ECEN315 Assignment 4

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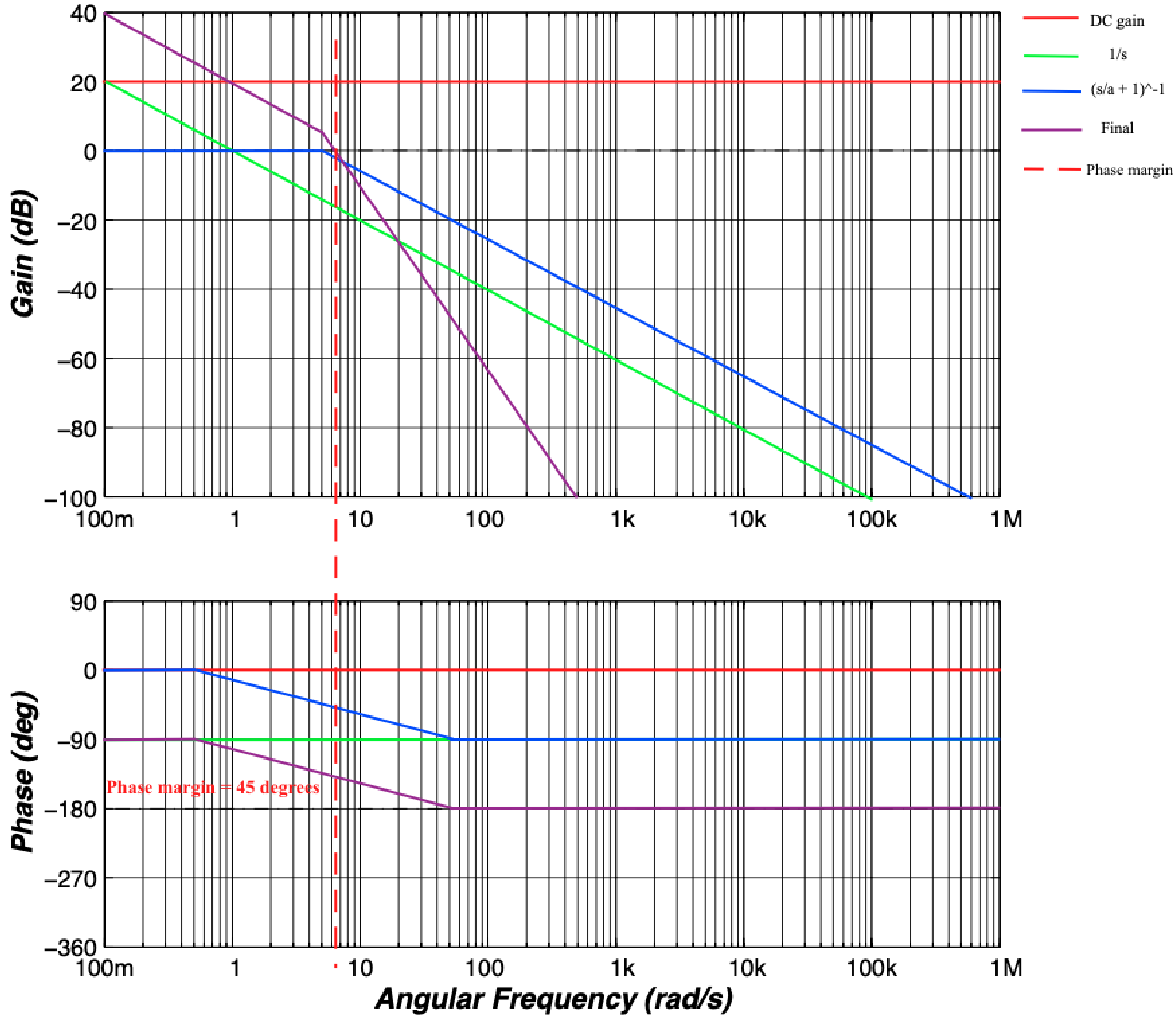
Bode Plots

1.

(1) $\frac{50}{s^2+5s} = \frac{50}{s(s+5)} = \frac{50}{5s(\frac{s}{5}+1)} = 10 \times \frac{1}{s} \times \frac{1}{\frac{s}{5}+1}$

DC gain = 20log(10) = 20

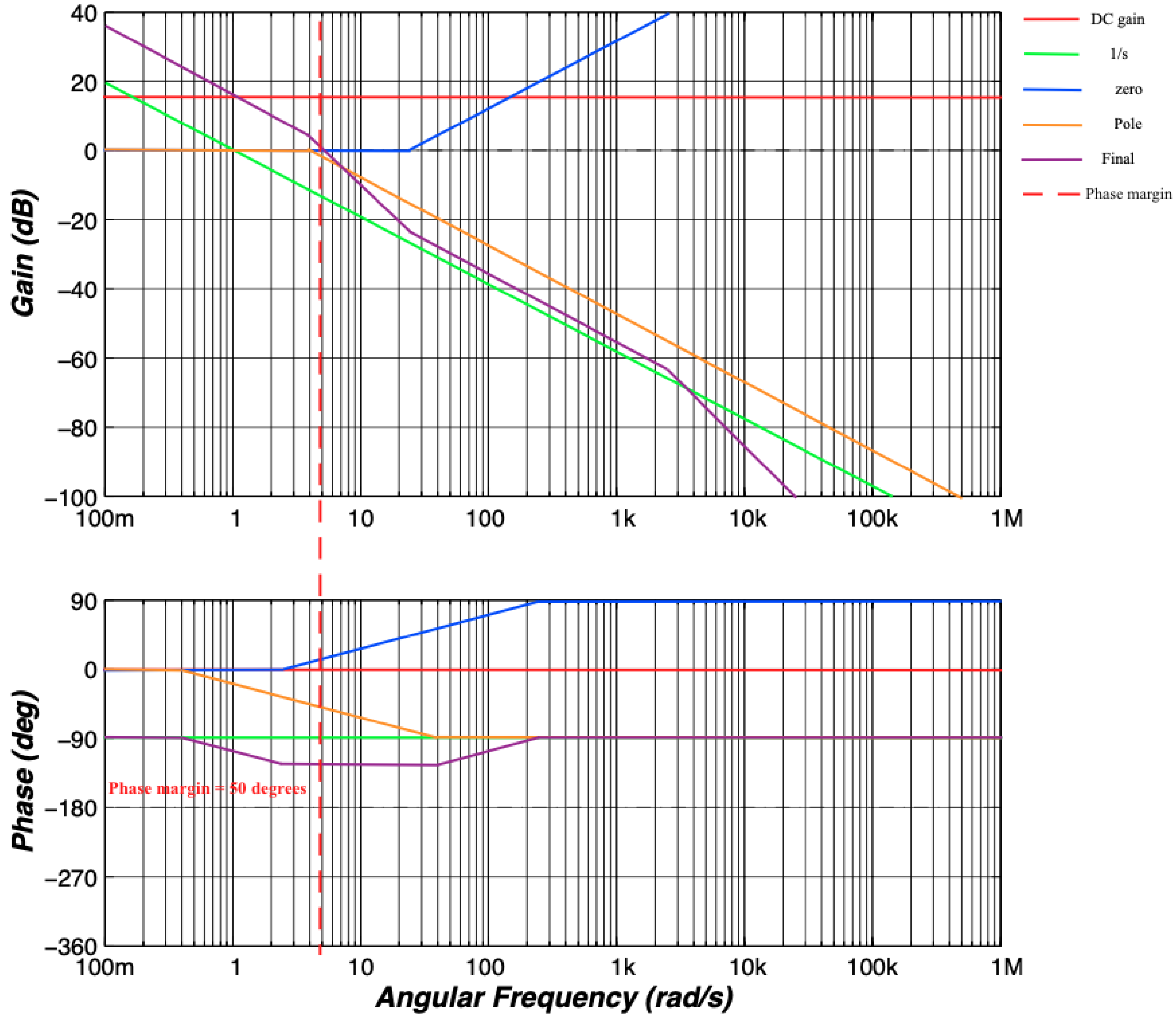


From the bode plot above we can see, there is no point that the phase will cross -180°, so the closed loop system will not be unstable for all values of K. Gain margin = inf dB

At the point of Gain = 0db, the phase is about -135°, so the phase margin = 180 – 135 = 45°. Thus, the closed loop system's phase can be decreased about 45° before it becomes unstable.

(2) $\frac{s+25}{s^2+4s} = \frac{s+25}{s(s+4)} = \frac{25(\frac{s}{25}+1)}{4s(\frac{s}{4}+1)} = \frac{25}{4}\frac{1}{s}\frac{(\frac{s}{25}+1)}{(\frac{s}{4}+1)}$

DC gain = 20log(25/4) = 16



From the bode plot above we can see, there is no point that the phase will cross -180°, so the system will not be unstable for all values of K. Gain margin = inf dB.

At the point of Gain = 0db, the phase is about -130°, so the phase margin = 180 – 130 = 50°. Thus, the closed loop system's phase can be decreased about 50° before it becomes unstable.

(3) $\frac{640}{s^3+16s^2+64s} = \frac{640}{s(s^2+16s+64)} = \frac{1}{s} \times \frac{640}{s^2+16s+64}$

Deriving the function as the form of

$$\frac{1}{s^2+2\varsigma\omega s+\omega^2}$$

Therefore, we have

$$\frac{640}{s^2+2\varsigma 8s+8^2}$$

The damping ratio: $2\varsigma 8 = 16, \varsigma = 1$

The break point ω = 8

For getting the DC gain, we need to do,

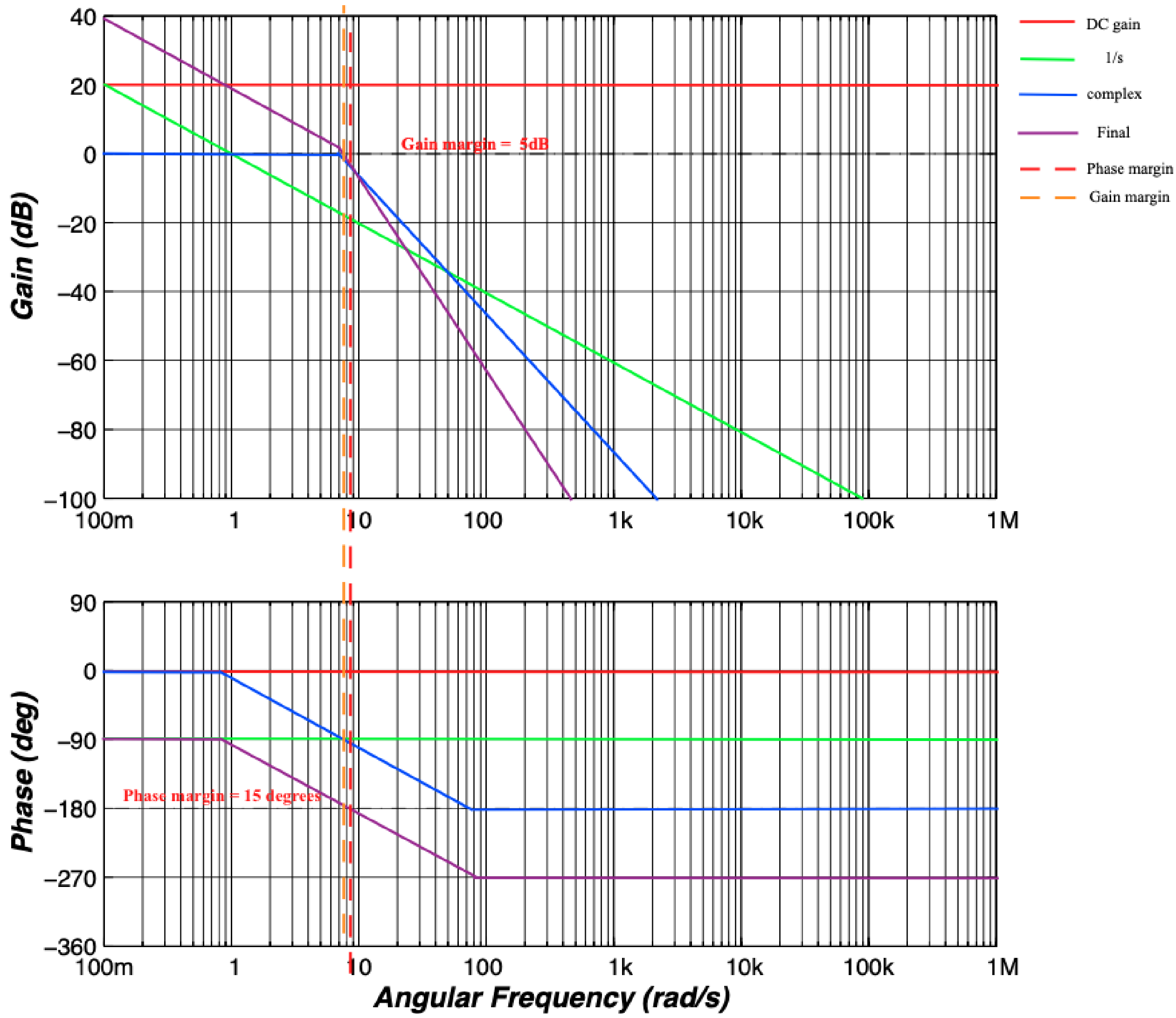
$$\frac{640}{s^3+16s^2+64s} = \frac{640}{s(s^2+16s+64)} = \frac{640}{64s(\frac{s^2}{8^2}+\frac{s}{4}+1)} = 10 \times \frac{1}{s} \times \frac{1}{\frac{s^2}{8^2}+\frac{s}{4}+1}$$

So DC gain = 20log(10) = 20 dB,

The complex roots system act same as repeated roots, the magnitude drops at

- 40dB/decade, when beyond the break point. And the phase drops from 0 to -180 degrees.

So we have following bode plot approximation,



From the bode plot above we can see, at the point of phase equals -180°, the gain is about -5 dB (1.78), so the gain margin is approximately 5dB. Thus, the closed loop system's gain can be increased about 5 dB before the system goes unstable.

At the point of Gain = 0db, the phase is about -165°, so the phase margin is around 15°. Thus, the closed loop system's phase can be decreased about 15° before it becomes unstable.

2. for the original system,

$$G(s) = \frac{5400}{s^2 + 60s + 900}$$

Deriving the function as the form of

$$\frac{1}{s^2 + 2\varsigma\omega s + \omega^2}$$

Therefore, we have

$$\frac{5400}{s^2 + 2\varsigma 30s + 30^2}$$

The damping ratio: $2\varsigma 30 = \ 60, \varsigma = 1$

The break point $\omega = 30$

For getting the DC gain, we need to do,

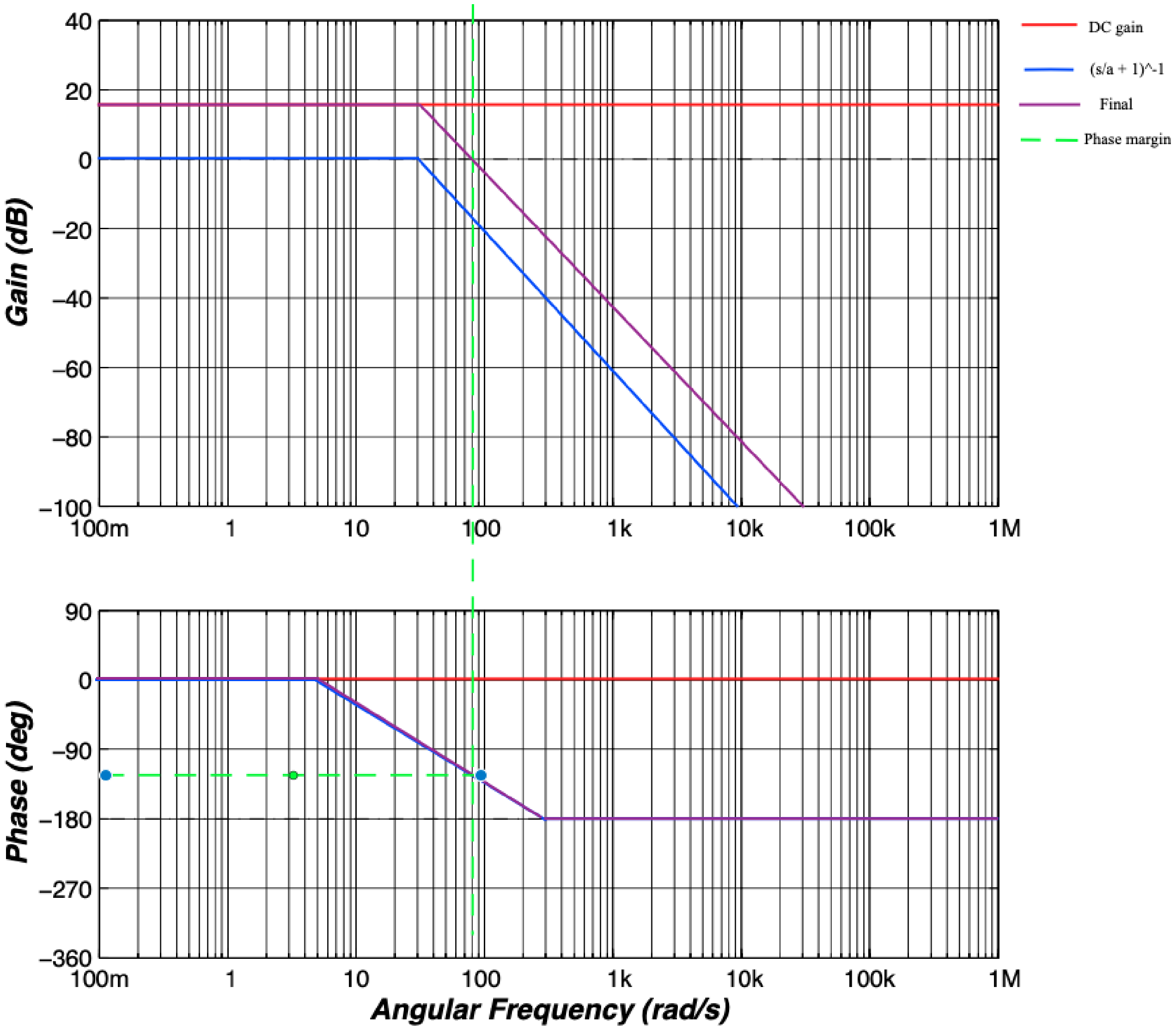
$$\frac{5400}{s^2 + 60s + 900} = \frac{5400}{900(\frac{s^2}{30^2} + \frac{s}{15} + 1)} = 6 \times \frac{1}{\frac{s^2}{30^2} + \frac{s}{15} + 1}$$

So DC gain = 20log(6) = 16 dB,

The complex roots system act same as repeated roots, the magnitude drops at

- 40dB/decade, when beyond the break point. And the phase drops from 0 to -180 degrees.

So we have following bode plot approximation for uncompensated plant,

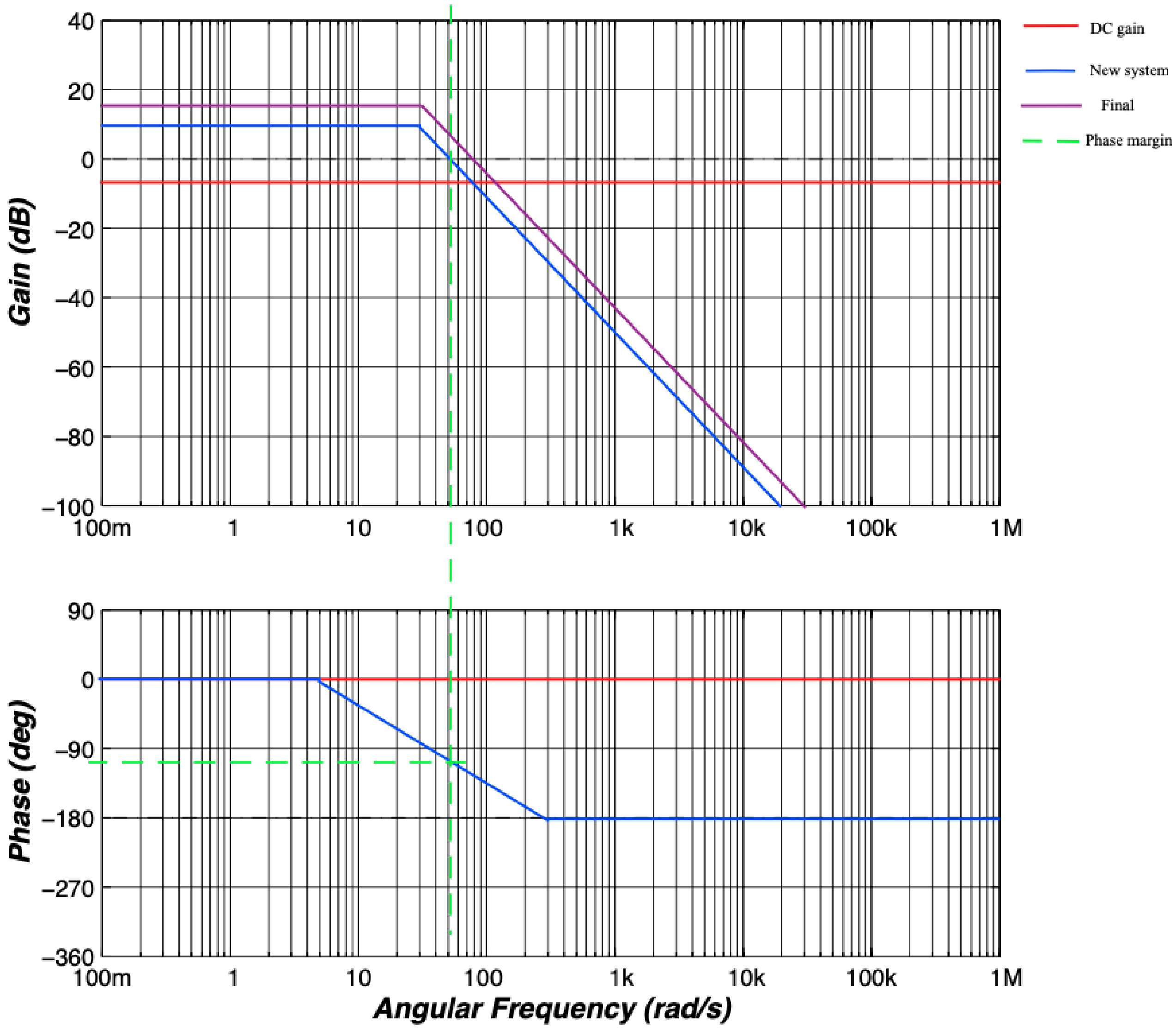


According to the matlab calculation, the phase margin of uncompensated system is 48.2 deg.

For the first step, adding a Gain between 0 to 1 to shift down to gain plot.

Set gain = 0.5, DC gain = 20log(0.5) = -6 dB

The bode plot of the system after adding gain of 0.5 shown below,



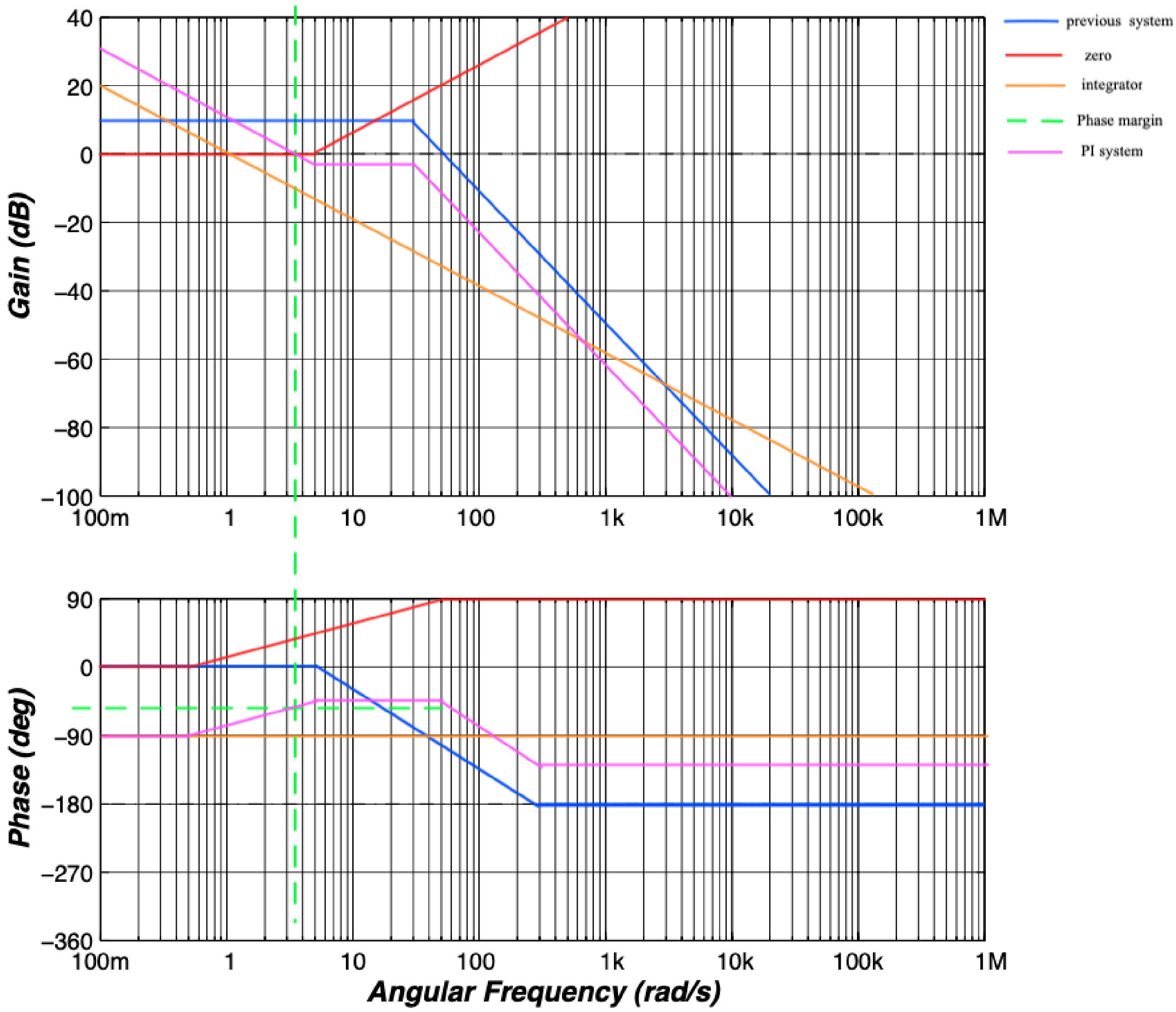
The original system has been shifted down and the phase margin has been increase to around 70°, which is greater than 50°

So new transfer function will be G(s) = $0.5 \times 6 \times \frac{1}{\frac{s^2}{30^2}+\frac{s}{15}+1} = 3 \times \frac{1}{\frac{s^2}{30^2}+\frac{s}{15}+1}$

It would be ideal to add a pole at the origin and a zero to cancel out the pole. However, if the zero is too far away from the pole the system will become unstable. This will remove the steady state error as it reduces the system to a type 1 system where a step input will have 0 steady state error. Thus a PI compensator will be introduce,

$$PI = \frac{s+5}{s}$$

Set the zero at -5, apply the PI to new transfer function, we have the bode plot below,



The compensated system now is type, which has zero steady state error for a step input. According to Matlab calculation, the new phase margin is 60°, which is over 50°.

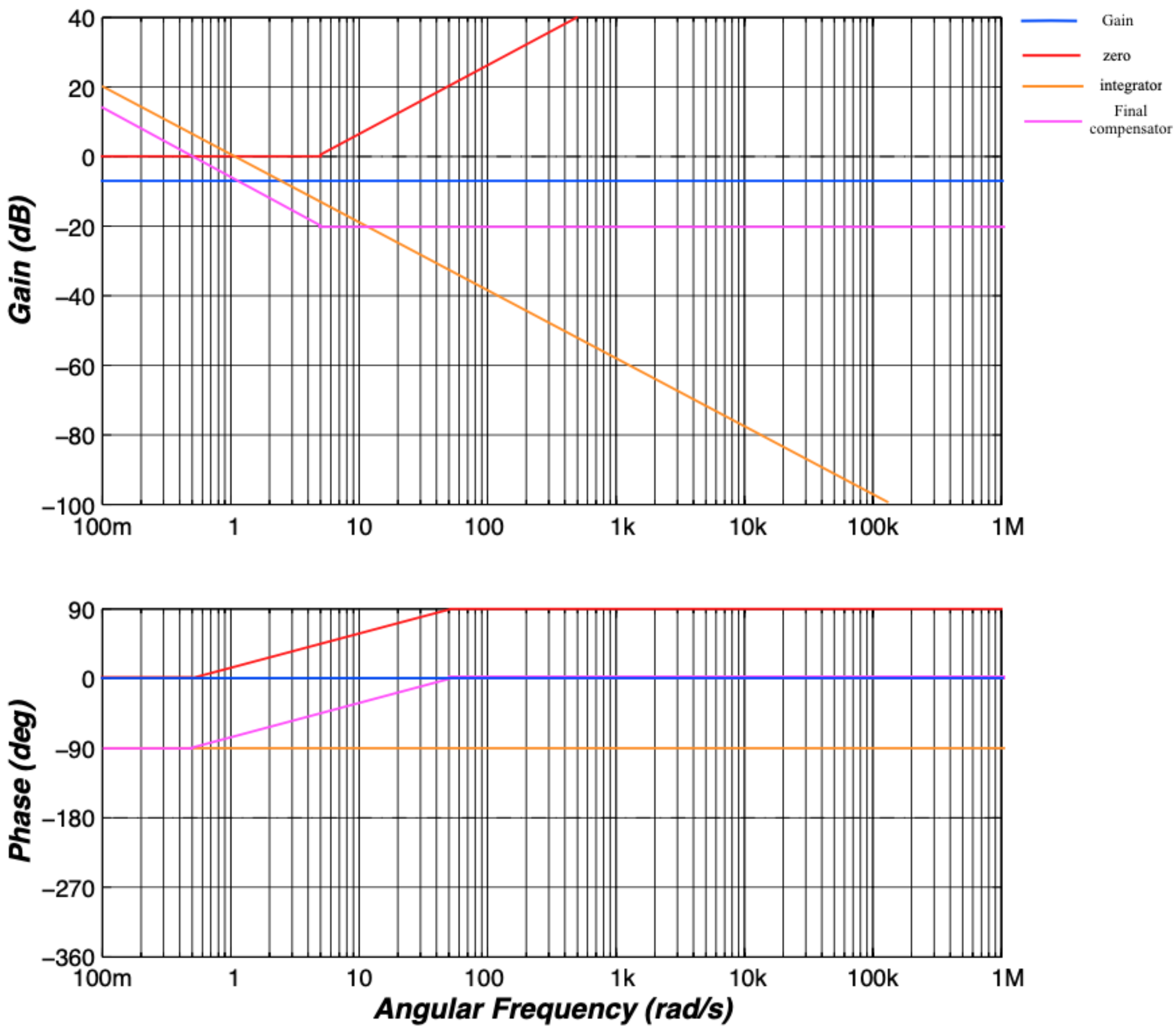
So the final transfer function is,

$$G(s) \times PI = \frac{5400}{s^2 + 60s + 900} \times 0.5 \times \frac{s+5}{s} = \frac{2700s + 13500}{s^3 + 60s^2 + 900s}$$

Therefore, our final PI compensator is,

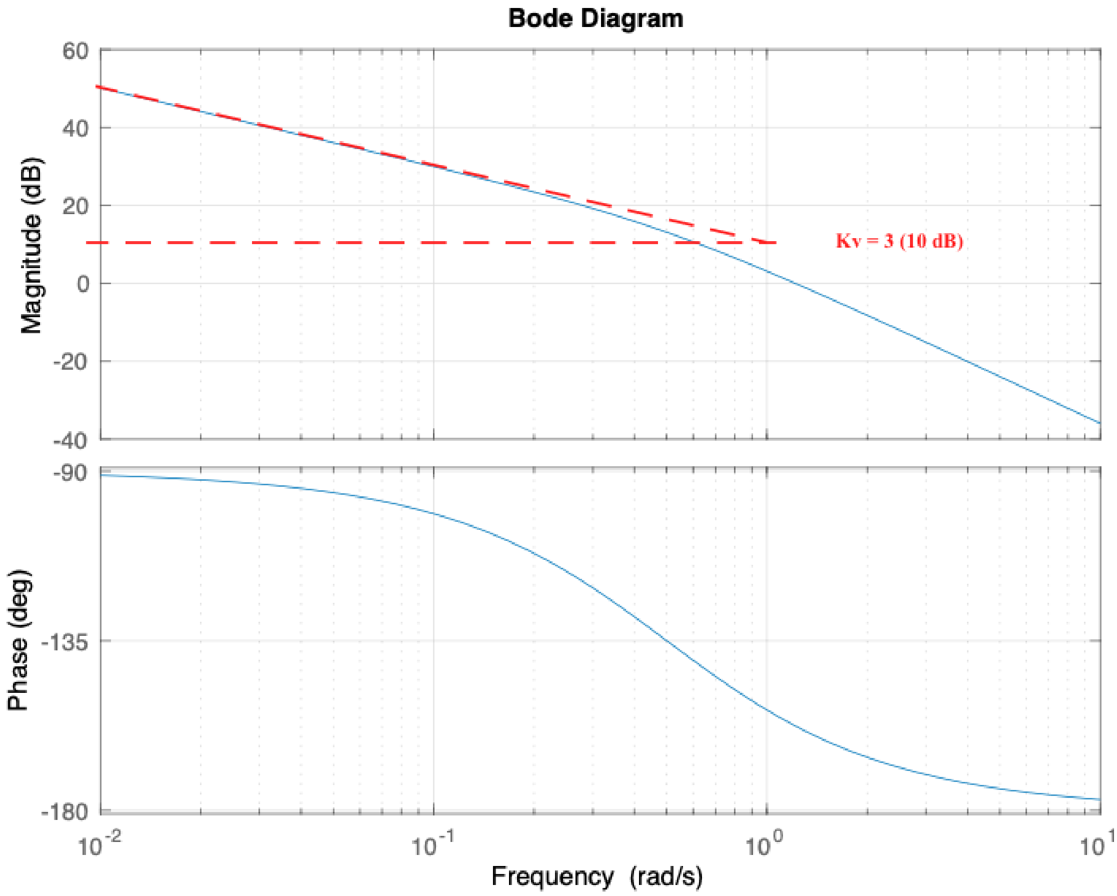
$$PI = 0.5 \times \frac{s+5}{s}$$

The bode plot of final PI compensator shown below,



40
20
0
−20
−40
−60
−80
−100
Gain (dB)
100m
1
10
100
1k
10k
100k
1M
Gain
zero
integrator
Final compensator
90
0
−90
−180
−270
−360
Phase (deg)
Angular Frequency (rad/s)

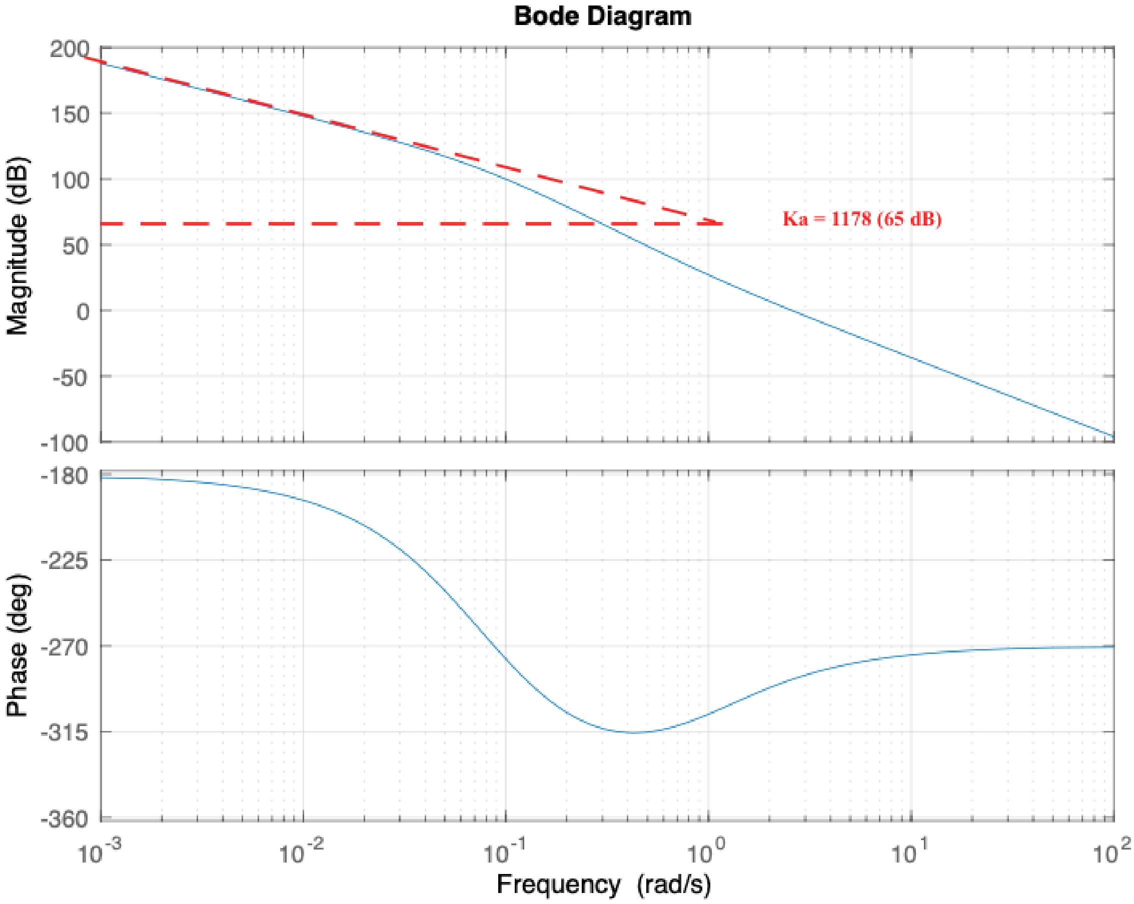
3. (1)



The system above has a slope of -20 dB/decade and a phase of -90° at low frequencies, therefore, it's a type 1 system.

It has zero steady state error for a step input, an infinite error for a parabola input, and error of 1/Kv = 1/3 for a ramp input that shown in the bode plot above.

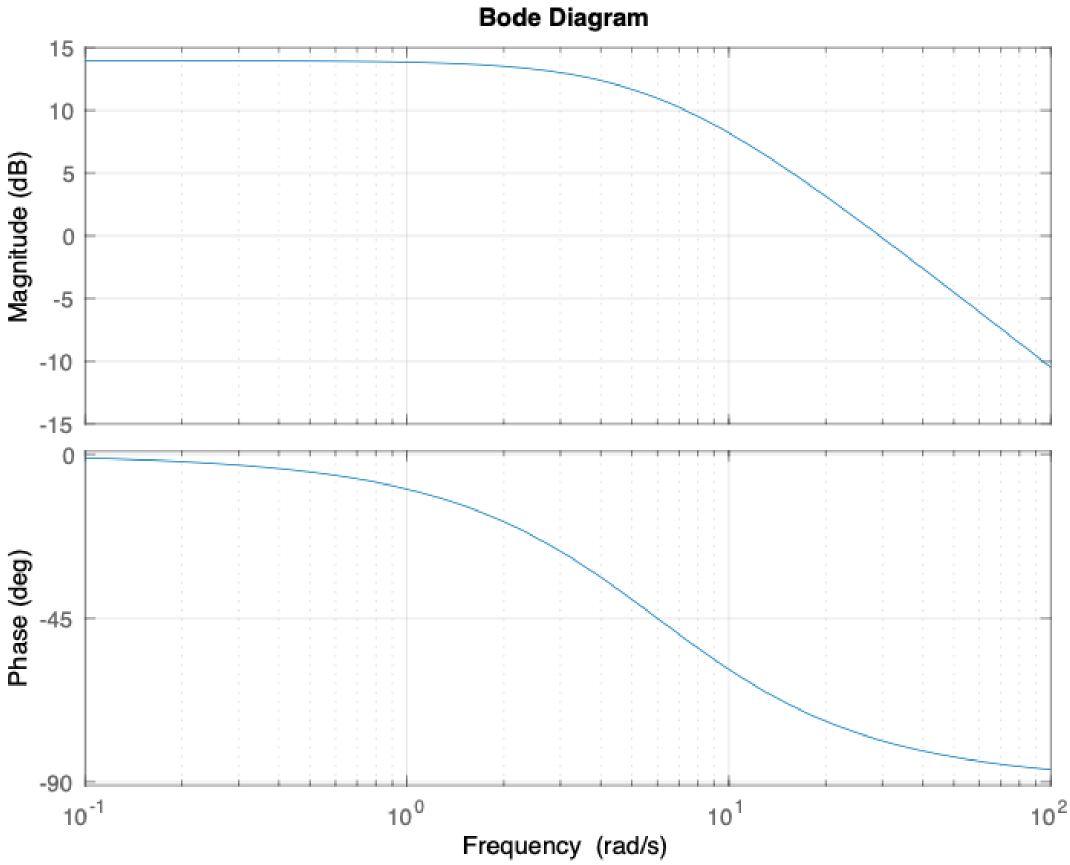
(2)



The system above has a slope of -40 dB/decade and a phase of -180° at low frequencies, therefore, it's a type 2 system.

It has zero steady state error for step input and ramp input, error of 1/Ka = 1/1178 for parabola input that shown in the bode plot above.

(3)



The system above has a slope of 0 and a phase of 0^o at low frequencies, therefore, it's a type 0 system.

It has infinite steady state error for ramp input and parabola input.

It's a straight line at the low frequency of gain plot,

therefore Gain = 14 dB,

20log(x) = 14, x = $10^{14/20}$ = 5, so Kp = 5

Steady state error for step input is $\frac{1}{1+Kp} = \frac{1}{6}$