Control of a Motorised Pendulum

Lab 2: The transfer function of a DC motor.

1. Previous – Identification of sub systems from Lab1

In the first lab you have tried to model this relatively complex system by breaking it down into smaller sub-systems. There are a number of ways in which this can be done (and not one single absolutely correct answer). One possible model for these sub-systems is shown below:

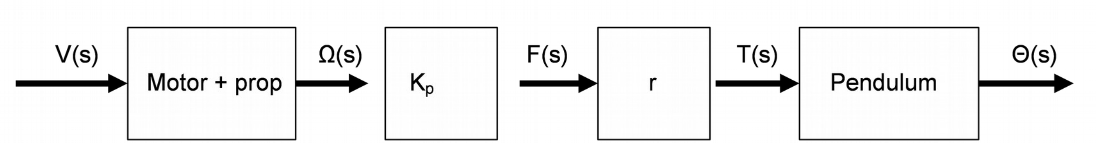


Figure 1: Block diagram of the system model.

1) The electric motor, together with the inertia and damping of the propeller will reach a certain angular velocity after application of a voltage to the motor.

2) Due to this angular velocity of the prop, together with its shape and size, it will produce a certain thrust (force).

3) This force will be converted to a torque by the length of the pendulum arm.

4) The torque on the pendulum will produce an angular displacement of the pendulum.

We can now model each of these subsystems and attempt to measure (estimate) the system characteristics of each.

2. Deriving motor transfer function

The block diagram representation of the standard model for an armature controlled, permanent magnet DC motor is given by [1]:

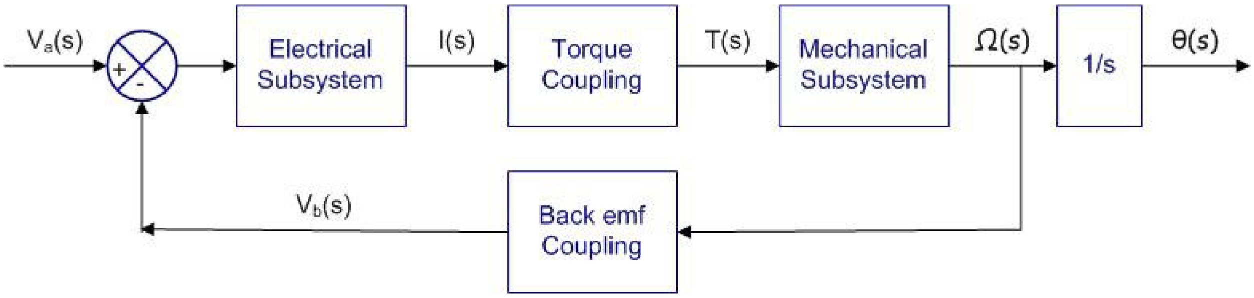
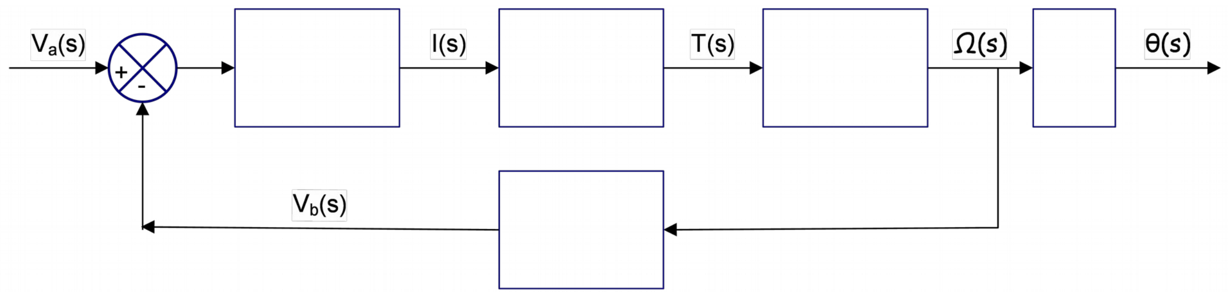


Figure 2: Sub-systems that can be identified in the electric motor.

a) For each of these block now write down the transfer that will describe the input-output characteristics of that block.



b) Show that the transfer function, relating angular velocity, Ω_m, to input voltage, V_a, of the motor will then be given by:

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{\dfrac{K_t}{J_m L_a}}{s^2 + \left[\dfrac{J_m R_a + D_m L_a}{J_m L_a}\right]s + \left[\dfrac{R_a D_m + K_t K_b}{J_m L_a}\right]}$$

where:

L_a is the armature inductance,
R_a is the armature resistance,
K_t is the torque constant,
J_m is the load inertia,
D_m is the damping coefficient of the load and
K_b is the back emf constant.

3) Measurement of motor parameters.
You should now try to get a relatively accurate estimate (measurement!) of each of the variables in the motor transfer function. In the following section a brief indication is given how you should measure/calculate each of these variables. More information can be found in the handout "Understanding small armature controlled DC motors" Remember that ideally you should do several measurements of each variable and average.

Each of your measurements should include full error analysis. That is, you are expected to consider equipment and experimental accuracy and propagate errors so that you can provide error bounds on each parameter of your model.

a) R_a - Measure the resistance of the armature with the motor rotated in several positions w.r.t. brushes and average over the measurements.

b) L_a – Put a 220 Ω resistor in series with armature and either clamp the motor or remove the armature and shaft from the housing. Measure the response of the current (voltage across 220 Ω resistor) to a step input in the voltage. From the observed response calculate the value of the inductance (Hint calculate the time constant of the response of this electrical system in order to extract the value of L_a).

c) K_b – At steady state conditions in the armature the value of K_b is given by:

$$K_b = \frac{v_b - i_b R_a}{\omega}$$

and a measurement of armature current and rotational velocity at a certain applied voltage will facilitate calculation of K_b. Thus need to vary the applied voltage while we measure the current and the rotational velocity at that voltage in order to calculate K_b.

d) K_t – In [1] it is shown that $K_t = K_b$, so we have calculated both these constants in c).

d). D_m – Can be shown that [1] $K_t\, i_a = D_m\, \omega$, so a measurement of the steady state rotational velocity and the current and from knowledge of K_t we can calculate the damping coefficient of the motor.

e). J_m – The mechanical part of the motor can be modelled as an inertia and a damping element, so that:

$$\frac{\Omega(s)}{T(s)} = \frac{1/J_m}{s + \frac{D_m}{J_m}}$$

which will be a first order response with time constant $\tau = J_m/D_m$. We can then estimate J_m by letting the motor run and then using a switch to break the electrical circuit (disconnect the electrical part). Monitor the rotational velocity during this off step by means of a light dependant resistor. From a calculation of the time constant of the off step we can calculate J_m.

4) Simulation of the motor response

a) Complete the table below with your estimated values of the different motor and then use these variables to express the motor transfer function:

Motor Parameters	**Average Value**
R_a	
L_a	
$K_b{=}K_t$	
D_m	
J_m	
$K_t/(J_m L_a)$	
$(J_m R_a + D_m L_a)/J_m L_a$	
$(R_a D_m + K_b K_t)/J_m L_a$	

The motor transfer function is then given by

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{\frac{K_t}{J_m L_a}}{s^2 + \left[\frac{J_m R_a + D_m L_a}{J_m L_a}\right]s + \left[\frac{R_a D_m + K_t K_b}{J_m L_a}\right]}$$

b) Simulate the response of this transfer to a unit step in the input voltage. Also do the simulation for steps of 2, 3, 4, 5 and 6V in the voltage. For each case calculate the time constant of the system, the settling time, steady state value of ω as well as the steady state gain. What is the unit for the steady state gain?

c) Now try to measure the actual response in the rotational velocity with a step input voltage. Perform this for steps of 1 – 6 V. One way you can do this is by means of an LDR to measure the rotational velocity during every half revolution. Can you suggest any other methods?

d) Compare the actual responses observed to that predicted by the simulation results, both in the dynamics of the response as well as the steady state values.

e) Explain any serious deviations between the experimental observation and the modelled response. Can you relate these to any assumptions or obvious shortcomings of your model or your measurements? Can either be improved ?

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Reference.

[1] Understanding small brushed DC motors, G.J. Gouws, August 2008