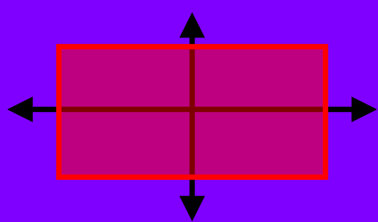
# Functions of Several Variables

Domain- set of all possible input values

Range- set of all possible output values

Domain of 2 variables



If doesn't include endpoints we use dotted lines

For two variable functions domain is set of ordered pairs (a region on the XY plane)

For two variable functions range is still a one value

We know z=f(x,y) is a surface so if we set z equal to a constant then the graph $z_0 = f(x, y)$ is called a level curve (basically a trace)

If $\lim_{x\to a} f(x) = L$ then the closer a gets to x the closer f(x) gets to L

Conditions of continuity

1. f(c) exists
2. $\lim_{x\to c} f(x)$ exists
   - (All paths lead to the same limit)
3. $\lim_{x\to c} f(x)$ =f(c)

If $\lim_{x\to a} f(x) = f(a)$ then f(x) is continuous at x=a

Polynomials, sine, cosine and exponential functions are continuous functions

Rational and log functions are continues where they are defined

$\lim_{(x,y)\to a,b} f(x, y) = L$

Approaching a two function limit is more complicated because their are infinite paths (along x=a, y=b along b/ax parabola exponential etc

If the limit of two different paths don't mach than the limit does not exist (but we can't check all)

Check the type of function it is and if its continuous at f(a,b) than $\lim_{(x,y)\to a,b} f(x, y) = f(a, b)$

For z=f(x,y) z is affected by x and y so we can not take the derivative with both changing
We can hold one variable constant and allow the other to change to get the partial derivative

Partial derivative- the rate of change of z with respect to the variable that is changing

$$\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$\frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

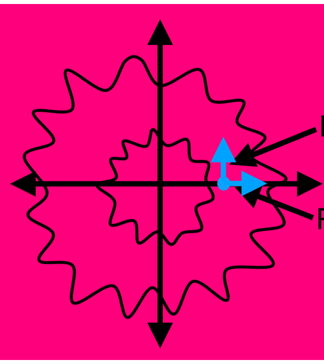
$F_x$ and $F_y$ tell us the effect of varying on variable at a time

Clairaut's Theorem- $F_{xy} = F_{yx}$ wherever f(x,y) is defined and $F_x$ and $F_y$ are continuous

$y = f(u)$ $u = g(t)$ and $f(g(t))$ then $y' = f'(g(t)) * g'(t)$

To find out how fast z is changing use the chain rule
$z = f(x, y)$ and $y = g(t)$ and $x = h(t)$
Then $\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

Implicit Differentiation
If we have F(x,y) we can differentiate implicitly to find $\frac{dy}{dx} = -\frac{F_x}{F_y}$




Directional Derivative

Let $\overrightarrow{U} = \langle a, b \rangle$ be a unit vector (giving direction). Than the rate of change in that direction is

$$D_{\overrightarrow{u}} f(x, y) = \lim_{h \to 0} \frac{f(x + ah, y + bh) - f(x, y)}{h}$$

$Gradient = \nabla = \langle F_x, F_y \rangle$

$D_{\overrightarrow{u}} f(x, y) = \langle F_x, F_y \rangle * \langle a, b \rangle$

Max directional derivative occurs when $\theta = 0$ and the value of $D_{\overrightarrow{u}} f(x, y) = \overrightarrow{\nabla} f$

Along the level curve the height does not change so must move along level curve (tangent to level curve)

Min directional derivative occurs when $\theta = 180$ and the value of $D_{\overrightarrow{u}} f(x, y) = -\overrightarrow{\nabla} f$

Gradient is perpendicular to the level curve

$D_{\overrightarrow{u}} f(x, y) = 0$ $if$ $\overrightarrow{\nabla} f \perp \overrightarrow{U}$ $\theta = \frac{\pi}{2}$ $or$ $\frac{3\pi}{2}$ is the only directions you go to see no change

If the level curve is $\vec{r}(t) = \langle x(t), y(t) \rangle$ then

$$(x_0, y_0) \langle F_x, F_y \rangle \cdot \frac{\langle \frac{dx}{dt}, \frac{dy}{dt} \rangle}{|\langle \frac{dx}{dt}, \frac{dy}{dt} \rangle|} = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

z=f(x,y) is a surface so instead of finding a tangent line at point $(x_0, y_0, z_0)$ we actually have infinitely meany tangent lines which form tangent plane

When very near to $(x_0, y_0, z_0)$ tangent plane roughly equals true surface

Infinitely many curves form curve$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ and their tangent vectors are are $\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$ and the normal vector is perpendicular to all the tangent vectors

The normal vector of the tangent plane is the gradient

Equation for tangent plane

$$z - z_0 = F_x(x - x_0) + F_y(y - y_0)$$

$$z - z_0 = F_x(\Delta X) + F_y(\Delta Y)$$

The smaller dx and dy are the closer dz is

$\% x = \frac{dx}{x}$

Critical points occur when $F_x$ and $F_y$ are both equal to zero

If D>0 and Fx<0 its a maximum

Second derivative test $D = F_{xx} * F_{yy} - F_{xy}^2$

If D>0 and Fx>0 its a minimum

If D=0 test is inconclusive and you have to check points around it

If D<0 its a saddle point

To find absolute max and main find all critical points in regions find all end critical points and find all corner points

Plug them all into the equation and see which yields the largest and smallest value

Domain and Range | Directional Derivatives | Limits | Continuity | Chain Rule

Partial Derivatives | Tangent Plane and Partial Derivative | Max and Mins