CSC 343 Assignment 3

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1 Dependencies, Decompositions, Normal Forms

1. (a) **Step 1:** Split the RHSs to get our initial set of FDs, $S1$:
   (a) $M \to I$
   (b) $M \to J$
   (c) $M \to L$
   (d) $J \to L$
   (e) $J \to I$
   (f) $JN \to K$
   (g) $JN \to M$
   (h) $M \to J$
   (i) $KLN \to M$
   (j) $K \to I$
   (k) $K \to J$
   (l) $K \to L$
   (m) $IJ \to K$

   Step 2: For each FD, try to reduce the LHS. Since FDs with a singleton LHS cannot be reduced, only the FD with two or more LHS attributes need be considered:

   (f) $J^+ = IJ\underline{K}L$, so $JN \to K$ can be reduced to $J \to K$.
   (g) $J^+ = IJKL$ and $N^+ = N$, so $JN \to M$ cannot be reduced.
   (i) $K^+ = IJKL$, $L^+ = L$, and $N^+ = N$, so $KLN \to M$ cannot be reduced to an FD with a singleton LHS.
   $KN^+ = IJKL\underline{M}N$, so $KLN \to M$ can be reduced to $KN \to M$.
   (m) $J^+ = IJ\underline{K}L$, so $IJ \to K$ can be reduced to $J \to K$.

   The new set of DFs, denoted $S2$, is

   (a) $M \to I$
   (b) $M \to J$
   (c) $M \to L$
   (d) $J \to L$
   (e) $J \to I$
   (f) $J \to K$
   (g) $JN \to M$
   (h) $M \to J$
   (i) $KN \to M$
   (j) $K \to I$

(k) $K \to J$
(l) $K \to L$
(m) $J \to K$

Step 3: Try to eliminate each FD:

(a) $M^+_{S2-(a)} = \underline{I}JKLM$. We can remove this FD.
(b) $M^+_{S2-(a),(b)} = \underline{J}LM$. We can remove this FD.
(c) $M^+_{S2-(a),(b),(c)} = IJK\underline{L}M$. We can remove this FD.
(d) $J^+_{S2-(a),(b),(c),(d)} = IJK\underline{L}$. We can remove this FD.
(e) $J^+_{S2-(a),(b),(c),(d),(e)} = \underline{I}JKL$. We can remove this FD.
(f) $J^+_{S2-(a),(b),(c),(d),(e),(f)} = \underline{K}$. We can remove this FD.
(g) $JN^+_{S2-(a),(b),(c),(d),(e),(f),(g)} = IJKL\underline{M}N$. We can remove this FD.
(h) $M^+_{S2-(a),(b),(c),(d),(e),(f),(g),(h)} = M$. We need this FD.
(i) $KN^+_{S2-(a),(b),(c),(d),(e),(f),(g),(i)} = IJKLN$. We need this FD.
(j) $K^+_{S2-(a),(b),(c),(d),(e),(f),(g),(j)} = JKL$. We need this FD.
(k) $K^+_{S2-(a),(b),(c),(d),(e),(f),(g),(k)} = IKL$. We need this FD.
(l) $K^+_{S2-(a),(b),(c),(d),(e),(f),(g),(l)} = IJK$. We need this FD.
(m) $J^+_{S2-(a),(b),(c),(d),(e),(f),(g),(m)} = J$. We need this FD.

The minimal basis of R is:

$$\boxed{\begin{array}{ll} \text{(a)} & J \to K \\ \text{(b)} & K \to I \\ \text{(c)} & K \to J \\ \text{(d)} & K \to L \\ \text{(e)} & KN \to M \\ \text{(f)} & M \to J \end{array}} \tag{1}$$

(b) Grouping the RHSs of FDs in (1) with common LHSs, the following is obtained:

$$\begin{array}{ll} \text{(a)} & J \to K \\ \text{(b)} & K \to IJL \\ \text{(c)} & KN \to M \\ \text{(d)} & M \to J \end{array} \tag{2}$$

Since O and P do not appear in any of the FDs in (2), O and P must be in every key of R.
Since N does not appear on the RHS of any of the FDs in (2), N must also be in

every key of R.
Since I and L only appear on the RHS of the FDs in (2), I and L must not be in any key of R.
The remaining attributes J, K, and M do not fall into any of the three categories above, so every combination of J, K, and M must be considered. For each combination, N, O, and P, must be added in, since they are in every key:

- $JNOP^+ = IJKLMNOP$. So $JNOP$ is a key.
- $KNOP^+ = IJKLMNOP$. So $KNOP$ is a key.
- $MNOP^+ = IJKLMNOP$. So $MNOP$ is a key.
- All other possibilities include $JNOP$, $KNOP$, or $MNOP$.

$\boxed{JNOP, KNOP, \text{and } MNOP}$ are all the keys of R.

(c) Using (2), a set of relations where each relation contains the union of the LHS and RHS of an FD in (2) is as follows:

- $R1(J, K)$
- $R2(I, J, K, L)$
- $R3(K, M, N)$
- $R4(J, M)$

Since the attributes JK occur within $R2$, the relation $R1$ can be discarded.
Neither $R2$, $R3$, nor $R4$ contain a superkey for R, so a relation that includes a key, such as $R5(J, N, O, P)$, must be added. The resulting schema in 3rd Normal Form is:

$$\boxed{R2(I, J, K, L),\ R3(K, M, N),\ R4(J, M),\ R5(J, N, O, P)} \tag{3}$$

(d) Each relation in (3) was formed from an FD, so the LHS of those FDs must be superkeys for their corresponding relations. However, other FDs may violate BCNF and therefore allow redundancy.
Projecting the provided set of FDs S_P onto $R3(K, M, N)$, one of the FDs obtained is $M \to K$. The closure of M with respect to $R3$ is $M^+ = KM$. M^+ does not include N, so M is not a superkey of $R3$. Since M is on the LHS of an FD of $R3$ but is not a superkey, BCNF is violated, which means the schema in (3) $\boxed{\text{allows redundancy}}$.

2. (a) If the LHS of an FD in S_T is not a superkey of T, then BCNF is violated by that FD. To determine whether the LHS of an FD is a superkey of T, one can check the attributes in the closure of the LHS. If the closure of the LHS includes all the attributes of T, then the LHS is a superkey; otherwise, the LHS is not a superkey and BCNF is violated. The closures of each FD's LHS are as follows:

- $C^+ = CDEFGHIJ$

- $DEI^+ = DEFI$
- $F^+ = DF$
- $EH^+ = CDEFGHIJ$
- $J^+ = DFGIJ$

Therefore, $\boxed{DEI \rightarrow F,\ F \rightarrow D, \text{ and } J \rightarrow FGI}$ violate BCNF, since their LHSs are not superkeys of T.

(b) **Step 1:** $DEI \rightarrow F$ violates BCNF, so the decomposition step may be applied to it:

- $DEI^+ = DEFI$

Replacing T with $T1 = DEI^+$ and $T2 = T - (DEI^+ - DEI)$, the following is obtained:

- $T1 = DEFI$
- $T2 = CDEGHIJ$

Note: When projecting previous FDs onto a new relation, the projection may be stopped if a single resultant FD violates BCNF. BCNF decomposition can be continued via the violating FD.

Step 2: Projecting the provided set of FDs S_T onto $T1$, one of the FDs obtained is $F \rightarrow D$. The closure of F with respect to $T1$ is $F^+ = DF$. F^+ does not include E or I, so F is not a superkey of $T1$. Since F is on the LHS of an FD of $T1$ but is not a superkey, BCNF is violated, which means $T1$ must be further decomposed.
Replacing $T1$ with $T3 = F^+$ and $T4 = T1 - (F^+ - F)$, the following is obtained:

- $T3 = DF$
- $T4 = EFI$

Step 3: Projecting the provided set of FDs S_T onto $T2$, one of the FDs obtained is $J \rightarrow G$. The closure of J with respect to $T2$ is $J^+ = DGIJ$. J^+ does not include C, E, or H, so J is not a superkey of $T2$. Since J is on the LHS of an FD of $T2$ but is not a superkey, BCNF is violated, which means $T2$ must be further decomposed.
Replacing $T2$ with $T5 = J^+$ and $T6 = T2 - (J^+ - J)$, the following is obtained:

- $T5 = DGIJ$
- $T6 = CEHJ$

The collection of relations

- $T3 = DF$
- $T4 = EFI$
- $T5 = DGIJ$
- $T6 = CEHJ$

satisfies BCNF. This can be shown by projecting the provided set of FDs S_T onto the final set of relations above.
Projecting S_T onto $T3 = DF$:

D	**F**	**Closure**	**FDs**
✓		$D^+ = D$	Nothing
	✓	$F^+ = DF$	$F \rightarrow D$

Projecting S_T onto $T4 = EFI$:

E	**F**	**I**	**Closure**	**FDs**
✓			$E^+ = E$	Nothing
	✓		$F^+ = DF$	Nothing
		✓	$I^+ = I$	Nothing
✓	✓		$EF^+ = DEF$	Nothing
✓		✓	$EI^+ = EI$	Nothing
	✓	✓	$FI^+ = DFI$	Nothing

Projecting S_T onto $T5 = DGIJ$:

D	**G**	**I**	**J**	**Closure**	**FDs**
✓				$D^+ = D$	Nothing
	✓			$G^+ = G$	Nothing
		✓		$I^+ = I$	Nothing
			✓	$J^+ = DFGIJ$	$J \rightarrow DGI$
✓	✓			$DG^+ = DG$	Nothing
✓		✓		$DI^+ = DI$	Nothing
✓			✓	Since J is a key, supersets of J can only yield weaker FDs	
	✓	✓		$GI^+ = GI$	Nothing
	✓		✓	Since J is a key, supersets of J can only yield weaker FDs	
		✓	✓	Since J is a key, supersets of J can only yield weaker FDs	
✓	✓	✓		$DGI^+ = DGI$	Nothing
✓	✓		✓	Since J is a key, supersets of J can only yield weaker FDs	
✓		✓	✓	Since J is a key, supersets of J can only yield weaker FDs	
	✓	✓	✓	Since J is a key, supersets of J can only yield weaker FDs	

Projecting S_T onto $T6 = CEHJ$:

C	E	H	J	Closure	FDs
✓				$C^+ = CDEFGHIJ$	$C \to EHJ$
	✓			$E^+ = E$	Nothing
		✓		$H^+ = H$	Nothing
			✓	$J^+ = DFGIJ$	Nothing
✓	✓			Since C is a key, supersets of C can only yield weaker FDs	
✓		✓		Since C is a key, supersets of C can only yield weaker FDs	
✓			✓	Since C is a key, supersets of C can only yield weaker FDs	
	✓	✓		$EH^+ = CDEFGHIJ$	$EH \to CJ$
	✓		✓	$EJ^+ = DEFGIJ$	Nothing
		✓	✓	$HJ^+ = DFGHIJ$	Nothing
✓	✓	✓		Since C is a key, supersets of C can only yield weaker FDs	
✓	✓		✓	Since C is a key, supersets of C can only yield weaker FDs	
✓		✓	✓	Since C is a key, supersets of C can only yield weaker FDs	
	✓	✓	✓	Since EH is a key, supersets of EH can only yield weaker FDs	

The final set of relations and their corresponding FDs are shown below:

Relations	FDs
$T6 = CEHJ$	$C \to EHJ$, $EH \to CJ$
$T3 = DF$	$F \to D$
$T5 = DGIJ$	$J \to DGI$
$T4 = EFI$	Nothing

It can be shown that all the relations and FDs above satisfy BCNF:

- The closure of C with respect to $T6$ is $C^+ = CEHJ$. Therefore, C is a superkey of $T6$ and thus satisfies BCNF.
- The closure of EH with respect to $T6$ is $EH^+ = CHEJ$. Therefore, EH is a superkey of $T6$ and thus satisfies BCNF.
- The closure of F with respect to $T3$ is $F^+ = DF$. Therefore, F is a superkey of $T3$ and thus satisfies BCNF.
- The closure of J with respect to $T5$ is $J^+ = DGIJ$. Therefore, J is a superkey of $T5$ and thus satisfies BCNF.
- $T4$ has no FDs, which satisfies BCNF.