Control of a Motorised Pendulum

Lab 6: Closed loop control – PID compensation.

1. Previous

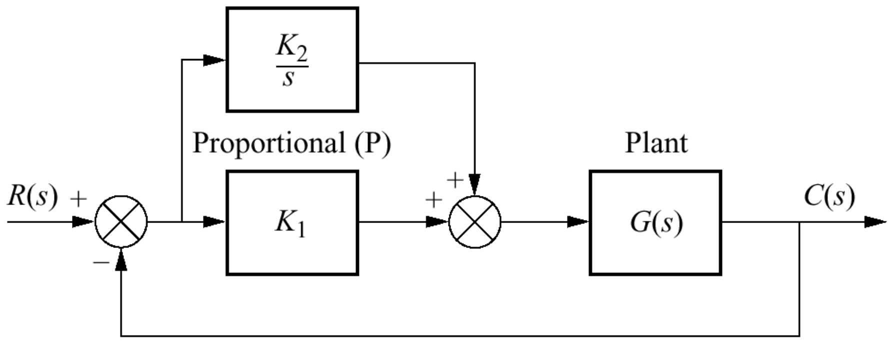
In the previous lab we constructed a closed loop system and added a proportional gain element in the forward path. From the results two drawbacks of a proportional gain element become apparent:

- Increasing the gain value tend to make the system less damped and may drive the system towards instability.
- There will always be a steady state error associated with a pure proportional gain. In our pendulum this steady state error was significant at the small values of K_p that was allowed in the system.

In addition to the above two drawbacks, it is clear that the response of the system will be limited to a position on the root locus as determined by the chosen gain value, i.e. we cannot arbitrarily select an operating point for our system. All this makes a pure proportional gain controller appear rather useless and we indeed need a more sophisticated control scheme. This can be achieved by the addition of proportional and/or derivative elements to our controller.

2. Integral control - Improving the steady state error.

In class we have seen that a pure integrator in the transfer function will ensure that we have a zero steady state error to a step input. The addition of such an integrator to the transfer function of the controller will then solve the steady state error, but will have the additional complication that the root locus will change due to this new pole at the origin. This can prevented by placing the integrating element in parallel with our proportional gain as shown below.



The resultant controller transfer function will be given by

$$G_c(s)=K_P+\frac{K_I}{s}=\frac{K_P\left(s+\dfrac{K_I}{K_P}\right)}{s}$$

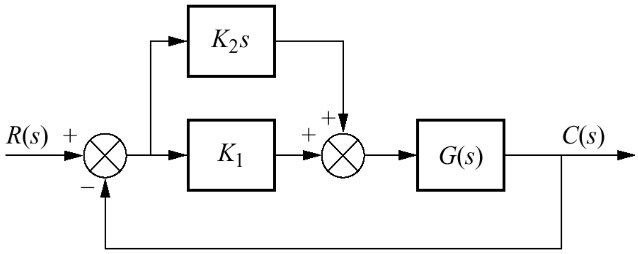
which will produce a new pole at the origin plus a new zero into the transfer function. The position of the zero will be determined by the ration of K_I/K_P. If this ratio is chosen to be small (close to zero) the new pole and new zero will be close together. This will lead to pole-zero cancellation, and the path of the root locus will be essentially unchanged from the uncompensated system. However, the steady state error should be eliminated.

3. Design of a PI controller.

Modify your Sumlink system so that it incorporates a PI compensator. Choose a value of K_p for your system based on your previous experimentation. Now use this value of K_p to calculate a value for K_I that will produce a zero close to the origin. Test your system response to these gain values. Experiment with different values for K_I and K_P and record the response. Select the most appropriate values. What has been the effect of this control scheme on both the steady state error as well as the system response?

4. Changing system response by derivative compensation.

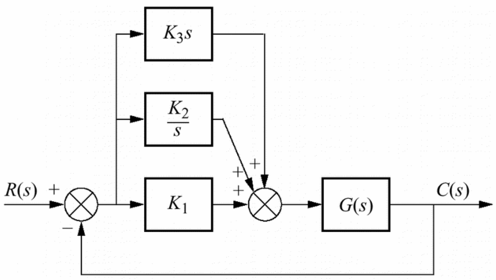
The response of a system can be changed by selecting a different path for the root locus in order to force it through a selected operating point on the s-plane. This operating point will be the point where our system has the desired characteristics (%OS, T_s, T_p). The change in path of the root locus is achieved by the addition of a new open loop zero from the compensator, with the zero placed to force the root locus to go through the desired operating point. The new zero is the result of a derivative action on the error, as shown below.



The transfer function of the compensator is then given by: $G(s)=K_D s+K_P=K_D\left(s+\frac{K_P}{K_D}\right)$

The ratio of K_P/K_D then represents the position of the new zero in the system and will thus determine the path of the new root locus.

In similar fashion, a combined PID compensator will have a transfer function as shown below:



$$G_c(s)=K_P+\frac{K_I}{s}+K_D s=\frac{K_P s+K_I+K_D s^2}{s}=\frac{K_D\left(s^2+\frac{K_P}{K_D}s+\frac{K_I}{K_D}\right)}{s}$$

5. Design of a PID controller.
The open loop response of you pendulum will typically show a settling time of approximately 6 seconds for a step input. Assuming that the % OS is acceptable, try to design a compensator that will allow the system to settle with at least a three fold improvement in the settling time and have a zero steady state error at the same time. The set point should be approximately 20º.

Explain how the values of the gain constants in the PID compensator leads to the introduction of new zeros and poles on the root locus and how the values chosen for the gains will influence the position of the zeros end poles. Plot the new root locus (compensator + system) and compare the expected response to that actually obtained in the compensated system.

6. Tuning a PID controller.
The Ziegler Nicholl's rules provide us with an empirical method of tuning a PID controller with no knowledge of the transfer function of the system. Use the ultimate cycle method as discussed in the text to obtain values for K_P, K_I and K_D. How do these values compare to your values previous obtained in section 5?