Vector Calculus

Vector field is a function that assigns a vector to a point (so it has a direction and a magnitude) Scalar field is a function that assigns a scalar value to each individual point

If the vector field F is the gradient field of some other scalar field then we call the scalar field the potential function of the U vector field Not every vector field has a potential function (because its not always the gradient of a function)

If the vector field F represents a force and it has a potential function then the force in conservative

Conservative- the path doesn't matter only the endpoints do. (The work done is path independent like gravity)

The length of $\vec{r}(t)$ over $a \le t \le b$ is $\int_{a}^{b} |\vec{r}(t)| dt = \int_{a}^{b} ds$

The line integral of $\vec{r}(t)$ over $a \le t \le b$ is $\int_C f(x, y) ds$

If f(x,y) is the density of a wire with the shape given by C then $\int_C f(x,y)ds$ is the mass of the wire

In vector field the line integral $a \le t \le b$ is $\int_a^b \overrightarrow{F} \cdot \overrightarrow{r}' dt = \int_a^b \overrightarrow{F} \cdot d\overrightarrow{r} = \int_a^b f dx + g dy$

Vector fields

Scalar functions

Line Integrals

Vector Field Line integral