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ECEN315 LABORATORY REPORT ONE

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Submitted in partial fulfilment of the requirements for
the degree of Bachelor of Science

Abstract

This document is about the report of the first three labs of the entire lab project. Decomposing a complex motorised pendulum system into a motor system and a pendulum system, which reduced the difficulty of modelling the system. In each sub-system, using the block diagram technique to further decompose the sub-system, found its transfer function, and then simulated its step response by Matlab. Finding the actual response and compare it with the simulated response.

1. Introduction

This ECEN315 lab project was based on controlling motorised pendulum system. The first three labs aim to study the system characteristic, derive the system's mathematical model and measure the open-loop response.

There were some problem that need to be solved in this project:

1. The system was a complex system, which was difficult to analyse.
2. Transfer function needed to be found.
3. The numeric values of the system variables need to be determined to model the system response.

First, we modelled this motorised pendulum system by breaking it down into various sub-systems. Then we demonstrated the motor's block diagram and derived its transfer function. Next, we measured each of the parameters in the transfer function and simulated the motor's step response. Last, we also derived the pendulum's transfer function and measured the transfer function parameters to test the system's open-loop response.

2. Background

The motorised pendulum system was provide which shown below.

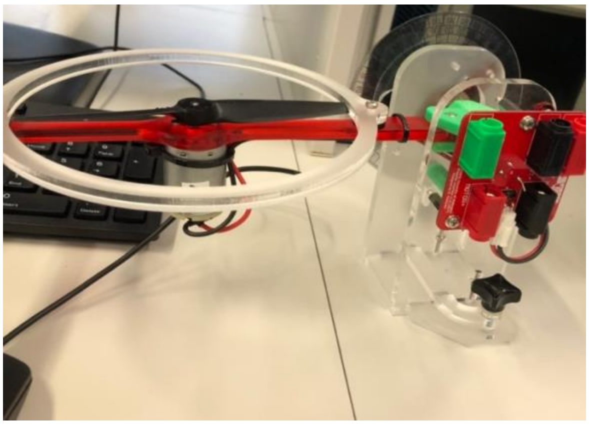


Figure 1: real motorised pendulum

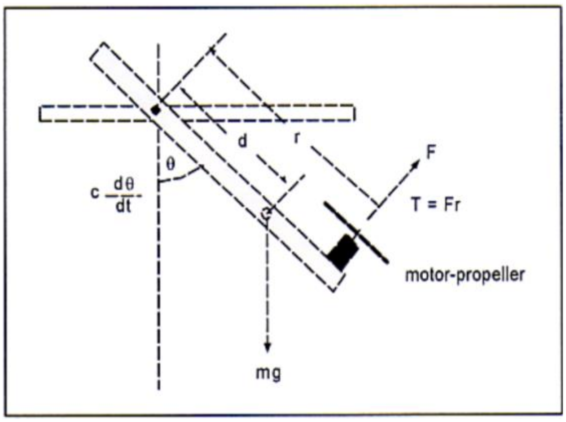


Figure 2: motorised pendulum draft [1]

A propeller was driven by a small electric motor which provided the torque of the system. A step of the voltage input would make the pendulum swing out of a certain angle, oscillate for some time, then stabilized at a steady-state angle. Used the rotary potentiometer to monitor the system's angular displacement in order to obtain the record of the angular position over time.

Some control theories were required for the reader: block diagram, transfer function and system's step response.

3. Methods and Results

1. Identification of sub-systems

A complex Motorised Pendulum system consist of electrical system and mechanical system. In order to model the system response, we broke it down into smaller sub-system, modelled each of them and measured the system characteristics. The model of sub-systems was shown below:

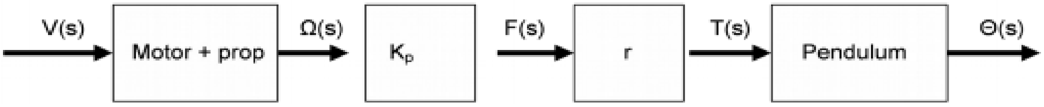


Figure 3: Block diagram of the system model [2]

2. Deriving motor transfer function

The motor can be represented in block diagram form as shown below [3]:

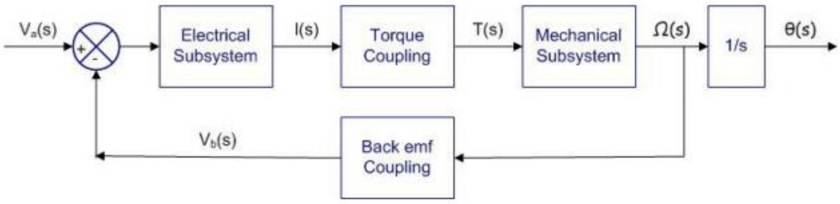


Figure 4: Block diagram of DC motor

The derivation of the motor's transfer function was demonstrated in section 3 of [3], which was:

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{K_t/(J_m L_a)}{s^2 + \left(\frac{J_m R_a + D_m L_a}{J_m L_a}\right)s + \left(\frac{R_a D_m + K_t K_b}{J_m L_a}\right)}$$

where:

La is the armature inductance,

Ra is the armature resistance,

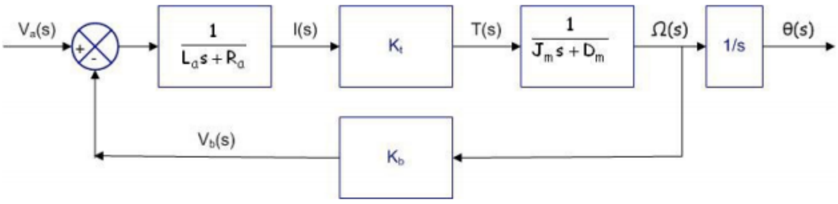
Kt is the torque constant,

Jm is the load inertia,

Dm is the damping coefficient of the load and

Kb is the back emf constant.

And it was produced by the following block diagram:



3. Measurement of motor parameters

Next, we measured each of the variables in the motor transfer function.

(a) Ra - the armature resistance. Measured the resistance with the motor rotated in several positions and average over the values. Directly connected motor input with DMM input and chose resistance measurement function.

We selected 4-wire measurement at the beginning, which was a mistake, so the Ra value was twice larger than the expected value.

After choosing the correct 2-wire measurement, which is for measuring small resistance, the Ra values shown in the table below:

Ra position 1	14.96Ω
Ra position 2	7.87Ω
Ra position 3	6.88Ω
Ra position 4	19.5Ω
Ra position 5	17.53Ω
Ra position 6	14.47Ω
Ra position 7	13.84Ω
Average	13.58Ω

Table 1: measurement of Ra

(b) La - the armature inductance. Put a 220 Ω resistor in series with armature, measured the response of the current (voltage across 220 Ω resistor) by putting oscilloscope on it. In order to extract the value of La, we needed to calculate the time constant of the response. We switched the power on and off to create a step input, then stop running the oscilloscope to capture the response plot.

We first mistakenly measured the time constant of the falling edge, which is much larger than the expected value. The time constant should be the time for the system's step response to reach $1 - 1/e \approx 63.2\%$ of its final value (say from a step increase).

The actual resistor value we measured was 233.58 Ω instead of 220 Ω. So we then measured the increasing system's time constant under different voltage input, which shown in the table below:

V	t	R	L
10.0	1.20ms	233.58 Ω	(1.20/1000)*233.58 = **0.280H**
5.0	1.24ms	233.58 Ω	(1.24/1000)*233.58 = **0.289H**
2.5	1.24ms	233.58 Ω	(1.24/1000)*233.58 = **0.289H**

Table 2: measurement of La

So the average value of La = 0.286H.

Due to the limitation of time, we just measured three sets of value, which might cause the inaccuracy of the result of La.

(c) Kb - the back emf constant. In the armature at steady state conditions the value of Kb can be calculated as:

$$K_b = \frac{v_a - i_a R_a}{\omega}$$

We used tachometer to measure RPM under different input voltage. (1 rpm = $2\pi/60$rad/s). The RMP values were divided by 2, because the propeller had two blades. Shown in the table below:

Voltage (Va)	**Current (Ia)**	**RMP**	ω	**Ra**	**Kb**
2.5	0.24	1075	112.57	13.58	-0.00674
3.7	0.42	1129	118.23	13.58	-0.0169
5	0.56	2100	219.91	13.58	-0.0118
6.2	0.82	4998	523.39	13.58	-0.00943

7.5	0.99	6000	628.32	13.58	-0.00946

Table 3: measurement of Kb

The average value of Kb = | -0.010866 | = 0.010866

Sometimes the tachometer did not work well, and sometimes it did not even have reading. So some of the RPM values accuracies did not guarantee. However, we measured many sets of values to offset this inaccuracy.

(d) Kt - the torque constant. In [3] it was proved that Kt = Kb, so Kt = 0.010866.

(e) Dm - the damping coefficient of the load. As shown in [3], $K_t i_a = D_m \omega$. So we calculated the damping coefficient directly, shown in table below:

$$Dm = (kt * ia)/\omega$$

Kt	Ia	ω	**Dm**
0.00674	0.24	112.57	1.437E-5
0.0169	0.42	118.23	6.003E-5
0.0118	0.56	219.91	3.004E-4
0.00943	0.82	523.39	1.477E-4
0.00946	0.99	628.32	1.491E-5

Table 4: measurement of Dm

The average value of Dm = 2.685E-05.

(f) Jm - the load inertia. To get the motor inertia, we needed to measure the mechanical time constant of the system. It could be achieved by breaking the electrical circuit. Then the time constant could be measured from the decay in angular velocity with time.

So we put a switch and a voltage divider with the motor in series to disconnect the circuit when needed. The voltage divider consisting of a resistor and an LDR. LDR connected to the oscilloscope as output. We pointed the LDR probe to the propeller then switched off the motor; the rotation velocity decreased. Whenever the blade passed the LDR, there was a step input to the oscilloscope, the plot shown in the figure below:

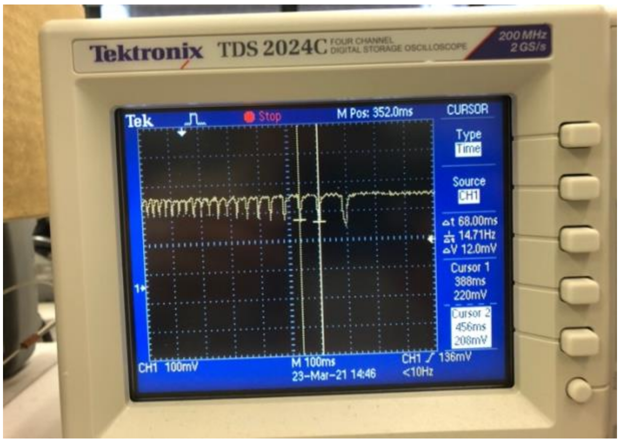


Figure 5: oscilloscope outcome

We picked a period of off step with good looking and easy to measure as shown in figure 5. Then we measured the time interval of each step and calculated the current overall time of each stage, which shown in the table below:

	Δt(ms)	Overall time (ms)
1	32	0
2	36	32
3	38	32+36=68
4	42	68+38=106
5	48	106+42=148
6	54	148+48=196
7	66	196+54=250
8	90	250+66=316

Table 5: time of off step

Overall time was calculated as from the first point to the last point of the off step. When the time came to 63% of the total, 316*63% = 199.08ms, it close to 196ms, so the corresponding Δt = 54ms was the time constant.

In [3], the equation of Jm was given by

$\tau_m = \frac{J_m}{D_m}$, so Jm = 0.054s * 2.685E-05 = 1.4499E-06.

There was a deviation of our Jm value compared with the expected value (2.41E-06). It might be because the deviation of RMP caused by the tachometer, which could affect Kb and Kt, then generated the error of Dm, then led to the Jm deviation.

4. Simulation of the motor response

(a) The transfer function parameters was shown the table below:

Motor Parameters	Average Value
R_a	13.58
L_a	0.286
$K_b=K_t$	0.010866
D_m	2.68E-5
J_m	1.4499E-6
$K_t/(J_mL_a)$	2.62E+4
$(J_mR_a+D_mL_a)/J_mL_a$	1.95E+1
$(R_aD_m+K_bK_t)/J_mL_a$	1.16E+3

Table 6: transfer function parameters

(b) We simulated the step response with the input of 1, 2, 3, 4, 5 and 6V, Matlab print out shown in the figures below:

```
clear all
clf
Ra = 13.58;
La = 0.286;
Kb =0.010866;
Kt = 0.010866;
Dm = 2.685e-5;
Jm = 1.4499e-6;
% Top
top = Kt/(Jm*La);
% Bottom Left
BL = (Jm*Ra+Dm*La)/(Jm*La);
% Bottom right
BR = (Ra*Dm+Kt*Kb)/(Jm*La);

% The step response of 1,2,3,4,5,6V input.
hold on;
for i = 1:6
    num=[i*top];
    den=[1 BL BR];
    sys=tf(num,den)
    stepResults = stepinfo(sys);
    % Print out settling time
    settlingTime = stepResults.SettlingTime;
    disp(['When input = ', num2str(i), 'V', ', settling time = ',
 num2str(settlingTime)])
    % Print out time constant
    damp(sys);
    stepplot(sys);
end

grid on
hold off
legend('1V','2V','3V','4V','5V','6V')
```

```
sys =

       2.62e04
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 1V, settling time = 0.16002
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-3.30e+01 + 8.66e+00i	9.67e-01	3.41e+01	3.03e-02
-3.30e+01 - 8.66e+00i	9.67e-01	3.41e+01	3.03e-02

```
sys =

      5.241e04
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 2V, settling time = 0.16002
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-3.30e+01 + 8.66e+00i	9.67e-01	3.41e+01	3.03e-02
-3.30e+01 - 8.66e+00i	9.67e-01	3.41e+01	3.03e-02

```
sys =

      7.861e04
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 3V, settling time = 0.16002
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-3.30e+01 + 8.66e+00i	9.67e-01	3.41e+01	3.03e-02
-3.30e+01 - 8.66e+00i	9.67e-01	3.41e+01	3.03e-02

```
sys =

      1.048e05
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 4V, settling time = 0.16002
```

Pole	Damping	Frequency (rad/seconds)	Time Constant (seconds)
-3.30e+01 + 8.66e+00i	9.67e-01	3.41e+01	3.03e-02
-3.30e+01 - 8.66e+00i	9.67e-01	3.41e+01	3.03e-02

```
sys =

       1.31e05
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 5V, settling time = 0.16002

         Pole               Damping      Frequency      Time Constant
                                       (rad/seconds)     (seconds)

 -3.30e+01 + 8.66e+00i     9.67e-01      3.41e+01        3.03e-02
 -3.30e+01 - 8.66e+00i     9.67e-01      3.41e+01        3.03e-02

sys =

      1.572e05
  -----------------
  s^2 + 66 s + 1164

Continuous-time transfer function.

When input = 6V, settling time = 0.16002

         Pole               Damping      Frequency      Time Constant
                                       (rad/seconds)     (seconds)

-3.30e+01 + 8.66e+00i     9.67e-01      3.41e+01        3.03e-02
-3.30e+01 - 8.66e+00i     9.67e-01      3.41e+01        3.03e-02
```

The Matlab outcome showed that the step response with six different step input had the same **time constant** (0.0303s) and **settling time** (0.16002s).

Steady state value was shown in the step response plot below:

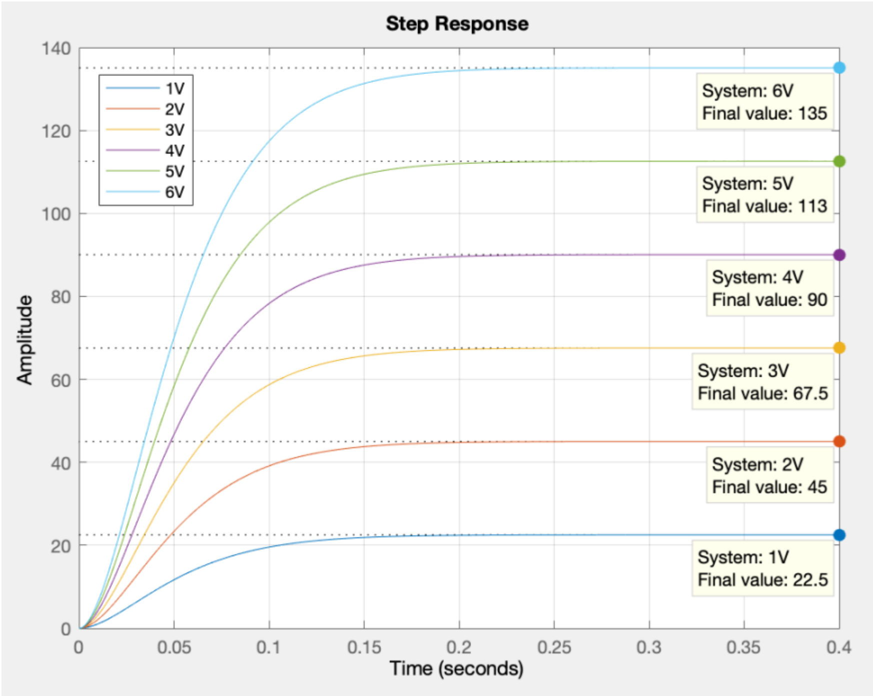


Figure 6: step response and steady state value

The figure proved that higher step input had a higher steady state value.

In order to calculate **steady state gain**, we set s=0 into the transfer function, which shown in the Matlab print out below:

```
clear all
Ra = 13.58;
La = 0.286;
Kb =0.010866;
Kt = 0.010866;
Dm = 2.685e-5;
Jm = 1.4499e-6;
% Top
top = Kt/(Jm*La);
% Bottom Left
BL = (Jm*Ra+Dm*La)/(Jm*La);
% Bottom right
BR = (Ra*Dm+Kt*Kb)/(Jm*La);

% The step response of 1,2,3,4,5,6V input.
for i = 1:6
    s = tf('s');
    % substituting s=0 into a transfer function
    s = 0;
    sys = (i*top)/(s^2 + BL*s + BR);
    disp(['When input = ', num2str(i), 'V, ', 'steady state gain is ',
 num2str(sys)])
end

When input = 1V, steady state gain is 22.5112
When input = 2V, steady state gain is 45.0224
When input = 3V, steady state gain is 67.5336
When input = 4V, steady state gain is 90.0448
When input = 5V, steady state gain is 112.556
When input = 6V, steady state gain is 135.0672
```

The Matlab outcome showed that higher step input voltage had a higher steady state gain.

The **unit of steady state gain** depended on the type of the system. For example, if the system were a standard pneumatic instrument loop, then the unit would be psig/unit

(c) The design method to measure the actual response in the rotational velocity with a step input voltage was:

Measuring the mechanical time constant of the system. It could be achieved by turn on the motor. Then the time constant could be measured from the increase in angular velocity with time.

It was similar to measuring Jm. We planned to put a switch and a voltage divider with the motor in series, the voltage divider consisting of a resistor and an LDR. LDR connected to the oscilloscope as output. Pointing the LDR probe to the propeller then switched on the motor, the rotation velocity increased. Whenever the blade passed the LDR, there was a step input to the oscilloscope.

Very similar but opposite to the La part, we would see a plot with a series of steps but with decay delta t in the oscilloscope monitor. The expected outcome plot shown below:

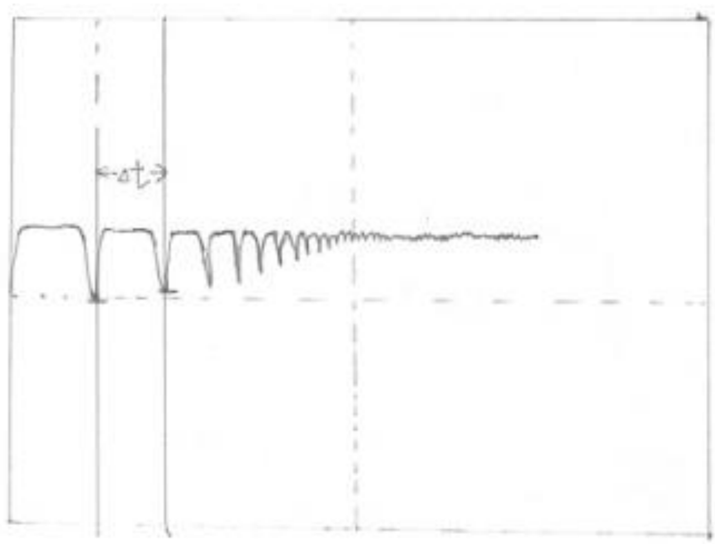


Figure 7: expected plot of outcome

We could pick a period of on step with good looking and easy to measure. Then measure the time interval of each step and calculated the current overall time of each stage. Using a tachometer to measure the RPM at the final steady state, calculate the rest of the RPM according to the overall time proportion, and then convert the RPM value to the angular velocity. The estimate measuring value shown in the table below:

<table>
<tr><th></th><th>Δt(ms)</th><th>Overall time (ms)</th><th>RPM</th><th>W(rad/s)</th></tr>
<tr><td>1</td><td rowspan="10">Numbers decrease then tend to be stable.</td><td rowspan="10">Numbers increase rapidly at the beginning, then tend to increase slowly, finally the rise step will remain same.</td><td rowspan="9">Similar pattern with Overall time.</td><td rowspan="9">Same pattern with RPM.</td></tr>
<tr><td>2</td></tr>
<tr><td>3</td></tr>
<tr><td>4</td></tr>
<tr><td>5</td></tr>
<tr><td>6</td></tr>
<tr><td>7</td></tr>
<tr><td>8</td></tr>
<tr><td>9</td></tr>
<tr><td>10</td><td>Final Steady state RPM</td><td>Final angular velocity</td></tr>
</table>

Table 7: estimate value of measurement

The actual measured value might not be the same as the value above but should have the same increasing or decreasing pattern. Then the step response could be plotted with the number of step at x and angular velocity at y.

However, due to the limitation of time, we could not complete the expected measurement. We only measured the RPM at final steady state, then convert it to angular velocity, which shown in the table below:

Step input (V)	W (rad/s)
1	72.5
2	149.75
3	268.86
4	356
5	418

6	471

Table 8: angular velocity of final steady state

5. Modelling the pendulum part of the system

(a) Modeling the rotational velocity to thrust conversion

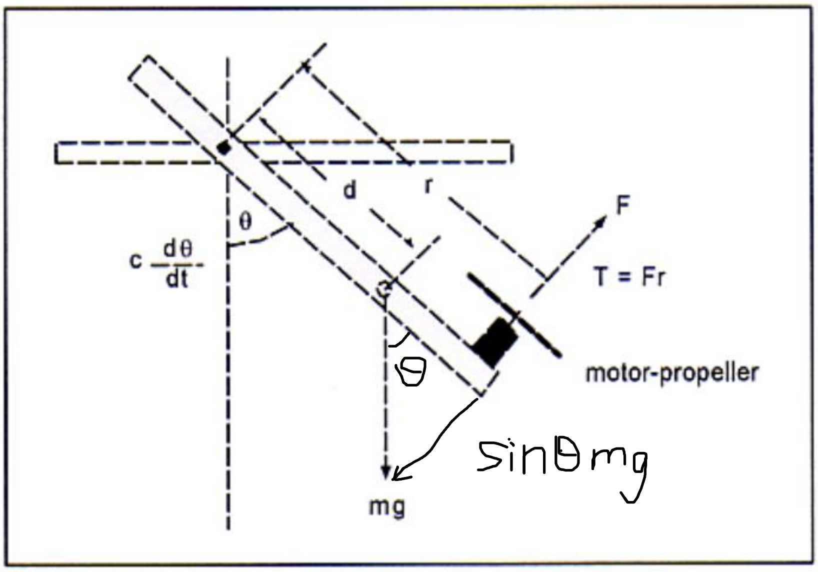


Figure 8: thrust

As shown in the figure above, the thrust F = sinθ*mg.

Assume θ is a small angle, so sinθ ≅ θ. Given that m = 0.168kg, So F = θ*mg = θ*0.168*g.

(b) Modelling the rigid pendulum

The angular displacement of the pendulum was produced by a gravitational force **mg*r*sinθ** and the thrust **F.** Corresponding to the equation of applied torque:

$$mr^2(\frac{d^2\theta}{dt^2}) = F - mg \cdot r\sin\theta = mg \cdot \theta - mg \cdot r \cdot \theta$$

$$r^2(\frac{d^2\theta}{dt^2}) = g\theta(1-r)$$

(c) Measurement of system parameters for the pendulum

Given the ‘Data.csv’ file, we read the data and drew the pendulum’s damping plot as shown below:

```
clear all
T = readtable('Data.csv');
Tx = T.Var1
Ty = T.Var2
plot(Tx,Ty)
grid on
```

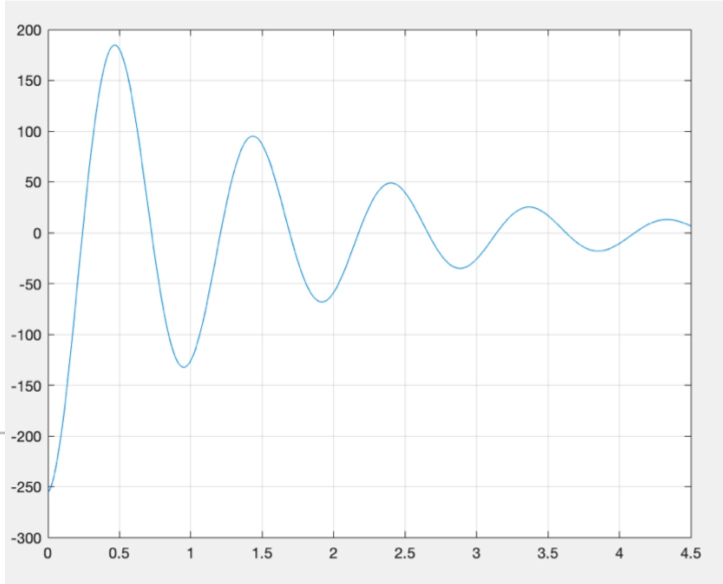


Figure 9: pendulum damping

The figure showed that there were n = 10 damping cycle (include positive and negative). In the given data, the first positive peek value was Xi = 184.663196410543, so we wrote the following code to calculate damping coefficient c:

```
n = 10; % number of cycles
Xi = 184.663196410543; % first peak
% Logarithmic decrement
nDelta = log(Xi/((1-0.6)*Xi)) % (1-0.6) means 60% reduction after 10 cycles
delta = nDelta/n
% Dampling factor
zeta = delta/sqrt((2*pi)^2+delta^2)
% Dampling Coefficient
c = zeta*50 % 50 is the critical damping constant
```

The outcome was nDelta = 0.9163, delta = 0.0916, zeta = 0.0146 and damping coefficient **c = 0.7291.**

Given in the lab handout, $y(t) = Ae^{-Bt}\cos(\omega t + \varphi)$, we had

A = -256 (the first y value in the Data.csv),

y(t) = 184.663196410543 (the y value of the first positive peek),

t = 0.47 (the x value of the first positive peek),

w = 2 π /T = 6.545,

T = 0.96 (interval between two adjacent positive peek) and φ = 0.

So according to the WolframAlpha's calculation:

$$184.663196410543 = -256\, e^{-B\times 0.47}\cos(6.545\times 0.47)$$

$184.663196410543 = 255.452\, e^{-0.47\, B}$

$e^{0.47\, B} = 1.38334$

B = 0.69

Given another equation in the lab,

$$\omega = \sqrt{\frac{mgd}{Jp} - B^2}$$ (g = 6.67408*10^-11)

We got

$$6.545 = \sqrt{\frac{0.168 \cdot 6.67408 \times 10^{-11} \cdot 0.14}{Jp} - 0.69^2}$$

$$6.545^2 = \frac{0.168 \cdot 6.67408 \times 10^{-11} \cdot 0.14}{Jp} - 0.69^2$$

42.837025 + 0.4761 = (0.1569743616*10^-11) / Jp

So $Jp = 3.62418 \cdot 10^{-14}$

The system transfer function for the driven pendulum is

$$\frac{22.913}{0.206\, s^4 + 13.6244\, s^3 + 241.9\, s^2 + 48.27\, s + 268.302}$$

4. Discussion

- Before simulating the motor's step response, we did not expect that the time constant and the settling time under different voltage would be the same.
- The most significant finding in this lab is the transfer function of the sub-systems because we could simulate the systems and compare them to their actual response.
- For modelling the motor system, even if the Jm value was different from the expected value, the simulation results work well. So this model could be used for the following lab.
- We had only a limited time to complete it for modelling the pendulum system, so that model was not very useful.
- When modelling a complex system, a handy method was to break the system into several sub-systems, then modelling them.

- Two essential techniques when modelling the system were the block diagram and the transfer function. Drawing the block diagram could help derive the transfer function.
- To control the drone or the helicopter hover at a certain altitude, somewhat similar to this lab project. For example, if we needed to model the complex drone system, we must decompose it into serval sub-systems, then derived the transfer function.

5. Conclusions

In this lab, we decomposed a complex motorised pendulum system into a motor system and a pendulum system, and used block diagram technique to further decompose each of the sub-system, and obtain its transfer function, then simulated its step response by Matlab. Also, compare it with the system's actual response.

References

[1] Ecs.wgtn.ac.nz. 2021. [online]. Available: https://ecs.wgtn.ac.nz/foswiki/pub/Courses/ECEN315_2021T1/Laboratories/Lab_Project_Lab1.pdf

[2] Ecs.wgtn.ac.nz. 2021. [online]. Available: https://ecs.wgtn.ac.nz/foswiki/pub/Courses/ECEN315_2021T1/Laboratories/Lab_Project_Lab2.pdf

[3] Understanding small brushed DC motors, G.J. Gouws, August 2008

Appendices

The experimental apparatus used in this ECEN315 project poses several particular hazards. To avoid the occurrence, a risk assessment shown below might be helpful:

No.	Hazard	Cause	Probability	Severity	Mitigation
1	Fan	Putting finger in Fan	6/10	5/10	Avoid putting your finger in the fan

2	Fan	Getting your hair stuck in the fan	5/10	5/10	Tie your hair up or wear a hat
3	Fan	Getting the paper suck into the fan while running	6/10	2/10	Press your paper while fan is running
4	Pendulum fall	When clamping the pendulum on the table, it might fall if the screw is not tightened.	1/10	1/10	Tighten the screw while clamping the pendulum
5	Electrocution	Touching exposed wires	2/10	6/10	Make sure wires are insulated, and avoid touching that area
6	Burn	Touching hot motor	4/10	7/10	Make sure to not touch this area of the device
7	Cut	Touching sharp parts on the circuit board	3/10	6/10	Keep your fingers clear of the circuit board
8	Motor	Motor might be broken if apply large voltage which over limit	3/10	5/10	Check the voltage of power supply whenever you turn on
9	Cables	When pull out the cables, equipment might be broken if apply large force	2/10	3/10	Pull out slightly
10	Fingers	Fingers might get stuck in between the pendulum and circuit board	1/10	1/10	Don't put the finger in between