ECEN315 Assignment 2

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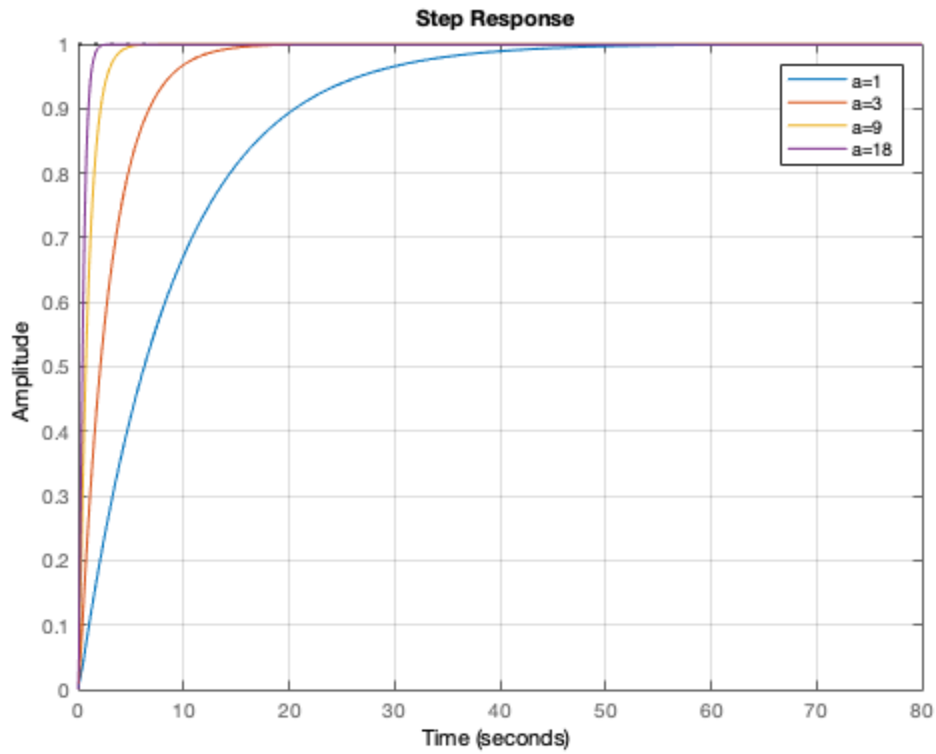
Plot the responses when a=1,3,9,18.

```
clear all;
clf;
hold on;
for a = [1 3 9 18]
    num = [a];
    den = [1 9 a];
    sys = tf(num,den)
    stepplot(sys);
end
grid on;
hold off;
legend('a=1','a=3','a=9','a=18');
```

```
sys =

        1
  -------------
  s^2 + 9 s + 1

Continuous-time transfer function.


sys =

        3
  -------------
  s^2 + 9 s + 3

Continuous-time transfer function.


sys =

        9
  -------------
  s^2 + 9 s + 9

Continuous-time transfer function.


sys =

        18
  --------------
  s^2 + 9 s + 18

Continuous-time transfer function.
```



Plot the poles for each case

```
clear all;
clf;

% When a=1
a1 = 1;
num = [a1];
den = [1 9 a1];
sys = tf(num,den);
stepResults = stepinfo(sys);
% Print out settling time
settlingTime = stepResults.SettlingTime;
X = ['When a=1, settling time = ',num2str(settlingTime)];
disp(X);
% Print out time constant
damp(sys);
disp(' ');
% Plot poles
pzmap(sys);
legend('a=1');
grid on;

% When a=3
figure;
```

```
a2 = 3;
num = [a2];
den = [1 9 a2];
sys = tf(num,den);
stepResults = stepinfo(sys);
% Print out settling time
settlingTime = stepResults.SettlingTime;
X = ['When a=3, settling time = ',num2str(settlingTime)];
disp(X);
% Print out time constant
damp(sys);
disp(' ');
% Plot poles
pzmap(sys);
legend('a=3');
grid on;

% When a=9
figure;
a3 = 9;
num = [a3];
den = [1 9 a3];
sys = tf(num,den);
stepResults = stepinfo(sys);
% Print out settling time
settlingTime = stepResults.SettlingTime;
X = ['When a=9, settling time = ',num2str(settlingTime)];
disp(X);
% Print out time constant
damp(sys);
disp(' ');
% Plot poles
pzmap(sys);
legend('a=9');
grid on;

% When a=18
figure;
a4 = 18;
num = [a4];
den = [1 9 a4];
sys = tf(num,den);
stepResults = stepinfo(sys);
% Print out settling time
settlingTime = stepResults.SettlingTime;
X = ['When a=18, settling time = ',num2str(settlingTime)];
disp(X);
% Print out time constant
damp(sys);
disp(' ');
% Plot poles
pzmap(sys);
legend('a=18');
grid on;
```

When a=1, settling time = 34.8829

Pole	*Damping*	*Frequency (rad/seconds)*	*Time Constant (seconds)*
-1.13e-01	*1.00e+00*	*1.13e-01*	*8.89e+00*
-8.89e+00	*1.00e+00*	*8.89e+00*	*1.13e-01*

When a=3, settling time = 11.4023

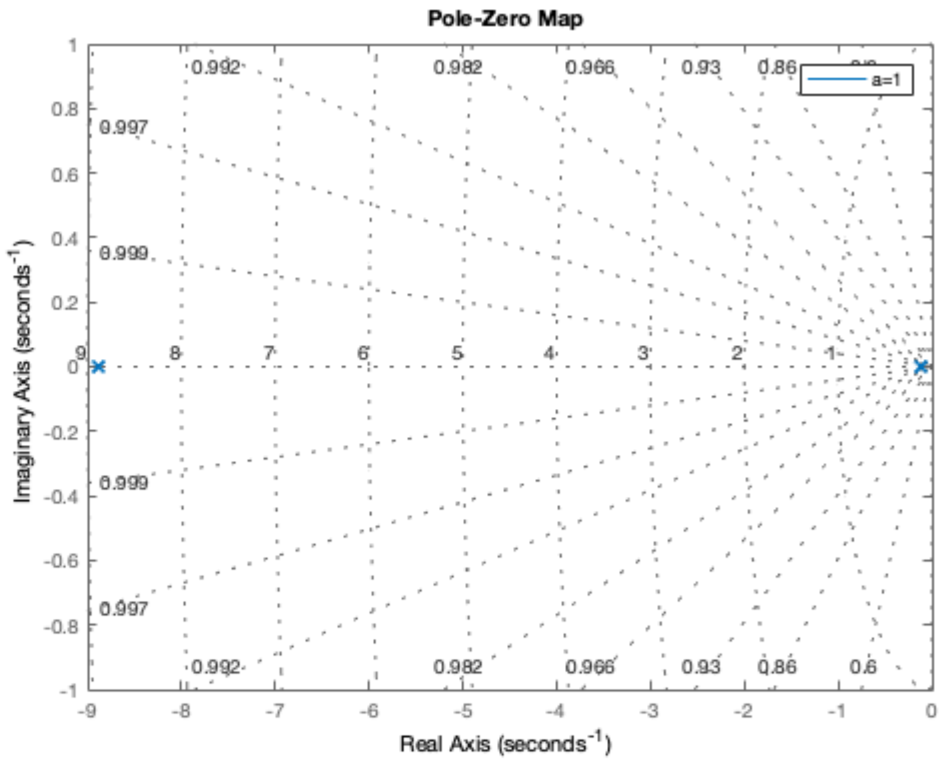
Pole	*Damping*	*Frequency (rad/seconds)*	*Time Constant (seconds)*
-3.47e-01	*1.00e+00*	*3.47e-01*	*2.88e+00*
-8.65e+00	*1.00e+00*	*8.65e+00*	*1.16e-01*

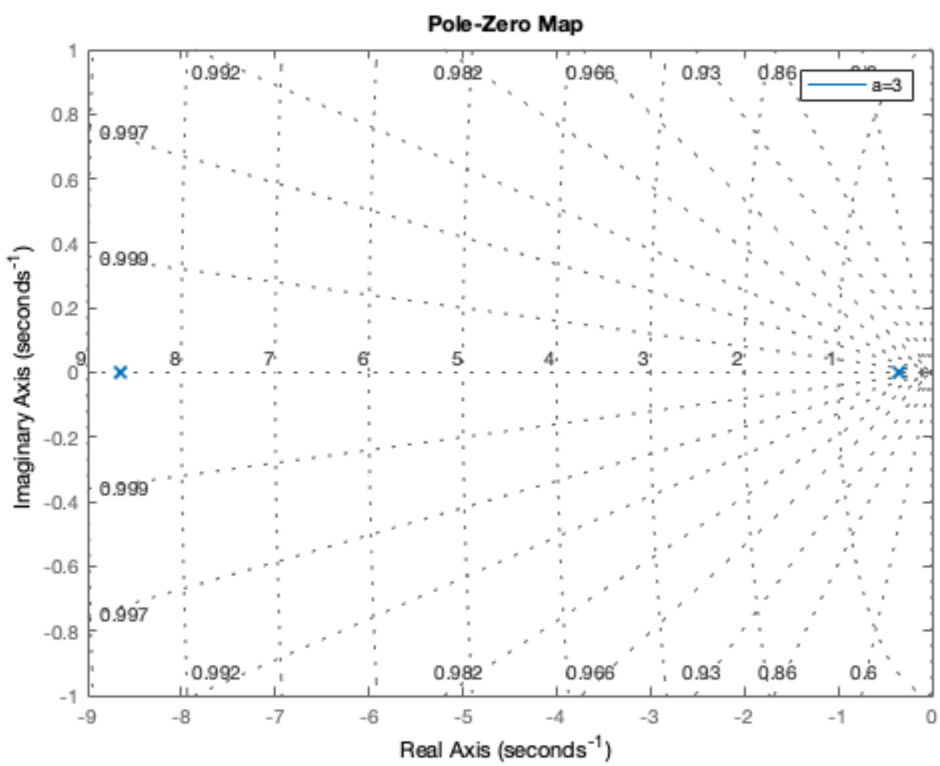
When a=9, settling time = 3.5516

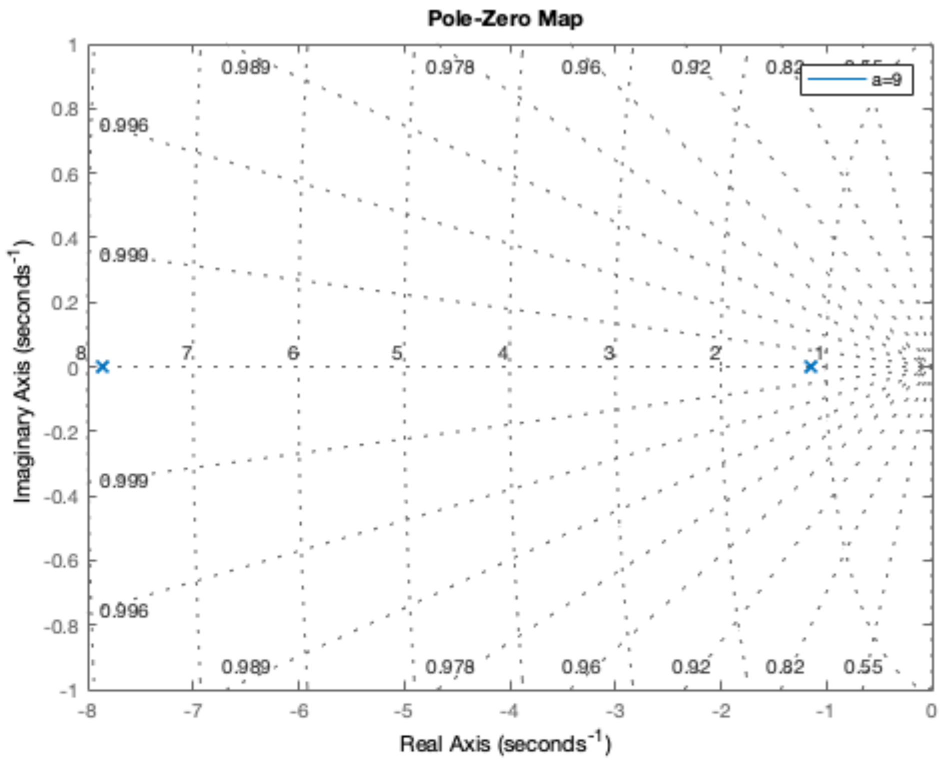
Pole	*Damping*	*Frequency (rad/seconds)*	*Time Constant (seconds)*
-1.15e+00	*1.00e+00*	*1.15e+00*	*8.73e-01*
-7.85e+00	*1.00e+00*	*7.85e+00*	*1.27e-01*

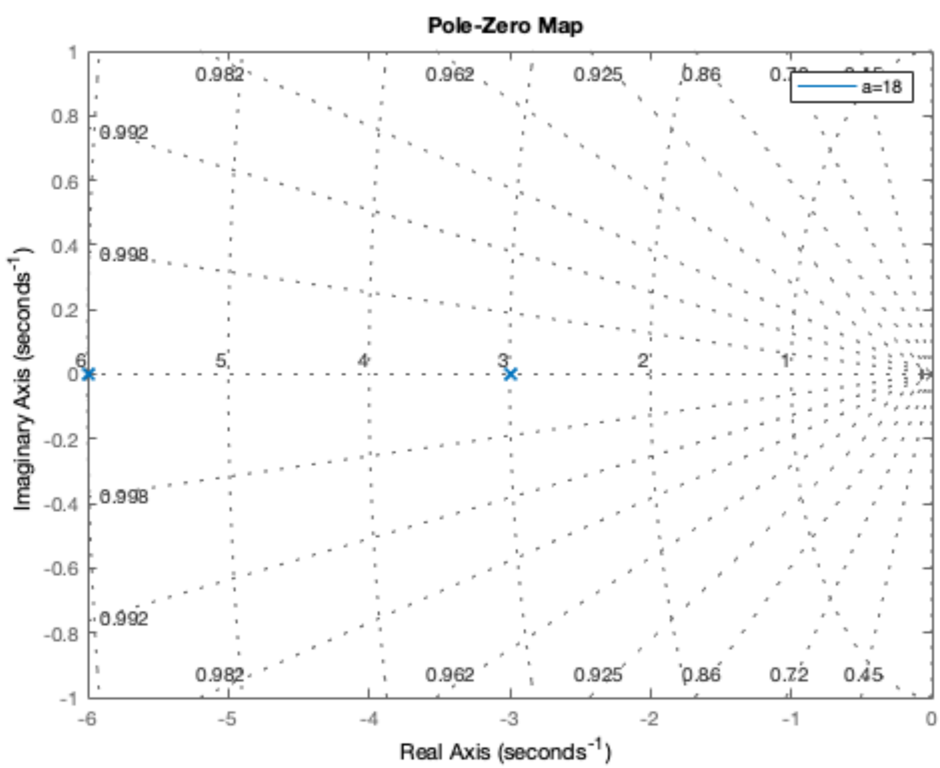
When a=18, settling time = 1.5334

Pole	*Damping*	*Frequency (rad/seconds)*	*Time Constant (seconds)*
-3.00e+00	*1.00e+00*	*3.00e+00*	*3.33e-01*
-6.00e+00	*1.00e+00*	*6.00e+00*	*1.67e-01*


Pole-Zero Map
a=1
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.992
0.982
0.966
0.93
0.86
0.997
0.999
0.6


Pole-Zero Map
a=3
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.992
0.982
0.966
0.93
0.86
0.997
0.999
0.6

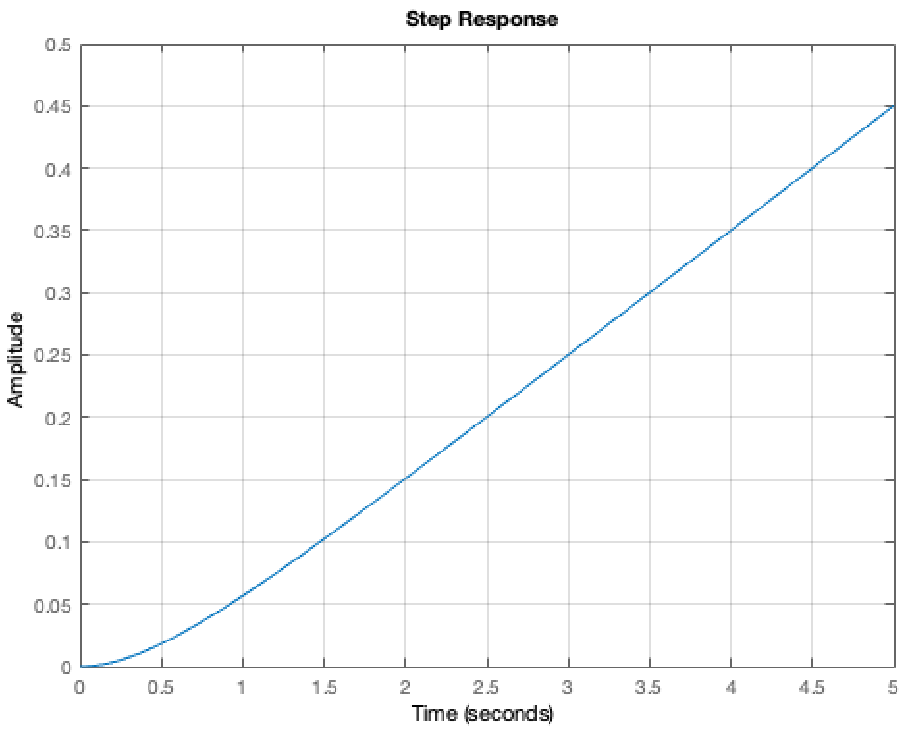

Pole-Zero Map
a=9
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


Pole-Zero Map
a=18
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})

2) (a)

$$\frac{\theta_L(s)}{E_a(s)} = \frac{0.04}{s\,(s+2.0)} = \frac{0.04}{s^2+2.0s}$$

```
% 2)
% (a)
clear all
clf
T = 0:0.01:5;
step = 5;
num = step*0.04;
den = [1 2.0 0];
sys = tf(num,den)
stepplot(sys,T)
grid on


sys =

     0.2
  ---------
  s^2 + 2 s

Continuous-time transfer function.
```

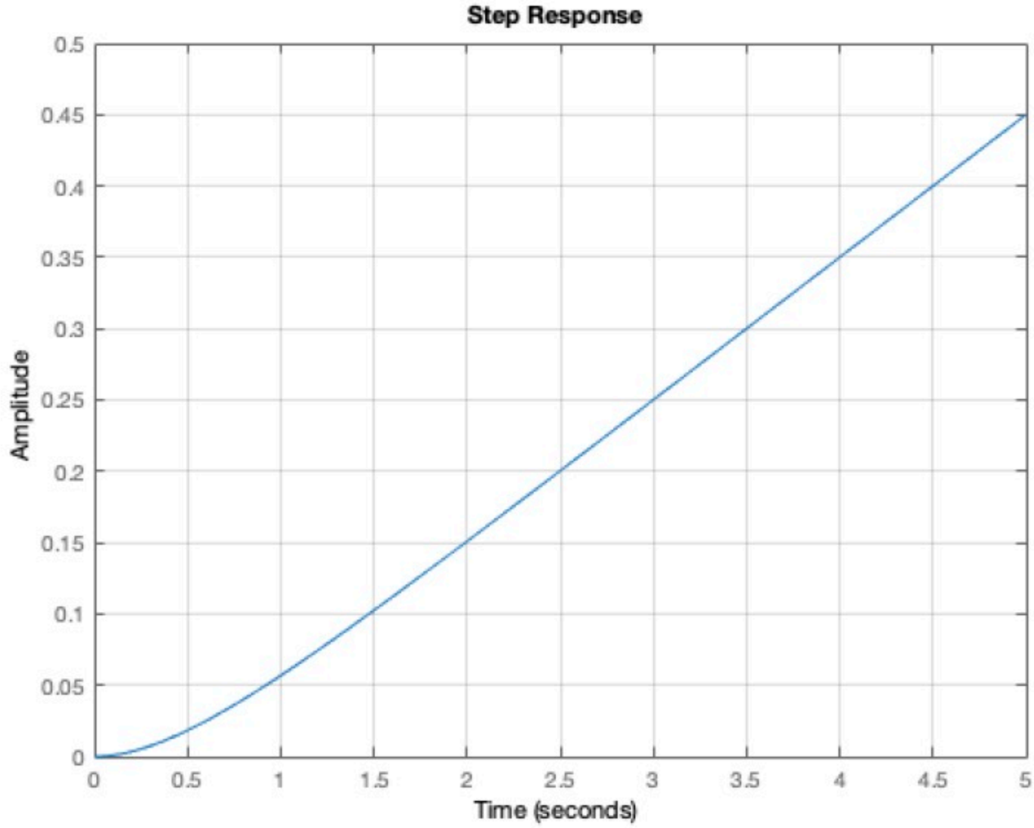


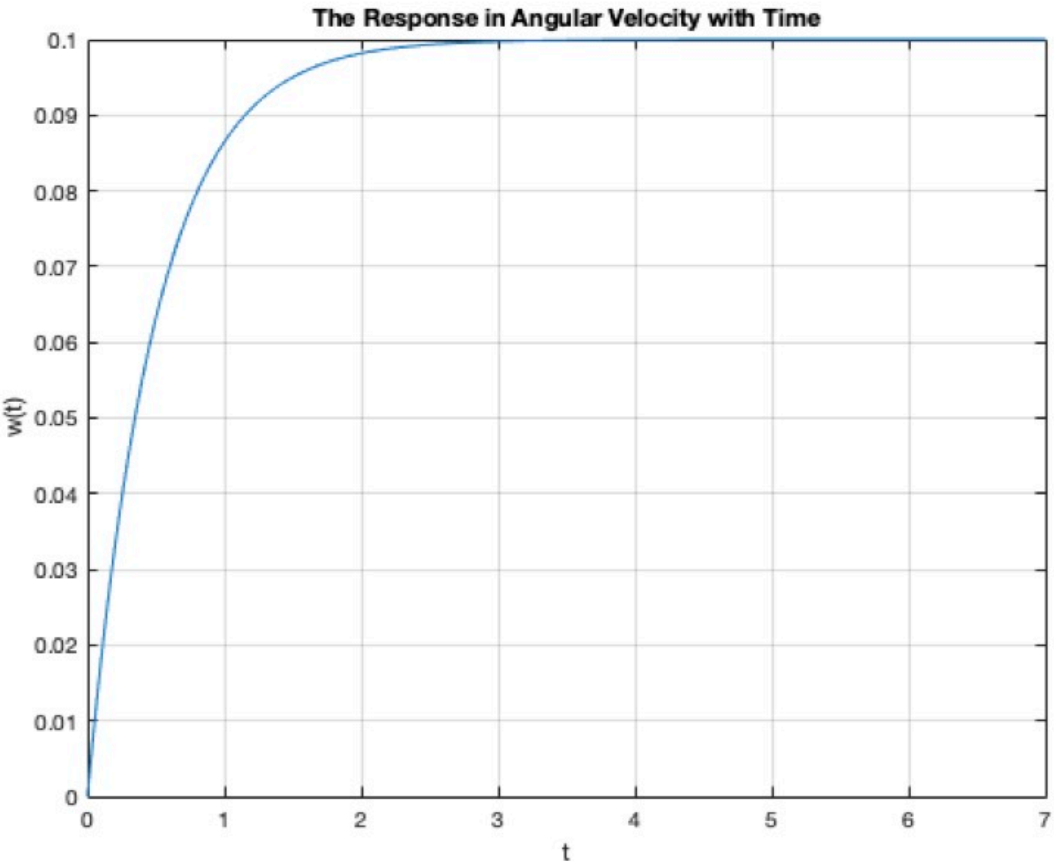
It gives a ramp output as expected from a 2nd order system.

(b)

```
% 2)
% (a)
clear all
clf
T = 0:0.01:5;
step = 5;
num = step*0.04;
den = [1 2.0 0];
sys = tf(num,den)
stepplot(sys,T)
grid on

%(b)
figure;
syms s t
snum = poly2sym(num,s); % Symbolic Numerator Polynomial
sden = poly2sym(den,s);  % Symbolic Denominator Polynomial
SysTimeDomain = ilaplace(snum/sden) % Inverse Laplace transform
f = matlabFunction(SysTimeDomain); % converts the symbolic expression
 to a MATLAB function
t = 0:0.01:7;
plot(t,f(t))
xlabel('t');
ylabel('w(t)');
title('The Response in Angular Velocity with Time');
grid on
```

```
sys =

      0.2
  ---------
  s^2 + 2 s

Continuous-time transfer function.


SysTimeDomain =

1/10 - exp(-2*t)/10
```

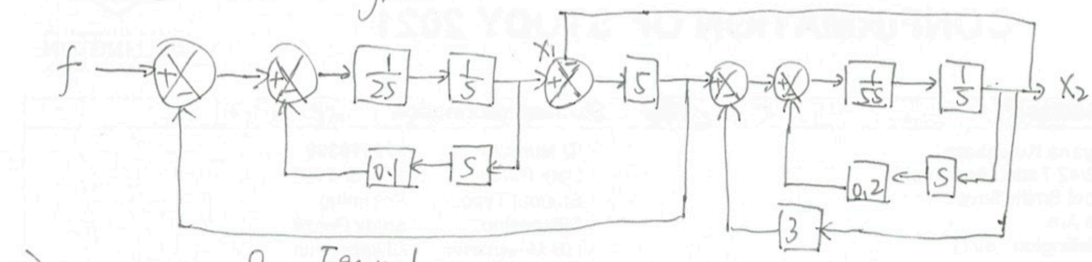

Step Response
Amplitude
Time (seconds)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


The Response in Angular Velocity with Time
w(t)
t
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
1
2
3
4
5
6
7

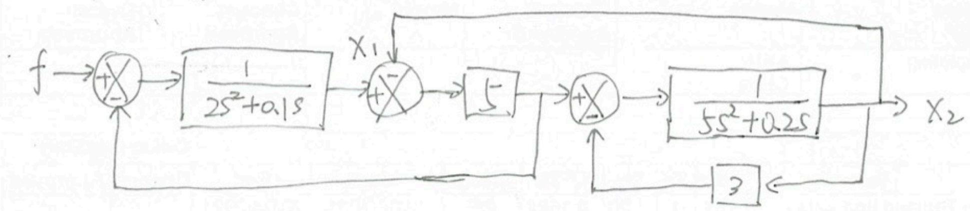
3)

X2/f

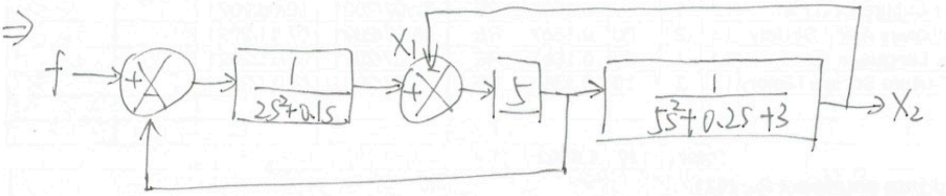
reduce block diagram



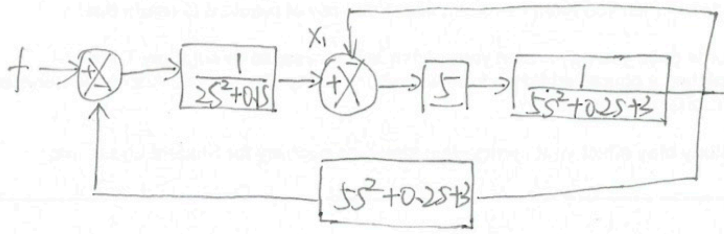
$\Rightarrow$ $\frac{O}{I} = \frac{Forward}{1-loop}$



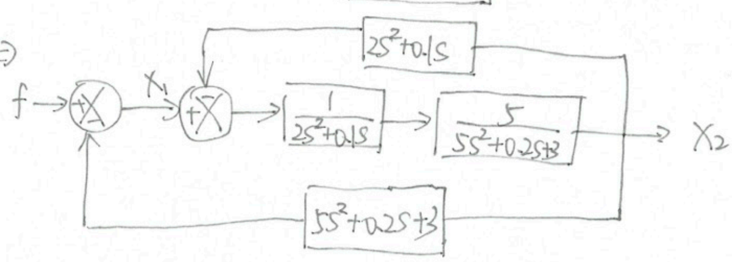
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$

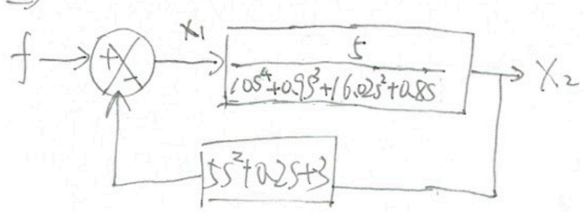


$$\frac{O}{I} = \frac{Forward}{1-loop}$$

$$= \frac{\frac{1}{2s^2+0.1s}\cdot\frac{5}{5s^2+0.2s+3}}{1+\frac{1}{2s^2+0.1s}\cdot\frac{1}{5s^2+0.2s+3}\cdot 2s^2+0.1s}$$

$$= \frac{\frac{1}{2s^2+0.1s}\cdot\frac{5}{5s^2+0.2s+3}}{\frac{5s^2+0.2s+8}{5s^2+0.2s+3}}$$

$\Rightarrow$



$$= \frac{5}{5s^2+0.2s+8}\times\frac{1}{2s^2+0.1s}$$

$$= \frac{5}{10s^4+0.9s^3+16.02s^2+0.8s}$$

$$TF = \frac{X_2(s)}{f(s)} = \frac{\text{Forward}}{1-\text{loop}}$$

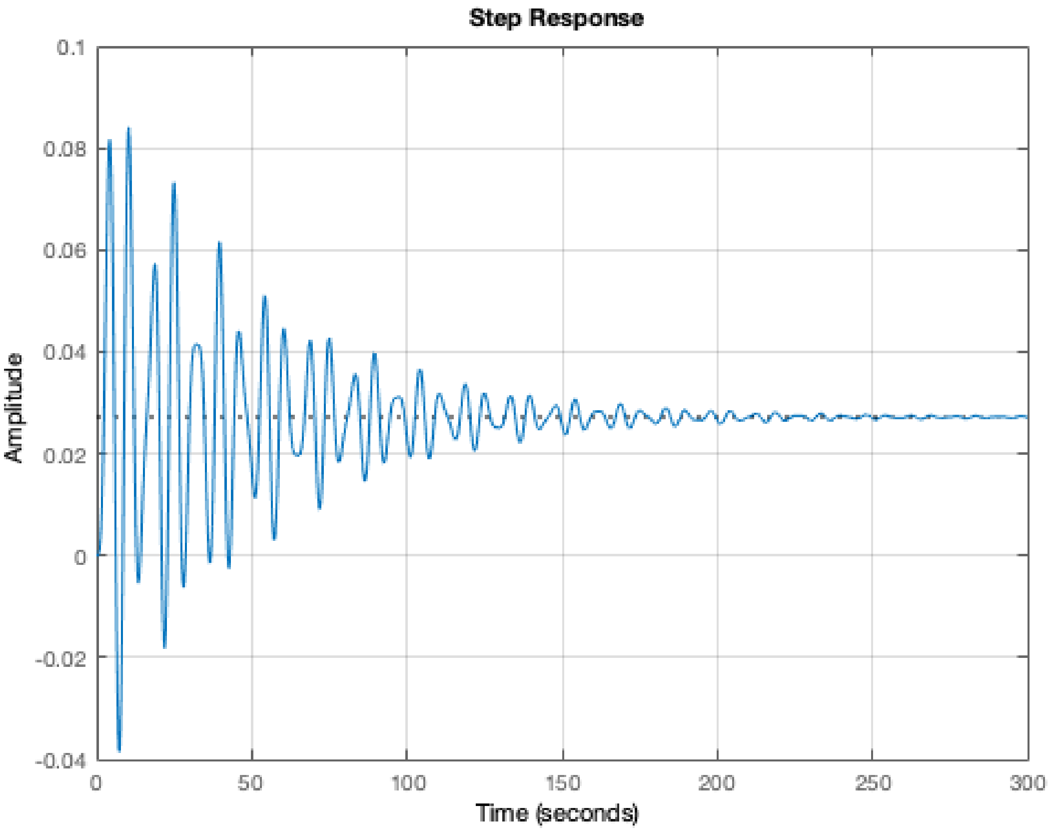
$$= \frac{\frac{5}{10s^4+0.9s^3+16.02s^2+0.8s}}{1+\frac{5}{10s^4+0.9s^3+16.02s^2+0.8s}\times(5s^2+0.2s+3)}$$

$$= \frac{\frac{5}{10s^4+0.9s^3+16.02s^2+0.8s}}{\frac{10s^4+0.9s^3+16.02s^2+0.8s+25s^2+s+15}{10s^4+0.9s^3+16.02s^2+0.8s}}$$

$$= \frac{5}{10s^4+0.9s^3+41.02s^2+1.8s+15}$$

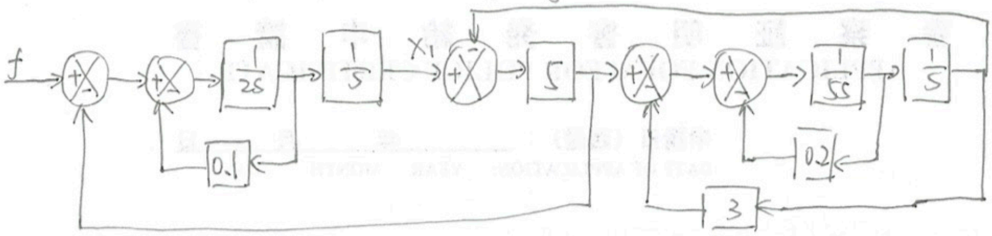
Step response:

```
% 3)
clear all
clf
num = 5;
den = [10 0.9 41.02 1.8 15];
sys = tf(num,den)
stepplot(sys)
grid on


sys =

                    5
  ----------------------------------------
  10 s^4 + 0.9 s^3 + 41.02 s^2 + 1.8 s + 15

Continuous-time transfer function.
```


Step Response
Amplitude
Time (seconds)
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
0
50
100
150
200
250
300

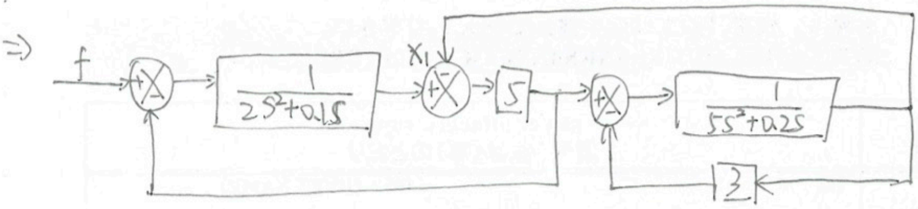
X1/f

Find the transfer function of $\frac{X_1}{f}$, let $X_2 = 0$

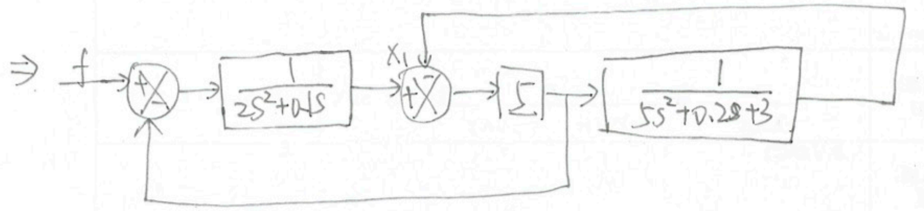


$$\frac{O}{I} = \frac{\text{Forward}}{1-\text{loop}} = \frac{\frac{1}{2s}\cdot\frac{1}{s}}{1+\frac{1}{2s}\cdot\frac{1}{s}\cdot s\cdot 0.1} = \frac{1}{2s^2+0.1s} \qquad \frac{O}{I} = \frac{\frac{1}{5s^2}}{1+\frac{1}{5s^2}\cdot s\cdot 0.2} = \frac{1}{5s^2+0.2s}$$

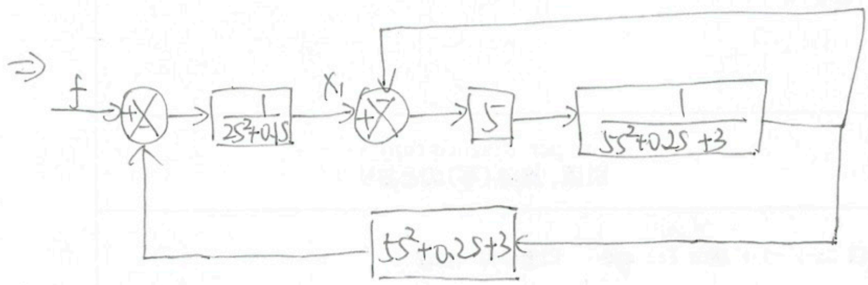
$\Rightarrow$



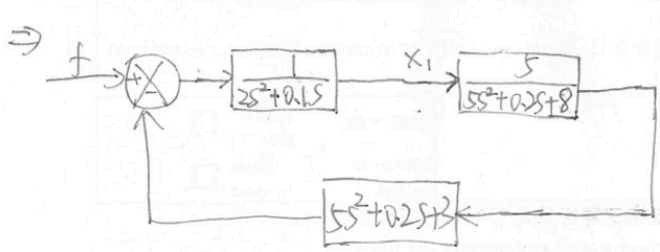
$\Rightarrow$

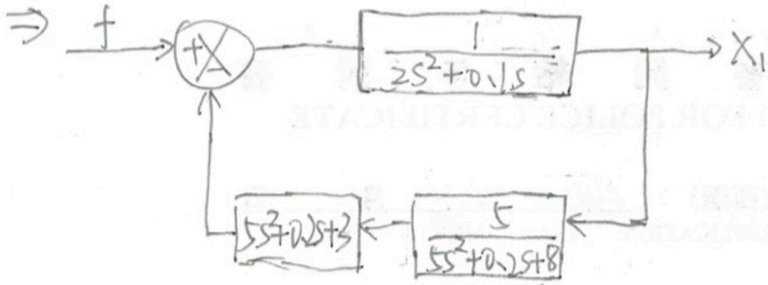


$\Rightarrow$



$\Rightarrow$





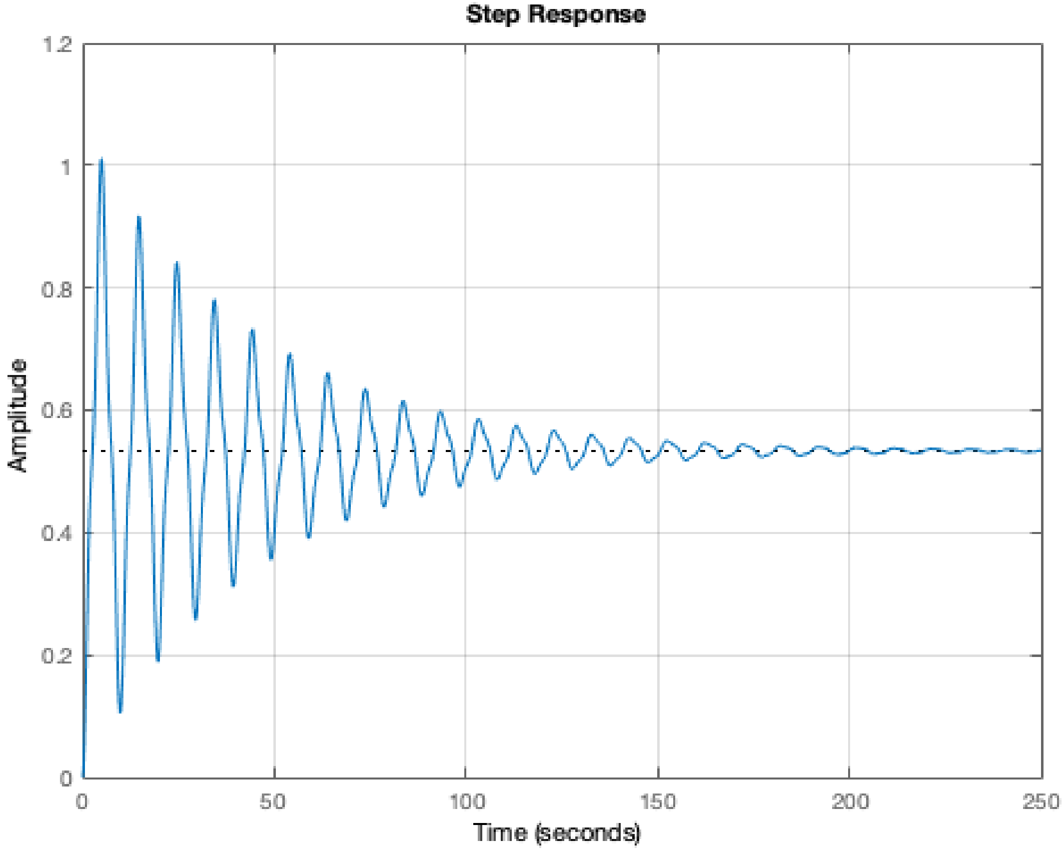
$$\therefore \frac{X(s)}{f(s)} = \frac{\text{Forward}}{1-\text{loop}} = \frac{\frac{1}{2s^2+0.1s}}{1+\frac{1}{2s^2+0.1s}\times\frac{(5s^2+0.2s+3)5}{5s^2+0.2s+8}}$$

$$= \frac{1}{2s^2+0.1s} \times \frac{(2s^2+0.1s)(5s^2+0.2s+8)}{(2s^2+0.1s)(5s^2+0.2s+8)+25s^2+s+15}$$

$$= \frac{5s^2+0.2s+8}{10s^4+0.9s^3+41.02s^2+1.8s+15}$$

```
% 3)
% x1/f
clear all
clf
num = [0 0 5 0.2 8];
den = [10 0.9 41.02 1.8 15];
sys = tf(num,den)
stepplot(sys)
grid on
```

```
sys =

                 5 s^2 + 0.2 s + 8
  -----------------------------------------
  10 s^4 + 0.9 s^3 + 41.02 s^2 + 1.8 s + 15

Continuous-time transfer function.
```



5)

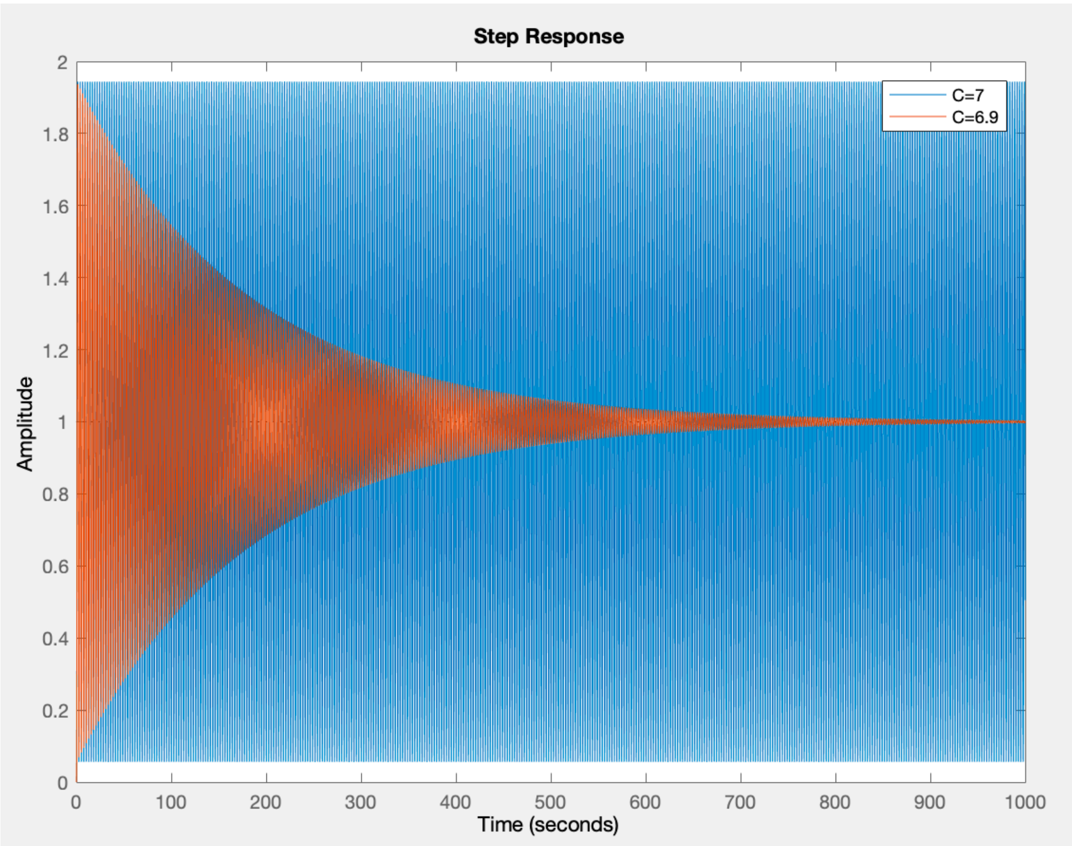
(a)

$$G(s) = \frac{6}{s(s+6)(s+1)} = \frac{6}{s^3 + 7s^2 + 6s}$$

(a) Finding Minimum Value which cause system become unstable

```
clear all
clf
for C = 0.1:0.1:10 % Gain Controller
    Gnum = 6;
    Gden = [1 7 6 0];
    G = tf(Gnum,Gden); % Plant
    sys = feedback(C*G,1);
    B = isstable(sys); % Chenck if the system is stable
    if B==0 % B=0 means system is unstable
        break
    end
end
disp(['The minimum vaule of gain is ',num2str(C)])
sys
step(sys) % When gain = 7
hold on
step(feedback(6.9*G,1)) % When gain = 6.9
hold off
legend('C=7','C=6.9')
```

```
The minimum vaule of gain is 7

sys =

             42
  ----------------------
  s^3 + 7 s^2 + 6 s + 42

Continuous-time transfer function.
```

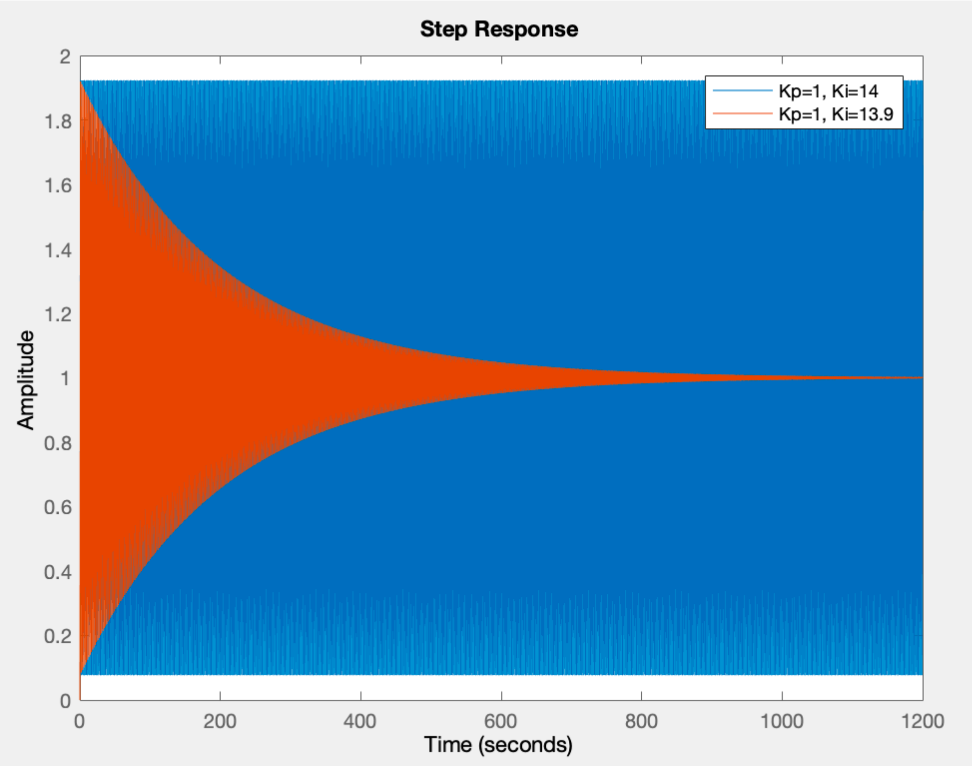


(b)

Replace the gain controller with a P-I controller:

```
% (b)
clear all
clf
for Ki = 0.1:0.1:20; % Integral gain
    Kp = 1; % Proportional gain
    s = tf('s');
%      C = (s+K)/(s+K');
    C = Kp + Ki/s;
    Gnum = 6;
    Gden = [1 7 6];
    G = tf(Gnum,Gden);
    sys = feedback(C*G,1);
    B = isstable(sys);
    if B==0
        break
    end
end
sys
disp(['The minimum value of integral gain is ',num2str(Ki)])
step(sys,feedback((1+13.9/s)*G,1))
legend('Kp=1, Ki=14','Kp=1, Ki=13.9')
```

```
sys =

         6 s + 84
  -----------------------
  s^3 + 7 s^2 + 12 s + 84

Continuous-time transfer function.

The minimum value of integral gain is 14
```

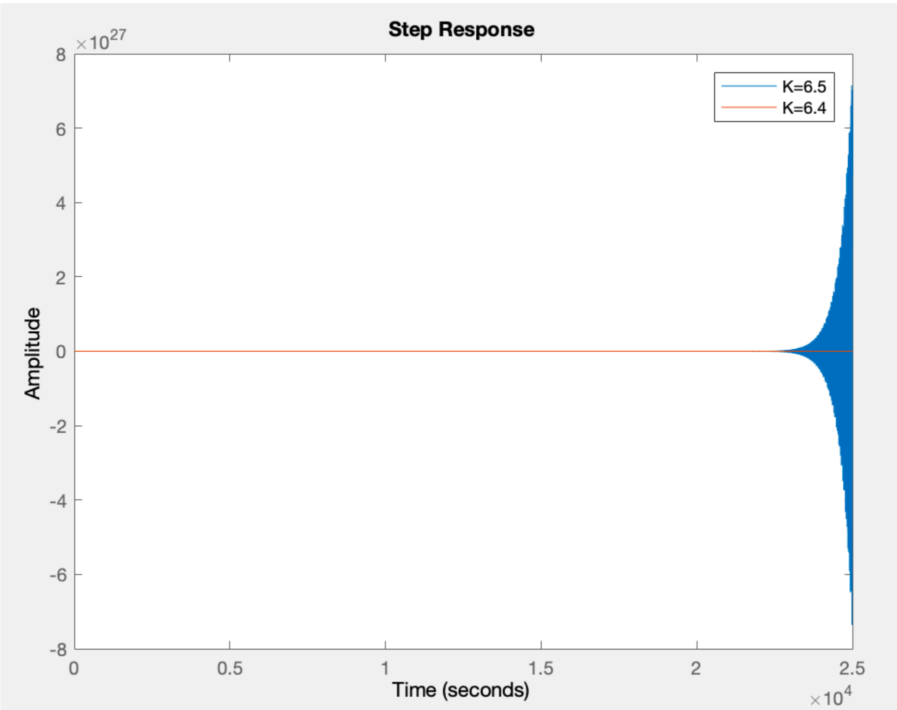


The minimum value of integral gain is 14, which is larger than gain controller's.

Replace the gain controller with a lead lag controller.

```
% (b) Lead Lag
clear all
clf
for K = 0.1:0.1:10; % Integral gain
    Zc = 20; % Zc < -6
    Pc = 4; % -6 < Pc <-1
    s = tf('s');
    C = K*(s+Zc)/(s+Pc);
    Gnum = 6;
    Gden = [1 7 6];
    G = tf(Gnum,Gden);
    sys = feedback(C*G,1);
    B = isstable(sys);
    if B==0
        break
    end
end
sys
disp(['The minimum value of gain is ',num2str(K)])
stepplot(sys)
hold on
stepplot(feedback((6.4*(s+Zc)/(s+Pc))*G,1))
legend('K=6.5','K=6.4')
```

```
sys =

         39 s + 780
  -------------------------
  s^3 + 11 s^2 + 73 s + 804

Continuous-time transfer function.

The minimum value of gain is 6.5
```



The minimum value of gain of the Lead Lag controller is 6.5, which is less than gain controller (7).