Vector Valued Functions

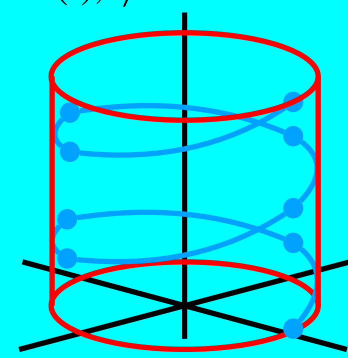
An objects position in 3D space is defined by $\vec{r} = \langle x, y, z \rangle$

Full trajectory of an object is $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

All 3 variables x, y, and z change with respect to a single variable t

$\vec{r}(t) = \langle cos(t), sin(t), t \rangle$

T	X	Y	Z
0	1	0	0
$\pi/2$	0	1	$\pi/2$
π	-1	0	π
$3\pi/2$	0	-1	$3\pi/2$
2π	1	0	2π

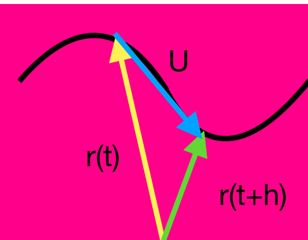


Circle with radius r is $\vec{r}(t) = \langle rcos(t), rsin(t) \rangle$

Circle with radius r is to see if surfaces interest put one equation into another

Domain of a vector valued function must work for x(t), y(t) and z(t)

For f(t) $f'(t) = \lim_{h \to 0} \frac{f(h+t) - f(t)}{h}$



r(t)+u=r(t+h)

For r(t) $\vec{r}'(t) = \lim_{h \to 0} \frac{\vec{r}(h+t) - \vec{r}(t)}{h}$ So $\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$

Integral $\int \vec{r}(t)dt = \vec{R} + \vec{c}$

$\int_a^b \vec{r}(t)dt = \vec{R}(b) - \vec{R}(a)$

Integral is an accumulation of all the tangent vectors over inverval [a,b]

if $\vec{r}(t)$ is a position vector than $\vec{r}'(t)$ is the velocity vector

if $|\vec{r}'(t)|$ is speed (scalar)

$\vec{r}''(t)$ is acceleration vector

$f(x) = \frac{k}{x}$ then $f'(x) = -\frac{k}{x^2}$

$f(x) = kx$ then $f'(x) = k$

$f(x) = k$ then $f'(x) = 0$

$f(x) = x$ then $f'(x) = 1$

$f(x) = k\sqrt{x}$ then $f'(x) = \frac{k}{2\sqrt{x}}$

Quotient Rule

$f(x)\frac{u}{v}$ then $f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$f'(x) = \frac{LoDeHi - HiDeLo}{(Lo)^2}$

Chain Rule

$h(x) = f(g(x))$ then $h'(x) = f'(g(x)) * g'(x)$

Power Rule

$f(x) = x^n$ then $f'(x) = nx^{n-1}$

Product Rule

$f(x) = uv$ then $f'(x) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$f(x)a^x \; then \; f'(x) = a^x ln(a)$

$f(x) = ln(x) \; then \; f'(x) = \dfrac{1}{x}$

$f(x) = e^x \; then \; f'(x) = e^x$

$\dfrac{dy}{dt} = ky$

$\int a^x dx = \dfrac{a^x}{lna} + C$

$\int e^x dx = e^x + C$

$\int \dfrac{dx}{x} = ln|x| + c$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$

$\int k dx = kx + C$

$\int u(dv) = uv - \int v(du)$

$\int ln(x)dx = xln(x) - x + C$

Trig Functions

function	**Derivative**
$sin(x)$	$cos(x)$
$cos(x)$	$-sin(x)$
$tan(x)$	$sec^2(x)$
$sec(x)$	$sec(x)tan(x)$
$csc(x)$	$-csc(x)cot(x)$
$cot(x)$	$csc^2(x)$

Estimated Length of curve

$$\sum_{i=1}^{n} |\vec{r}'(t)| \Delta t$$

Actual Length of Curve

$$\lim_{n \to \infty} \sum_{i=1}^{n} |\vec{r}'(t)| \Delta t = \int_a^b |\vec{r}'(t)| dt$$

$s(t) = \int_a^t |\vec{r}'(t)| dt$ Shows how length and time are related

$r(s)$ tells us where we are after a given length but to do this we need s(t)

Unit tangent vector $T(t) = \dfrac{r'(t)}{r(t)}$ T dies bot change based on magnitude so a change in T means direction changes

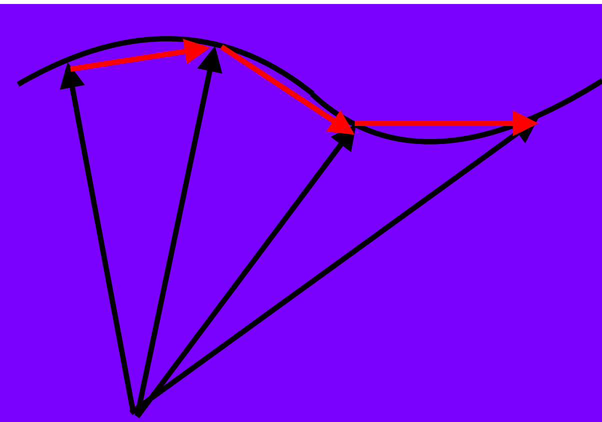
So $|\dfrac{dT}{ds}|$ is how much the curve turns

If $|\dfrac{dT}{ds}|$ its big then the curve turns a lot on very sharply

If $|\dfrac{dT}{ds}|$ its small then the curve turns very gradually

If $|\dfrac{dT}{ds}| = 0$ then the function is a straight line

$$\frac{ds}{dt} = |r'(t)| \text{ so } \frac{dT}{ds} = \frac{\frac{dT}{ds}}{\frac{ds}{dt}} = |\frac{T'(t)}{r'(t)}| = \kappa$$



Vector functions	Derivatives	Integrals	Derivative rules	Special derivatives
unit tangent vector	Length	Domain of function	Motion in space	