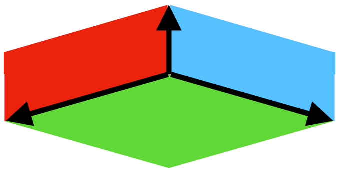
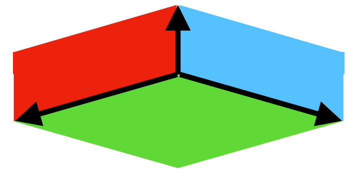
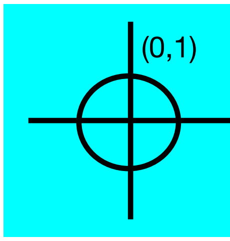


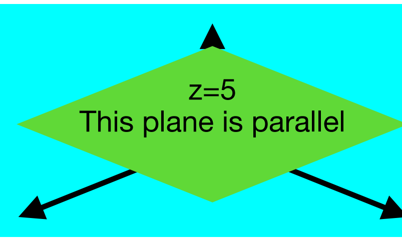
3D Surfaces

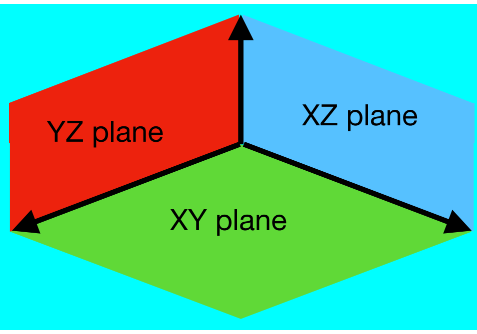


Coordinates are needed to write equations

Coordinates are geometric visual representations
(ie circle with a radius of 1)







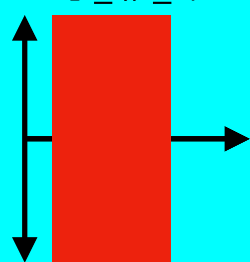
3 numbers (a,b,c) mean there is a point with coordinates a,b,c

Vectors have 2 properties a direction and a magnitude

Equations are algebraic
(ie $x^2 + y^2 = 1$)

Equation for sphere of radius r centered at (a,b,c)
$x^2 + y^2 + z^2 = 1$

Use inequalities to represent region
$1 \le x \le 4$



Ball is $x^2 + y^2 + z^2 \le 1$

Disk is $x^2 + y^2 \le 1$

Circle is $x^2 + y^2 = 1$

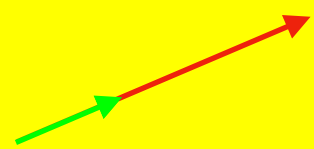
Upper half of sphere
$x^2 + y^2 + z^2 \le 4, z \ge 0$

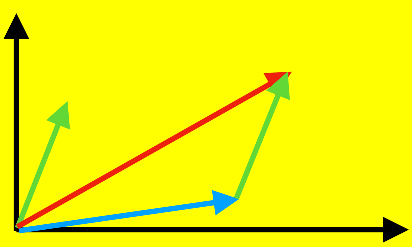
Vectors- 2 ordered points $\langle$a,b,c$\rangle$

$\overrightarrow{PQ}$ is a vector from point P to point Q

Adding Vectors
$\langle x, y, z\rangle + \langle x', y', z'\rangle = \langle x + x', y + y', z + z'\rangle$

Scalars
$c\langle x, y, z\rangle = \langle cx, cy, cz\rangle$





$\vec{V} + \vec{U} = \vec{U} + \vec{V}$

$0 + \vec{U} = \vec{U} + 0$

$\vec{V} - \vec{V} = 0$

$1 * \vec{V} = \vec{V}$

$c(\vec{V} + \vec{U}) = c\vec{U} + c\vec{V}$

$(c + d)\vec{U} = c\vec{U} + d\vec{U}$

$c|\vec{U}| = |c\vec{U}|$

IJK form
$\langle x, y, z\rangle = xi = yj + zk$

Unit Vectors- magnitude is 1
$$\vec{U_v} = \frac{1}{|v|}\vec{V}$$

Cross product (on 3D)
$\vec{U} \times \vec{V} = |\vec{U}||\vec{V}|sin\theta$

$$\langle x, y, z\rangle \times \langle x', y', z'\rangle = \begin{pmatrix} i & j & k \\ x & y & z \\ x' & y' & z' \end{pmatrix}$$

Magnitude (or norm) = $|\vec{U}|$

θ = angle between vectors

Dot product
$$\vec{U} \cdot \vec{V} = |\vec{U}||\vec{V}|cos\theta$$
$$cos\theta = \frac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|}$$
$$\vec{U} \cdot \vec{V} = xi * x'i + yj + y'j + zk * z'k$$

Cross product gives the vector perpendicular to the plane contains the other two vectors

If dot product equals 0 then the vectors are orthogonal

$\vec{U} \times (\vec{V} \times \vec{W}) = (\vec{U} \cdot \vec{W}) \cdot \vec{V} - (\vec{U} \cdot \vec{V}) \cdot \vec{W}$

$(c\vec{U}) \times \vec{V} = c(\vec{U} \times \vec{V})$

$(c\vec{U}) \times \vec{V} = c(\vec{U} \times \vec{V})$

$(\vec{V} + \vec{U}) \times \vec{W} = \vec{U} \times \vec{W} + \vec{V} \times \vec{W}$

$\vec{U} \cdot (\vec{V} \times \vec{W}) = \vec{W} \cdot (\vec{V} \times \vec{U})$

$\vec{V} \times \vec{U} = -\vec{V} \times \vec{U}$

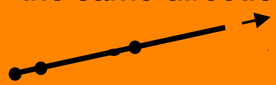
$\vec{U} \cdot \vec{V} = \vec{V} \cdot \vec{U}$

$(c * \vec{U}) \cdot \vec{V} = c(\vec{U} \cdot \vec{V})$

$\vec{U} \cdot \vec{U} = |\vec{U}|^2$

$(\vec{U} + \vec{V}) \cdot \vec{W} = \vec{U} \cdot \vec{W} + \vec{V} \cdot \vec{W}$

Lines- collection of points that all lie along the same direction

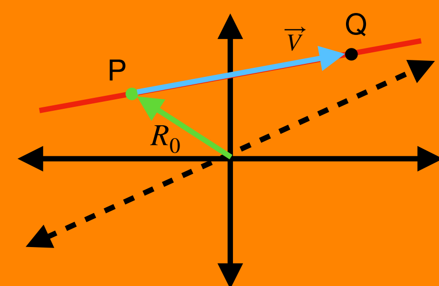


In 2D lines either intersect or are parallel

Lines are only parallel in 3D when there direction vectors are the same

In 3D lines either intersect are parallel or pas by each other

Finding the equation of a Line



R_0- position vector one point on line

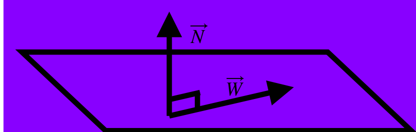
Move along $\vec{V}$ to find next point

$\vec{R}(t) = R_0 + t\vec{V}$

$\vec{R}(t) = \langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

$\vec{R}(t) = \langle x, y, z \rangle = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$

Equation of Plane



$\vec{N}$ is normal vector to plane

$\vec{W} = \langle x - x_0, y - y_0, z - z_0 \rangle$

$0 = \langle a, b, c \rangle \langle x - x_0, y - y_0, z - z_0 \rangle$

$0 = \langle a, b, c \rangle \langle x - x_0, y - y_0, z - z_0 \rangle$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$ax + by + cz = d$

Find the line where 2 planes intersect

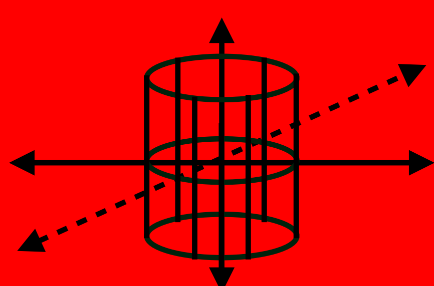
Use the cross product of the two normal vectors to find $\vec{V}$

Find R_0 by setting one variable to zero and solving the system of equations

Angle between two planes- dot product between the two normal vectors

Free variable- in 3D sometimes a variable is missing not tied to any other variable

In equation $x^2 + y^2 = 1$ is a free variable



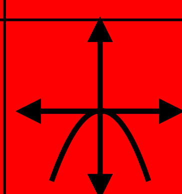
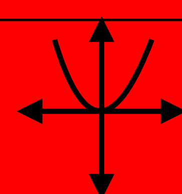
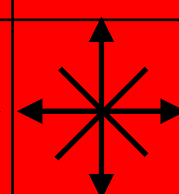
Traces- cross section view of 3D shape to help people identify what the shape is

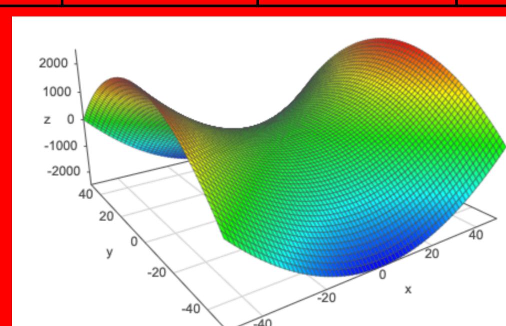
When trying to draw shape find X Y and Z intercepts by setting other variables to zero

Draw traces of XY YZ and XZ plane

If shape is still unclear do additional traces at x,y,z=k

$x^2 - y^2 = z$

	X	Y	Z
Intercept	0	0	0
Trace equation (var=0)	$z = -y^2$	$z = x^2$	$x = \pm y$
Trace picture			



3D space | Vectors | Dot product | Cross product | Lines | Planes | Surfaces