

## Objectives

The primary object of this lab was to do Fourier transform to several different signals by using MATLAB.

## Methodology

### 1.

(a) Plotting  $x(t) = \cos(w_0 t)$  on  $t = [-5, 5]$  for  $w_0 = 2\pi/10$ , with step size  $1/50$  (if the step size was small, cos wave would not be smooth).

(b) Then plotted Fourier transform of cosine  $X(w)$ . Because  $X(w)$  is complex, I plotted two graph: Magnitude and Phase of Fourier transform. To have a clear graph of Magnitude and Phase, I used `fftshift()` to shift the impulses of Magnitude (on the right and left) to the centre.

Matlab code and graph are shown in figure below.

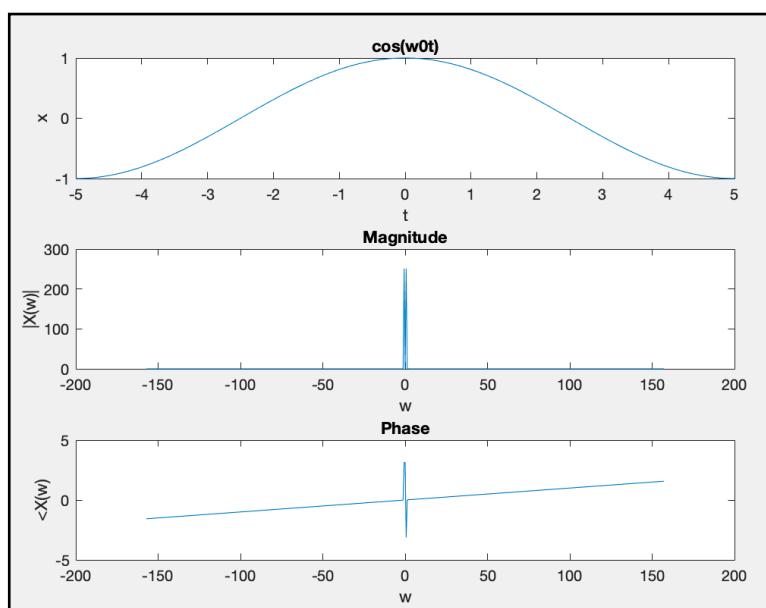


fig. 1

```

2 - clear all
3 - t = -5:1/50:5;
4 - w0 = 2*pi/10;
5 - x = cos(w0*t);
6 - subplot(311)
7 - plot(t,x)
8 - title('cos(w0*t)')
9 - xlabel('t')
10 - ylabel('x')
11 -
12 - % (b)
13 - y = fftshift(fft(x));
14 - f = (t*w0/(1/50));
15 - subplot(312)
16 - plot(f,abs(y))
17 - title('Magnitude')
18 - xlabel('w')
19 - ylabel('|X(w)|')
20 - subplot(313)
21 - plot(f,angle(y))
22 - title('Phase')
23 - xlabel('w')
24 - ylabel('<X(w)')

```

fig. 2

(c) The theoretical Fourier transform of cosine  $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$  is , it isn't a complex number. However, in Matlab, after doing `fft()`, it became a complex number. Which shown in figure 3.

fig. 3

y					
1x501 complex double					
	1	2	3	4	5
1	3.8810e-...	3.4930e-...	9.7039e-...	1.9023e-...	3.1452e-...

(d) when I reduced the step size to 1/100, the impulse of magnitude and phase became narrower. Which shown together with Matlab code in figure 4.

```

26 %% (d)
27 clear all
28 t = -5:1/100:5; % step size reduced to 1/100
29 w0 = 2*pi/10;
30 x = cos(w0*t);
31 subplot(311)
32 plot(t,x)
33 title('cos(w0t)')
34 xlabel('t')
35 ylabel('x')
36
37 y = fftshift(fft(x));
38 f = (t*w0/(1/100));
39 subplot(312)
40 plot(f,abs(y))
41 title('Magnitude')
42 xlabel('w')
43 ylabel('|X(w)|')
44 subplot(313)
45 plot(f,angle(y))
46 title('Phase')
47 xlabel('w')
48 ylabel('<X(w)')

```

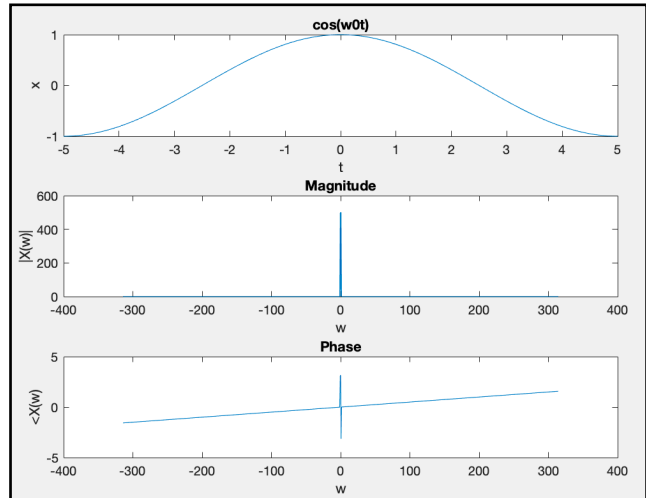


fig. 4

Because after I reduced the step size, sampling point 't' was increased, and 'f' was also increased. It needed to do Fourier transform with more sampling points, so the graph looked narrower.

## 2.

(a)  $x(t) = \sin(\omega_0 t)$  (similar with cosine)

```

1 %% Question 2 (a) 1.sine
2 clear all
3 t1 = 0:1/50:5;
4 w0 = 2*pi/10;
5 x1 = sin(w0*t1);
6 subplot(311)
7 plot(t1,x1)
8 title('sin(w0t)')
9 xlabel('t1')
10 ylabel('x1')
11
12 y1 = fftshift(fft(x1));
13 f1 = (t1*w0/(1/50));
14 subplot(312)
15 plot(f1,abs(y1))
16 title('Magnitude')
17 xlabel('w')
18 ylabel('|X(w)|')
19 subplot(313)
20 plot(f1,angle(y1))
21 title('Phase')
22 xlabel('w')
23 ylabel('<X(w)')

```

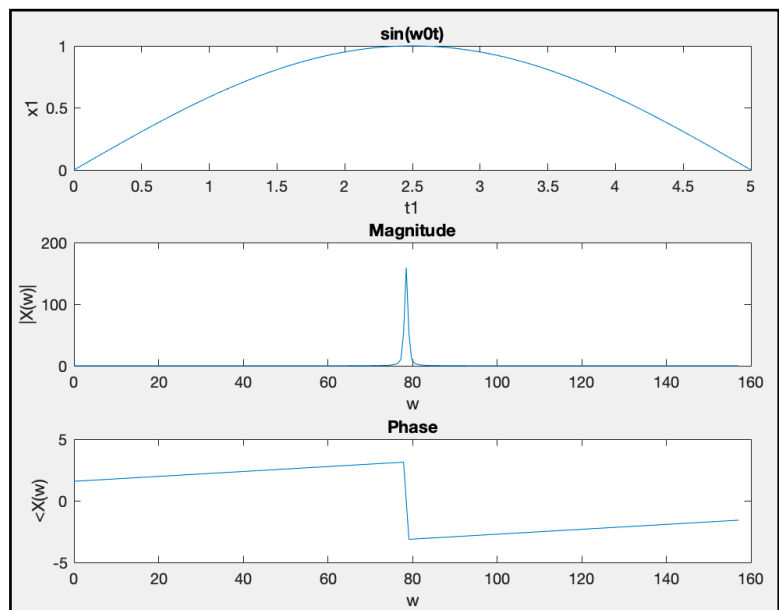


fig. 5

However, the Fourier transform of sine wave had only one impulse over one period, different with cosine which had two impulse.

$$x(t) = e^{j\omega_0 t}$$

```

25 %% 2.exponential
26 clear all
27 t2 = 0:1/50:10;
28 w0 = 2*pi/10;
29 x2 = exp(j*w0*t2);
30 subplot(411)
31 plot(t2,real(x2))
32 title('Real')
33 xlabel('t2')
34 ylabel('x2')
35 subplot(412)
36 plot(t2,imag(x2))
37 title('Imaginary')
38 xlabel('t2')
39 ylabel('x2')
40
41 y2 = fftshift(fft(x2));
42 f2 = (t2*w0/(1/50));
43 subplot(413)
44 plot(f2,abs(y2))
45 title('Magnitude')
46 xlabel('w')
47 ylabel('|X(w)|')
48 subplot(414)
49 plot(f2,angle(y2))
50 title('Phase')
51 xlabel('w')
52 ylabel('<X(w)')

```

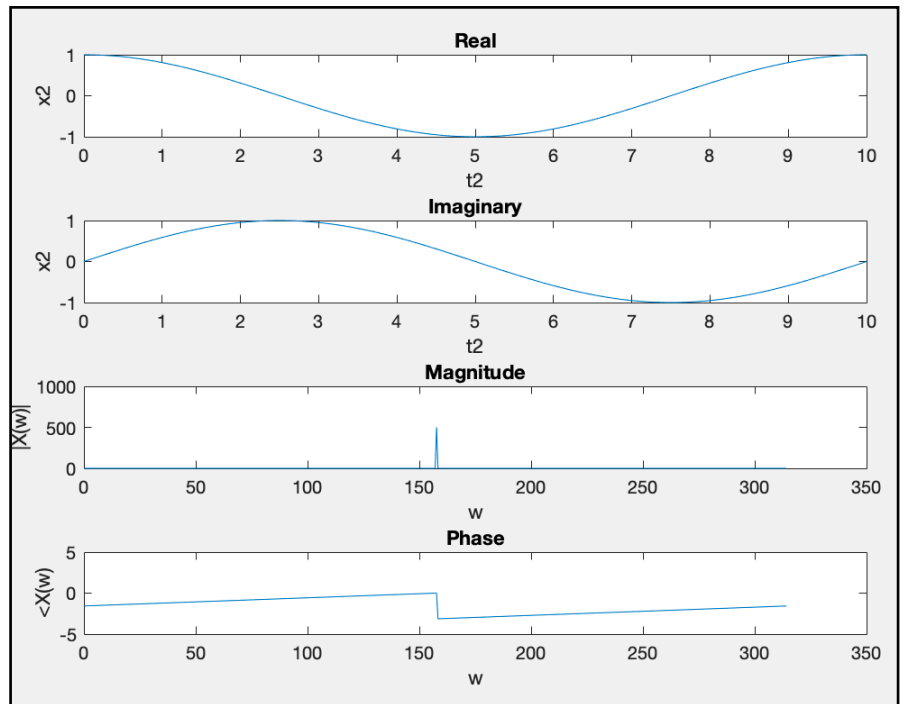


fig. 6

Exponential signal was a complex number, so I plot real and imaginary part separately.

Shown in figure 6.

Fourier transform of  $e^{j\omega_0 t}$  is  $2\pi\delta(\omega - \omega_0)$ , there was an impulse on the plot of magnitude, the position is more right than the Fourier transform of sine. Because the value

of  $2\pi\delta(\omega - \omega_0)$  is larger than  $\frac{\pi}{f}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$

$$x(t) = \delta(t)$$

```

54 %% 3.delta
55 clear all
56 t3 = -1:0.1:1;
57 x3 = dirac(t3);
58 idx = x3 == Inf; % find Inf
59 x3(idx) = 1; % set Inf to finite value
60 subplot(311)
61 stem(t3,x3)
62
63 y3 = fft(x3);
64 f3 = (t3/0.1);
65 subplot(312)
66 plot(f3,abs(y3))
67 title('Magnitude')
68 xlabel('w')
69 ylabel('|X(w)|')
70 subplot(313)
71 plot(f3,angle(y3))
72 title('Phase')
73 xlabel('w')
74 ylabel('<X(w)')

```

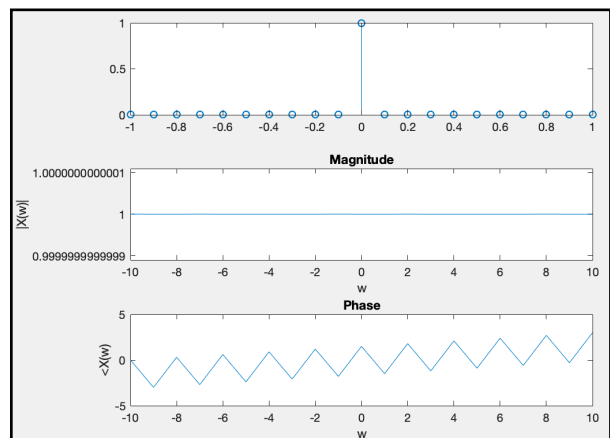


fig. 7

Figure 7 showed the code and plot of  $\delta(t)$  Fourier transform. The Fourier transform of  $\delta(t)$  is 1, which match the magnitude plot.

$$x(t) = u(t)$$

```

77 - clear all
78 - t4 = -1:0.0001:1;
79 - x4 = heaviside(t4);
80 - subplot(311)
81 - plot(t4,x4)
82 - title('u(t)')
83 - xlabel('t4')
84 - ylabel('x4')
85
86 - y4 = fftshift(fft(x4));
87 - f4 = (t4/0.0001);
88 - subplot(312)
89 - plot(f4,abs(y4))
90 - title('Magnitude')
91 - xlabel('w')
92 - ylabel('|X(w)|')
93 - subplot(313)
94 - plot(f4,angle(y4))
95 - title('Phase')
96 - xlabel('w')
97 - ylabel('<X(w)')

```

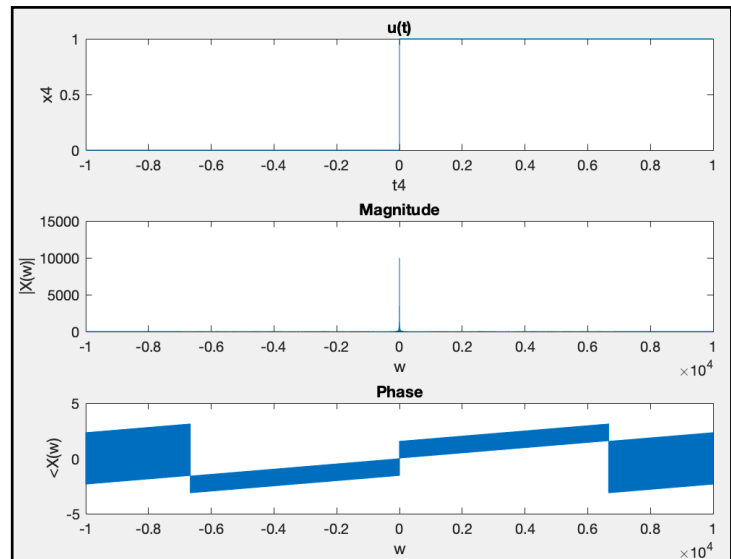


fig. 8

For unit step function, I set step size into a small number (0.0001) was because the unit step graph would be more accurate if there were enough points.

$$x(t) = \delta(t - t_0)$$

```

99 % 5.delta(t-t0)
100 - clear all
101 - t5 = -1:0.1:1;
102 - x5 = dirac(t5-0.6);
103 - idx = x5 == Inf; % find Inf
104 - x5(idx) = 1; % set Inf to finite value
105 - subplot(311)
106 - stem(t5,x5)
107 - title('delta(t-0.6)')
108 - xlabel('t5')
109 - ylabel('x5')
110
111 - y5 = fft(x5);
112 - f5 = (t5/0.1);
113 - subplot(312)
114 - plot(f5,abs(y5))
115 - title('Magnitude')
116 - xlabel('w')
117 - ylabel('|X(w)|')
118 - subplot(313)
119 - plot(f5,angle(y5))
120 - title('Phase')

```

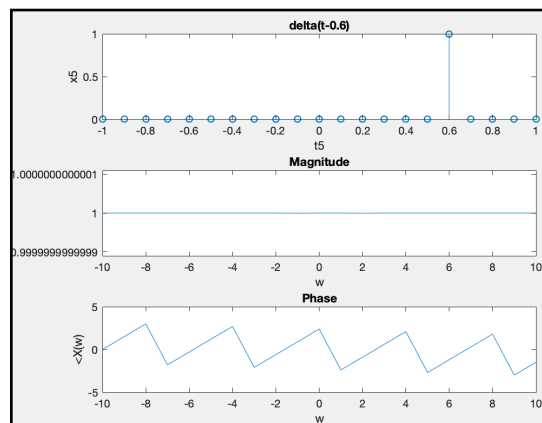


fig. 9

Shift  $\delta(t)$  to  $\delta(t-0.6)$ , the impulse shift from 0 to 0.6. Which shown in figure 9.

Theoretical Fourier transform of  $\delta(t-t_0)$  is  $e^{-j\omega t_0}$ , however, the magnitude graph showed the value is constant 1.

$$x(t) = e^{-at} * u(t), \operatorname{Re}\{a\} > 0$$

```

124 %% x(t) = e^-at*u(t)
125 - clear all
126 - t6 = -5:0.0001:5;
127 - ut = heaviside(t6);
128 - x6 = exp(-t6).*ut;
129 - subplot(311)
130 - plot(t6,x6)
131 - title('exp(-t6).* u(t)')
132 - xlabel('t6')
133 - ylabel('x6')
134
135 - y6 = fftshift(fft(x6));
136 - f6 = t6/0.0001;
137 - subplot(312)
138 - plot(f6,abs(y6))
139 - title('Magnitude')
140 - xlabel('w')
141 - ylabel('|X(w)|')
142 - subplot(313)
143 - plot(f6,angle(y6))
144 - title('Phase')

```

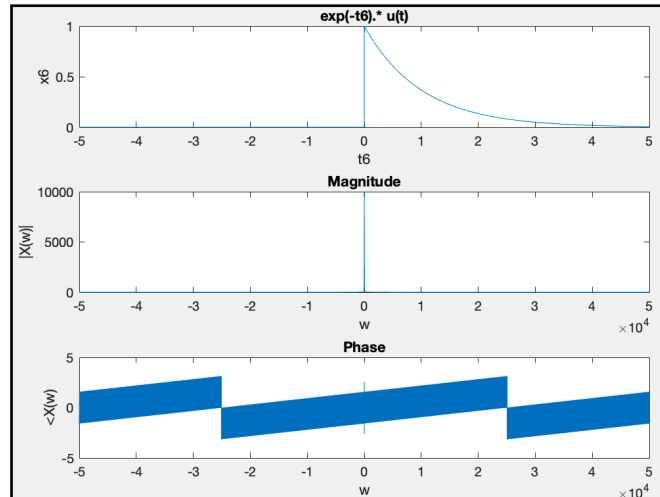


fig. 10

Set  $a = 1$ , when  $t < 0$ ,  $u(t) = 0$ ,  $x(t) = 0$ ;  $t > 0$ ,  $u(t) = 1$ ,  $x(t) = e^{-t}$ . The graph shown in figure 10. Theoretical Fourier transform of  $e^{-at}$  is  $1/(a+j\omega)$ , however, according to the magnitude graph I plotted, it only had value when  $\omega = 0$ .

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T}{2} \end{cases}$$

I tried to use for loop and if statement to plot  $x(t)$ , however it didn't work..

Code shown in figure 11.

```

148 %% x(t) = 1, |t| < T1; 0, T1 < |t| < T/2
149 - clear all
150 - for t7 = -10:10
151 -     T = 4;
152 -     T1 = 1;
153 -     if abs(t7) < T1
154 -         x7 = 1;
155 -     else
156 -         T1 < abs(t7) && abs(t7) < T/2
157 -         x7 = 0;
158 -     end
159 - end
160 - plot(t7, x7*ones(size(t7)))

```

fig. 11

Instead of using that method, I used the other way to plot square wave with 30% duty cycle. Although it was little bit different with the required square wave. Shown in figure 12.

```

162 - t7 = -10:0.01:10;
163 - x7 = square(t7,30); % 30% duty cycle
164 - subplot(311)
165 - plot(t7,x7,'linewidth',2)
166 - title('square wave')
167 - ylabel('x7')
168 - xlabel('t7')
169 -
170 - y7 = fftshift(fft(x7));
171 - f7 = t7/0.01;
172 - subplot(312)
173 - plot(f7,abs(y7))
174 - title('Magnitude')
175 - xlabel('w')
176 - ylabel('|X(w)|')
177 - subplot(313)
178 - plot(f7,angle(y7))
179 - title('Phase')
180 - xlabel('w')
181 - ylabel('<X(w)')

```

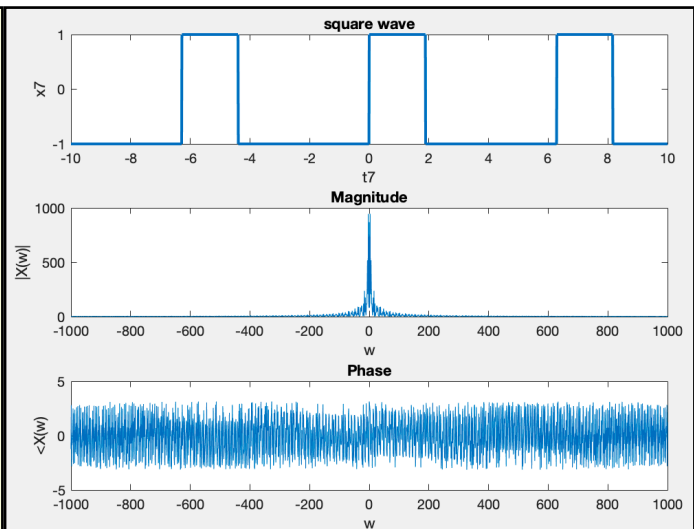


fig. 12