Executive Summary

Title: Beating the S&P 500

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Background: Portfolio allocation helps investors reduce risk through diversification. Market conditions that lead to some assets outperforming can lead to other assets underperforming. As such, efficient portfolio allocation results in less volatility in the value of portfolios since these opposite movements can offset each other.

Objective: To determine the extent to which different statistical modelling frameworks, with focus on stochastic and regime switching volatility models, impact the performance of an optimal portfolio allocation strategy for stocks of S&P 500.

Methodology: We will focus exclusively on portfolio allocation for listed stocks of the S&P 500. We will estimate the expected return and standard deviation of each stock independently using 3 different statistical models, namely: ARMA, GARCH and Markov regime switching GARCH (MSGARCH). We will determine the optimal portfolio allocation by maximizing the Sharpe ratio using a one-step ahead forecast. We will use rolling-window to iteratively compute one-step ahead returns. We will compare our different allocation strategies at the end of the out-of-sample period using different financial metrics of risk and return.

Potential Importance: We compared the Max Sharpe Portfolio and the Equal Weights (EW) Portfolio to find which portfolio performs better. Through our project, we found that the EW Portfolio performed better in terms of rolling returns. However, this project could be extended to compare the portfolios on more financial metrics such as Sharpe Ratio and Information Ratio. With these metrics, our project can help provide investors with more information to construct their portfolio.

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1. Introduction

Financial portfolio allocation is an important challenge faced by investors. Markowitz (1952) laid the grounds for modern portfolio theory (MPT) showing that an asset's risk contribution to a portfolio is not only dependent on its own risk but also on its correlation with other assets in the portfolio. However the Markowitz model was notorious for producing extreme weights that fluctuate substantially over time and performing poorly out of sample.

Subsequent research tried to improve on the Markowitz models by using Bayesian approaches to reduce estimation errors (Barry, 1974; Bawa, Brown, and Klein, 1979; Jobson, Korkie, and Ratti, 1979; Jobson and Korkie, 1980; Jorion, 1985, 1986). Other methods focused on reducing the error in estimating the covariance matrix (Best and Grauer, 1992; Chan, Karceski, and Lakonishok, 1999; Ledoit and Wolf, 2004a, 2004b). DeMiguel et al. (2007) compares these portfolio allocation strategies based on different asset-allocation models against the EW portfolio and concludes that there is no single portfolio allocation method that consistently performs better than the EW portfolio. However, he believes that one of the better allocation methods can be discovered by improving the estimation of the moments of asset returns and covariance matrix.

As such, we seek to extend DeMiguel et al. by **considering additional statistical models** to have better estimations of moments of asset returns and covariance matrix. Specifically, we will use three models - ARMA, GARCH, MSGARCH. After considering the statistical models, we will build a portfolio that maximises the Sharpe ratio, which we will compare against a baseline and the EW portfolio.

2. Data

In our study, we will be using historical data from S&P 500. The S&P 500 includes 500 companies that combine to represent about three-quarters of the stock market in terms of market capitalization. Hence, the S&P 500 is representative of the US stock market, which is why it's the benchmark so many investors and fund managers compare their performance against.

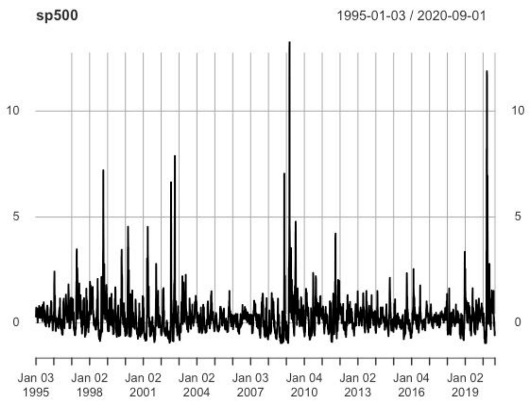
2.1 Data Sourcing

The data used in this project is sourced from Yahoo! Finance using the R library quantmod. The data consists of all S&P 500 securities (except for Voltier which was only recently added as the result of a corporate spin off) and the S&P 500 index itself. The data includes historic prices and log monthly returns from 1995-01-01 to 2020-10-01.

2.2 Data Analysis

We perform exploratory data analysis on the returns for the S&P 500 index and the stocks to examine stationarity. Firstly, we can visualise the monthly returns to examine mean reverting behaviour. Additionally, we can also perform the ADF tests where the null hypothesis is that the time series is not stationary.

2.2.1 Stationarity of S&P 500



Test	p-value
ADF Test	< 0.01 (Reject Null)

Figure 1: Stationarity Tests (S&P 500)

From the time series plot of monthly returns against time in Figure 1 above, we can observe that the monthly returns are mean-reverting, indicating stationarity. This is also supported by the ADF and KPSS tests shown in Figure 1.

2.2.2 Stationarity of S&P 500 Securities

Figure 2: Stationarity Tests (S&P 500 Securities), each point in the x-axis represent a single stock.

As shown in Figure 2 above, the p-values for the ADF test are all less than or equal to 0.01 (due to how adf.test returns p-value), which leads us to reject the null hypothesis and conclude that the monthly returns are stationary

3. Methodology

3.1 Forecasting Method

Using the models that we have decided on above, we will attempt to predict the returns and volatility for the index and each stock in the S&P 500 index using the rolling forecast method. As shown in Figure 3, we forecast one month ahead with the past 10 years of data. This simulates an investor who will only invest in stocks that have withstood the test of time (i.e traded for at least 10 years) and uses only the most up-to-date information.

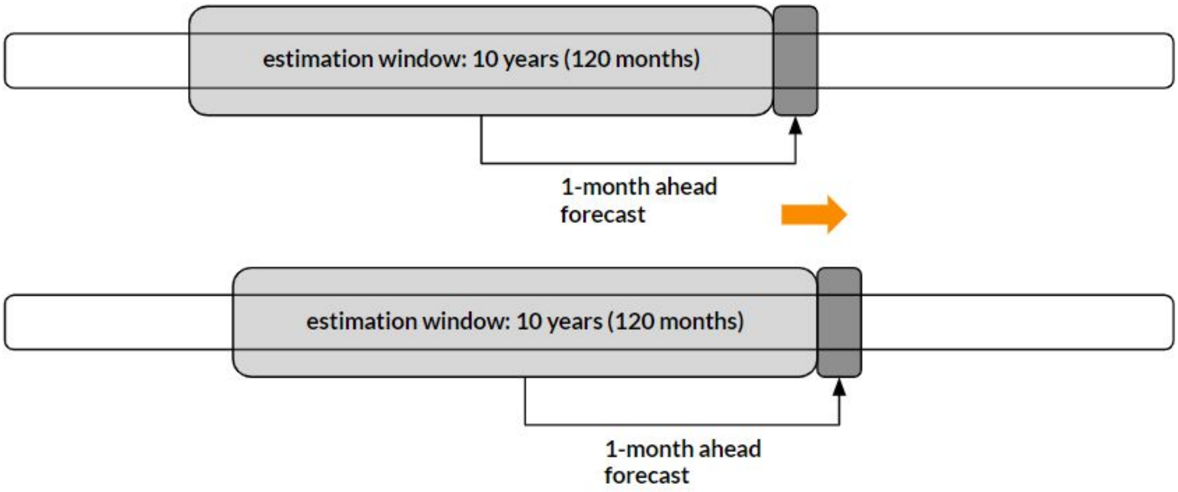


Figure 3: Rolling Forecast Method

3.2 Covariance Estimator

As we estimate each stock independently, we would not get estimates for the cross-correlation terms. Instead, we propose a work-around by estimating the off-diagonal terms as follows:

1. Compute the forecast error.

$$\varepsilon_i = y_{i,true} - y_{i,pred}$$

2. Compute covariance between errors ($cov(\varepsilon_i, \varepsilon_{□})$) as our off-diagonal estimator.

3.3 Model Parameters and Specifications

In our forecasts, we use the following parameters for our models:

1. <u>ARIMA</u>: 'auto.arima' was used to determine the optimal parameters for each model fitted for each set of stock data. Additionally, we set stationarity and seasonality as TRUE.
2. <u>GARCH</u>: We used GARCH (1,1) for our GARCH model. This uses a lag 1 residual errors and lag 1 variances.
3. <u>MSGARCH</u>: We assumed a 2-state volatility regime for our MSGARCH model. 1 regime assumes a normally distributed volatility while other regime assumes a Student-t distributed volatility. In each regime, volatility follows the GARCH (1,1). Using these two regimes allows us to account for both normal periods and periods of financial instability. The model parameters are found using the Maximum Likelihood Estimator.

3.4 Model Evaluation

We evaluated our models using mean squared error. With the predicted values from each model, we backtest and compare with the historical data to calculate MSE. For each model, we calculate the MSE of all included stocks and then we take the average. The average MSE is calculated as such: $\text{Average}_{\text{MSE}} = (1/n_{\text{stock}})\ \Sigma_i\ \text{MSE}_i$.

Across all 3 models, they have very similar MSE as seen in the Figure 5 below. We also performed a Kruskal-Wallis H test to compare the MSE of all 3 models and found that there is no statistically difference between their MSE, with a p-value of 0.688.

Model	Number of stocks	Average period length	Average absolute log-return	Average of MSE	Standard deviation of MSE
ARIMA	452	286 months	0.06030038	0.008068208	0.006007860
GARCH				0.007740972	0.005576524
MS-GARCH				0.007740557	0.005508433

Figure 5: MSE of Models

3.5 Portfolio Allocation

With the results from our forecasted models, we created a portfolio allocation that aims to maximise Sharpe Ratio. Our portfolio consists of choosing the weights that maximises Sharpe Ratio. This is the portfolio which lies on the efficient frontier and is tangent to the Capital Market Line. Following the same assumption as the rolling forecast method, we only selected the 452 stocks which had been traded for at least 10 years to be added into our portfolio.

$$\max_{w} \; h(w) = \frac{\hat{\mu}^T w - r_f}{w^T \hat{\Sigma} w}$$
$$s.t.$$
$$w^T \hat{\mu} = \overline{r}$$
$$w^T 1 = 1$$

Figure 6: Max Sharpe Portfolio

We will compare the Max Sharpe Portfolio against the baseline S&P 500 index and the EW Portfolio.

4. Findings

As seen in Figure 7 below, our proposed portfolios outperform the S&P 500 from 2005 to 2020. However, it did not manage to beat the EW Portfolio. Nevertheless, our method seems to be less volatile than the EW portfolio, with less drastic drops in the portfolio value as compared to the EW portfolio

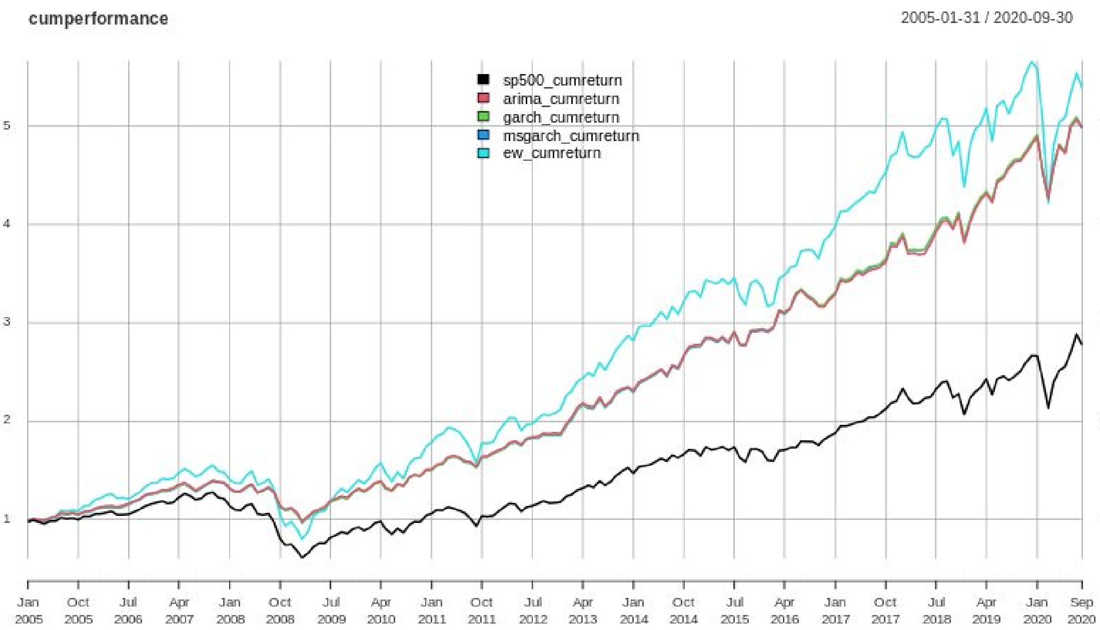


Figure 7: Comparison of Portfolios' Performance

The findings above are also corroborated by Figure 8 below which compares the performance between our method (Labelled as Sharpe) against the S&P 500 and the EW portfolio. Our method enjoys higher returns along the 25th, 50th and 75th percentile rows, with the smallest difference between the min and max rows. As such, we can say that our method generates higher returns than S&P 500 but not as high as the EW portfolio. The advantage of our method against the EW portfolio would be lower volatility, albeit with lower returns.

	yearly			5-yearly			10-yearly		
	S&P500	**Sharpe**	**EW**	**S&P500**	**Sharpe**	**EW**	**S&P500**	**Sharpe**	**EW**
min	**-44.7%**	**-23.6%**	**-41.8%**	**-3.3%**	4.7%	4.2%	4.2%	9.6%	10.17%
25 perc.	2.6%	8.5%	3.8%	0.7%	7.3%	8.5%	5.2%	10.5%	11.36%
50 perc.	11.2%	13.2%	15.7%	9.1%	11.9%	11.8%	6.1%	11%	12.6%
75 perc.	15.5%	17.3%	22%	12.2%	14.7%	16.7%	10.9%	13.8%	14.6%
max	50.3%	34.8%	77.7%	20.4%	19.6%	29.9%	14.2%	15.5%	20%
n	933	933	933	885	885	885	825	825	825

Figure 8: Comparison of Portfolios' Returns

5. Conclusion

In conclusion, in our project, we extended DeMiguel et al.'s paper which compared modern portfolios against the EW portfolio. Specifically, we constructed the Max Sharpe Portfolio which is the portfolio that lies on the efficient frontier and is tangent to the Capital Market Line. To construct that portfolio, we used estimates for expected return and covariance matrix that were forecasted by three different statistical models, namely ARIMA, GARCH and MSGARCH. From our results, we found that our portfolio had on average a lower return compared to the EW portfolio, but with comparatively lower variance as expected by our optimization strategy that focused on the Sharpe ratio. In addition to that, our strategy outperformed the S&P 500 index both in terms of returns and variance.

Due to time constraints, we were not able to compute additional financial metrics to evaluate the analysed portfolios. Rather, we focused on describing the distribution of returns which provided a good indication of its variability. Additional metrics such as drawdown and turnover would have helped us analysing additional risks and costs in implementing the proposed portfolio. Transaction fee could substantially reduce the return of our portfolio depending on the turnover of stocks included. Hence, these additional metrics could provide a more comprehensive evaluation of the portfolios.

6. References

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