ROWS):
    for j in range(0, COLS):
        if MAT[i][j] == NBR:
            FOUND = True
            (ROW, COL) = (i, j)
            break
    if FOUND:
        break
```

reporting the result

```
# we report the result
if FOUND:
    print(’found %d at [%d][%d]’ % (NBR, ROW, COL))
else:
    print(’%d does not occur in the matrix’ % NBR)
```

Assignments

1. Use a stack to store the bits in the binary expansion to print the bits *after* the loop in the correct order.
2. Given a list of numbers between 0 and 100, define the algorithm to assign a letter grade to each number: $\geq$ 90: A, $\in [80, 89]$: B, $\in [70, 79]$: C, etc.
   Report at the end how many As, Bs, Cs, etc.
   Write the algorithm in words and draw a flowchart.
3. Implement exercise 2 in Python.
4. Write a Python program that generates *n* numbers uniformly distributed in $[0, 1]$ and counts how many numbers are < 0.5.
5. Use turtle graphics to visualize the Monte Carlo method to estimate π. Represent the unit circle by a circle of radius equal to half of the width of the turtle window. Mark samples inside the disk by green circles of radius equal to 2 pixels, centered at the sample point. Use red circles for the points outside the disk.

Summary

We covered more of

- section 2.6 of *Python Programming in Context*,
- section 5.4 in *Computer Science, an overview*.