Assignment 1. ECEN315

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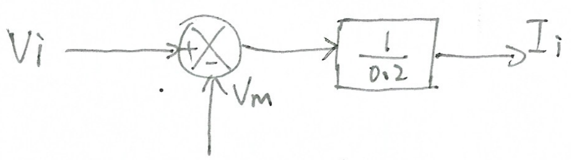
Q1: In s-domain:

We have ① $V_i - V_m = 0.2 I_i$, ② $I_m = I_i - I_o$, ③ $V_m = 4s I_m$,

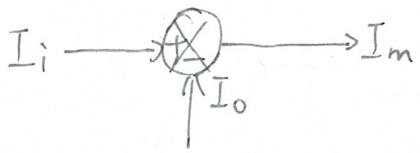
④ $V_m - V_o = 0.1 I_o$, ⑤ $V_o = \frac{1}{s} I_o$

For ① $V_i - V_m = 0.2 I_i \Rightarrow (V_i - V_m)\frac{1}{0.2} = I_i$

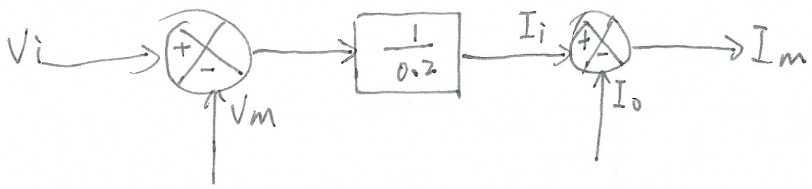
drawing block diagram:



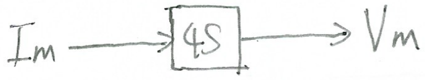
For ② $I_m = I_i - I_o$



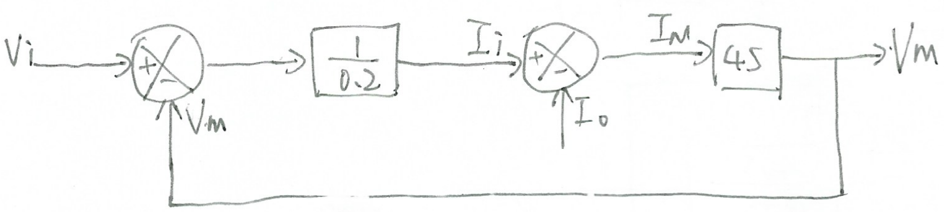
⇒ Combine block diagram ① & ②:



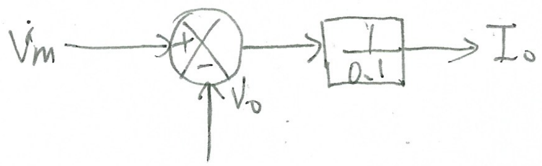
For ③ $V_m = 4s I_m$



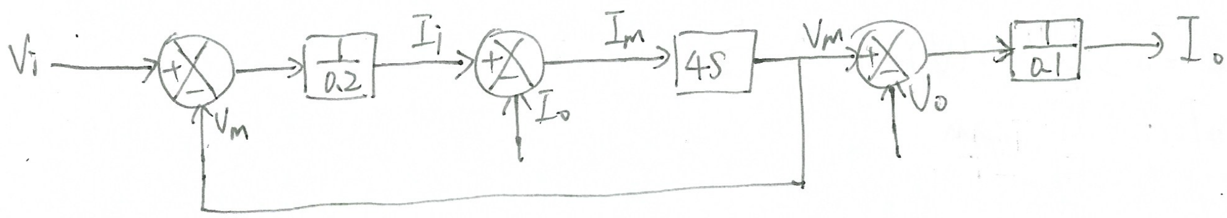
⇒ redrawing block diagram:



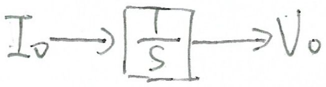
For ④ $V_m - V_o = 0.1 I_o \Rightarrow (V_m - V_o)\frac{1}{0.1} = I_o$



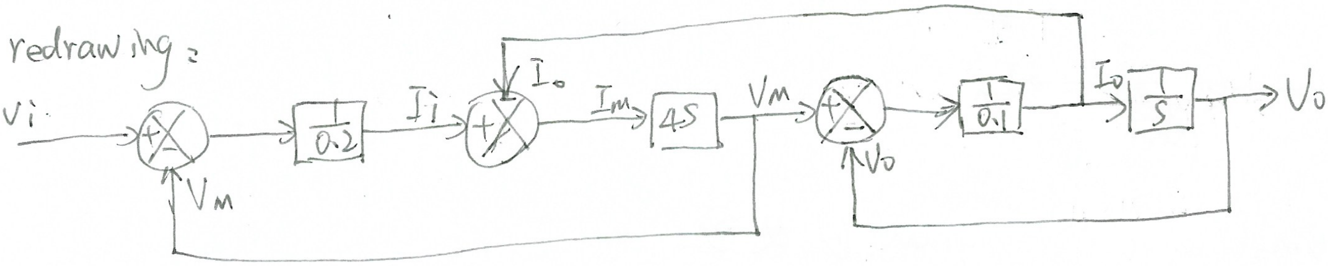
redrawing diagram:



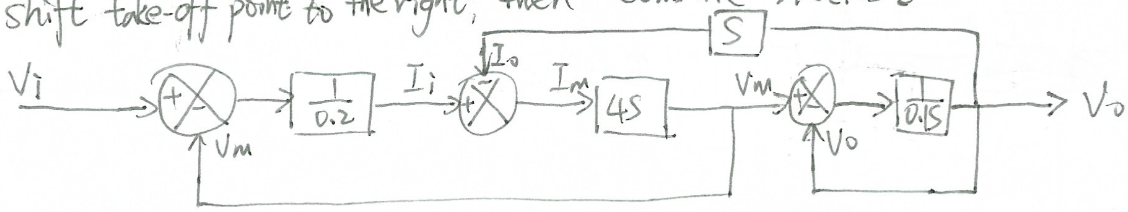
For ⑤ $V_o = \frac{1}{s} I_o$



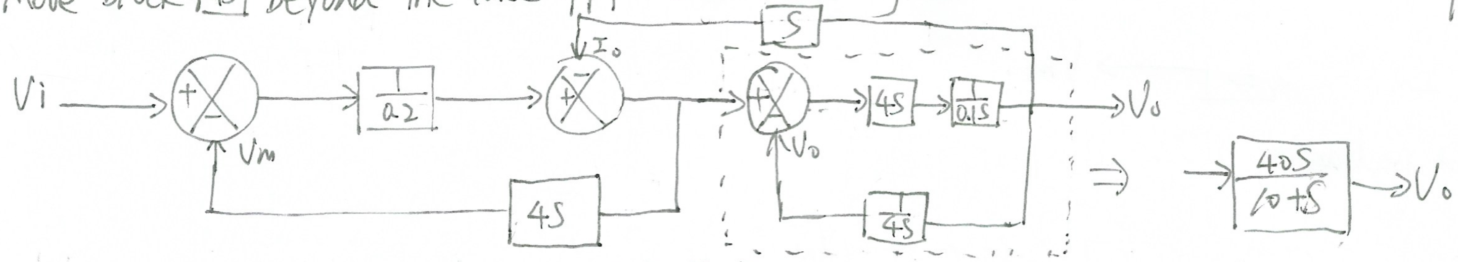
redrawing:

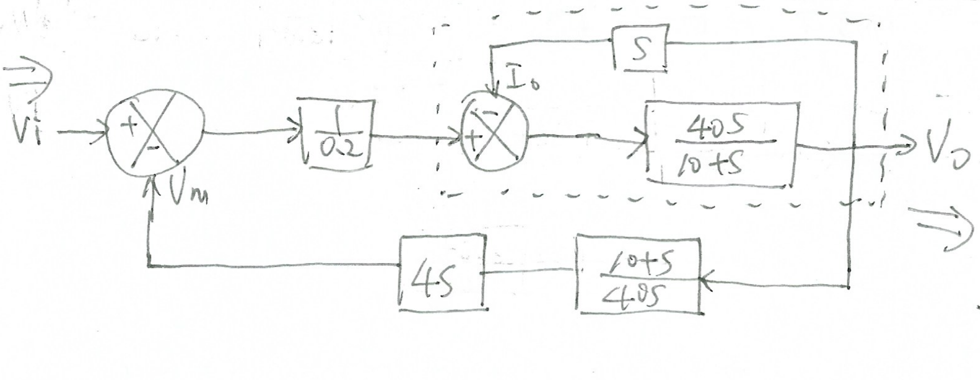


shift take-off point to the right, then combine blocks:

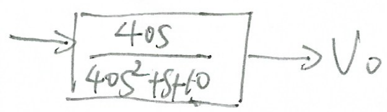


Move block $\boxed{4s}$ beyond the take-off point and summing junction, then we can reduce loop:

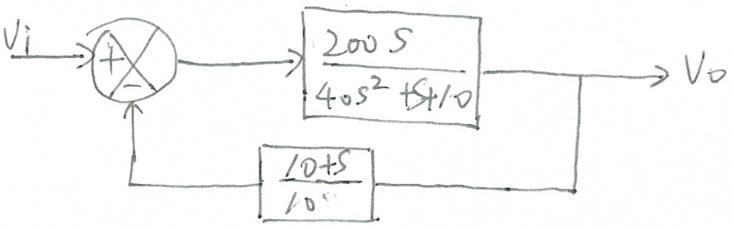




$\frac{\text{Forward}}{1-\text{Loop}}$



Combine the rest of blocks:



$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{\text{Forward}}{1-\text{loop}} = \frac{\frac{200s}{40s^2+s+10}}{1+\frac{200s}{40s^2+s+10}\cdot\frac{10+s}{10}} = \frac{\frac{200s}{40s^2+s+10}}{1+\frac{20s^2+200s}{40s^2+s+10}} = \frac{20s}{6s^2+20.1s+1}$$

A unit step input $V_i(s) = \frac{1}{s}$,

$$\Rightarrow V_o(s) = \frac{20s}{6s^2+20.1s+1}\cdot\frac{1}{s}$$

$$= \frac{20}{6s^2+20.1s+1}$$

$$= \frac{\left(\frac{1}{3}\right)}{(s+0.0505129)(s+3.29949)}$$

$b^2 - 4ac$

$= 20.1^2 - 4\times 6\times 1 > 0$

$\therefore$ System is overdamp

$$V_o(s) = \frac{1.02596}{s+0.0505129} - \frac{1.02596}{s+3.29949}$$

$$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\}$$

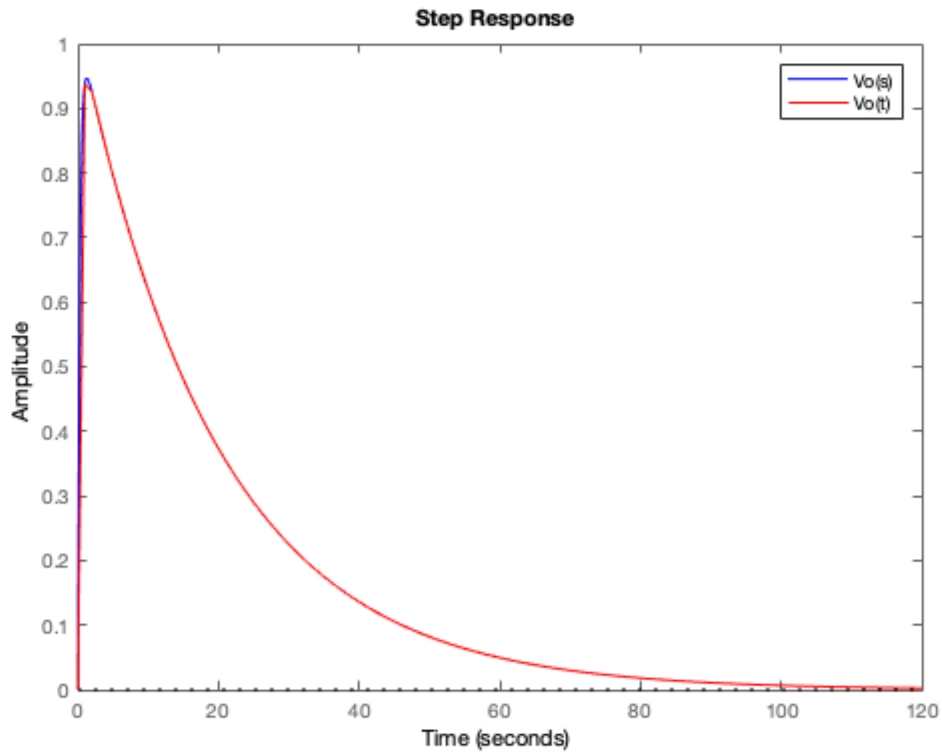
$$= 1.02596\left(e^{-0.0505129t} - e^{-3.29949t}\right), \quad t \geq 0$$

Matlab Code shown below.

```
t=0:150
% Vo(s) transfer function
num=[20 0];
den=[6 20.1 1];
sys=tf(num,den)
stepplot(sys,'b')
hold on
% Vo(t) when input is a unit step
F=1.02596*(exp(-0.0505129*t)-exp(-3.29949*t));
plot(t,F,'r')
legend('Vo(s)','Vo(t)')
hold off
```

```
t =

  Columns 1 through 13

     0     1     2     3     4     5     6     7     8     9    10
 11    12

  Columns 14 through 26

    13    14    15    16    17    18    19    20    21    22    23
 24    25

  Columns 27 through 39

    26    27    28    29    30    31    32    33    34    35    36
 37    38

  Columns 40 through 52

    39    40    41    42    43    44    45    46    47    48    49
 50    51

  Columns 53 through 65

    52    53    54    55    56    57    58    59    60    61    62
 63    64

  Columns 66 through 78

    65    66    67    68    69    70    71    72    73    74    75
 76    77

  Columns 79 through 91

    78    79    80    81    82    83    84    85    86    87    88
 89    90

  Columns 92 through 104
```

```
    91     92     93     94     95     96     97     98     99    100    101
  102    103

   Columns 105 through 117

   104    105    106    107    108    109    110    111    112    113    114
  115    116

   Columns 118 through 130

   117    118    119    120    121    122    123    124    125    126    127
  128    129

   Columns 131 through 143

   130    131    132    133    134    135    136    137    138    139    140
  141    142

   Columns 144 through 151

   143    144    145    146    147    148    149    150


sys =

         20 s
  ------------------
  6 s^2 + 20.1 s + 1

Continuous-time transfer function.
```



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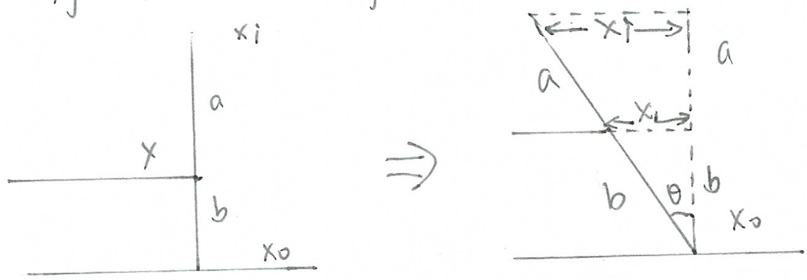
Q2.

if the lever fix at x_o joint, when move lever, we can get:



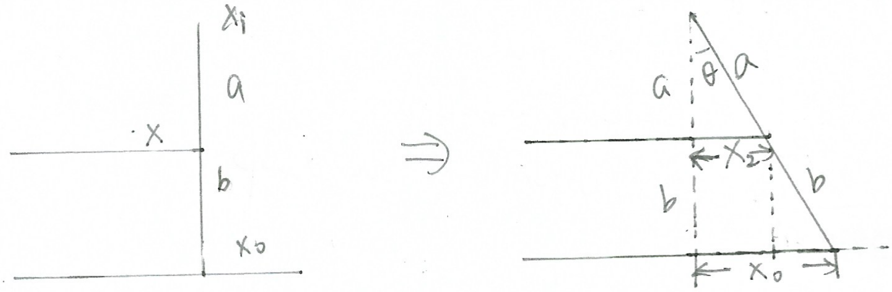
$x_i = \sin\theta(a+b)$

$x_1 = \sin\theta\, b.$

$\theta = \sin^{-1}\left(\frac{x_1}{b}\right) = \sin^{-1}\left(\frac{x_i}{a+b}\right)$

$\Rightarrow \frac{x_1}{b} = \frac{x_i}{a+b}, \quad x_1 = \left(\frac{b}{a+b}\right)x_i$

if the lever fix at x_i point, when move it, we can get:



$x_o = \sin\theta(a+b)$

$x_2 = \sin\theta\, a$

$\theta = \sin^{-1}\left(\frac{x_2}{a}\right) = \sin^{-1}\left(\frac{x_o}{a+b}\right)$

$\Rightarrow \frac{x_2}{a} = \frac{x_o}{a+b}, \quad x_2 = \left(\frac{a}{a+b}\right)x_o$

For example if we move lever to the left, it will cause displacement (x), Then oil goes in, push piston back, which will cause the other displacement (x) with opposite direction. So the actual displacement (x) = the first displacement (x) − the second displacement (x)

$\therefore x = x_1 - x_2 = \left(\frac{b}{a+b}\right)x_i - \left(\frac{a}{a+b}\right)x_o$

we have $a = 4cm$, $b = 12cm$, $A = 10cm^2$ and $k_v = 20cm^3/s/cm$

$$\Rightarrow X_{(s)} = \left(\frac{b}{a+b}\right) X_{i(s)} - \left(\frac{a}{a+b}\right) X_{o(s)}$$

$$= \left(\frac{12}{4+12}\right) X_{i(s)} - \left(\frac{4}{4+12}\right) X_{o(s)}$$

$$= \frac{3}{4} X_{i(s)} - \frac{1}{4} X_{o(s)}$$

$\because$ flow rate q is proportional to x

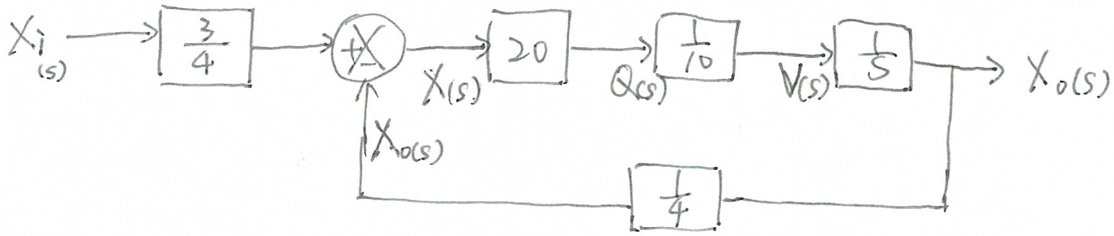
$\therefore Q(s) = k_v X(s) = 20\, X(s)$

$\because Q(s) = A \times V(s)$

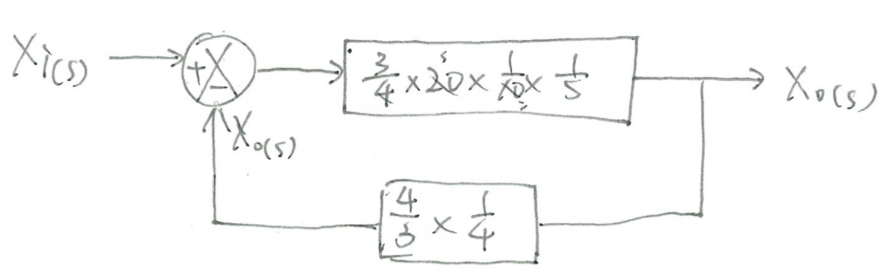
$\therefore V(s) = Q(s) \cdot \frac{1}{A} = Q(s) \times \frac{1}{10}$

$\because \frac{dx_o}{dt} = v \quad \therefore \int v\, dt = x_o \xrightarrow{\text{S domain}} \frac{V(s)}{s} = X_o(s)$

Drawing block diagram:



$\Rightarrow$



$$\therefore \frac{X_{o(s)}}{X_{i(s)}} = \frac{\text{Forward}}{1 - \text{loop}} = \frac{\frac{3}{2s}}{1 - \frac{1}{2s}} = \frac{3}{2s} \cdot \frac{2s}{2s-1} = \frac{3}{2s-1}$$