Multivariable Integrals

Lagrage Multipliers- used when finding the max and min of constrained optimization problems

f(x,y) is the objective

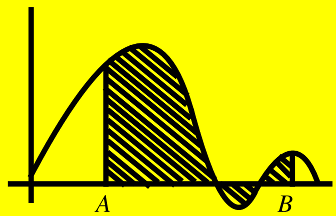
g(x,y)=0 is the constraint

At critical point the level curve and the constraint curve are tangent to each other

At the point where f and g are tangent the $\vec{\nabla} f \,||\, \vec{\nabla} g$

To find max and mins of f(x,y) subject to g(x,y) we solve for location where $\vec{\nabla} f = \lambda \vec{\nabla} g$

If $f(x)$ $a \leq x \leq b$ then $\int_a^b f(x)dx$ gives us the area bounded by f(x) and the x axis

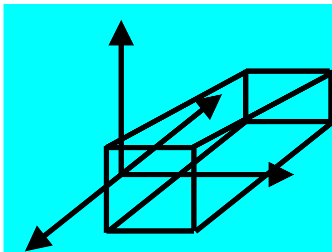


$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta X$$

Total volume

$$\lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{n} f(x_i, y_j)\Delta A$$

$z = f(x, y)$ is a surface, we can use the same idea to find the volume under f(x,y) between $a \leq x \leq b$ and $c \leq y \leq d$



Switching the order of integration can make it easier

$$\int_a^b \int_c^d f(x, y)dydx = \int_c^d \int_a^b f(x, y)dxdy$$

When both x and y are bounded by constants

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

$$f_{avg} = \frac{1}{b-a}\frac{1}{d-c}\int_a^b \int_c^d f(x, y)dydx$$

When regions are non rectangular swapping order is not so easily

Integrate variable with constants bounds last

We can still swap order but we need to reformulate R

Type I- x bounded by constants y bounded by vars

Type II- y bounded by constants x bounded by vars

In Cartesian form point is expressed by (x,y) distance from y axis and distance from x axis

In polar form point is expressed by (r, θ) where r is displacement from origin and theta is the angle of the point to the origin from x axis

$x^2 + y^2 = r^2$

$x = rcos(\theta)$

$y = rsin(\theta)$

$\int\int_R f(r, \theta)dA = \int\int_R f(r, \theta)rdrd\theta$

$\int\int\int_D dV$ accumulates. Volume over volume D

Start with a projection on a plane (generally XY)

Integrate var bounded by constants last and the variable bounded by the other variables first

$x = \rho sin\phi cos\theta$	$dV = \rho^2 sin\phi$
$y = \rho sin\phi sin\theta$	$0 \leq \rho \leq \infty$
$z = \rho cos\phi$	$0 \leq \phi \leq \pi$
$\rho^2 = z^2 + x^2 + y^2$	$0 \leq \theta \leq 2\pi$

$\bar{x} = \frac{1}{m}\int\int xpdA$	$\bar{x} = \frac{1}{m}\int\int\int xpdV$
$\bar{y} = \frac{1}{m}\int\int ypdA$	$\bar{y} = \frac{1}{m}\int\int\int ypdV$
$m = \int\int pdA$	$\bar{z} = \frac{1}{m}\int\int\int zpdV$

Cylindrical

$$\int\int\int f(z, r, \theta) * rdzdrd\theta$$

$$dV = rdzdrd\theta$$

Spherical

$$\int\int\int f(\rho, \phi, \theta) * \rho^2 sin\phi d\rho d\phi d\theta$$

LaGrange Multipliers | Integrations | Triple integrals | Spherical Integral | Cylindrical Integral | Averages

Integral General Region | Integral Rectangular Region | Integral Polar Region | Center of Mass