Jervis Muindi Tuesday, February 26, 2013
Numerical Algorithms and Complexity
Homework 3

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Part A

Evaluate Using Monte Carlo :

$$\int_{0}^{2}\int_{-1}^{1}\int_{1}^{1.5} x^2y + e^{-z}\,dxdydz$$

I wrote a simple python program called montecarlo.py[1] that evaluated this definite triple integral using the Monte-Carlo Technique. Running the python program, I got the following result:

Sample Output:

```
For N = 100, Value is 0.756653. Error = 0.108012

For N = 1000, Value is 0.837359. Error = 0.027306

For N = 10000, Value is 0.868746. Error = 0.004081

For N = 100000, Value is 0.862611. Error = 0.002054

For N = 1000000, Value is 0.862637. Error = 0.002028
```

Thus, the value of the triple integral obtained is from running the program is `0.862637` with an (actual) error of `0.002028`

Part B

To generate the random inputs which were utilized while running Monte-Carlo, I made use of the built-in pseudorandom number generator that comes along with Python. The name of the exact method is *random.uniform()*.

Part C

I used an online Mathematical system called Wolfram Alpha to evaluate the true value of this integral. From this I got the following result:

$$\int_{0}^{2}\int_{-1}^{1}\int_{1}^{1.5} x^2y + e^{-z}\,dxdydz = 0.864665$$

[1] See https://github.com/jervisfm/W4142_HW3 to see the source code. This will be accessible after HW due date.

This result can be verified by seeing this URL : http://goo.gl/eLNDk which will re-do the above computation.

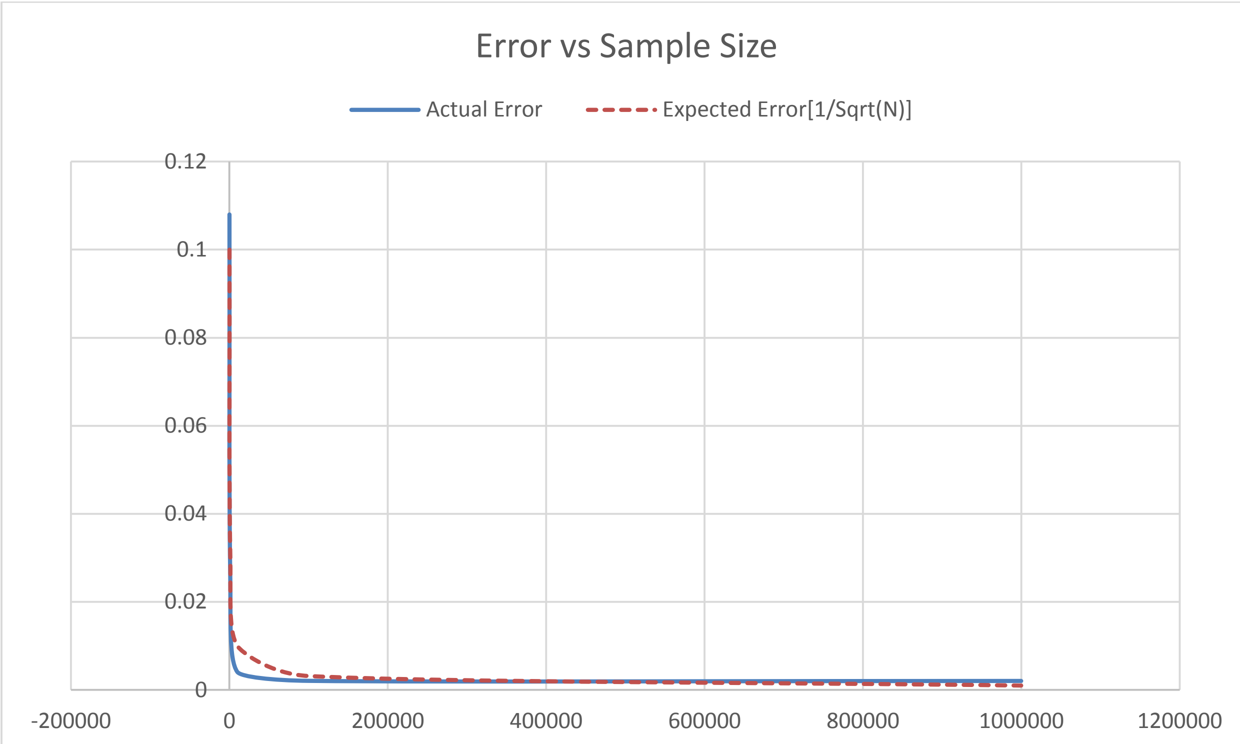
Part D

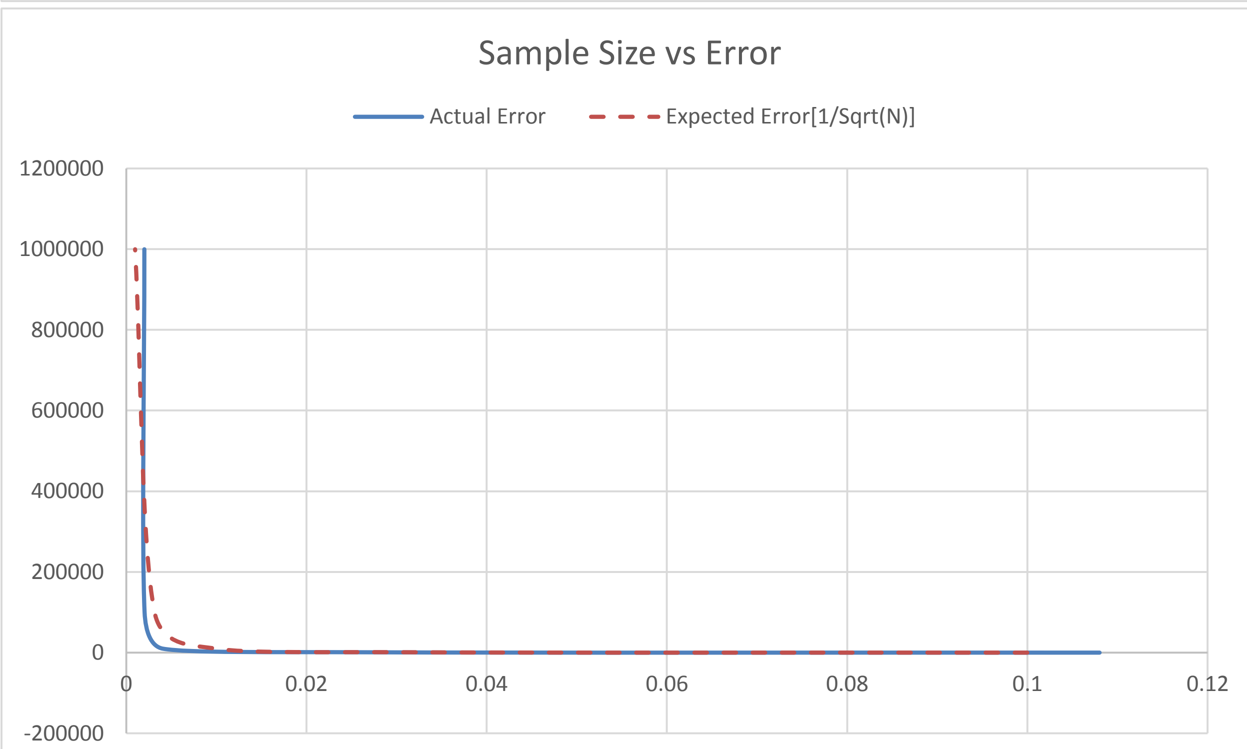
Analyzing the data obtained from Part A in the table below, it is clear that it is indeed the case that the convergence of the error to 0 is like $\frac{1}{\sqrt{N}}$

N	Actual Error	Expected Error
100	0.108012	0.10000
1000	0.027306	0.03162
10000	0.004081	0.01000
100000	0.002054	0.00316
1000000	0.002028	0.00100

So as we increase the number of points that we sample, it is self-evident that the actual error does indeed (approximately) converge to zero as by $1/\sqrt{N}$ where N is the sample size.

To see this relationship more clearly I plotted the data above in two graphs on the next page.





As you can see, the plotted data very clearly follows the 1/Sqrt(N) curve and this is further evidence that the convergence is like 1/Sqrt(N).